INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) I Year : 1994-95

BACK-PAPER SEMESTRAL-II EXAMINATION

Computational Techniques and Programming II

Date: 23.6.95 Maximum Marks: 100 Time: 3 hours

1.(a) Describe clearly Jacobi's method for computing all the eigen values and eigen vectors of a real symmetric matrix.

(b) Show that the above method converges.

[20]

2. Describe clearly the method of bordering for computing the inverse of a nonsingular matrix. Discuss the limitations of this method and explain how these can be overcome.

[20]

3. Suppose f is a real-valued function defined on an interval I satisfying the following conditions:

(i) f and f' are continuous on I,

(ii) the equation $x = f(x)$ has a solution s (located in the interior of I) such that $f'(s) = f^{(2)}(s) = 0$,

(iii) $f^{(3)}$ exists, is continuous and does not vanish on I.

Show that the iteration sequence $x_n = f(x_{n-1})$ converges to s cubically, provided x_0 is sufficiently close to s.

[20]

4.(a) Derive Taylor's polynomial approximation of a function $f(x)$.

(b) Find the smallest value of n such that the (n-1)st degree Taylor's polynomial of $\sin(x)$ approximates $\sin(x)$ with an error not exceeding $0.5 / 10^{-6}$ for all x in [0,2].

[20]

5. Define Aitken's Δ^2-method. Show that the sequence obtained by the method converges faster than the original sequence.

[20]

INDIAN STATISTICAL INSTITUTE

B.STAT.(HONS) I YEAR : 1994-95

SEMESTRAL II BACK-PAPER EXAMINATION

PROBABILITY THEORY AND ITS APPLICATIONS II

Date: 21.06.95 | Maximum Marks: 100 | Time: 3 Hours

Answer all questions

1. Consider the union of the line segments in the plane, L_1 joining the pts (0,1) to (1,1) and L_2 joining (1,0) to (1,1), $L = L_1 \cup L_2$. Let P be the probability distribution corresponding to the experiment of choosing a point of L at random. (a) Write down the associated distribution function $F(a,b) = P\Big((-\infty,a] \times (-\infty, b]\Big)$, $(a,b) \in R^2$.
   (b) If $X(x,y) = x$, $(x,y) \in R^2$, find the distribution function of X.
   (c) Find the points of discontinuity of F (no proof of discontinuity is needed). [8+4+4=16]

2.(a) Let X and Y be positive random variables. Prove that XY is a random variable.
   (b) Let X be uniformly distributed on (0,1). Find the density of the random variable $\cos^2 2\pi X$. [8+8=16]

3.(a) Let X be a random variable such that $E(|X|^\alpha) < \infty$ where $\alpha > 0$. Show that if $0<\beta<\alpha$, then $E(|X|^\beta) < \infty$.
   (b) Let $\mu_r = E(|X|^r)$, $r > 0$. If μ_{2r+2} exists, show that $\mu_{2r+1}^2 \le \mu_{2r} \cdot \mu_{2r+2}$ [8+8=16]

4.(a) Let (X,Y) have the joint density

$$f(x,y) = \frac{1}{2} e^{-y}, \quad |x|<y$$
$$= 0 \qquad \text{otherwise}$$

   Find the Probability $P(|X|<\frac{Y}{2})$.
   (b) Let(X,Y) be independent uniform random variables on (0.1). Find the conditional distribution of X given $Z = X \wedge Y$. [8+12=20]

5. (a) Let X and Y be independent random variables having gamma distribution with parameters (λ,α) and (λ,α') respectively. Find the density of X+Y. (Here λ is the scale parameter).
   (b) Let X have gamma distribution with parameter (λ,n) where n is a positive integer. Compute F_X. [8+8=16]

6. (a) If X and Y are independent positive random variables having density f and g respectively, find the density of $Z = X/Y$.
   (b) If you and a friend of yours are waiting at a bus stop for different buses, find the probability that your waiting time is more than three times that of your friend's – assuming that the waiting times are independent and identical exponential random variables. [8+8=16]

INDIAN STATISTICAL INSTITUTE

B.STAT.(HONS) I YEAR : 1994-95

SEMESTRAL II BACK-PAPER EXAMINATION

STATISTICAL METHODS II

Date: 20.6.95 | Maximum Marks:100 | Time: 3 Hours

Answer all questions.

1.(a) Lengths of iron ingots Y obtained in a factory are modelled by a statistician to have a Normal distribution with mean 165 and variance 9.

(i) Write down the probability density function of Y.

(ii) The engineer of the factory is not satisfied with the model since Y, here, cannot assume negative values while a normally distributed variable may assume positive as well as negative values. How do you justify the model assumed?

(b) The electricity board of a State has found that the percentage of households consuming low amounts of electricity is small; the percentage of households consuming very high amounts of electricity is very small; and most households consume moderate amounts of electricity.

Explain what statistical model would apply for this case and describe the model (mathematically).

(c) In a game an 'over' consists of six balls. The umpire can declare any one of them as a 'no ball' if it does not satisfy certain conditions. While analysing the data on 'no balls' a statistician postulates that the chance of having none of the balls being declared as a 'no ball' is usually quite high and this falls off very rapidly – in fact, very rarely all the six balls turn out to be 'no balls'. Suggest a suitable statistical model for this problem stating your assumptions clearly.

(7+7+7)=21

2. (a) With the usual notation, show that

$\sigma^2_{0.12..p} = \sigma^2_0 (1-\rho^2_{01})(1-\rho^2_{02.1})(1-\rho^2_{03.12}) .. (1-\rho^2_{0p.123..(p-1)})$.

Explain clearly what you understand by the situation when p=2.

(b) An anthropologist has predicted the index Y by the relation $Y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$ where X_1, X_2, X_3 are 3 measurements on 56 subjects (all measurements X_i centred around their means). The corrected sums of squares and products matrix is calculated as:

Y	X_1	X_2	X_3
0.12692	0.03030	0.04410	0.03629
	0.01875	0.00848	0.00684
		0.02904	0.00878
			0.02886

He is now interested in the (i) multiple correlation coefficient of Y on the other variables and (ii) the partial correlation coefficient between the variables X_2 and X_3, eliminating the effect of X_1 and Y. Obtain these values. (14)+(14+16)=44

3. (a) It has been observed in a hostel that there is 10% chance that the phone does not work on a day. Calculate the probability that in a month, you have to use an outside phone on atmost 3 days.

(b) On the basis of the performance in examinations, students lose their stipends (L) if they score less than 400 and continue to get them (C) if the scores are between 400 and 500 and get a prize in addition to the stipend (P) if they score 500 or more. In one semester, the percentages of the categories L, C and P were respectively 23,62 and 15. Under a suitable model for the scores, find the mean and s.d. of the scores of (i) all students and (ii) students of the category(P).

(7)+(14)=21

4.(a) X and Y are two independently distributed variables with N(2,9) and N(3,16) distributions respectively. If $144\ Z = 16X^2+9Y^2 - 64X - 54Y+145$, find the Expection and Variance of Z. Precisely state the theorems you have used (without proof).

(b) Draw a random sample of size three from the distribution whose density is

$$f(x) = \frac{1}{\pi}\,\frac{1}{1+x^2}\,,\quad -\infty < x < \infty.$$

Justify your method. (7+7) =14

INDIAN STATISTICAL INSTITUTE

B.STAT.(HONS) I YEAR : 1994-95

SEMESTRAL II BACK-PAPER EXAMINATION

VECTORS AND MATRICES II

Date: 19.6.95 | Maximum Marks:100 | Time: 3 Hours

1.(a) Let A and B be two matrices such that AB is a square matrix. Express the determinant of AB in terms of majors of A and B.

(b) Consider the partitioned square matrix $X = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ where A is a nonsingular matrix.

Show that $|X| = |A|\,|D - CA^{-1}B|$. [15+5=20]

2.(a) Show that the symmetric matrix A and $B'AB$ are of same definiteness where B is any nonsingular matrix.

(b) For a non-indefinite matrix $A=[a_{ij}]$, show that $a_{ii}=0$ implies i^{th} column of A is null.

(c) Show that for any symmetric matrix A of order n there exists a nonsingular matrix B such that $B'AB$ is a diagonal matrix D. Further if A is non-indefinite B can be chosen such that leading principal minor of order k of A is equal to leading principal minor of order k of D for $k=1,2,\cdots,n$.

(d) Show that if A is nonnegative definite, there exists a triangular matrix T such that rank of A is equal to rank of T and $A = T'T$. [5+5+10+5=25]

3.(a) Show every reflexive g-inverse of a matrix A is also Moore-Penrose inverse A^{+}_{MN} for some suitable positive definite matrices M and N (that is, for some suitable norms involved).

(b) Define group inverse of a matrix. Obtain a necessary and sufficient condition for the existence of group inverse of a matrix A. [9+1+5=15]

4.(a) State and Prove the Cayley-Hamilton theorem.

(b) Show that algebraic multiplicity of any eigen value λ of a matrix is greater than its geometric multiplicity.

(c) Show that for a matrix A, algebraic multiplicity of the eigen value zero is equal to its geometric multiplicity implies rank of A=rank of A^2. [10+5+5=20]

5. State and prove the Courant - Fischer min-max theorem on eigen values of a real symmetric matrix.

6. Prove or disprove the following

(a) Let $A=UV'$ be a rank factorization of A then rank of (A^2)= rank of $(V'U)$.

(b) A is Jordan block of order n with diagonal elements all 0 then A' is a A^{-}.

(c) For a real symmatric matrix maximum eigen value is equal to maximum singular value.

(d) $|A+B| \le |A|+|B|$ if A and B are nonsingular.

(e) A and B are nonnegative definite implies $A+B$ is also non-negative definite. [5 × 3=15]

INDIAN STATISTICAL INSTITUTE

Semester - II, 1994-95
B.Stat.-I
Semestral - II Examination

COMPUTATIONAL TECHNIQUES AND PROGRAMMING - II

Date: 4.5.1995 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions

1.(a) Describe Jacobi's and Gauss-Seidel methods for the solution of linear equations bringing out their similarities and dissimilarities.

(b) Consider $Ax = b$ where A is a square matrix with all diagonal elements nonzero. Show that neither Jacobi's method nor Gauss-Seidel method converges if A is singular. (Hint : if R is a singular n x n matrix then SR is singular for all n x n matrices S.)

[20]

2. The subroutine EIG(B,n,X,P) computes all the eigen values and eigen vectors of the real symmetric matrix B of order n x n and gives the eigen values (in nonincreasing order) as the elements of the vector X and the eigen vectors as the columns of P such that the i-th column of P corresponds to x_i.
If you are given a p x q matrix A, describe a detailed algorithm (by using the above subroutine) to compute (i) a singular value decomposition of A, (ii) P_A, the orthogonal projector projecting vectors into column space of A, (iii) $P_{A'}$ and (iv) Moore-Penrose inverse of A.

[20]

p.t.o.

3.(a) The inverse of the nonsingular matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix}$$

is $$A^{-1} = \frac{1}{4}\begin{bmatrix} 4 & -4 & -16 & 12 \\ -1 & -1 & 5 & 0 \\ -1 & 3 & 9 & -8 \\ -2 & 2 & 6 & -4 \end{bmatrix}$$

Obtain the inverse of B where $b_{ij} = a_{ij}$ whenever $(i,j) \neq (2,3)$ and $b_{23}=3$, by modifying A^{-1} suitably.

(b) Show that the polynomial

$$1 + x + x^2 + \ldots + x^n$$

is a good approximation to $1/(1-x)$ for any x satisfying $0 < |x| < 1$.

(c) Find the value of $\sqrt{2}$ to five decimal figures by the scant method. Pick your own starting points.

(7+6+7) = [20]

OR

Let n be a positive integer. Consider the polynomial P interpolating the function $f(x)=e^x$ at the points $x = 0, 1, \ldots, n$. Suppose one has to compute e^{n+1} by evaluating P at $x=n+1$.

(a) Indicate a lower bound and an upper bound for the error $|e^{n+1}-P(n+1)|$.

(b) Obtain an exact expression for this error.

(8+12) = [20]

4. Write a subroutine for finding the value of a polynomial for a given value of the variable by Horner's method. Using this subroutine, write another subroutine for finding a root of a polynomial equation by Newton-Raphson method.

[20]

OR

By the method of linear iteration together with Aitken's Δ^2-method, compute the root of

$$x^3 + 2x^2 + 10x - 20 = 0$$

in [1,2] correct to 6 decimal places.

[20]

5. Let a be a positive real number and $f(x) = (x^3+bx)/(cx^2+d)$. Consider the iteration $x_n = f(x_{n-1})$. Determine the constants b, c, d such that x_n converges cubically to $\sqrt{a}$.

[20]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) I Year : 1994-95
SEMESTRAL-II EXAMINATION
Vectors and Matrices II

Date: 2.5.1995 Maximum Marks: 60 Time: 3 Hours

Note: This paper contains a max. of 75 marks.
You can answer any part of any question.
Max. you can score is 60.

1. Show that

$$\begin{vmatrix} A & B & B & \cdots & B \\ B & A & B & \cdots & B \\ B & B & A & \cdots & B \\ \vdots & \vdots & \vdots & & \\ B & B & B & \cdots & A \end{vmatrix} = |A-B|^{n-1} \, |A + (n-1) B|$$

Where A and B are square matrices of order k and n is the number of blocks in each row and each column.

[5]

2. Show that for any two matrices A and B, A^+ is a B^-, and B^+ is a A^- implies $A = B$.

[10]

3. Define Jordan block $J(n, \lambda)$. Show that for a Jordan block $J(n, 0)$, then exists a g-inverse with "eigen value property" and "power property". Hence or otherwise show that every square matrix has a g-inverse with eigen value property and power property. (You can assume Jordan Canonical Form of the matrix).

(1+10+4) = [15]

4.(a) Show that every square matrix A can be expressed as

$$A = PTP^*$$

Where P is unitary and T is Triangular.

(b) Derive Spectral decomposition of a real symmetric matrix.

(c) Derive singular value decomposition of a matrix.

(8+6+6) = [20]

p.t.o.

5. Prove the following:

   (a) A is real skew symmetric matrix implies the real part of each eigen value of A is zero.

   (b) Arithmetic multiplicity of the eigen value 0 of A is equal to its geometric multiplicity implies rank of A = rank of A^2.

   (c) For a real square matrix A, the sum of squares of its eigen values is equal to sum of squares of it's singular values implies A is symmetric.

(5+5+5) = [15]

6. Express the given matrix A as

$$A = L.U$$

Where L is a lower triangular matrix with all it's diagonal elements equal to unity and U is an upper triangular matrix.

$$A = \begin{bmatrix} -1 & 0 & -1 & 4 \\ 2 & 6 & 0 & -15 \\ -4 & -18 & 6 & 40 \\ 9 & 0 & -19 & -33 \end{bmatrix}$$

[10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) I Year : 1994-95
Statistical Methods II
Semestral-II Examination

Date : 28.4.1995 Maximum Marks : 100 Time : 3 Hours.

Answer all questions.

1(a) The breaking strength X of a bottle is modelled by a statistician to have a normal distribution with mean 165 and variance 9 in some units. However, the engineer of the factory is worried that X cannot assume negative values, while a normally distributed random variable may assume all positive and negative values. How do you justify the model assumed ?

(b) The accountant of an Institute prepares the frequency distribution of the salaries paid during the tax year 1994-95. He finds that the percentage of workers with low salary is small; the percentage of workers with very high salary is very low; and most workers are in the intermediate salary ranges. Explain what statistical model would apply to this situation and describe the model mathematically.

(c) On locating an albino-type child a geneticist goes to the family and records how many other children in the family are of albino-type. He has collected data on 32 families of 5 children each and tabulated the frequencies. Suggest a suitable distribution for these data and justify your answer.

(7+7+7)=(21)

2.(a) With the usual notation (to be explained by you), show that

$$\sigma^2_{0.12\ldots p} = \sigma^2_0 (1-\rho^2_{01})(1-\rho^2_{02.1})(1-\rho^2_{03.12})\ldots(1-\rho^2_{0p.123\ldots(p-1)}).$$

Explain briefly what do you understand by the situation when p=2.

p.t.o.

2.(b) A Testing Service predicted an index Y based on 3 tests (x_1, x_2, x_3) given to 86 candidates (all scores X_i are centred around their means). The corrected sums of squares and products matrix is calculated by them as :

Y	X_1	X_2	X_3
0.12692	0.03030	0.04410	0.03629
	0.01875	0.00848	0.00684
		0.02904	0.00878
			0.02886

The service is now interested to compute (i) multiple correlation coefficient of Y on the other variables and (ii) the partial correlation coefficient between the variables X_3 and X_2, eliminating the effect of Y and X_1. Obtain these values.

14+(14+16) = (44)

3. EITHER

From extensive optometric data it was found that the distribution of inter pupillary distance (i.p.d.) of adult males is Normal with mean 64.90mm. and s.d. 3.81 mm. A company manufacturing optical instruments plans to manufacture the following standard types of field glasses :

type	A	B	C	D	E
distance between the centres of eye pieces in mms.	54	58	62	66	70

Assuming that persons having i.p.d. differing by not more than 2 mm. from the distance between the centres of eye pieces of a field glass have no difficulty in using it, find how many of each type of field glass should be manufactured in a lot of 10,000 ?

3. OR

On the basis of the performance in examinations, students get an ordinary Pass (P) if they score between 400 and 500, fail(F) if they got below 400 and obtain an Honours(H) if they score above 500. In one particular semester, the percentage of the categories P, F and H were respectively 65, 25 and 10. Under a suitable model for the scores, find the mean and the s.d. of the scores of (i) all students and (ii) of students of category H.

(14)

Contd.....

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR:1994-95
SEMESTRAL-II EXAMINATION
PROBABILITY THEORY AND ITS APPLICATIONS II

Date:26.4.95 Maximum Marks:60 Time: 3 Hours

Note: Answer any five questions.

1.(a) Define a 'Borel subset of $\mathbb{R}$'

(b) Prove that Q, the set of rational numbers, is a Borel subset of $\mathbb{R}$.

(c) Under uniform distribution on $[0,1]$, find the probability of the set

$\{ x \in [0,1]$: the digit 5 does not occur in the regular decimal representation of x $\}$ [2+4+6]

2. Let F(a,b) be the distribution function of the pair of random variables (X,Y).

(a) Prove that

$F(c,d) - F(a,d) - F(c,b) + F(a,b) \geq 0$ whenever $a \leq c$ and $b \leq d$.

(b) Let $Z = \max(X,Y)$ and $W = \min(X,Y)$. Show that

$$F_Z(z) = F(z,z) \text{ and } F_W(w) = F_X(w) + F_Y(w) - F(w,w)$$

(c) If F has the density f(x,y), find the density of Z. [4+4+4]

3.(a) Let the probability distribution P of a pair of random variables (X,Y) be such that

$$P((a,b] \times (c,d]) \leq K(b-a)(d-c)$$

for all a,b,c,d, where $K > 0$ is a constant. It is given that $P((0,1] \times (0,1]) = 1$. Show that

$$P(X = Y) = 0$$

(b) From the definition of 'Expectation' prove that if $0 \leq X \leq Y$ everywhere on the sample space, then

$$E(X) \leq E(Y)$$ [6+6]

4.(a) Let X and Y be independent uniform random variables on (0,1). Find the distribution function and the density of the random variable $Z = X^2 - Y$.

(b) Let $X_1, \ldots, X_n$ be independentand uniformly distributed random variables on (0,1). Find the distribution of

$$Y = -\sum_{i=1}^{n} \log X_i$$ [6+6]

p.t.o.

5. (a) Let X_1, X_2, X_3 be independent uniform random variables on $(0,1)$. Find the density of the random variable

$$X_1 + X_2 - X_3$$

(b) Let X have the gamma distribution with parameter(λ, n) where n is a positive integer. Compute the distribution function of X. [6+6]

6. (a) If X is a positive random variable, show that

$$E\left(\frac{1}{X}\right) \geq \frac{1}{E(X)}$$

(b) If $E(|X|^r)$ and $E(|Y|^r)$ are finite $(r > 1)$, show that

$$E(|X+Y|^r) < \infty.$$ [6+6]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) I YEAR:1994-95
SEMESTRAL-II EXAMINATION
CALCULUS II

Date:24.4.95 Maximum Marks:100 Time: 4 Hours

Note: Question 1 is compulsory. Answer any five questions from the rest.

1. State whether the following statements are true or false. Give reasons for your answer

(a) Let $f \in C^1[a,b]$ i.e. $f^1 = \frac{df}{dx} \in C[a,b]$.

$f(a) = f(b) = 0$ and $\int_a^b f^2(x)dx = 1$

Then $\int_a^b x f(x) f^1(x)dx = -\frac{1}{2}$

(b) The derivative ^{1} of the function has the property that $^1(s) < 0$ $s \in (0,\infty)$

(c) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series whose radius of convergence $= \infty$. suppose that f is a bounded function. Then is constant.

(d) Let $g(x) = \sum_{n=0}^{\infty} b_n x^n$ be a power series whose radius of convergence is R where $0 < R < \infty$. Then $M > 0 \ni |g(x)-g(y)| \leq M|x-y| \cdot x_1 y \in (-R,R)$.

(e) Let $f \in C^2(a,b)$ be a convex function. Then the function $g(x) = e^{f(x)}$ $x \in (a,b)$ is not convex. [20]

2. If f, g, f^2, g^2 are all Riemann integrable on $[a,b]$, prove that

$$\frac{1}{2}\int_a^b \left[\int_a^b \begin{vmatrix} f(x) & g(x) \\ f(y) & g(y) \end{vmatrix}^2 dy \right] dx$$

$$= \left(\int_a^b f^2(x)dx\right)\left(\int_a^b g^2(x)dx\right) - \left(\int_a^b f(x)g(x)dx\right)^2$$

and deduce the Cauchy-schuvartz inequality,

$$\left(\int_a^b f(x)g(x)dx\right)^2 \leq \left(\int_a^b f^2(x)dx\right)\left(\int_a^b g^2(x)dx\right).$$ [16]

p.t.o.

3. Let for $x \in \mathbb{R}$,

$$f(x) = \left(\int_0^x e^{-t^2}\,dt\right)^2 \quad ; \quad g(x) = \int_0^1 \frac{e^{-x^2(t^2+1)}}{t^2+1}\,dt$$

Assuming that differentiation under the integral sign is allow show that $g^1(x)+f^1(x) = 0$ for all x and deduce that $g(x)+f(x)$.

Use the above result to prove that

$$\underset{x\to\infty}{\mathrm{Lt}} \int_0^x e^{-t^2}\,dt = \frac{\pi}{2}.$$

4. Let for $n = 1,2,3,\ldots$ $f_n : \mathbb{R} \to \mathbb{R}$ be the

function $f_n(x) = \frac{x}{1+nx^2}$

show that f_n converges uniformly to a function, f say and that the equation $f^1(x) = \mathrm{Lt}\, f_n^1(x)$ is correct if $x \neq 0$, but false if $x = 0$. [1

5. Let $f \in C[0,1]$ have the property that $\int_0^1 f(x)\, P(x)dx=0$ for all polynomials P of even degree. Using the Weierstrass approximati theorem show that $f = 0$. [1(

6. Let n be a positive integer. Show that the family of polynomial $P(x) = a_0+a_1x+\ldots+a_nx^n$ of degree $\leq n$ whose coefficients a_i sati the inequalities $m \leq a_i \leq M$ $(i = 0,1,\ldots,n)$ for some constants m, M is equicontinuous on any $[\alpha,\beta]\subset\mathbb{R}$. [:

7. Which of the following improper Riemann integrals exist ? Justify your answer.

(a) $\int_{0}^{\pi/2} \frac{\tan x}{x}$ (b) $\int_{0}^{1} \cos x.\ x^{-3/2}dx$ [1

8. Prove that $(2m) = \dfrac{2^{2m-1}\ (m)\ (m+\frac{1}{2})}{\pi}$

for $m = 1,2,3,\ldots$

INDIAN STATISTICAL INSTITUTE

B. Stat. (Hons.) I Year : 1994-95

Computational Techniques and Programming II

Semestral - I Examination

Date : 07.12.1994 Maximum Marks : 100 Time : 3 Hours

1. Write a program to compute the binomial co-efficients $\binom{n}{i}$, $i = 0, 1, 2, \ldots, n$ for a given positive integer n.

OR

Write a program to find all the prime factors of a given positive integer n.

[15]

2. A is an MxN matrix and B an NxK matrix. Write a program to compute the product AB.

[15]

3. Show the storage configuration of the variables in COMMON statements in the two modules given below :

In main programme : REAL A(10), B(5,2), C, D
COMMON A, B, C, D

In a subprogramme : REAL X, Y(8), Z(3,3)
COMMON X, Y, Z

[10]

4. Suppose $x_R - x_L = 1.0$. Find the minimum number of bisection steps needed to assure that the approximation error of x_m is less than 10^{-9} ?

OR

Derive the error in Simpson's 1/3 rd rule.

[12]

6. Estimate the value of the definite integral $\int_0^4 (x^2+2x+2)\,dx$ using both Simpson's 1/3rd rule as well as trapezoidal rule. (Use 8 intervals for Simpson's rule and 4 intervals for trapezoidal rule).

[18]

7. Give the output of the following programs :

(a) [7]

```
      n(1) = 23
      n(2) = 10
      n(3) =  8
      n(4) = 45
      n(5) = 11
      n(6) = 31
      m=6
      do 10 i = 1, m-1
      do 10 j = i+1, m
      if (n(i).le.n(j)) go to 10
      k = n(i)
      n(i) = n(j)
      n(j) = k
10    continue
      write(*,*) (n(i), i=1,m)
```

(b) [6]

<u>Main Program</u>

```
      do 10 i=3,30
      call sub(i,k)
      if(k.eq.0) go to 10
      write(*,*) i
10    continue
      stop
```

<u>Subroutine</u>

```
      subroutine sub(j,n)
      n = 0
      do 20 m = 2, j/2
      mm = mod(j,m)
      if(mm.eq.0) go to 30
20    continue
      n = 1
30    return
```

8. Write a FORTRAN programme to compute the sum of the first 50 terms of the following series for a given x and print the final sum.

[17]

$$x - x^3/3! + x^5/5! - x^7/7! + \dots$$