INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) I Year : 1990-91

Probability II

Semestral-II Backpaper Examination

Date : 1.7.1991 Maximum Marks : 100 Time: 3 Hours.

Answer all the questions.

1. A ten-digit number is formed by picking digits for the 10 places at random. Letting X denote the sum of digits in the number thus formed, use the method of probability generating functions to find the distribution of X. [15]

2. Let X be a random variable distributed uniformly on the interval (-1,1). Find a function ϕ on (-1,1) such that $Y = \phi(X)$ has the density

$$g(y) = \frac{1}{2(1+|y|)^2} \text{ for } -\infty < y < \infty.$$ [15]

3. Derive the density function of the F-distribution with parameters (m,n). Find out which moments exist and what they are. [8+7 =15]

4. Let (X,Y) be jointly normal with EX=EY=0, V(X)=V(Y)=1 and Cov (X,Y)=ρ. Denoting $E(X^rY^s)$ by μ_{rs}, show that

$$\mu_{rs} = (r+s-1)\rho\, \mu_{r-1,s-1} + (r-1)(s-1)(1-\rho^2)\mu_{r-2,s-2}.$$

Deduce that $\mu_{rs} = 0$ whenever r+s is odd.

Find μ_{31} and μ_{22}. [10+5+5=20]

5. Let X be a random variable, not identically zero with probability 1, such that for all t in an interval I, $|X|^t$ has finite expectation. Show that $\phi(t) = \log E(|X|^t)$ is a convex function on I. [15]

6. For a pair of independent random variables X and Y, each distributed exponentially with parameter α, denote ~~min~~ min (X,Y) and max (X,Y) by U and V respectively.

Show that U and V-U are independent exponential random variables. [20]

[1990-91][118(b)]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Calculus II
Semestral-II Backpaper Examination

Date : 28.6.1991 Maximum Marks : 100 Time: 3 Hours.

Note : Answer all the questions which carry equal marks.

1.(a) Show that if f is continuous on [a,b] and α is a monotone increasing function then

$$\int_a^b f\,d\alpha$$

exists

(b) Evaluate the following limit by writing it as an integral and then using the fundamental theorem of calculus

$$\lim_{n\to\infty} \sum_{k=1}^{n} \text{Sin}\left(\frac{k\pi}{n}\right).\frac{\pi}{n}.$$

2.(a) Given a function g, continuous everywhere, such that $g(1) = 5$ and $\int_0^1 g(t)\,dt = 2$. Let $f(x) = \frac{1}{2}\int_0^x (x-t)^2 g(t)\,dt$.

Prove that

$$f'(x) = x\int_0^x g(t)\,dt - \int_0^x t\,g(t)\,dt,$$

then compute $f''(1)$ and $f'''(1)$.

(b) Show that

$$\int_0^\pi x\,\text{Sin}^6 x\,\text{Cos}^4 x\,dx = \frac{3}{512}\pi^2.$$

3.(a) Show that $\int_0^\infty \frac{\text{Sin}\,x}{x}\,dx$ converges but not absolutely.

(b) Show that $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \frac{\pi}{3}.$

4.(a) Show that if $\{f_n(x)\}$ converges uniformly to $f(x)$ on an interval S, and if each $f_n(x)$ is continuous at a point p in S, then the limit function is also continuous at p.

p.t.o.

4.(b) Show that $\Sigma\, x^n(1-x)$ converges pointwise but not uniformly on [0,1]; whereas $\Sigma(-1)^n\, x^n\,(1-x)$ converges uniformly on [0,1].

5.(a) If R is the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n\, x^n$, then show that the radii of convergence of $\sum_{n=1}^{\infty} n\, a_n\, x^{n-1}$ and $\sum_{n=0}^{\infty} a_n\, x^{n+1}/(n+1)$ are also R.

(b) Show that

$$\int_0^1 \frac{\log(1+x)}{x}\, dx = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$

justifying each step.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Computational Techniques and Programming II
Semestral-II Backpaper Examination

Date : 26.6.1991 Maximum Marks : 100 Time: 3 Hours.

Note : Answer any five questions. You may have with you interpolation formulae and a list of Bernoulli numbers but no other material.

1. Derive an algorithm for piecewise cubic interpolation through n given data points. Derive the corresponding quadrature formula. [14+6=20]

2. A rocket is launched vertically upwards from the ground. Its acceleration is registered during the first 80 secs and is given in the table below :

t(sec)	0	10	20	30	40
$a(m/sec^2)$	30.00	31.63	33.44	35.47	37.75

(the acceleration registered takes care of the effect of gravity)

t(sec)	50	60	70	80
$a(m/sec^2)$	40.33	43.29	46.69	50.67

Find the velocity and the height of the rocket at time t = 80 sec. [20]

3. Obtain a difference equation for

$$y_n = \int_0^1 \frac{x^n}{1+x+x^2}\,dx$$

and use this equation for computing y_n for $n \leq 12$. Also find a value of n such that $y_n = 0.04$.

[6+8+6=20]

4. Describe an algorithm for computing f(x) for a given x by interpolation from a given table of values of f(x), for $x = x_0, x_0 + h, \ldots, x_0 + nh$.

Apply this algorithm to compute f(3.636) from the following table :

p.t.o.

Q.4. contd.

x	f(x)
3.60	0.112046
3.61	0.120204
3.62	0.128350
3.63	0.136462
3.64	0.144600
3.65	0.152702
3.66	0.160788
3.67	0.168857
3.68	0.176908

[8+12 = 20]

5. For a given system of linear equations $AX = b$, Construct the Jacobi and Gauss-Seidel iterative methods.

Derive the condition for convergence of an iterative method

$$X^{(m+1)} = B\, X^{(m)} + C$$

where B is an n x n matrix and X, C are column vectors of n elements. Show that the Jacobi iterative method is convergent if the matrix A is strictly diagonally dominant

[4+4+6+6=20]

6. Describe the mathematical basis of the Jacobi's method for computing the eigenvalues and eigenvectors of a symmetric matrix. Discuss the implementation of the method.

[12+8=20]

7.(a) Explain the Monte Carlo method for numerical integration. How would you apply the method to compute

$$\int\int_{x^2+y^2 \leq 1} e^{-(x^2+y^2)}\, dx\, dy \;?$$

(b) Describe the Romberg Principle and draw a flow-chart for Romberg Integration algorithm.

[10+10=20]

[1990-91][128(b)]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Vectors and Matrices II
Semestral-II Backpaper Examination

Date : 25.6.1991 Maximum Marks : 100 Time: 3 Hours.

Note : This paper contains 105 marks. Answer as many as you can. Max. marks you can get is 100.

1.(a) Let A and B be matrices of order mxn and nxm respectively such that $m \leq n$. Show that

$$|AB| = \Sigma\ |A_m|\ |B_m|$$

where A_m is a major of A and B_m is the corresponding major of B and the summation is over all majors.

(b) Show that $\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A|.|D - CA^{-1}B|$ when $|A| \neq 0$.

[10+5=15]

2.(a) Define eigen values and eigen vectors of a square matrix.

(b) Define algebraic and geometric multiplicities of an eigen value and show that algebraic multiplicity of any eigen value is greater than or equal to its geometric multiplicity.

(c) For a square matrix A, show that algebraic multiplicity of the eigen value zero is equal to its geometric multiplicity if and only if $r(A) = r(A^2)$.

(d) Using (c) show that a real symmetric matrix has complete set of eigen vectors .

(e) Derive spectral decomposition of a real symmetric matrix.

[3+(3+6)+4+4+5 = 25]

p.t.o.

3.(a) Show that $A^2 = A \Leftrightarrow r(A) + r(I-A) = r(I)$.

(b) Let $A_1 + A_2 + \dots + A_k = I$. Then show that

$$A_i^2 = A_i \text{ for } i = 1,2 \dots k \Leftrightarrow \sum_1^k r(A_i) = r(I).$$

(c) Let $A_1 + A_2 + \dots + A_k = A = A^2$. Then show that

$$A_i^2 = A_i \text{ for } i = 1,2 \dots k \Leftrightarrow \sum_1^k r(A_i) = r(A).$$

[5+5+5 = 15]

4.(a) Let A be a square matrix such that $r(A) = r(A^2) = r$. Then show that there exists an orthogonal matrix P such that

$$A = P\begin{pmatrix} T & TC \\ O & O \end{pmatrix} P'$$

where T is an upper triangular nonsingular matrix of order r.

(b) Let A be an idempotent matrix. Then show that singular values of A are greater than or equal to 1, and all singular values are equal to 1 if and only if A is symmetric.

[10+5 = 15]

5.(a) Show that A is positive definite if and only if leading principal minors of all orders are positive.

(b) Show that if A is positive definite matrix of order n product of eigen values of A $\leq$ product of diagonal elements of A.

[10+5 = 15]

6. Prove or disprove the following statements.

(a) A is real skew symmetric matrix if and only if $x'Ax = 0$ for all real vectors x.

(b) Nonzero eigen values of a real skew symmetric matrix are pure imagenary numbers.

(c) For a real square matrix A sum of squares of eigen values is equal to sum of squares of its singular values if and only if A is symmetric.

(d) The sets of nonzero eigen values of AB and BA are same.

[4x5 = 20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Statistical Methods II
Semestral-II Backpaper Examination

Date : 24.6.1991 Maximum Marks : 100 Time: 4 Hours.

Group A

1.(a) Consider a correlation matrix $R = (r_{ij}) : p \times p$ with $r_{ij} = r$ for all $i \neq j$. Show that $-1/(p-1) \leq r \leq 1$. Suppose R is non-singular; obtain $R_{1(23..p)}$ and $r_{1p.23...(p-1)}$.

(b) Consider n observations on a set of p variates $(X_1, .., X_p)$. Show that $R_{1(23...p)}$ is unchanged, if the variates are transformed as follows :

$$X_1 \to aX_1, \quad a \neq 0$$

$$\begin{pmatrix} X_2 \\ \vdots \\ X_p \end{pmatrix} \to B \begin{pmatrix} X_2 \\ \vdots \\ X_p \end{pmatrix}$$, where B is non-singular.

[12+6]

2. Describe methods for generating random variates from the following probability models using only the random number generator.

(a) Poisson distribution $p(\lambda)$

(b) Normal distribution $N(\mu, \sigma^2)$

(c) Standard Cauchy distribution

(d) Chi-square distribution with integral d.f.m.

[24]

3.(a) What is a confidence interval for a parameter ?

(b) Suppose $X \sim B(n,p)$, and

$$\text{Prob} \left[-2\sqrt{\frac{p(1-p)}{n}} \leq \frac{X}{n} - p \leq 2\sqrt{\frac{p(1-p)}{n}} \right] \approx .95.$$

Based on the above find a confidence interval for p.

(c) Would you always question the postulated model if the observed Chi-square statistic for goodness-of-fit is high ? Explain.

3.(d) What do we mean when we say that observed Chi-square value is significant at 1% level ? Does it mean that the probability that the model is correct is 1% ? Explain.

(e) What is the objective of stratified random sampling ? Is it always better than the random sampling (both with SRSWOR) for estimating the population mean ? Discuss.

[2+5+4+4+13]

Group B

1. The following table gives the frequency distribution of the number of RBC per cell of a haemocytometer. Fit a Poisson distribution to the data and test for goodness-of-fit.

No. of RBC	No. of cells
0	143
1	156
2	68
3	27
4	5
5	1
Total	400

[10]

2. 374 lines of rice were raised with the results shown below:

	no. of lines
All plants resistant	97
Mixed : some plants resistant, some susceptible	184
All plants susceptible	93

According to the IRRI model, the lines are independent; each line has a 25% chance to be resistant, a 50% chance to be mixed, and a 25% chance to be susceptible. Are the facts consistent with the model ?

[8]

3. Examine the following correlation matrix obtained by a computer for internal consistency.

1.00	0.46	0.59	0.81
0.46	1.00	0.38	0.67
0.59	0.38	1.00	0.98
0.81	0.67	0.98	1.00.

[10]

* *

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) I Year : 1990-91

Computational Techniques and Programming II

Semestral-II Examination

Date : 10.5.1991 Maximum Marks : 100 Time: 3 Hours.

Note : Answer any five questions. You may have with you interpolation formulae and a list of Bernoulli numbers but no other material.

1. Construct an algorithm to compute the following two matrices

$$L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ c_2 & 1 & 0 & \cdots & 0 \\ 0 & c_3 & 1 & \cdots & 0 \\ \vdots & \vdots & & & \\ 0 & 0 & & c_n & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & 0 \\ \vdots & & & & \\ 0 & & & \cdots & d_n \end{bmatrix}$$

such that $A = L D L^T$ where

$$A = \begin{bmatrix} a_1 & 1 & 0 & \cdots & 0 \\ 1 & a_2 & 1 & \cdots & 0 \\ 0 & 1 & a_3 & \cdots & 0 \\ \vdots & & & & \\ 0 & \cdots & & 1 & a_n \end{bmatrix}$$

Using this factorization of A, how would you solve the system of equations $AX = b$ where $b = [b_1, b_2, \ldots, b_n]^T$?

Use this method to solve the following system of equations:

$$\begin{aligned} x_1 + x_2 &= 3 \\ x_1 + 2x_2 + x_3 &= 8 \\ x_2 + 3x_3 + x_4 &= 15 \\ x_3 + 4x_4 + x_5 &= 24 \\ x_4 + 5x_5 + x_6 &= 35 \\ x_5 + 6x_6 &= 41 \end{aligned}$$

[8+6+6 = 20]

2.(a) Show that the general eigenvalue problem $AX = \lambda BX$ where A is symmetric, and B is symmetric positive-definite can be solved by using an algorithm for the symmetric eigenvalue problem $QX = \lambda X$, where Q is a symmetric matrix.

p.t.o.

2.(b) Assuming that a subroutine JACOBI (N,Q,V) is available for computing the eigenvalues and eigenvectors of a symmetric matrix with the following meaning of the parameters :

N = order of the given symmetric matrix

Q = the input symmetric matrix whose eigenvalues are to be computed

V = the two - dimensional array whose columns store the eigenvectors of Q.

Describe, by means of a flowchart, the computing process for the general eigenvalue problem stated in 2(a). [10+10=20]

3.(a) Show that the quadrature formula

$$\int_a^b w(x)\, f(x)\, dx \simeq \sum_{i=1}^{n} A_i\, f(x_i)\ ,\quad w(x) \geq 0,$$

will be exact for all polynomials of degree $\leq 2n - 1$ if the x_i's are chosen to be the zeros of $P_n(x)$ where $P_n(x)$ is a polynomial of degree n and belongs to the set of orthogonal polynomials $P_i(x)$ defined for the inner produce $\langle P_i, P_j \rangle = \int_a^b w(x)\, P_i(x)\, P_j(x)\, dx$.

(b) Find a,b,c and α such that the quadrature formula

$$\int_{-1}^{+1} f(x)dx \simeq a[f(-1) + f(1) + b[f(-\alpha) + c\, f(c)$$

may be exact for all polynomials upto a highest possible degree (to be determined by you).

Use this method to compute the value of

$$\int_0^5 [(5x + 13)/2]^{1/2}\ dx.$$

[8+(8+4)=20]

4. The solution of the integral equation $y(x) = 1+\int_0^x f(t)\ y(t)dt$, where f is a given function, is to be computed at a given number of points : 0, h, 2h, ..., nh.

(i) Derive an algorithm for tabulating y(x).

(ii)For the following function f(x) compute y(x) :

x	0	0.25	0.50	0.75	1
f	0.5000	0.4794	0.4594	0.4398	0.4207

at the values of x given above. [10+10=20]

OR

Contd.....

OR

5. The function $y = f(x)$ which is given in the table below has a minimum in the interval $2 < x < 1.4$. Find x and $f(x)$ corresponding to the minimum.

x	$y = f(x)$
0.2	2.10022
0.4	1.98730
0.6	1.90940
0.8	1.86672
1.0	1.85937
1.2	1.88737
1.4	1.95063

[12+8 = 20]

6.(a) Compute the value of

$$C = \lim_{n \to \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} - \log n\right)$$

Correct to 6 decimal places.

6.(b) Compute $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ correct to 6 decimal and use this result to compute the value of

$$\frac{1}{1.1} - \frac{2}{3.3} + \frac{1}{5.5} + \frac{1}{7.7} - \frac{2}{9.9} + \frac{1}{11.11} + \frac{1}{13.13} - \frac{2}{1515}$$

[10+10 = 20]

7. Let $y_r = \int_0^1 e^{4/3\,(x-1)}\, x^{r+3}\, dx$, $r = 0, 1, 2, \ldots$

Show that

$$y_r = \frac{1}{0.75(r+4)}\,(0.75 - y_{r+1})$$

Discuss the ill-conditioning of the problem for forward and backward recurrence for computing y_r's.

[8+12 = 20]

8. Describe mathematically the two methods of solving a set of n non-linear equations in n unknowns known as Newton-Gauss Seidel and Gauss-Seidel-Newton methods. Solve the following equations by one of these methods

$$x - 0.1y^2 + 0.05\, z^2 = 0.7$$
$$0.3x^2 + y - 0.1\, xz = 0.5$$
$$0.4y^2 + 0.1xy + z = 1.2$$

Correct to three decimal places.

(No credit will be given if the problem is solved by methods other than indicated).

[5+5+10 = 20]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Vectors and Matrices II
Semestral-II Examination

Date : 8.5.1991 Maximum Marks : 60 Time : 3 Hours.

Note : This paper carries 75 marks. You can answer as many questions you like. Maximum marks you can get is only 60.

1. For a square matrix, define the determinant and the cofactor of an element. Derive an expression for the determinant in terms of the elements of a row and their cofactors. [10]

2. Let A be a real symmetric matrix of order n. Show that there exists a nonsingular matrix Z such that $A = ZDZ'$ where D is a diagonal matrix such that when A is positive definite, kth order leading principal minors of A and D are equal for all k. Hence show that if A is positive definite then it can be written as BB' for some nonsingular matrix B. [10]

3. State and prove the Courant-Fischer min-max theorem on eigen values of a real symmetric matrix. [10]

4.(a) Let A be a square matrix of order n. Show that there exists a unitary matrix P such that $A = PTP^*$ where T is upper triangular.

(b) Deduce spectral decomposition for a real symmetric matrix and using it, Obtain singular value decomposition of any matrix. [5+5 = 10]

5. Let $A = BC$ be a rank factorization of A. Write down the expressions for (a) A^+ and (b) $P(A)$, the orthogonal projection operator onto column space of A in terms of B and C. Prove that these expressions given by you indeed are A^+ and $P(A)$. [5+5 = 10]

6. Let $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \left(\begin{array}{cc|cc} 4 & 0 & 2 & 8 \\ 0 & 1 & 2 & 3 \\ \hline 2 & 2 & 14 & 4 \\ 8 & 3 & 4 & 33 \end{array}\right)$

Obtain an upper triangular matrix T such that $A = T'T$ using square root method and discuss the uniqueness of T. Using the above computation write down $A_{22} - A_{21} A_{11}^{-1} A_{12}$ where A_{11} and A_{22} are 2x2 matrices. [10]

7. Show that

   (a) A and B are of same order and $A'A = B'B$ if and only if $A = PB$ for some orthogonal matrix P.

   (b) A has a commuting g-inverse if and only if $r(A) = r(A^2)$ and in such a case there exists a unique reflexive commuting g-inverse of A.

   (c) Let H and E be Hermite canonical forms of matrices A and B then H = E if and only if $\mathcal{R}(A) = \mathcal{R}(B)$.

[3x5 = 15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Probability II
Semestral-II Examination

Date : 6.5.1991 Maximum Marks : 100 Time: $3\frac{1}{2}$ Hours.

Note : This paper carries questions worth a total of 125 marks. Maximum you can score is 100 marks. Answer as much as you can.

Important : Priority should be on answering correctly rather than answering all.

1. For a random variable X taking non-negative integer values only, denote by ϕ the probability generating function of X and by Q the generating function of the sequence $\{q_k\}$ where $q_k = P(X>k)$ for $k\geq 0$.

   (a) Show that $Q(s) = \frac{1-\phi(s)}{1-s}$ for $-1<s<1$.

   (b) Derive formulas for $E(X)$ and $V(X)$ in terms of Q and its derivatives.

   (c) Deduce, in particular, that EX^2, if it exists, equals $\sum_{k=0}^{\infty}(2k+1)P(X>k)$.

   [6+4+2]=[12]

2. (a) Show that if X is a random variable with a continuous distribution function F, then the random variable $Y = F(X)$ has the uniform distribution over (0,1).

   (b) Show that if X and Y are independent random variables with a common continuous distribution function, then $P(X=Y) = 0$.

   [8+4] = [12]

3. Let X be exponentially distributed with parameter α. Show that the random variable $Y = X(\text{mod } 1) \stackrel{\text{def}}{=} X-[X]$ has a density.

   [8]

4. (a) Show that if F is a bivariate probability distribution function, then it must satisfy $F^2(x,y) \leq H(x)G(y)$ for all $(x,y) \in \mathbb{R}^2$, where H and G are the marginals of F.

   (b) Let X,Y be independent U(0,1) random variables, and, let $V = \max(X,Y)$ write down the joint distribution function of (X,V). Show that this joint distribution function is continuous. Does it admit a joint density ? Justify your answer.

   [4+(5+3+4)]=[16]

p.t.o.

5. Find the probability that the quadratic equation $\lambda^2 - 2X\lambda + Y = 0$ in λ has real roots, if X and Y are random coefficients having joint density

$$f(x,y) = \begin{cases} \frac{\alpha}{2|x|} e^{-\alpha|x|} & \text{if } 0 < y < |x| \\ 0 & \text{otherwise} \end{cases} \quad (\text{Here } \alpha > 0).$$

[10]

6. Derive the density function of the t-distribution with parameter k, and, find a general formula for its moments.

[8+7] = [15]

7. Let (X,Y) have a joint normal density with zero means, unit variances and covariance ρ. Denoting by (R,Θ) the usual polar transformation, find the density of Θ.

Hence show that $P(XY>0) > P(XY<0)$ if and only if $\rho>0$.

[9+7] = [16]

8. Let X,Y be independent random variables. For every $n\geq 1$, denote by X_n the discrete random variable defined as

$$X_n(w) = \frac{k}{2^n} \text{ if } \frac{k}{2^n} \leq X(w) < \frac{k+1}{2^n},\ k = \ldots,-2,-1,0,1,2,\ldots$$

Show that for any $x \in \mathbb{R}$. $P(X_n + Y \leq x) = E(G(x-X_n))$, where G is the distribution function of Y.

Deduce that $P(X+Y\leq x) = E(G(x-X))$ for every $x \in \mathbb{R}$.

[8+4] = [12]

9. A point is picked at random from the unit interval (0,1), thus partitioning the interval into two subintervals.

Denoting by Y the length of the longer subinterval and by Z the shorter one, show that Y/Z does not have finite expectation.

[12]

10. Let X be a non-negative random variable such that $Y=[X]$ has Poisson distribution with parameter $\lambda(\lambda>0)$, $Z=X(\text{mod } 1)$ is uniformly distributed over (0,1) and X,Z are independent. Show that X has density and find the density.

[12]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Calculus II
Semestral-II Examination

Date : 2.5.1991 Maximum Marks : 100 Time: 3 Hours.
Note : Answer all questions.

1.(a) If f is Riemann-Stieltjes integrable on [a,b] w.r to α, prove that $\lim_{\Delta(P)\to 0} S(P,f,\alpha)$ exists, and equals $\int_a^b f\,d\alpha$, where $S(P,f,\alpha)$ is the Riemann-Stieltjes sum of f for the partition P with respect to α.

(b) Evaluate the following limit by writing it as an integral and then using the fundamental theorem of calculus.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(n+2k)^4}{n^5}.$$

[7+7 = 14]

2.(a) If φ'' is continuous and non-zero on [a,b], and if there is a constant m > 0 such that $\varphi'(t) \geq m$ for all t in [a,b], then use the Second Mean-value Theorem for integrals to prove that

$$\left| \int_a^b \sin \varphi(t)\,dt \right| \leq \frac{4}{m}$$

[Hint: Write the integrand as $[\sin \varphi(t)\cdot \varphi'(t)].\frac{1}{\varphi'(t)}$]

If a > 0, show that $\left|\int_a^x \sin(t^2)dt\right| \leq \frac{2}{a}$ for all x > a.

(b) Show that

$$\int_0^a x\sqrt{\frac{a^2-x^2}{a^2+x^2}}\,dx = \frac{a^2}{2}\left(\frac{\pi}{2}-1\right).$$

[7+7 = 14]

3.(a) Show that $\int_0^\infty \frac{\cos x}{1+x}dx$ converges but not absolutely.

(b) Define

$$f(x) = \left(\int_0^x e^{-t^2}dt\right)^2 \text{ and } g(x) = \int_0^1 \frac{e^{-x^2(t^2+1)}}{t^2+1}\,dt$$

(i) Show that $f'(x) + g'(x) = 0$, and deduce $f(x)+g(x)=\frac{\pi}{4}$.

(ii) Use (i) to prove $\int_{-\infty}^{\infty} e^{-t^2}dt = \sqrt{\pi}$

[7+7 = 14]

4.(a) Show that if $\Sigma f_n(x)$ converges uniformly, and $\{g_n(x)\}$ is uniformly bounded on $S \subset \mathbb{R}$, and $\{g_n(x)\}$ is monotonic for every fixed $x \in S$, then $\Sigma f_n(x)\, g_n(x)$ converges uniformly on S.

p.t.o.

4.(b) Let $f_n(x) = nx(1-x)^n$. Show that $\{f_n(x)\}$ converges pointwise but not uniformly on the interval [0,1]. Verify that term-by-term integration leads to a correc result in this case.

[7+7=14]

5.(a) Determine the interval of convergence of the power se

$$x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1.3}{2.4} \cdot \frac{x^5}{5} + \frac{1.3.7}{2.4.6} \cdot \frac{x^7}{7} + \ldots$$

(b) Show that the series $\sum_{n=1}^{\infty} (\sin nx)/n^2$ converges for every real x, and denote its sum by f(x). Prove that f is continuous on $[0,\pi]$, and that

$$\int_0^{\pi} f(x)dx = 2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}.$$

[7+7 =14]

6. Class Tests.

[30]

[1990-91][158]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1990-91
Statistical Methods II
Semestral-II Examination

Date : 29.4.1991   Maximum Marks : 100 Time: 4 Hours.

Group A

Note : Answer any three questions.

1.(a) Define orthogonal polynomials over n distinct values $x_1, x_2, \ldots, x_n$.

(b) Show that the orthogonal polynomials are essentially obtained by applying Gram-Schmidt process over $\underset{\sim}{u}_0, \underset{\sim}{u}_1, \ldots, \underset{\sim}{u}_{n-1}$, where $\underset{\sim}{u}_j = (x_1^j, x_2^j, \ldots, x_n^j)$. Show that u_j's are linearly independent.

(c) Briefly describe the use of orthogonal polynomials for fitting polynomial curve over a scatter of n points $(x_1, y_1), \ldots, (x_n, y_n)$.

[3+7+10]

2.(a) Consider n observations on a set of p variates $(X_1, \ldots, X_p)$. Define the multiple correlation coefficient $R_{1(23..p)}$ between X_1 and $(X_2, \ldots, X_p)$. Show that $0 \leq R_{1(23..p)} \leq 1$, and discuss the limiting cases.

(b) Show that for any linear combination $\sum_j^p c_j X_j$ with non-zero variance.

$$\text{Corrln.}\left(X_1, \sum_j^p c_j X_j\right) = \left[\text{Corrln.}\left(X_1, \sum_j^p b_{1j} X_j\right).\right.$$
$$\left.\text{Corrln.}\left(\sum_j^p b_{1j} X_j, \sum_j^p c_j X_j\right)\right],$$

where the linear regression coefficients are given by b_{1j}'s and $(s_{12}, \ldots, s_{1p})$ is assumed to be non-zero, $S = [s_{ij}]$ being the covariance matrix of $X_1, \ldots, X_p$.

Use this to show that

$$R_{1(23..p)} = \sqrt{\frac{\sum_j^p b_{1j} s_{1j}}{s_{11}}}$$

(c) Show algebraically or geometrically

$$1-R^2_{1(23..p)} = (1-R^2_{1(23..(p-1))})(1-r^2_{1p.23...(p-1)}).$$

p.t.o.

3.(a) What are the advantages of using probability methods of sampling ?

(b) What is selection bias ? What is non-response bias ?

(c) Consider a population of N units $u_1, \ldots, u_N$ with $X(u_i)=x_i$. Let $\mu = \frac{1}{N}\sum_1^N x_i$, $\sigma^2 = \frac{1}{N}\sum_1^N (x_i - \mu)^2$. Let $X_1, \ldots, X_n$ be the X-values for a sample of n units drawn from the population by SRSWOR. Obtain Var $(\frac{1}{n}\sum_1^n X_i)$.

Suppose, furthermore, the population has been divided into K strata. The size of the ith stratum being N_i. A sample of size n_i is taken from the ith stratum by SRSWOR. What would be the variance of $\overline{X}_{st}$ under proportional allocation ($\sum_1^k n_i = n$).

Compare this estimate $\overline{X}_{st}$ with $\overline{X}$, obtained under the first sampling scheme, when $n_i << N_i$ for all i.

[3+5+12]

4. Describe methods for generating random variates from any three of the following probability models using only random number generators. Give theoretical justification in each case

(a) Normal distribution $N(\mu,\sigma^2)$

(b) Poisson distribution $P(\lambda)$

(c) Standard logistic distribution

(d) Negative binomial (m,p).

[20]

Group B

1. The following table gives the means and (ns_{ij}) matrix for four variables.

X_0 = gasoline yield percent

X_1 = crude oil gravity (API)

X_2 = crude oil vapour pressure

X_3 = crude oil 10% point ASTM

based on 32 samples of crude oil. Obtain linear prediction formula for predicting X_0 on the basis of X_1, X_2, X_3.

$$(ns_{ij}) = \begin{pmatrix} 3564.1 & 461.4 & 334.4 & -3931.0 \\ & 984.5 & 264.0 & -4591.9 \\ & & 212.8 & -2763.0 \\ & & & 43690.0 \end{pmatrix}$$

$i,j = 0,1,2,3$

Means $\overline{X}_0 = 19.66$, $\overline{X}_1 = 39.25$, $\overline{X}_2 = 4.18$

$\overline{X}_3 = 241.5$

Contd.....

Group B Contd.....

1. Contd.....

   Also, obtain $R^2_{0(123)}$. and $r^2_{03.12}$;
   interpret the results obtained.

   [22]

2. The following table gives the frequency distribution of systolic blood pressure of 1206 women, aged 17-24 years.

Blood pressure (mm)	Percentage frequency
85-95	1
95-100	3
100-105	10
105-110	11
110-115	15
115-120	20
120-125	13
125-130	10
130-135	8
135-140	4
140-145	3
145-155	2
Total	100

   Fit a suitable normal distribution to this frequency distribution. Draw the histogram with end-points of class-intervals being standardized, and draw the appropriate normal curve over the histogram . Test for goodness of fit.

   [18]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR:1990-91
SEMESTRAL-I BACKPAPER EXAMINATION
COMPUTATIONAL TECHNIQUES-I

Date:2.1.91 Maximum Marks:100

Note: From Group A, answer Qn.1 and any two from the rest. From Group B, answer any three questions. All questions in Group B are of equal value.

GROUP-A Max.Marks:50

1. You are given two sorted arrays A and B of size N_1 and N_2, respectively. The array A is sorted in ascending order, while B is in descending order. Write a FORTRAN program to generate a new array C of size (N_1+N_2) from the elements of A and B such that C is sorted in ascending order. [20]

(a) Rewrite the following program segment without using any IF statement.

```
INTEGER   PROD
READ(*,*) PROD
IF (PROD.EQ.1) THEN
   PRICE = 10.0
ELSE
   IF (PROD.EQ.2) THEN
      PRICE=20.0
   ELSE
      IF (PROD.EQ.3) THEN
         PRICE=15.0
      ELSE
         IF (PROD.EQ.4) THEN
            PRICE=25.0
         ENDIF
      ENDIF
   ENDIF
ENDIF
STOP
END
```

(b) In the main program you have two unlabelled COMMON statements as follows.

```
COMMON    A,B,C
COMMON    D,E
```

In a subroutine the order of the two COMMON statements has been changed as

```
COMMON    D,E
COMMON    A,B,C
```

In the subroutine, will you get the desired values for the common variables? Explain your answer. [10+5=15] p.t.o.

3.(a) Write a FORTRAN program to read an integer of atmost four digits and then generate the integer in which the digits are in reverse order. Print both the input and output integers. For example, if the input integer is 1359 then the generated integer would be 9531. The integer should be read as a single number. You are not allowed to read the digits separately.

(b) You have a function named PLOT (A,B,N,FX); when A,B are two real numbers (A,B > 0, A < B), N is a positive integer and FX is the name of the function to be plotted. Write the main program to plot SIN and COS functions using PLOT for a set of A,B and N (which are to be read). Note that PLOT evaluates FX at N equispaced points over [A,B]. [10+5=15]

4. Write a FORTRAN program to find the largest and smallest elements of a symmetric matrix by utilising the property of a symmetric matrix. [15]

GROUP-B Max.Marks:50

5. For a computer with number base d and t-digit mantissa for a floating-point number, find the upper bounds on the relative round-off errors of u=3(ab) and v=(a+a+a)b. Illustrate with a=0.4299 and v=0.6824, doing all computations in decimal arithmetic with 4 digits.

6. Write two subroutines as follows:

(i) a subroutine to find an initial approximation to a root of a non-linear equation f(x)=0 in the interval (a,b) by the bisection method;

(ii) a subroutine to compute the final approximate value of the root with a chosen precision using the initial approximation provided by the subroutine in (i) by the linear interative method.

Write a 'main programme' to compute the approximate value of the root in (2,3) of the equation

$$x^4-0.486\ x^3-5.792\ x^2+0.486\ x + 4.792 = 0$$

with error $\leq 10^{-5}$.

7. Find the order of convergence of the secant method.

contd. 3/-

8. Describe an algorithm in the form of a flowchart for computing the approximate solution of a system of linear equations by Gaussian elimination method with pivoting.

For implementing the algorithm in a FORTRAN programme, describe how you should organize the entire computing process in appropriate subroutines/functions.

9. Describe an algorithm for computing the approximate value of

$$x+\frac{x^3}{\text{[illegible]}}+\frac{1\cdot 3x^5}{\text{[illegible]}}+\frac{1\cdot 3\cdot 5\,x^7}{\text{[illegible]}\,4\cdot 6\cdot 7}+\ldots\ldots\ldots$$

Write a FORTRAN programme to implement the algorithm.

:ss:

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR:1990-91
SEMESTRAL-I BACKPAPER EXAMINATION
CALCULUS-I

Date: 31.12.90 | Maximum Marks:100 | Time: 3 Hours

Note: Answer all questions.

1.(a) If $a < b$ are two real numbers then show that there is a rational number r such that

$$a < r < b.$$

(b) If $\{a_n\}$ and $\{b_n\}$ are sequences of real numbers, then does the equality

$$\overline{\lim}\,(a_n+b_n) = \overline{\lim}\,(a_n) + \overline{\lim}\,(b_n).$$

always hold? Justify your answer. [14+6]

2.(a) If $\{a_n\}$ is a sequence of positive real numbers such that

$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists then show that

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} a_n^{1/n}.$$

(b) Let $\{x_n\}$ be the sequence of reals defined inductively by

$$x_1 = 2$$

$$x_{n+1} = \sqrt{2 + x_n}$$

Show that $\lim\limits_{n\to\infty} x_n$ exists. [14+8]

3.(a) State and prove the root test for the convergence of a series of real numbers.

(b) Show that if Σa_n is a series of real numbers with bounded partial sums and $\{b_n\}$ is a sequence decreasing to 0 then $\Sigma a_n b_n$ is convergent. [8+10]

4.(a) Let $f(x) = x^3$ if x is rational

$= \frac{1}{x}$ if x is irrational.

Determine the set of all real numbers a such that $\lim\limits_{x\to a} f(x)$ does exist.

contd. 2/-

4.(b) Show that the composition of two continuous functions is continuous.

(c) Show that every continuous function $f : [0,1] \to \mathbb{R}$ is uniformly continuous.

(d) Let

$$f(x) = x^{\alpha} \sin 1/x \quad \text{if } x \neq 0$$
$$= 0 \quad \text{if } x = 0$$

(where α is a positive real number).

Show that f is differentiable at 0 if and only if $\alpha > 1$. [8+8+16+8]

INDIAN STATISTICAL INSTITUTE
B.STAT(HONS.) I YEAR: 1990-91
SEMESTRAL-I EXAMINATION
COMPUTATIONAL TECHNIQUES AND PROGRAMMING I

Date: 12.12.90 Maximum Marks: 100 Time: 3 Hours

1. Suppose your FORTRAN Compiler supports only one dimensional array; i.e., you cannot use an array of more than one dimension in your FORTRAN programme. Write a program to read and store a lower triangular matrix and then print it in a matrix form. [20]

2. Answer either question (a) or question (b).

(a) (i)

```
    DØ  10   I = 1,10
    DØ  10   J = 1,15
    IF ( IA(I,J). EQ.0) GO TO 100
10  CONTINUE
    WRITE (*,*) 'all elements are non zero'
    GO TO  120
100 WRITE (*,*) I,J, 'th  element is the first non zero element'
120 CONTINUE
```

Rewrite the above program segment without using any GO TO and IF - THEN - ELSE statements (IA is an array of size 10x15).

(ii)

```
    SUBROUTINE   TRNSPS (A)
    INTEGER      A(10,10)
    DØ  10   I = 1,10
    DØ  10   J = 1,10
    M = A(I,J)
    A(J,I) = A(I,J)
    A(I,J) = M
10  CONTINUE
    RETURN
    END
```

The above subroutine has been written for transposing an array of size 10x10. Does it work correctly? If not, correct it. [5+10=15]

(b) Comment on and/or justify the following statements.

a) "STOP and END serve same purpose."

b) "There is no difference between EQUIVALENCE and COMMON statements."

c) "A FORTRAN subroutine can transfer control to more than one points of the calling program." [5+5+5=15]

contd. 2/-

3. Answer any one:

(a) What will be the output of the followings FORTRAN program (Read the program very carefully)? Justify your answer.

```
      DIMENSION     IX  (3,2)
      K = 0
      DØ  10  I = 1,3
      DØ  10  J = 1,2
      K = k + 1
10    IX  (I,J) = k
      CALL  TEST (IX)
      STOP
      END
      SUBROUTINE TEST (IX)
      DIMENSION IX  (2,3)
      WRITE (*,*) IX  (1,2)
      RETURN
      END
```

[15]

(b) Find the values of J and K after each of the following program segments are executed.

(i)
```
J = 5
K = 10
IF (J.LE.10) THEN
    K = K+5
    J = J+K
ENDIF
J = J+K
J = 2*J
```

(ii)
```
J = 5
K = 10
IF (J.LE.10) THEN
    K = K+5
    J = J+K
ELSE
    J = J+K
ENDIF
J = 2*J
```

(iii)
```
J = 5
K = 10
IF (J.GT.10) THEN
    K = K+5
    J = J+K
ENDIF
J = J+K
J = 2*J
```

(iv)
```
J = 5
K = 10
IF (J.GT.10) THEN
    K = K+5
    J = J+K
ELSE
    J = J+K
ENDIF
J = 2*J
```

[15]

contd. ...3/-

4. Answer any one:

(a) Show that the maximum relative round-off error in the result of a floating point arithmetic operation does not depend on the sizes of the numbers and that in a computer with a number system of radix b and t - digit mantissa, the maximum relative round-off error is b^{-t+1}.

Consider the n floating - point numbers $x_1, x_2, x_3, \ldots, x_n$ with

$$0 < x_1 < x_2 < \ldots\ldots < x_n.$$

Find the upper bound on the round-off error in $x_1+x_2+\ldots\ldots+x_n$ and in $x_n+x_{n-1}+\ldots+x_1$. Use the result in finding the best way of adding n given floating-point numbers. [5+4+4+2=15]

(b) Construct an algorithm in the form of a flow-chart for computing the approximate value of the infinite series

$$1 - \frac{(x/2)^2}{1^2} + \frac{(x/2)^4}{1^2.2^2} - \frac{(x/2)^6}{1^2.2^2.3^2} + \ldots\ldots\ldots\ldots$$

for a given value of x and a given precision. Write a FORTRAN programme to implement the algorithm. [7+8=15]

5. Answer any one:

(a) Discuss completely the derivation and implementation of Newton-Raphson method for computing the approximate value of a root of a given non-linear equation in one variable. How would you modify the method to have quadratic convergence for a multiple root? [4+4+7=15]

(b) If, in an iterative method; $x_{n+1}=f(x_n)$, for finding a root of the equation $x = f(x)$, the error relation $e_{n+1}=K\ e_n$ holds exactly ($K < 1$ is a constant independent of e_n) then it is possible to find the exact value of the root from three consecutive approximations: x_{n+2}, x_{n+1} and x_n. Construct the expression in x_{n+2}, x_{n+1} and x_n which gives the exact root. [15]

6. Answer any one:

(a) Derive the LU factorization technique of a given square matrix A. Show how this factorization may be utilized in computing the solution of the system of linear equations: $AX = \beta$. How would you compare this method with the Gaussian elimination method? [10+5+5=20]

(b) Describe an algorithm for computing the inverse of a square matrix A (if it exists) in the A array itself without using a separate array for storing the inverse. [20]

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR: 1990-91
SEMESTRAL-I EXAMINATION
STATISTICAL METHODS-I

Date: 10.12.90 Maximum Marks:100 Time: 4 Hours

Note: Attempt all the questions.

1. Consider the following devices which have been developed in experiments: (a) Blocking (b) Randomization (c) Replication (d) Sample Survey (e) Control group and blind allocation. How does these devices strengthen experiments? Briefly discuss any three of the above with illustrations. [21]

2. The following table gives the frequency distribution of systolic blood pressures of a sample of 1747 women aged 35-44 years.

Blood Pressure (mm)	Percentage relative frequency
85-95	3
95-100	5
100-105	9
105-110	11
110-115	15
115-120	16
120-125	9
125-130	10
130-135	8
135-140	5
140-145	4
145-150	2
150-165	3
Total	100

(a) Draw the histogram and the box and whisker chart.
(b) Find approximately the proportion in the group with systolic blood pressures within

(i) mean $\pm$ SD , (ii) mean $\pm$ 2 SD. [24]

3. Answer any two from the following.

(a) Prove by calculus that the arithmetic mean of n positive numbers is greater than or equal to their geometric mean.

(b) Let $\bar{x}$, m and S denote the mean, median and the S.D. of a data set. Show that

$$-S \le \bar{x}-m \le S.$$

When are the equalities attained?

cont'd. 2/-

3. (c) Pearson and Lee obtained the following results in a large study of body measurements:

average height of men	68 inches	SD	2.7 inches
average forearm length	18 inches	SD	1 inch

correlation = 0.80

The scatter diagram was oval shaped.
Find approximately the percentage of men having forearms 18" long to the nearest inch. Of the men who are 68" tall, what percentage (approximately) have forearms which are 18" long to the nearest inch ? [15]

4. Answer any one from the following:

(a) Two bivariate data sets have the same SD line and the same correlation r . Show that the correlation in the pooled data is at least r .

(b) Let e^2_{yx} be the correlation ratio of y on x, and r be the correlation coefficient for a bivariate frequency distribution. Show that

$$r^2 \leq e^2_{yx} \leq 1$$

When are the equalities attained ? [8]

5. The following tables show body weight and kidney measurement for normal and diabetic ten-month old mice.

Normal mice

Body weight (g)	Kidney (mg)
34	810
43	480
35	680
33	920
34	650
26	650
30	650
31	560
31	620
27	740
28	600

Diabetic mice

Body weight (g)	Kidney (mg)
42	1030
44	1240
38	1150
52	1280
48	1240
46	1100
34	1040
44	1080
38	870

(a) Plot the scatter diagrams of the two data sets on the same graph. (Denote points by ⊙ for the normal set and by ⊗ for the diabetic set. Take x as the body weight and y as the kidney weight).

5. (b) Derive the least squares lines (y on x) for the two data sets and draw them on the scatter diagram. Which line provides a better fit to the corresponding data ?

(c) Pool the two data sets. Use the following results with the usual notations:

$$n\ S_{xy} = n_1\ S_{xy}^{(1)} + n_2\ S_{xy}^{(2)} + \frac{n_1 n_2}{n_1+n_2}(\bar{x}^{(1)}-\bar{x}^{(2)})(\bar{y}^{(1)}-\bar{y}^{(2)}).$$

Similar results hold for S_{xx} and S_{yy}.

Derive the least-squares line (y on x) for the pooled data set, and draw it on the scatter diagram. Compare the total sum of squares of errors for fitting the above single line with the total sum of squares of errors for fitting the data sets by two separate lines as in (b). [5+18+9]

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR: 1990-91
SEMESTRAL-I EXAMINATION
PROBABILITY-I

Date: 7.12.90 Maximum Marks: 100 Time: 3 Hours

Note: The paper carries question worth 120 points. Answer as many as you can. Maximum you can score is 100 points. Your answers should be as self contained and precise as possible.

1. From a deck of 52 bridge cards, a random sample of 10 cards is drawn without replacement. What is the probability of having cards from all four suits in the sample? [15]

2. There are m urns containing m balls each. In the kth urn, k of the m balls are black and the rest are white (k=1,2,...,m). One of the urns is selected at random and then balls are drawn from that urn one by one with replacement. Let $p_{n,k}$ be the conditional probability that the kth urn was selected given that the first n draws resulted in black balls. Find $p_{n,k}$. Show that $\lim_{n \to \infty} p_{n,k} = 0$ if $k < m$ while $\lim_{n \to \infty} p_{n,k} = 1$ if k=m. [10]

3. $A_1, A_2, \ldots, A_n$ are independent events, each having the same probability α $(0 < \alpha < 1)$. Let B denote the event that exactly one of the A_i's occur and C the event that exactly two of them occur. If $P(B) = P(C)$, what is α? [10]

4. Consider a game involving repeated trials, at each of which one of the two sides win. The side that first accumulates 21 victories wins the whole game (Recognise the game?). Suppose side A has probability $p (0 < p < 1)$ of winning a trial. Denoting by X the number of victories accumulated by the losing side when the game ends, find the distribution of X. [15]

5. Let n balls be distributed at random in r boxes. (Everything is distinguishable — balls among themselves, boxes among themselves and also balls from boxes!) Find the expected value and the variance of the number of boxes that are occupied. [15]

6. Let X and Y be independent random variables, each having geometric distribution with parameter p.
   (a) Find the conditional distribution of X given X+Y=n.
   (b) Let V= max(X,Y) and W=min(X,Y). Prove that V and V-W are independent. (10+10)=[20]

contd. 2/-

7. (a) Prove that for a random variable X taking non negative integer values only, $E(X)$ exists if and only if the series $\sum_{n=0}^{\infty} P(X > n)$ converges, and, in that case $E(X)$ equals $\sum_{n=0}^{\infty} P(X > n)$.

   (b) Let X_1 and X_2 be independent random variables, both having the same discrete distribution given by the probability mass function

$$p(k) = \frac{1}{k(k+1)} \quad \text{for } k \in \{1,2,\ldots\}$$

   Show that the random variable $X = \min(X_1, X_2)$ has finite expectation. (10+10)=[20]

8. An urn contains n balls of which k are black and the rest white where $k > \frac{n}{2}$. Balls are removed one by one from the urn until it becomes empty,

   (a) Let $p_{k,n}$ denote the probability that during the entire process the urn always contained more black balls than white. Show that $p_{k,n}$ satisfies the recursive relation

$$p_{k,n} = \frac{k}{n} p_{k-1,n-1} + \frac{n-k}{n} p_{k,n-1}$$

   Show by induction that $p_{k,n} = \frac{2k-n}{n}$.

   (b) If $q_{k,n}$ denotes the probability that at any stage there are more black balls than white among the balls that have been withdrawn, then argue that $q_{k,n} = p_{k,n}$. Hence answer the following well-known (Ballot) problem:

   " If in a ballot candidate A secures a votes and candidate B secures b votes, where a > b, what is the probability that throughout the counting A was always ahead of B? (10+5)=[15

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INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR:1990-91
SEMESTRAL-I EXAMINATION
CALCULUS-I

Date:14.12.90 Maximum Marks:100 Time: 4 Hours

Note: The paper carries 116 marks. You can answer as many questions as you wish. Maximum score 100.

1.(a) Show that there is a real number x such that $x^2 = 7$.

(b) Let A and B be non-empty sets of real numbers,

$$A + B = \{x+y : x \in A \text{ and } y \in B\},$$

and

$$A.B = \{x.y : x \in A \text{ and } y \in B\}.$$

Which of the following statements is always correct?

(i) Sup (A+B) = Sup (A) + sup (B)

(ii) Sup (A.B) = Sup (A) . Sup (B).

Justify your answer. [8+7]

2.(a) Find

$$\lim_{n\to\infty} \left(\frac{1}{n^2+1} + \dots + \frac{1}{n^2+n} \right).$$

(b) Determine the set of all real numbers x

such that $\lim_{n\to\infty} n^{100}x^n$ exists.

(c) Show that every bounded sequence of real numbers has a convergent subsequence. [5+5+12]

3.(a) Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers such that

$\limsup \frac{a_n}{b_n}$ is finite and Σb_n is absolutely convergent. Prove

that Σa_n is also absolutely convergent.

(b) Test for the convergence of the following series:

(i) $\sum_{n=2}^{\infty} \sin \frac{\pi}{n}$, (ii) $\sum_{n=1}^{\infty} \frac{1}{\log(e^n+e^{-n})}$.

(c) If Σa_n^2 and Σb_n^2 are convergent then show that $\Sigma a_n b_n$ is absolutely convergent. [8+(6+6)+8]

4.(a) Let

$$f(x) = x \quad \text{if } x \text{ is rational}$$
$$= x^2 \quad \text{if } x \text{ is irrational.}$$

Show that $\lim_{x\to 1} f(x)$ exists and $\lim_{x\to 2} f(x)$ does not exist.

contd. ...2/-

4(b) If $f : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $f(x) = 0$ for every rational number x then prove that $f(x) = 0$ for all real x.

(c) Let $f : [0,1] \to \mathbb{R}$ be continuous; then show that f is bounded above and that there is a $x_C \in [0,1]$ such that

$$f(x_C) = \text{Sup}\left\{f(x) : 0 \leq x \leq 1\right\}.$$ [7+8+10]

5.(a) If $f : \mathbb{R} \to \mathbb{R}$ is continuous and takes only rational values i.e. $f(x)$ is rational for every x, then show that f is a constant map.

(b) Determine which of the following functions are uniformly continuous ?

(i) $f(x) = \sin x$, $-\infty < x < \infty$

(ii) $f(x) = x^2$, $-\infty < x < \infty$.

(c) Give an example of a continuous map $f : \mathbb{R} \to \mathbb{R}$ which is not differentiable precisely at $x = 1$ and $x = -1$. [8+(6+6)+6]

:ss:

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR: 1990-91
SEMESTRAL-I EXAMINATION
VECTORS AND MATRICES-I

Date: 5.12.90 Maximum Marks: 60 Time: 3 Hours

Note. Answer as many questions as you can.

1.(a) Define a linear transformation from V to W where V and W are vector spaces over a field F. Show that 'f' is a linear transformation from R^n to R^m if and only if $f(x) = Ax$ for all x in R^n, for some real matrix A of order $m \times n$.

(b) Let S and T be subspaces of R^n such that $S + T = R^n$. Show that f is a projection operator from R^n into S along T if and only if $f(x) = Ax$ for all x in R^n where A is a square matrix of order n satisfying (i) $A^2 = A$, (ii) $\mathcal{C}(A) = S$ and (iii) $N(A) = T$.
Further if $T = S^{\perp}$, show that A is symmetric.
[(2+5)+(2+8+3)=20]

2. Show that a) $r(AB) = r(B) - d\{\mathcal{C}(B) \cap N(A)\}$

b) $r(AB) + r(BC) \leq r(B) + r(ABC)$

c) $r(A^m) = r(A^{m+1}) \Rightarrow r(A^m) = r(A^n)$ for all $n \geq m$. [5+3+2=10]

3. Define idenpotent matrix. Prove that the following are equivalent
(a) A is idenpotent
(b) CB=I where A=BC is any rank factorization of A
(c) $\mathcal{C}(I-A) = N(A)$
(d) $r(A)+r(I-A)=r(I)$
(e) $Ax=x$ for all x in $\mathcal{C}(A)$. [10]

4. Define A^-, a generalized inverse of a matrix A through linear equations. Show that A^- exists for any given matrix A. Show the equivalence of
(a) G is a A^-
(b) $AGA = A$
(c) AG is idenpotent and $r(AG) = r(A)$ [10]

5. Prove or disprove the following (all matrices are real)
(a) $r(A) = r(A^2) \Leftrightarrow CB$ is nonsingular where $A = BC$ is any rank factorization of A.
(b) $AB = B$ and $BA = A \Rightarrow A$ and B are idenpotent.
(c) A and B are symmetric $\Rightarrow AB$ is segmmetric.
(d) $ABB' = CBB' \Leftrightarrow AB = CB$.
(e) For any square matrix A $\mathcal{C}(A) \geq N(I-A)$. [10]

contd. 2/-

6. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 1 & 1 & 4 \end{bmatrix}$

(a) Compute a g-universe of A.

(b) For the system of linear equations Ax=b, discuss the consistency and obtain general solution if consistent in each of the following cases when

(i) $b = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

(ii) $b = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$

[6+2+2=10]

:SS:

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons) I Year 1990-91

FIRST SEMESTER EXAMINATION

REMEDIAL ENGLISH

Date :   Maximum Marks : 100   Time : 3 Hours.

Section I : Reading and Vocabulary

Task 1   (20 marks)

Read the passages and tick off the right answer. Give reasons for your answer.

a) The scientist and the watches.

b) The Sultan and the cheat.

c) The Prisoners.

d) Four Women.

The scientists and the watches

One night, a crazy scientist got involved in a rather silly argument with a fellow scientist. They were arguing about whose watch was the better, the Swiss one or the Japanese one. Being scientists, they decided to do an experiment to test the watches. The first part of the test was to see if both were waterproof. (They were both so convinced of the quality of their watches that they were willing to risk ruining them.)

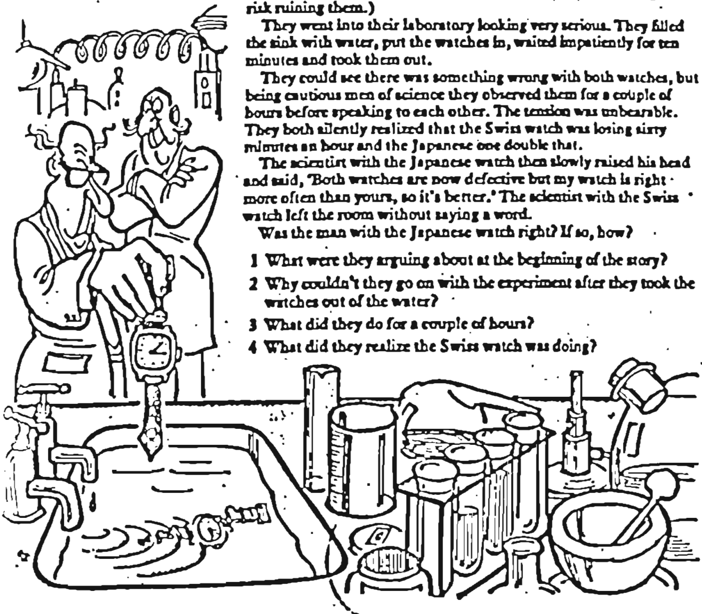
They went into their laboratory looking very serious. They filled the sink with water, put the watches in, waited impatiently for ten minutes and took them out.

They could see there was something wrong with both watches, but being cautious men of science they observed them for a couple of hours before speaking to each other. The tension was unbearable. They both silently realized that the Swiss watch was losing sixty minutes an hour and the Japanese one double that.

The scientist with the Japanese watch then slowly raised his head and said, 'Both watches are now defective but my watch is right more often than yours, so it's better.' The scientist with the Swiss watch left the room without saying a word.

Was the man with the Japanese watch right? If so, how?

1 What were they arguing about at the beginning of the story?
2 Why couldn't they go on with the experiment after they took the watches out of the water?
3 What did they do for a couple of hours?
4 What did they realize the Swiss watch was doing?



5 If the Swiss watch was losing sixty minutes in sixty minutes, was it
   a going forwards?
   b stopped?
   c going backwards?
6 So how often in every 12-hour period would the Swiss watch show the right time?
   a Once.
   b Twice.

7 How many minutes was the Japanese watch losing every hour?

8 If a watch loses 120 minutes every sixty minutes, is it
   a going forwards?
   b stopped?
   c going backwards?

9 How often in every 12-hour period will the Japanese watch show the correct time?
   a Once?
   b Twice?

10 Was the scientist right when he said, 'But my watch is right more often than yours'?

11 Why is this absurd?

The sultan and the cheat

A sultan ordered ten goldsmiths to make ten coins each. Each coin was to weigh exactly ten grams of pure gold.

One of the goldsmiths was a bad man. He decided to cheat. He made all his ten coins one gram short. Now the sultan heard that one of the goldsmiths had cheated. He also heard that this man had made each of his coins one gram short.

The sultan was a very clever person. He took a certain number of coins from each of the smiths, weighed them together once only and found their weight to be 540 grams. This was enough for him to find out which one of the goldsmiths had cheated.

How did he do it and who was the cheat?

1 How many goldsmiths were there?

2 How many of them were cheats?

3 The cheat, like the others, made ten coins. How many grams short was each coin?

4 Did the sultan find the cheat
   a by looking each man in the eye?
   b by weighing coins?
   c by asking his mother?

5 How many times did he weigh the coins he took from them?

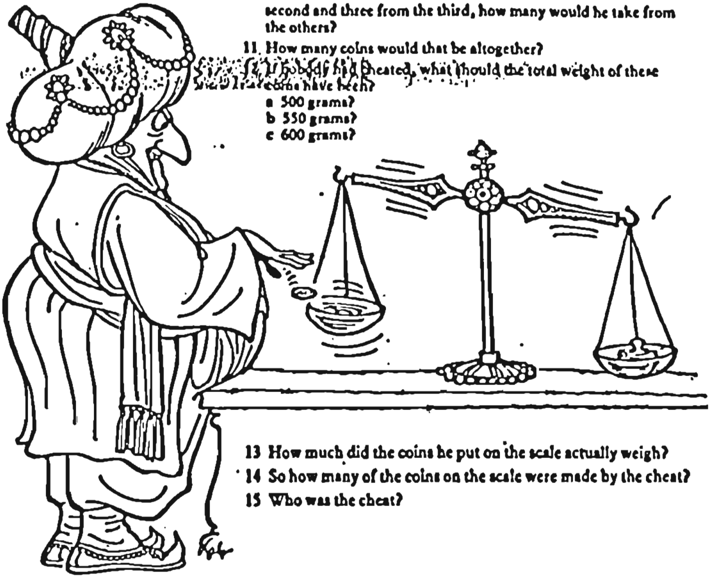
6 Did he take the coins to weigh from
   a one goldsmith?
   b some of the goldsmiths?
   c all of them?

7 Suppose he had taken all ten coins from each smith and put them together on the scales. When he weighed them how many grams short would they have been?

8 Would he have known that one of the smiths had cheated?

9 Would he have known which smith had cheated?

10 Suppose he took one coin from the first smith, two coins from the



second and three from the third, how many would he take from the others?

11. How many coins would that be altogether?

12 If nobody had cheated, what should the total weight of these [illegible] coins have been?

a 500 grams?
b 550 grams?
c 600 grams?

13 How much did the coins he put on the scale actually weigh?

14 So how many of the coins on the scale were made by the cheat?

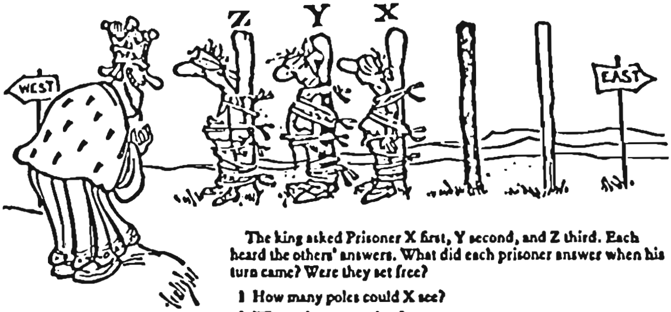
15 Who was the cheat?

The prisoners

The king of an unnamed country never tries his prisoners in a courtroom. Instead, he puts them to a test which he makes up himself. During a riot in the capital, three men were taken prisoner and brought to the king. This was the test he devised. He had the prisoners blindfolded and taken to a field where there were five poles, three white and two black. The poles were in a straight line from east to west. The prisoners were tied to the three poles nearest the west. All three were facing west. When the blindfolds were removed, each prisoner could see only the poles in front of him.

The king said, 'If one of you can tell me the colour of the pole he is tied to, I will set all three of you free. If none of you can tell me, you will have to stay in prison for ten years. If any of you guess wrong, you will all be shot. There are three white poles and two black ones.

I will now ask each of you if he can tell the colour of his pole. You may answer only yes or no or that you don't know.'



The king asked Prisoner X first, Y second, and Z third. Each heard the others' answers. What did each prisoner answer when his turn came? Were they set free?

1 How many poles could X see?
2 What colour were they?
3 If X had seen two black poles instead, what would he have known
4 As it was, what answer did X give the king?
5 When Y heard X's answer, he knew X had not seen two black poles. There are two other colour combinations X might have seen, Y thinks. What are they?
6 What colour pole could Y see?
7 Could Y tell whether his pole was black or white?
8 When the king asked him, what did he have to say?
9 When Z heard Y's answer, he had to think hard and fast. Like Y, he realized that X had not seen two black poles. So Z knew that at most only one of the two poles (his and Y's) could be . . . what colour?
10 Z also knew this: if Y had seen that Z's pole was black, Y would've known his own pole was . . . what colour? But Y didn't.
11 Did Z know the colour of his pole?
12 What happened to the three prisoners?

Four women

In a remote mountain village in the East, there is always one wise woman who is both feared and respected by her people. From the time she is chosen until her death she plays a very important part in the lives of the villagers. When she feels her death is near, she calls her four apprentices to her house to choose her successor. Her test has been a secret for many generations.

. The four women are asked to sit around a table. The wise woman tells them to close their eyes tightly and cover them with their hands. She then tells them she is going to put a mark on each of their foreheads. The mark may be either white or black. Then she marks them and tells them to open their eyes. They look at each others' foreheads. Any woman who sees more black than white marks must stand up. The first woman who can say what colour mark is on her own forehead becomes the successor.

The last time the test took place, all four women stood up. No one spoke for what seemed a long time. Finally one of the women identified her mark.

What was her reasoning?

1 The women were told to stand up only if . . .?
2 How many of the women stood up?
3 And how many of them saw more black than white marks?
4 If the wise woman had made three white marks and one black, how many would have stood up?
5 Suppose she'd made two white and two black marks., Would anyone have stood up? Which ones?
6 If she'd made one white mark and three black, would any of them have stood up? How many?
7 What if she'd made four black marks?
8 So which of the possible colour combinations could be the right one?
9 Would all four women have realized this when they saw that all of them were standing up?
10 If one of the marks was white, which of the women would've seen it?
11 What would they have known about their own marks then?
12 Wouldn't all three of them have said this immediately then?
13 Did any of them say anything?
14 So could any of them have seen a white mark?
15 When one of them spoke, what had she realized and what did she say?

Follow Up

1 Since no-one had a white mark, did all of them have an equal chance?
2 If one had had a white mark, would she have had an equal chance?
3 What other colour combination could the wise woman have used to make it a fair test?
4 Is this a good test of wisdom?
5 What, if anything, would you use it for?

Task 2 (5 marks)

Here are some phrases from a text. The text is not given to you. Each short line stands for one missing letter. The words in brackets will help you.

a) The oper - - - - - of biological clocks is manifest in a number of phenomena. (working)

b) Is there just one ba - - - mechanism or are there a number of systems for telling the time ? (fundamental)

c) There are two es - - - - - - - features in circadian rhythms. (fundamental)

d) There is a clear advantage in being able to ant - - - - - - - the cyclic changes. (predict)

e) Throughout the day there is a ser - - - of activities. (sequence)

Task 3 (5 marks)

Complete the box where possible.

exclude	exclusive	exclusion
	complex	
		proportion
exceed		
		sophistication
	adjustable	
		minimum
	parasitic	
		problem
reduce		

Task 4 (5 marks)

======

The expressions which help to outline the argument in this text are missing. Replace them with the help of words and expressions in the list at the bottom.

Rather than exploiting the environment, shouldn't we be in partnership ?

_____________ we continue to waste the earth's sources as if there were no tomorrow, there could well be no tomorrow. _____________ the year 2000, one third of the world's cropland will have turned to dust. One million species will have become extinct and hundreds of millions of people will face starvation. All this is happening ___________ our civilization has kept on expanding, on the assumption that the world's resources are limitless __________ merely stopping growth is not the answer. ____________ we need is development that works in partnership with the environment, that uses the earth's resources more productivityly and ___________ at the same time is sustainable. This is _____________ Earthlife exists. At the moment our approach is unique among conservation groups. ___________ the business community is the sector of society which has the greatest influence on our future, we're aiming to change the way is it thinks and operates. We want it to accept conservation as an integral principle of economic development. We're trying to do this working together with governments, businesses and international agencies to show that conservation can be commercially successful. ___________, if its commercially successful how can the business community afford to ignore it ?

after, what if, consequently, but, since, by, after all, yet, bacause, the, reason, actually

Section II : Writing and Grammar

Task 1 (5 marks)

Write instructions for the use of your cassette recorder (with the help of a diagram if necessary) so that some one else can operate it effectively. (any 5)

Task 2 (15 marks)

Write out a short story (150 - 200 words) using the visual given.

Fig. 4

Task 3 (5 marks)

Place an ad in the local newspaper to sell your car. (not more than 20 words including figures)

Task 4 (10 marks)

You have just been on an organised holiday with a group, and despite the advertisement the hotel you were staying in was very inadequate. Make a list of points to be included in the letter and write a letter of complaint to the tour operators. Make your case and say what you want done: for example, a refund, compensation, etc.

Name
Class
Roll. No.
Date

A diagnostic cum Proficiency Test

Total Marks 100
Total Time 90 Mts

Section I : Listening

Task 1 (5 marks)

Listen carefully to the passage being read out and write the items of Food and Drinks that you hear in two separate columns.

Food	Drink

Task 2 (10 marks)

Listen to the passages carefully. Make a list of the mistakes that you hear.

a) First Passage   b) Second Passage

Task 3 (5 marks)

Listen to the passage. Identify the nonsense words and replace them with the correct words.

nonsense words	correct words

Task 4 (5 marks)

You will hear part of a lecture. As you listen, try to decide what the talk is about. Three alternatives will be read out. Write only the answer.

Section II : Reading

Task 1 (10 marks)

Read the passage on page 3 and say whether each of the following statements is True or False.

1. The technical personnel have always been administrators. (True/False)
2. The adminsitrator and the technologist never had any differences of opinion. (True/False)
3. Technocracy resolves the unhealthy conflicts in industry. (True/False)
4. High efficiency means maximum results through minimum efforts. (True/False)
5. The technocrat has great influence over the administrator. (True/False)
6. It is not difficult to identify a technocrat. (True/False)
7. Techniques of management and methods of technology are not different. (True/False)
8. Technocracy is the administration of technical concerns by technical personal. (True/False)
9. Technologists occupy a higher rank in the industry than technocrats. (True/False)
10. There are five levels of working men in modern industry. (True/False)

Technical and administrative elements were considered to be far apart in technical establishments. The administrator was entrusted with the responsibility of the overall management of an industry while the technical personnel remained advisers to the administrators on technical affairs. With the rapid strides of technological progress in the world it was felt that the management could not keep pace with the technical advancement causing friction and defects in the system as a whole. The pure administrator found it difficult to see eye to eye with the technologist in matters of improvement in the industry. The technologist was deprived of the benefits of his creative thinking and research products for want of better understanding on the administrator's part. Such unhealthy conflicts landed the industry in a very different situation. People began to think of several ways and means of setting the industry on the right road for further progress. The result was the birth of technocracy.

The fundamental factor of technocracy is a faith in an efficiency so high that maximum results are obtained by minimum effort. A mixture of both the technical and the administrative elements in a single person was considered to be a successful proposition. Technocracy has dethroned the pure administrator and replaced him by the technocract, or rather the second has acquired a decisive influence over the first. Technocratic ideology strengthens the foundations and encourages the growth of such tendencies as the effective infiltration into the administrative machinery by the technocratic element.

The identification of the technocrat, the domain of technocratic intervention, and the significance of this transfer of power are the most important problems of technocracy. Technocrats are certainly not absolute masters of the administrative machinery; however, in many important sectors, they are guiding or directing the operation of the system. This is by no means a novelty, but at the present time this influence seems to be increasing. This aptitude enables technocrats to grasp a certain measure of authority over the administrators who are directly or indirectly chosen to assume control of the affairs of industry. The creative ideas and techniques of the technologists could then be easily translated into action to derive the maximum benefits of technological progress.

Management is a technique different from the usual methods of technology. This technique is not confined to a limited sector of society: it is society looked at from a certain angle and perspective. Apart from mechanical techniques, there are economic techniques, organizational techniques, human techniques and the techniques of intellectual work. It is a commonplace these days to think of almost all aspects of human life in terms of techniques and to consider the study of these aspects in terms of technology. That is to say, a scientific outlook and awareness has captivated the masses to develop a truly scientific method with definite objectives and clearly definable stages of the method for purposes of social amelioration. In these circumstances it is in the fitness of things to use the term technocracy for administration of technical enterprises, both the public and private sectors, by technical personnel well trained in the techniques of management. A pressing need of our times in our country is to look for better technocrats to man our flourishing industries. The technocrat will thus occupy a position higher than the technologist in the field of industry.

It is now customary to think of a five-fold hierarchy in the division of technical personnel into skilled workers, technicians, technologists, designers-researchers and technocrats. This only shows the developing trend in the field of science and technology. It is worthwhile for a technician to strive for his betterment by means of further education and training in order that he may lead the country towards industrial peace and progress.

Task 2 (10 marks)

Read the given passage and complete the following statements.

1. The technical personnel have been advising the _______________ in technical affairs.

2. There was need for the ___________ to keep pace with the _________.

3. The two benefits of the technologist in an industry are ___________ and ____________ .

4. The defects of the administration separated from the technologist are _____________ and _____________ .

5. Technocracy lays its faith in _____________ .

6. The three problems of technocracy are _____________, __________ and ____________

7. Techniques are of different kinds ____________, ____________ ____________ and ______________ .

8. The five levels of technical personnel are ____________, ___________ _____________, _____________ and ____________

9. A technician strives for his betterment by the two processes o ___________ and ____________ .

10. Technocracy encourages the effective infiltration of the ________ element into the ______________ machinery.

Task 3 (5 marks)

The text consists of five paragraphs. Each paragraph has a central point. The central point of the first paragraph is given below. Write one sentence each stating the main idea of the remaining four.

First paragraph :- Birth of Technocracy.

2nd paragraph

3rd paragraph

4th paragraph

5th paragraph

Section III : Vocabulary & Phrases

Task 1 (10 marks)

a) Complete the following groups.

technology	technocrat	technocratic
bureacracy	____________	____________
democracy	____________	____________

b) List from the text (3) words ending in '-ment' and '-ation' .

c) Write the opposites of the following words by addition of differe
suffixes or by the change of the main part of the word.

strenghthen important

encourage social

effective

Task 2 (15 marks)

a) Study the sentences from the reading passage which contain the following word-groups and write your own sentences using the same word-groups. (One sentence for each word group)

1) Keep pace with

2) In terms of

3) To agree on

4) For want of

5) Contrast with

6) To see eye to eye with

7) As a whole

8) Ways and means

9) In the form of

10) Act on

b) Certain words are followed mostly by some other small words 'eg. entrust' is followed by 'with'.
Write the small words that follow the words given below.

Deprive __________ Different __________

Land __________ Confine __________

Translate __________

Section IV : Writing and Grammar

Task 1 (5 marks)

a) Study the following sentence :

The administrator was entrusted with the responsibility of the overall management of an industry while the technical personnel remained advisors to the administrators on technical affairs.

The two parts of the sentence are connected by 'while' because the two activities occur at the same time and in contrast to to each other. Combine the following activities like that.

1. Technological progress took rapid strides in the world.
2. Industrial management lagged behind.

b) Combine the following sentences using "so........ that".

1. The efficiency is very high.
2. Maximum reports are obtained by minimum effort.

c) Combine the following sentences using 'but'.

1. Technocratic guidance is no novelty.
2. This influence seems to be increasing.

d) Combine the two sentences using 'however'.

1. Technocrats are not absolute masters of the administrative machinery.
2. In many important sectors, they are guiding the system.

e) Combine the two sentences using 'while'.

1. The pure administrator found it difficult to see eye to eye with the technologist in matters of improvement in the industry.

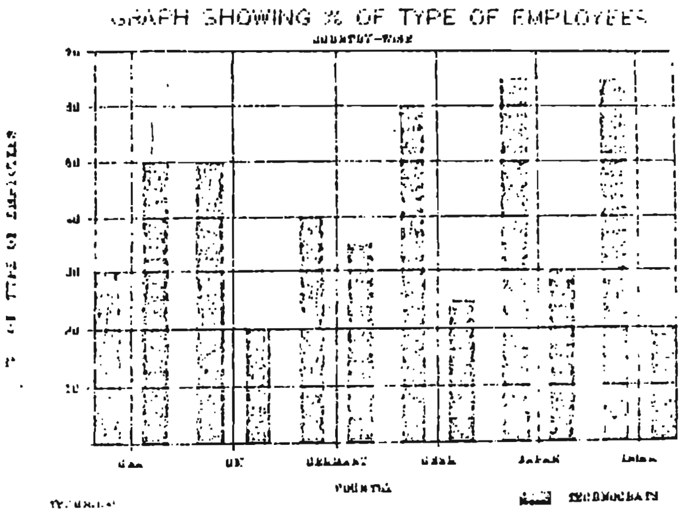
2. The technologist was deprived of the benefits of his creative thinking for want of better understanding on the administrator's part.

Task 2 (15 marks)

Write a paragraph each in about 200 words presenting the information contained in the following ~~two~~ chart~~s~~ (Chart 1 ~~and Chart 2~~).

Chart 1

Chart I



Task 3 (5 marks)

Present the following information in a circle chart.

There were 470 students in a polytechnic. 188 students came from villages and the rest from small towns. Among them were 47 girls from villages and 94 girls from small towns. In the four courses offered at the polytechnic there were 21 boys from villages in Electronics and 40 each in Civil, Electrical and Mechanical Engineering. Similarly, there were 42 boys from towns in Electronics and 80 each in the other 3 courses. While the Electronic course had 7 girls from villages and 14 girls from towns, the other three courses had equal distribution of girls from villages and towns.