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Research and Training School
B.Stat. Part II: 1969-70

PERIODICAL EXAMINATION

Mathematics-2: Calculus

Date: 15.9.69 Maximum Marks: 100 Time: 3 hours

Note:- Answer as many questions as you can. Marks allotted for each question are given in brackets [ ].

1. With suitable assumptions on the function f and the numbers a and b define

$$\int_a^b f(x)\,dx$$ [6]

2. If $f(x) = \frac{1}{2^n}$ for $n-1 < x \leq n$, $n = 1,2,\ldots$ show that

$$\lim_{n\to\infty} \int_0^n f(x)\,dx = 1.$$ [9]

3. Let x be a real number > 0, and let k be a positive integer >1. Let a_0 be the largest integer $\leq x$ and assuming that $a_0, a_1, \ldots, a_{n-1}$ have been defined, let a_n be the largest integer such that

$$a_0 + \frac{a_1}{k} + \frac{a_2}{k^2} + \ldots + \frac{a_n}{k^n} \leq x.$$

(a) Show that $0 \leq a_i \leq k-1$ for each $i = 1,2,\ldots$

(b) Explain how the numbers can be obtained geometrically.

(c) Show that the infinite series $a_0 + \frac{a_1}{k} + \ldots$ converges to the sum x. [6 X3]=[18]

4. Find the supremum and the infimum of each of the following sets of real numbers:-

(a) The set S_1 of all numbers of the form $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ where p, q, r take on all positive (>0) integral values.

(b) $S_2 =$ x; $3x^2 - 10x + 3 < 0$

(c) $S_3 =$ x; $(x-a)(x-b)(x-c)(x-d) < 0$
$a < b < c < d$ fixed real numbers.

(d) $S_4 = \sum_{i=0}^{n} \frac{1}{i!}$; $n = 1,2,\ldots$ [5X4]=[20]

5.a) Show that in general

$$.a_1 a_2 \ldots a_n \dot{\alpha}_1 \alpha_2 \ldots \dot{\alpha}_n = \frac{a_1\ldots a_n\alpha_1\ldots\alpha_n - a_1\ldots a_n}{99\ldots900\ldots0}$$

the denominator on the right hand side containing n 9's and m 0's.

b) From (a) $.1\dot{9} = \frac{19-1}{90} = .2$. How do you explain it? [6X3]=[18]

6. Show that limit of a sequence is uniquely determined, if it exists. [7]

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7.a) Prove that a bounded increasing sequence of real numbers converges.

b) Let A be a set of real numbers bounded above, let B be the set obtained by considering the negative of each number in A i.e., $x \in B$ if and only if $-x \in A$. Show that B is bounded below and

$$\operatorname{Sup} A = \operatorname{Inf} B.$$

c) Show that a bounded decreasing sequence of real numbers converges. [6 X3]=[18]

8. If a sequence of real numbers converges, show that any subsequence converges to the same limit.
Is the converse true? Give reasons. [6+3]=[9]

9. Calculate the following limits

(i) $\lim_{n \to \infty} \theta^n$ where $0 < \theta < 1$

(ii) $\lim_{n \to \infty} \frac{1}{n^n}$

(iii) $\lim_{n \to \infty} f(n)$ where $f(n) = \frac{1}{n+1}$ when n is odd

$= \frac{1}{n-1}$ when n is even.

(iv) $\lim_{n \to \infty} \left(\frac{1}{n+1} + \frac{1}{n^2-1}\right)$. [4X4]=[16]

10. State which of the following statements are true (no proof is necessary).

a) If f and are convergent sequences, and $f(n) < g(n)$ for all n, then $\lim_{n \to \infty} \leq \lim_{n \to \infty} g(n)$.

b) A sequence f converges to zero iff $|f|$ converges to zero.

c) If a sequence f diverges to infinity and $\lim_{n \to \infty} g(n) = 0$ then $\lim_{n \to \infty} f(n)g(n) = 0$.

d) The number $\sqrt{2}$ cannot be obtained as the limit of an increasing sequence of rational numbers.

e) If for a sequence f, $|f|$ converges then so does f.

f) If a and b are the supremum and the infimum of a set of real numbers, then $b < a$.

g) Given any sequence f, by changing a finite number of $f(n)$'s, f can be made to converge.

h) Any real number can be obtained as the limit of a decreasing sequence of rational numbers. [8]

11. Define the following:

a) Infimum of a set of real numbers

b) A sequence f diverging to $-\infty$

c) An increasing sequence. [10]

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PERIODICAL EXAMINATION

Statistics-2: Numerical Analysis - Theory and Practical

Dated: 22.9.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Define the forward difference (Δ), the backward difference (∇) and the shift (E) operators.

b) Establish the following relations

(i) $\nabla = 1 - E^{-1}$, (ii) $E\nabla = \Delta = \nabla E$,

(iii) $E = (1 - \nabla)^{-1}$, (iv) $(1 + \Delta)(1 - \nabla) = 1$.

c) For any positive integer n establish the following

$$E^n f_x = (1 + \Delta)^n f_x = \sum_{i=0}^{n} \binom{n}{i} \Delta^i f_x,$$

where f_x is any real function of x.

d) If n is not a positive integer, under what conditions the above binomial expansion is valid? [3+6+8+3]=[20]

2.a) Let f_x be any real function of x defined on an interval I. If $x_0, x_1, \ldots, x_n$ are any $n+1$ distinct points in I show that there exists a unique polynomial p_x of degree not exceeding n such that $p_{x_i} = f_{x_i}$, $i = 0, 1, \ldots, n$.

b) If f_x itself is a polynomial then what can you say about the polynomial p_x in relation to f_x? Give reasons.

c) Justify the consideration of various formulae of the unique interpolating polynomial p_x of 2(a) from the computational point of view. [10+5+5]=[20]

3.a) Define the divided difference and the central difference operators.

b) Prove that for any positive integer k the k-th divided difference of a function f_x is unaltered by any permutation of the corresponding arguments.

c) Obtain Newton's divided difference formula. When is this interpolation exact?

d) Obtain the relations of the divided differences with the central differences.

e) Obtain Gauss's forward formula from Newton's divided difference formula. [5+6+6+6+5]=[25]

4. Using a suitable formula of the interpolating polynomial compute the value of f(x) at $x = 0.98$ from the following table.

x	0.5	0.7	0.9	1.1	1.13
f(x)	0.47943	0.64422	0.78333	0.89121	0.96356

[20]

5. Home assignment and practical records. [15]

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PERIODICAL EXAMINATION

Economics-2

Date: 29.9.69 Maximum Marks: 100 Time: 3 hours

Note:- Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Macro-economics

Maximum Marks: 70 Suggested time: 2 hours

Attempt any three questions. One marks is reserved for neatness.

1.a) Distinguish between active and passive creation of deposits by commercial banks.

b) Do you think that bank deposits should be regarded as money? Give reasons for your answer.

c) Discuss the different forms of credit instruments. [7+8+8]=[23]

2. Give an analysis of the motives behind the demand for money. Explain the statement: 'The rate of interest is determined by liquidity preference in the private sector and the quantity of central bank money'. Also give your comments on this statement. [14+5+4]=[23]

3. Distinguish between ex ante and ex post analysis of national income.

Explain two alternate ways of determining the equilibrium level of national income. [5+18]=[23]

4.a) Discuss the minimum reserve and open market policies of the central bank. Are the two policies equally applicable? Give reasons for your answer.

b) Explain the interest effect of open market policy. [16+7]=[23]

5.a) Give an analysis of changes in national income resulting from changes in the propensities to invest and consume.

b) Explain the inflationary and deflationary gaps. [16+7]=[23]

Group B: Indian Economic Problems

Maximum Marks: 30 Suggested time: 1 hour

Answer any two questions.

1. Review the changes in the pattern of India's foreign trade after independence. [15]

2. Describe the major items of India's exports and imports indicating their relative shares. [15]

Indicate the major direction of India's foreign trade in recent years. [15]

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PERIODICAL EXAMINATION

Statistics-2: Probability

Date: 6.10.69 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [ ].

Instructions: Answer as much as you can. Maximum marks you can score is 100. For wrong answers to parts of question 2, marks will be deducted.

1. For each of the following three random experiments list the corresponding sample space and define what you consider to be a reasonable probability distribution:

(a) A symmetric die is rolled once. If the score is not 3 the experiment is stopped. If the score is 3 it is rolled for the second time and the experiment is stopped.

(b) 100 persons are to be classified as smokers or non-smokers.

(c) A coin is tossed until a tail appears. [12]

2. Consider a bridge hand (i.e., 13 cards selected at random from a standard deck of 52 cards) Assume equal probabilities to the $\binom{52}{13}$ sample points. Let A be the event 'The hand contains the ace of hearts', B the event 'The hand is a complete suit', C the event 'The hand contains all the aces', and D the event 'The hand contains no black cards'.

State whether the following assertions are true or false (No proof is needed):

(a) $P(D) < P(C)$

(b) $P(A \cup B \cup C \cup D) = 1$

(c) $C \cap (B \cup A)^c$ is an impossible event

(d) C and D are mutually exclusive

(e) $P(D) = \frac{1}{2}$

(f) $P(B \cap D \cap A^c) = \frac{1}{\binom{52}{13}}$

(g) $P(C) \leq P(A)$

(h) $P(A \cap D \cap C) > 0$. [16]

3. From a class of 52 students a committee of 7 is to be chosen at random. A student calculates the probability that he will be included in the committee as 7/52. How did he arrive at the answer? [8]

4. There were 34 presidents of the United States from 1789 to 1964. Before looking up their birthdays a student of probability theory asserts that there is a fair chance that at least two of the 34 presidents were born on the same day. How would you justify his evaluation of the probability as

$$1 - \frac{365!}{331!\,(365)^{34}}$$ stating the

assumptions he must have made? [10]

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5. There are 15 letters and 15 addressed envelopes and a careless secretary performs random matching of the envelopes and letters. What is the probability that there is no match? [10]

6. 10 balls numbered $1, 2, \ldots, 10$ are placed at random into 6 cells named $a_1, a_2, \ldots, a_6$. Your teacher claims that the probability of the event 'a_5 contains exactly 7 balls' is the same as the probability of getting 7 successes in 10 tosses of a coin with probability of success $1/6$. Supply a proof for his claim. [12]

7. An elevator starts with 5 passengers and stops at 8 floors. Enumerate the various configurations of discharge of these passengers and the corresponding probabilities (No proofs are required). [16]

8. Prove that

$$\binom{33}{0}\binom{27}{21} + \binom{33}{1}\binom{27}{20} + \binom{33}{2}\binom{27}{19} + \cdots + \binom{33}{21}\binom{27}{0} = \binom{60}{21}$$

by obtaining the hypergeometric distribution in a suitable fashion. [16]

9. An experiment consists of selecting an integer at random from the first 100 positive integers and repeating the same as long as there is no repetition. Describe the sample space. Define a suitable probability distribution on the sample space. You must, of course, prove that what you defined is a probability distribution. [10]

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FINAL EXAMINATION

General Science-3: Biology: Zoology Theory

Date: 13.10.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. What are the characteristics of the chordates? Draw comparative diagrams of the fundamental plans of a non-chordate and a chordate. [10+10]=[20]

2. Describe the anatomical peculiarities of *Balanoglossus*. [20]

3. Compare the external morphology of *Petromyzontia* with that of *Myxinoidea*. [20]

4. Give an account of the integumentary as well as the dental characters of mammals. [10+10]=[20]

5. Write short notes any four

A. Hepatic portal system

B. Retrograde metamorphosis

C. Notochord

D. Vertebrate skull

E. Bhorry fishery. [4X5]=[20]

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FINAL EXAMINATION
General Science: Biology: Zoology Practical

Date: 13.10.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe specimen A supplied, giving detailed labelled sketches of the different external organs. Assign the specimen into the class to which it belongs. [16+4]=[20]

2. Identify specimens B-E . [4X5]=[20]

3. Comment on specimens F-I. [4 X 3]=[12]

4. Draw specimens J and K and label their parts. [9+9]=[18]

5. Viva Voce. [10]

6. Practical records. [20]

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PERIODICAL EXAMINATION

Statistics-2: Statistics - Theory and Practical

Date: 3.11.69 Maximum Marks: 100 Time: 4 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Obtain the recurrence relation connecting moments of the binomial distribution and use it to find the γ_1- and γ_2-coeffidicents of the distribution. [8+6]=[14]

2. When and how does the binomial distribution approach the Poisson distribution? Demonstrate fully.

   Name some variables having the Poisson form of distribution. [8+4]=[12]

3. Define the moment generating function of a random variable. Find the moment generating function of the normal distribution. What does this tell us about the moments of the normal distribution? [4+8+4]=[16]

4. An unbiased coin is tossed repeatedly until 3 heads appear. What is the probability distribution of the number of tosses (x) required to give 3 heads? Find the mean and variance of x. [4+6]=[10]

5. Define the lognormal distribution and find its mode. [3+5]=[8]

6. A true die is thrown 200 times under uniform conditions. Find the probability of getting at least 30 sixes by using suitable approximations. [8]

7. Plot a suitable graph to examine whether the following distribution is approximately normal:-

right-hand grip (lb.)	upto 39.5	39.5-59.5	59.5-79.5	79.5-99.5	99.5-119.5	119.5 or more	Total
frequency:	1	14	151	156	22	1	345

[12]

8. Below is given the frequency distribution of the number of red blood corpuscles (rbc) per cell of a hemocytometer:-

No. of rbc (x):	0	1	2	3	4	5	Total
frequency (f):	143	156	68	27	5	1	400

Fit a Poisson distribution to the data. [20]

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PERIODICAL EXAMINATION
Mathematics-2: Linear Algebra

[208]

Date: 10.11.69 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can. The maximum marks you can score is 80. Twenty marks are reserved for assignments. Marks allotted for each question are given in brackets [ ].

1. Carefully explain the meaning of the following notions: (1) Basis for a vector space; (2) Span of a set of vectors; (3) Isomorphism; (4) Linear functional; (5) Natural correspondence; (6) Reflexive vector space. [6 X3]=[18]

2. For each of the following assertions, state if it is true or false. If an assertion is true, prove it; if false, give a counter-example.

(a) The real vector space C has dimension 1.

(b) If V is a vector space over Z_2 then $\{x,y,z\}$ is a linearly independent set in V $\Rightarrow$ $\{x+y, y+z, z+x\}$ is a linearly independent set.

(c) Every 3-dimensional vector space over Z_3 has 27 vectors.

(d) Let S and T be subspaces of a vector space V over a field F. Then $S \cup T$ is a subspace of V only if either $S \subset T$ or $T \subset S$ (or both).

(e) Let V and W be two vector spaces over a field F and let f be an isomorphism of V onto W. If $\{x_1, x_2, \ldots x_n\}$ is a basis for V then $\{f(x_1), f(x_2), \ldots, f(x_n)\}$ is a basis for W. [5 X5]=[25]

3.a) If A and B are finite dimensional subspaces of a vector space V over a field F, then show that A+B is a finite dimensional subspace and that

$$\dim (A+B) = \dim(A) + \dim(B) - \dim (A \cap B).$$

b) If L, M and N are subspaces of a vector space W over a field F then show that

$$L \cap (M + (L \cap N)) = (L \cap M) + (L \cap N).$$ [15 +10]=[25]

4. Consider the real vector space R^3.

a) Let $A = \{(x_1, x_2, x_3) \in R^3: \ x_1 + x_2 = 1, \ x_3 = 0\}$

Find the span of A.

b) Let $B = \{(x_1, x_2, x_3) \in R^3: \ x_1 + 2x_3 = 0\}$

Find the dimension of B.

c) Let B be as given in 4(b) above. Find a subspace of R^3 which is a complement of B. [7+5+8]=[20]

5.a) Consider the real vector space P of all real polynomials. Define

$$\lambda(x) = \int_0^1 x(t^3 + t^2 - 1)\,dt$$

for each $x \in P$. Is λ a linear functional on P?

b) Consider the real vector space R^2 and let $x_1 = (1,0)$ $x_2 = (1,2)$. Carefully define the linear functionals on R^2 which form the dual of the basis $\{x_1, x_2\}$.

[4+8]=[12]

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PERIODICAL EXAMINATION

General Science-2: Physics Theory

Date: 17.11.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Derive the following formula for an adiabatic process of an ideal gas, assuming γ to be the ratio of specific heats:

$$TV^{\gamma-1} = \text{constant.}$$ [10]

b) After detonation of an atom bomb, the ball of fire consisting of a sphere of gas was found to be 50 ft. radius at 3×10^5 degree absolute. Assuming adiabatic condition to exist, find the radius of the ball after 100 milli-seconds when its temperature is 3×10^3 degree absolute $\gamma = 1.666$. [15]

2.a) A straight thin weightless elastic beam of length λ and of uniform rectangular cross-section is rigidly clamped at one end and is loaded at the other with a weight W, the bending of the beam from the initial horizontal position remaining within elastic limits. Calculate the displacement of the loaded end. [15]

b) If the same beam is supported at both ends and the same load is placed at the mid-point what would be the depression of the mid-point compared with the previous displace of the loaded end? [10]

3. Prove that the moment of inertia of a uniform circular disc is

i) $\frac{1}{2} Ma^2$ about an axis through its centre and perpendicular to its plane.

ii) $\frac{5}{4} Ma^2$ about a tangent

where M and a are respectively the mass and radius of the disc. [9+7]=[16]

4.a) A particle is moving with simple harmonic motion in a straight line. When the distance of the particle from the equilibrium position has the values x_1 and x_2, the corresponding values of the velocity are u_1 and u_2. Show that the period is

$$2\pi \left\{ (x_2^2 - x_1^2)^2 / (u_1^2 - u_2^2) \right\}^{\frac{1}{2}}$$ [14]

b) A wheatstone's bridge has resistances of 1,2,3 and 4 ohms. A galvanometer of resistance 5 ohms. is connected to the junctions of 1 and 4 ohms. coils and of 2 and 3 ohms. coils. A 2-volt accumulator of negligible resistance is connected to other two junctions. Calculate (a) the current through the galvanometer (b) the resistance between the two points where the accumulator is connected. [10+10]=[20]

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MID-YEAR EXAMINATION

Mathematics-2: Calculus

Date: 19.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) State and prove 'Bolzano-Weierstrass Theorem'. [8]

b) What can you say about the following sets:

i) a bounded set S of real numbers which has no accumulation point. [4]

ii) a bounded set S for which sup S $\neq$ Inf. S.

2. ϕ and ϕ' are two convergent sequences. Show that the sequence $\phi.\phi'$ converges and

$$\lim_{n \to \infty} \phi.\phi'(n) = \lim_{n \to \infty} \phi(n).\ \lim_{n \to \infty} \phi'(n).$$ [6]

3. For the following sequences write either 'limit exists and is equal to ...' or 'limit does not exist' as the case may be (you need not give reasons for your choice.) [6]

i) $\phi(n) = (\frac{3}{4})^n$, $n = 1,2,\ldots$

ii) $\phi(n) = \frac{1}{n}$ if n is odd, $\phi(n) = \frac{n}{n+1}$ if n is even.

iii) $\phi(n) = (\frac{1}{2})^{1/n}$, $n = 1,2,\ldots$

iv) $\phi(n) = (1 + \frac{1}{n})^n$, $n = 1,2,\ldots$

v) $\phi(n) = \sqrt{n}$, $n = 1,2,\ldots$

vi) $\frac{3}{4}, \frac{1}{2}, (\frac{3}{4})^2, (\frac{1}{2})^2, (\frac{3}{4})^3, (\frac{1}{2})^3, \ldots$

4. Define:

i) an infinite sum $\sum_{n=1}^{\infty} a_n$.

ii) '$\lim_{x \to \infty} f(x) = \ell$' and from the definition show that

$$\lim_{x \to \infty} \frac{1+x}{x} = 1$$

iii) '$\frac{d}{dx}(f(x)) = f'(x)$'. [6]

5. Find any four of the following limits:

i) $\lim_{n \to \infty} \sqrt[n]{n}$; ii) $\lim_{n \to \infty} \frac{a_1 n^5 + a_2 n^4 + a_3 n^4}{b_1 n^5 + b_2 n^4}$ $(b_1 \neq 0)$

iii) $\lim_{x \to 0} \frac{\sin x}{ax}$ $a \neq 0$; (iv) $\lim_{x \to 0} e^x$;

v) $\lim_{x \to 2} x^4$; (vi) $\lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$. [20]

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6.a) Define continuity of a function f at a point x. When do you say that a function is continuous on a closed interval $[a,b]$?

b) Show that the function $f(x) = x^{a}$, where a is a positive constant is continuous at all points $x > 0$ [3+6]=[9]

7. Show from first principles any one of the following:

i) If $f(x) = x$, $\int_0^1 f(x)\,dx = \frac{1}{2}$.

ii) If $f(x) = a_i$, $t_{i-1} \leq x < t_i$, $i = 1,\dots,k$

then $\int_{t_0}^{t_k} f(x)\,dx = \sum_{i=1}^{k} a_i(t_i - t_{i-1})$. [9]

8. Let $f(x) = 0$ if $0 \leq x \leq 1$ and x is rational

$= 1$ if $0 \leq x \leq 1$ and x is irrational.

Show that $\int_0^1 f(x)\,dx$ does not exist. [7]

9. Find the primitives for any 4 of the following:

(a) $\int \frac{(1+x)^3}{x}\,dx$; (b) $\int \sin^3 x\,dx$; (c) $\int \frac{x^7}{(1-x^4)^2}\,dx$

(d) $\int \frac{e^x-1}{e^x+1}\,dx$; (e) $\int \frac{dx}{x^2+a^2}$; (f) $\int \sin^3 x\,dx$. [20]

10. Let f be continuous on $[a,b]$ and $f(a) < 0$, $f(b) > 0$. Show that $\exists\ \eta \in [a,b] \ni f(\eta) = 0$.

[Hints: Let $S = \{x: x \in [a,b]$ and $f(x) < 0\}$

Put $\eta = \operatorname{Sup} S$].

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MID-YEAR EXAMINATION

Mathematics-2: Linear Algebra

Date: 20.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer Q.1 and as many questions as you can from the remaining questions. Marks allotted for each question are given in brackets [ ].

1. Carefully define the following terms:

(a) Basis for a vector space
(b) Isomorphism
(c) Reflexive vector space
(d) Annihilator of a set of vectors
(e) Invertible linear operator. [5×3]=[15]

2. Show that every linearly independent set in a finite dimensional vector space can be extended to a basis for the vector space. [15]

3. If S is an m-dimensional subspace of an n-dimensional vector space V over a field F then show that S^o is an (n-m) dimensional subspace of V^*, the dual space of V. (Here S^o denotes the annihilator of S). [15]

4. Let T be a linear operator on a finite dimensional vector space V. Show that T is invertible if and only if $T(x) = 0$ implies that $x = 0$. [12]

5. Let M and N be subspaces of a vector space V over a field F. When do you say that V is the direct sum of M and N?

If V is the direct sum of M and N show that every vector z in V can be written in the form $z = x+y$ where x is in M, y is in N in one and only one way. Hence, or otherwise, show that if M and N are finite dimensional subspaces of V and V is the direct sum of M and N then V is finite dimensional and dim (V) = dim (M) + dim (N). [15]

6. What is $z = -3+2(5^{-1})$ in the field Z_{13}? [4]

7. Consider the real vector space R^4. Let $x_1 = (1,0,0,0)$ $x_2 = (1,1,0,0)$, $y_1 = (1,1,1,0)$ and $y_2 = (1,1,1,1)$. Find two bases A and B for R^4 such that A and B are disjoint sets, $x_1 \in A$, $x_2 \in A$, $y_1 \in B$ and $y_2 \in B$. [illegible]

8. Consider the real vector space R^3. Let $x_1 = (2,1,0)$, $x_2 = (1,0,3)$ and $x_3 = (-1,5,7)$. Then show that $\{x_1, x_2, x_3\}$ is a basis for R^3. Let $\{x_1^*, x_2^*, x_3^*\}$ be the dual basis of $\{x_1, x_2, x_3\}$ for the dual space of R^3. Then find $x_1^*(y) - \frac{1}{2} x_2^*(y) + x_3^*(y)$ where $y = (\frac{5}{2}, 1, -6)$. [8]

9. Let M and N be subspaces of a finite dimensional vector space W. Show that

$$(M+N)^o = M^o \cap N^o.$$ [8]

10. Let A be any linear operator on a vector space V over a field F. Let $M = \{x \in V: A(x) = 0\}$. Then is M a subspace of V? [4]

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11. Let P denote the real vector space of real polynomials. Let $T : P \to P$ defined by $(Tx)(t) = t\,x(t)$ for each $t \in R$ and $x \in P$.

Let $D: P \to P$ defined by $(Dx)(t_0) = \frac{d}{dt} x \Big|_{t = t_0}$.

Then show that T and D are linear operators on P and check that $DT - TD = 1$. [6]

12. Consider the real vector space R^4 and let B be the linear operator on R^4 defined by

$$B(x_1, x_2, x_3, x_4) = (x_1 + x_2 - x_3,\; x_3 + 2x_4 - x_1,\; x_4 - 2x_3,\; x_3 + x_2 + 3x_4 - 4x_1).$$

Is B invertible? If so, find $B^{-1}(y)$ where

$$y = (2, 1, -3, 0).$$ [12]

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MID-YEAR EXAMINATION

Economics-2: Economic Theory

Date: 23.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer any five questions. All questions carry equal marks.

1. Compare contrast the relative efficiencies of the Central Bank's open market and discount policies.

2. Briefly discuss the different viewpoints about the relationship of consumption to income.

3. Explain the process of income formation when an increase in consumption, brought about by a rise in income, in turn induces net investment. Outline the different time paths that the course of income may trace out.

4. Use the conditions of product market equilibrium and those of the money market equilibrium to derive graphically the IS and LM functions respectively. How would you find the solution to the problem of general equilibrium with the help of these functions?

5. Discuss the validity of the Quantity Theory of Money.

6. Examine the main differences between the classical and Keynesian theories of employment.

7. Is the rate of interest a purely monetary phenomenon? Give reasons for your answer.

8. Write short notes on:

   (a) the Pigou effect

   (b) the marginal efficiency of capital

   (c) the deflationary gap.

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MID-YEAR EXAMINATION

Economics-2: Indian Economic Problems

Date: 24.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions. Marks allotted for each question are given in brackets [ ].

1. Examine the main problems in the field of our foreign trade. Do you agree with the view that the failure to raise export is the main cause of the large deficit in India's balance of payments? [15+10]=[25]

2. Explain the consequences of British land-tenure systems on our rural economy. What are the main arguments for replacing the old systems by new ones? $[12\frac{1}{2}+12\frac{1}{2}]=[25]$

3. Critically review the progress of land reforms in India in the light of the recent official findings. Examine the main economic factors behind the present rural unrest spread throughout the country. [15+10]=[25]

4. Do you agree with the view that in the present context, expansion of co-operative farms can solve some of the urgent problems of the agricultural sector? Give reasons for your answer. Indicate the main drawbacks of the existing co-operative farms in India. [18+7]=[25]

5. Examine the main problems of agricultural finance in India, indicating the role played by the institutional agencies. Do you agree with the view expressed by the All India Rural Credit Review Committee that in the changed situation there is a vast scope for the Commercial Banks to provide farm credit in the rural areas? [15+10]=[25]

6. Write short notes on the following:

   a) Concentration of land-ownership in the agricultural sector;

   b) Report of the 'Kumarappa Committee';

   c) Integrated Scheme of Rural Credit. $[8\frac{1}{3}\times 3]=[25]$

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MID-YEAR EXAMINATION

Statistics-2: Numerical Analysis - Theory

Date: 25.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1.a) Obtain the Newton-Raphson scheme for solving an equation in one unknown.

b) Derive a sufficient condition for the convergence of the Newton-Raphson scheme.

c) Give an iterative scheme for computing the square root of a number and derive the bounds for the first approximation so that this scheme converges.

d) Obtain numerically or otherwise all the roots of the equation

$$3x^3 + 5x - 40 = 0$$

[5+6+6+13]=[30]

2.a) Define the inverse of a matrix. Is this unique? Give reason.

b) Prove that the necessary and sufficient condition for the existence of the inverse of a matrix A is that A is non-singular.

c) Indicate how to compute the inverse of a matrix using Gauss reduction technique. [2+5+5]=[12]

3.a) Define the determinant of a square matrix.

b) Prove that if the square matrix A' is obtained from the square matrix A by interchanging any two rows of A then $|A'| = -|A|$, where $|A|$ denotes the determinant of A.

c) Evaluate the determinant $|A|$ where

$$A = \begin{pmatrix} 6 & 8 & 6 \\ 11 & 13 & 10 \\ 23 & 27 & 20 \end{pmatrix}$$

[2+5+5]=[12]

4.a) Prove that

i) the solutions of a homogeneous linear equation $x_1\alpha_1 + \cdots + x_n\alpha_n = 0$, where the scalars x_i belong to a field F and the vectors α_i belong to a vector space V(F), considered as row-vectors $x = (x_1, \ldots, x_n)$ form a subspace S of the vector space of n-tuples $x = (x_1, \ldots, x_n)$ over F, and

ii) $d[S] = n - d[M(\alpha)]$, where $M(\alpha)$ = vector space spanned by $\alpha_1, \ldots, \alpha_n$ and the notation $d[X]$ stands for the dimension of X.

b) Find a polynomial

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

which agree with f(x) for the four values of x given in the following table.

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x	f(x)
4.80	60.7511
4.81	61.3617
4.82	61.9785
4.83	62.6015

Is this polynomial unique? Give reason. [4+8+20]=[32]

5. Given the integral $\int_1^2 \frac{dx}{x}$ find the values of n and h to assure four decimal places of accuracy in evaluating this integral by Simpson's three-eighth rule where h is the increment in x and n is the number of equidistant points to be taken in the domain of integration for the quadrature formula. Evaluate the integral and check the accuracy. [14]

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MID-YEAR EXAMINATION

General Science-2: Physics Theory

Date: 26.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. State and prove Carnot's theorem.

A Carnot's engine whose low temperature reservoir is kept at 12°C has an efficiency of 40 per cent. It is desired to increase the efficiency to 60 per cent. By how many degrees centigrade should the temperature of the reservoir at the higher temperature be increased? [4+10+9]=[23]

2. Prove that for any substance the ratio of the adiabatic and isothermal elasticities is equal to the ratio of the two specific heats. [6]

Prove the following from thermodynamical consideration:

(a) $C_p - C_v = - T(\partial p/\partial v)_T \, (\partial v/\partial T)_p^2$

(b) $(\partial C_v/\partial v)_T = T(\partial^2 p/\partial T^2)_v$

(c) $(\partial p/\partial T)_v = \dfrac{L}{T(v_2 - v_1)}$

where the symbols have their usual significance. [7+5+7]=[19]

3. An e.m.f. of E volts is suddenly applied to a circuit of an inductance and a resistance in series. Investigate the growth of current in the circuit.

What is the time constant of a circuit? [13+4]=[17]

4. Calculate the moment of inertia of a uniform rectangular lamina about a line passing through a corner and perpendicular to its plane.

A hole is drilled through the earth along a diameter and a particle is dropped into it. Show that the particle would execute S.H.M. Find its time period in terms of the radius of the earth and the acceleration due to gravity. Show that the time period remains unaltered even when the tunnel does not pass through the center of the earth. [8+7+8]=[23]

5. Select the correct answer from among those supplied.

i) The time required for a current to be established in a circuit depends upon

a. the magnitude of the current
b. the applied potential difference
c. its inductance only
d. its inductance and resistance

ii) A frictionless heat engine can be 100 per cent efficient only if its exhaust temperature is

a. equal to its input temperature
b. 0°C
c. less than its input temperature
d. 0°K . [10]

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MID-YEAR EXAMINATION

General Science-2: Physics Practical

Date: 27.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. Perform the experiment as indicated in Card A. [60]

2. Class work. [20]

3. Laboratory Note Book [10]

4. Oral test [10]

Distribution of
marks of Question No. 1.

Theory	7
Method	40
Calculation	6
Accuracy	7
	60

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MID-YEAR EXAMINATION

Statistics-2: Statistics Theory

Date: 29.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. Find the mode of the Poisson distribution. [8]

2. Suppose r balls are drawn one at a time without replacement from a bag containing m white balls and n black balls. Find the probability distribution of the number (x) of white balls found in the sample of r balls. Find also the mean and variance of x. [12]

3. Define cumulants.

   Find the constant of the distribution with density

$$f(x) = \text{const.} \; e^{-x} x^{p-1} \qquad x \geq 0$$

   where p is a positive-valued parameter.

   Find the cumulants of this distribution. Hence or otherwise find the mean and variance. [16]

4. Using m.g.f.'s prove that the sum of independently distributed normal variables is also normally distributed. [12]

5. Explain clearly the notion of independence of two r.v.'s x and y. Is it generally true that independence and absence of correlation mean the same thing? Discuss with examples. [12]

6. A joint distribution of 2 continuous r.v.'s is given by the density $f(x,y) = xe^{-x(y+1)}$, $x \geq 0$, $y \geq 0$.

   Find the marginal distribution of x, the conditional distribution of y given x, and the regression of y on x. [16]

7. Find the m.g.f. of the bivariate normal distribution and use it to interpret the parameters of this distribution. [12]

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MID-YEAR EXAMINATION

Statistics-2: Statistics - Practical

Date: 30.12.69 Maximum Marks: 100 Time: 3 hours

<u>Note:</u> Answer <u>all</u> questions. Marks allotted for each question are given in brackets [].

1. The following table shows the heights in centimeters of 1000 students;

x	155-157	158-160	etc.												
f	4	8	26	53	89	146	188	181	125	92	60	22	4	1	1

Fit a normal curve to the data. Superpose the fitted curve on the histogram, and calculate goodness of fit χ^2. [60]

2. In a certain examination, the percentages of passes and distinctions were 45 and 9 respectively. Estimate the average and s.d. of marks obtained by the candidates, and also the average and s.d. of marks obtained by those passing in the examination. You may assume that the distribution of marks is normal. The minimum pass and distinction marks were 40 and 75 respectively. [25]

3. An unbiased coin is tossed 10 times. Find the probability of getting

i) exactly 4 heads

and ii) at most 4 heads,

by using the normal approximation and also by the exact formula. [15]

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MID-YEAR EXAMINATION

General Science-A: Biology Theory

Date: 31.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer *any five* questions. Marks allotted for each question are given in brackets [].

1. Mention the systematic position of the family Compositae. Describe the characteristic features of this family with suitable illustrations and examples. [4+16]=[20]

2. Mention briefly the morphological peculiarities of the family Scitamineae. Draw and comment on the typical floral diagrams of its three sub-families. Write the names of two plants belonging to each sub-family. [5+9+6]=[20]

3. A. What is a hesperidium? Mention the chief morphological characters of the family where hesperidium is a common occurrence. [3+7]=[10]

B. Mention the names of ten plants belonging to Gramineae. [10]

4. A. Write an account on the aestivation in Malvaceae. [10]

B. Mention the special characteristics of the sub-family Leguminosae. [10]

5. Write short notes on *any four*:

a) Inflorescence in Euphorbiaceae,
b) Spikelet of Gramineae,
c) Androecium of Annonaceae,
d) Tendrils of Cucurbitaceae,
e) Gynaecium in Solanaceae. [4X5]=[20]

6. Write an illustrated account on the foliar spirals in palms. Enumerate the importance of palms. [12+8]=[20]

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MID-YEAR EXAMINATION

General Science-3: Biological Practical

Date: 31.12.69 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. Describe botanically specimen A and assign the plant to its family giving valid reasons. [15+5]=[20]

2. Identify specimen B and give the name of the family to which it belongs. Give an illustrated account of the various organs of the specimen emphasising on the characteristics specific to the family. [5+15]=[20]

3. Make a labelled drawing of specimen C and give its floral formula and floral diagram. [4+3+3]=[10]

4. Comment on specimens D to M. [10×3]=[30]

5. Practical records. [20]

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PERIODICAL EXAMINATIONS

Economics – 2

Date: 23.2.70 | Maximum Marks : 100 | Time : 3 hours

Note: Answer Groups A and B in separate answerscripts.
Marks allowed for each question are given in brackets ( ).

Group A

Economic Theory

Maximum Marks : 60 Suggested time: 2 hours

Attempt any three questions

1. (a) Explain the different sacrifice principles of taxation, stating the assumptions you would make for your analysis

   (b) Show that the principle of equal absolute sacrifice may justify (i) proportional, (ii) progressive or (iii) regressive taxation (14+6)

2. In a two-country model how will incomes in both change on account of an increase in autonomous exports in country I? Analyse the corresponding changes on account of an increase in autonomous investment in country II. (20)

3. (a) Work out the multiplier effect of a change in government expenditure when net taxes are a rising function of income

   (b) State and prove the Haavelmo theorem on balanced budget. (10+10)

4. Do you think that fiscal policy has to be supplemented by monetary policy in order to cope with a depression or an inflationary process? Give reasons for your answer. (20)

5. (a) State the causes of inflation

   (b) Describe the different types of inflation

   (c) Give an analysis of excess-demand inflation (4+6+10)

Group B

Indian Economic Problems

Maximum Marks 40 Suggested time: 1 hour

(answer any two questions)

1. What are the main findings of the All India Rural Credit Survey Committee? Critically examine the recommendations of the Committee. (20)

2. Examine the main problems of agricultural marketing in India. Indicate how the policy of direct purchase by the Government agencies or the activities of the State Trading Corporation can improve the situation. (20)

3. Analyse the background and objectives of the two Industrial Policy Resolutions of 1948 and 1956. (20)

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PERIODICAL EXAMINATION

Mathematics-2: Calculus and Matrix Algebra

Date: 2.3.70 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Answer all questions.

GROUP A Max. Marks: 50

1. State and prove Taylor's theorem for the expansion of a function $f(x)$ in a finite form with Lagrange's form for the remainder after n terms. Give Cauchy's form of the remainder in Taylor's expansion. [7+1]=[8]

2.a) Expand $\log(1+x)$ in an infinite series of ascending powers of x stating the conditions under which the expansion is valid.

b) If $y = \sin(m \sin^{-1} x)$, show that

$$(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} + (m^2 - n^2)y_n = 0$$ where y_n

has the usual meaning. Hence obtain the expansion of $\sin(m \sin^{-1} x)$. [6+6]=[12]

3. Evaluate the following limits:

i) $\underset{x \to 0}{\text{Lt}} \dfrac{a^x - b^x}{x}$ (ii) $\underset{x \to 1}{\text{Lt}} \left(\dfrac{x}{x-1} - \dfrac{1}{\log x}\right)$

iii) $\underset{x \to \infty}{\text{Lt}} \left(1+\dfrac{1}{x^2}\right)^x$ (iv) $\underset{x \to \infty}{\text{Lt}} \dfrac{x^n}{e^x}$ (n being positive) [$4 \times 2\frac{1}{2}$]=[10]

4.a) If $u = \dfrac{y}{z} + \dfrac{z}{x} + \dfrac{x}{y}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$$

b) State and prove Euler's theorem on homogeneous functions. [5+5]=[10]

5. Home assignments. [10]

GROUP B Max. Marks: 50

1.a) Define the terms 'rank' and 'nullity' with reference to a linear transformation on a finite-dimensional vector space and establish a relation between them. [2+2+10]=[14]

b) Let V be an n-dimensional vector space over the field F and let T be a linear transformation from V into V such that the range and the null space are identical. Prove that n is even. Also, give an example of such a transformation. [3+4]=[7]

2.a) When is a linear transformation said to be 'invertible'? [3]

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2.b) Examine if the following linear transformation σ is invertible:

V = the space of real polynomials of degree $\leq n-1$.

For $P(x) \in V$, $\sigma(P(x)) = \frac{d}{dx} P(x)$. [4]

3.a) Define 'a matrix over a field F'. [3]

b) Establish the correspondence between linear transformations and matrices. (State and prove your results clearly.) [8]

c) The following are some linear transformations σ on a vector space V. Find the matrix of σ in each case with respect to a basis in V, which you conveniently choose. (State clearly what this basis is.)

i) V is any finite dimensional vector space. σ is the identity transformation. [2]

ii) $V = R^3$. For $(x_1, x_2, x_3) \in V$, the image by σ is $(x_1, x_2, 0)$. [4]

iii) σ is the linear transformation in Question 2(b). [5]

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PERIODICAL EXAMINATION

Statistics-2: Probability

Date: 9.3.70 Maximum Marks: 100 Time: 2 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. A die is thrown as long as necessary for an ace to turn up. Assuming that the ace does not turn up at the first throw, what is the probability that more than three throws will be necessary. [16]

2. A symmetric die is rolled twice. Let A be the event 'The sum of the scores is odd', B the event 'First score is odd', and C the event 'The difference is odd. State whether the following statements are true or false. [20]

   a) A and B are independent
   b) B and C are independent
   c) $P(A^c \mid B \cap C) > 0$
   d) $P(A \mid C) = 1$
   e) $P(B \mid A \cap C) = P(B \mid A) . P(A \mid C)$.

3.a) State clearly Bayes' theorem.

b) Urn I contains 3 red balls and 1 white ball. Urn II contains 1 red ball and 3 white balls. A fair coin is tossed and if it falls 'heads' we draw a ball from Urn I; if 'tails' from Urn II we draw a ball. What is the probability that Urn II was selected if

   i) A red ball was drawn?
   ii) A white ball was drawn? [6+10]=[16]

4. A coin has an unknown probability p of success. It is known that 'p = 1/4 ' with probability 1/4 and 'p = 3/4 ' with probability 3/4 and a value is to be chosen. Two tosses are made with the coin and both are found to be successes. What value of p would you choose if you decide to use Bayes' theorem? Give reasons. [16]

5. In a certain school examination results showed that 10% of the students failed in Mathematics 12% failed in English, and 2% failed in both Mathematics and English. A student is selected at random from the school roll. Are the events 'student failed in Mathematics' and the event 'student failed in English' independent? [12]

6. A and B are two table-tennis players and if they play a game A has probability p of winning the game. You are given that p = 1/4 or 1/2 or 2/3 with probabilities 1/5, 1/10 and 7/10 respectively. They play 5 games and A wins twice. Assume that each game is independent of the others. If they play 5 more games what is the probability that A will win only twice again? [20]

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PERIODICAL EXAMINATION

Statistics-2: Statistics - Theory and Practical

Date: 16.3.70 Maximum Marks: 100 Time: 4 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. Define the multiple correlation coefficient $R_{1.23..p}$ and show that it is the highest possible correlation coefficient between x_1 and any linear function of $x_2, x_3, \ldots, x_p$. [12]

2. EITHER

Prove the formula

$$1 - R^2_{1.23...p} = (1 - r^2_{12})(1 - r^2_{13.2})\ldots(1 - r^2_{1p.23..p-1})$$

and discuss its significance. What happens when $R_{1.23...p} = 1$. [15]

OR

Explain the concept of partial correlation.

Prove the formula $r_{12.34..p} = -\dfrac{R_{12}}{\sqrt{R_{11} R_{22}}}$. [15]

3. Given the matrix of sums of squares and products, how do you solve for the partial regression coefficients by the pivotal condensation method, and how do you compute the multiple correlation coefficient? [12]

4. In a three-variable problem with the variables numbered 1,2 and 3, prove that $r_{12} + r_{13} + r_{23} \geq -3/2$. [7]

5. Define the multivariate normal distribution and state its properties. [12]

6. A true die is cast six times. How would you find the probability that the number of sixes exceeds the number of fives? (You need not carry out the computation.) [7]

7. In a three-variable correlation analysis, the following sums were found: $n = 18$, $\Sigma y = 581$, $\Sigma x_1 = 179$, $\Sigma x_2 = 66$; $\Sigma y^2 = 22293$, $\Sigma x_1^2 = 2133$, $\Sigma x_2^2 = 278$; $\Sigma yx_1 = 6636$, $\Sigma yx_2 = 2387$, $\Sigma x_1 x_2 = 715$.

Find the multiple regression of y on x_1 and x_2. Also compute the multiple correlation coefficient $R_{y.12}$. [25]

8. Practical Records. [10]

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PERIODICAL EXAMINATION

Statistics-2: Time Series

Date: 30.3.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

1. Describe the different components of a time series and explain the relationship among them. What purpose is served by analysing a time series? [15]

2. Briefly discuss the different types of growth curves and the corresponding methods of fitting these curves to observed data. [20]

3. The following table gives the consumer price index numbers (Base: July, 1914 = 100) for the years 1930-1945. Estimate the trend by the method of moving averages with a period of seven years and plot the original observations and the smoothed values on the same graph paper.

Year	Index number	Year	Index number
1930	158	1938	156
31	147	39	158
32	144	40	184
33	140	41	199
34	141	42	200
35	143	43	199
36	147	44	201
37	154	45	203

(Graph paper to be supplied) [32]

4. Fit a Gomportz curve to the following production data.

Year	Production (000 tons)
1951	310
52	332
53	326
54	365
55	396
56	415
57	431
58	511
59	661

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PERIODICAL EXAMINATION

General Science-C

Date: 6.4.70 | Maximum Marks: 100 | Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Physics Theory

Maximum Marks: 50. Answer all questions.

1. Find the impedance of an alternating current circuit containing a capacitance in series with a resistance. [9]

An alternating e.m.f. of $312 \cos 2\pi .50.t$ volts is applied to a circuit consisting of an unknown capacitance in series with a resistance of 100 ohms and the r.m.s. current is found to be 0.138 amp. On adding an inductance into the circuit in series, the r.m.s. current is observed to increase to 1.606 Amp.

a) What is the value of unknown capacitance?

b) What is the value of added inductance?

c) What value of the added inductance would have made the current in the circuit maximum?

d) What is the value of this maximum current? [4×4]=[16]

2. Derive the expressions for the position and width of the interference fringes produced by two monochromatic point sources. [7]

Draw a neat diagram (no description necessary) and properly-label it to show the formation of fringes in Lloyd's mirror. [5]

Compare the biprism system with the Lloyd's mirror in respect of the production of fringe pattern. [6]

Interference fringes are produced by biprism in the focal plane of a reading microscope which is 100 cm. from the slit. A lens placed between the biprism and the microscope gives two images of the slit in two positions. If the distance between the virtual slits be 4.05 mm. and 2.90 mm. in the two positions, calculate the fringe width of the bands formed with sodium light of wavelength 5893 A.U. [7]

Group B : Chemistry Theory

Maximum Marks: 50

Answer question No.1 and two others from the rest.

1. EITHER

In a series of six experiments with hydrogen iodide 0.96 gram of the latter in each experiment was directly converted into vapour at the given temperature and at constant pressure and then quickly cooled. The amount of iodine liberated in each experiment was determined by titration with 0.1 N sodium thiosulphate solution, and the volumes of the latter for corresponding temperatures were as follows:

Temp. (degree centigrade)	250	290	330	360	400	420
Volume of thio sulphate solution (c.c.)	13.25	12.4	12.0	12.9	14.6	15.7

Calculate the percentage of hydrogen iodide dissociated at each temperature and express your results in the form of a graph. What conclusion do you derive? [20]

OR

What do you mean by the 'order of a reaction'? What are the methods of determining the order of a reaction?

The following data were obtained in an experiment on the Inversion of cane-sugar:

Time (minutes)	0	7.20	18.00	27.00	∞
Angle of rotation (degree)	+24.09	+21.40	+17.73	+15.00	-10.74

Find out the order of the reaction.

2. What is the principle of Le Chatelier? Show how it may be utilised to forecast the result of the following changes -

a) Increase of pressure on the system

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

b) Increase of temperature on the reaction

$$H_2 + I_2 \rightleftharpoons 2HI + Q \text{ calories}$$

c) Increase of pressure on the reaction

$$N_2O_4 \rightleftharpoons 2NO_2 \ .$$ [15]

3. State and explain Hess' law of constant heat summation. Explain the terms: (a) Intrinsic heat, (b) Heat of formation, (c) Heat of combustion.

The heats of formation of carbon monoxide and steam are 26.40 and 58.00 kcal respectively. Calculate the heat of the reaction:

$$H_2O + C = CO + H_2.$$ [15]

4. The half-life of uranium (238) is 4.51×10^9 years. Calculate the age of a mineral in which the atomic ratio of lead (206) to uranium (238) is 0.291:1. Assume that all half lives in the decay-chain are small in comparison with that given and all lead has arisen out of the decay of uranium. [15]

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ANNUAL EXAMINATION

Mathematics-2: Calculus

Date: 18.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A

Answer questions 2 and 3 and any one of the questions 1 and 4.

1. State and prove Rolle's theorem.

   Prove that, if $\frac{a_0}{n+1} + \frac{a_1}{n} + \dots + \frac{a_{n-1}}{2} + a_n = 0$, then the equation $a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$ has at least one root between 0 and 1. [6+8]=[14]

2. If $u = f(x,y)$ be a function of two independent variables x and y and if $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x \partial y}, \frac{\partial^2 u}{\partial y \partial x}$ all exist and $\frac{\partial^2 u}{\partial y \partial x}$ (or $\frac{\partial^2 u}{\partial x \partial y}$) is continuous, then show that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$

   If

$$f(x,y) = xy\frac{x^2 - y^2}{x^2 + y^2}, \text{ when } x \neq 0 \text{ or } y \neq 0$$

$$= 0 \qquad \text{when } x = 0,\ y = 0$$

   show that at $x = 0$, $y = 0$, $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$. [10 +8=[18]

3. a) Find the primitives of

   i) $\int \frac{dx}{a + b\cos x}$

   ii) $\int (3x-2)\sqrt{x^2 - x + 1}\, dx$

   b) Show that $\int_0^{\pi} \log(1 + \cos x)dx = \pi \log \frac{1}{2}$. [6+6+6]=[18]

4. a) State and prove fundamental theorem of integral calculus.

   b) Find the area of the loop of the curve

$$xy^2 + (x+a)^2(x+2a) = 0$$

   [7+7]=[14]

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Group B

Answer any three questions.

5. Define the definite integral $\int_a^b f(x)dx$ as the limit of sum.

Prove by summation $\int_a^b \cos x \, dx = \sin b - \sin a$

Evaluate

$$\underset{n\to\infty}{\text{Lt}} \left[\frac{1}{\sqrt{2n-1^2}} + \frac{1}{\sqrt{4n-2^2}} + \frac{1}{\sqrt{6n-3^2}} + \dots + \frac{1}{n} \right].$$ [2+7+7]=[16]

6.a) If $\int_a^b f(x)dx$ exists, show that

$$\left| \int_a^b f(x)dx \right| \le \int_a^b |f(x)|dx .$$

b) When do you say a function $f(x)$ dominates another function $g(x)$?

Show that the following integrals

i) $\int_0^\infty e^{-x^2}dx$; (ii) $\int_0^1 x^{p-1}(1-x)^{q-1} dx$, $p > 0$, $q > 0$

converge.

[6+1+4½+4½]=[16]

7. If $y = e^{\tan^{-1}x} = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$

show that

i) $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$

ii) $(n+2)a_{n+2} + na_n = a_{n+1}$.

Determine the coefficients of a_0, a_1, a_2, a_3. [7+9]=[16]

8. i) Find the $\underset{x\to 1}{\text{Lt}}\ x^{\frac{1}{1-x}}$

ii) If V be a function of x and y, prove that

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}$$

where $x = r\cos\theta$, $y = r\sin\theta$. [5+11]=[16]

Neatness: 2 marks.

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ANNUAL EXAMINATIONS

Mathematics-2: Matrix Algebra

Date: 19.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer the two groups A and B in separate answer-scripts. Marks allotted for each question are given in brackets [].

Group A: Maximum Marks: 50

Answer as many as you can. The maximum marks you can score in this group is 50.

1.a) Define 'linear independence' of a set of vectors. [3]

b) If $\alpha_1, \alpha_2, \ldots, \alpha_k$ are linearly independent vectors in a vector space V, show that it can be extended to a basis of V. [7]

c) Find the number of linearly independent vectors in the set of n vectors

$$\{(a,b,\ldots,b),\ (b,a,b,\ldots,b),\ \ldots,\ (b,b,\ldots,b,a)\}$$ [10]

2.a) Consider the space V* of linear functionals on a vector space V of dimension n. Show that V* is a vector space of dimension n. Explain how to find a basis of V* from a basis of V. [7]

b) Let V be the vector space of all polynomial functions p(x) from R into R which have degree 2 or less. Define three linear functionals on V by

$$f_1(p) = \int_0^1 p(x)dx, \quad f_2(p) = \int_0^2 p(x)dx, \quad f_3(p) = \int_0^{-1} p(x)dx.$$

Show that $\{f_1, f_2, f_3\}$ is a basis for the dual of V by exhibiting the basis of V of which it is the dual. [13]

3.a) Derive the Hermite normal form for a matrix. [12]

b) Show that the row rank of a matrix is equal to its column rank. [8]

Group B: Maximum Marks: 50

Answer as many questions as you. The maximum marks you can score in the group is 50.

1.a) If matrices A and B are partitioned as follows:

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{pmatrix} \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{pmatrix}$$

where A_{ij} is a $n_i \times p_j$ matrix and B_{ji} is a $p_j \times n_i$ matrix, $i = 1,2;$ $j = 1,2,3,$ write down the expression for AB in terms of A_{ij}, B_{ji}. [6]

b) Given A, B, A^{-1}, B^{-1}, c find inverses of

(i) $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, (ii) $\begin{pmatrix} A & c \\ 0 & B \end{pmatrix}$. [10]

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c) Find the inverse of

$$\begin{pmatrix} 2 & 3 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$ [4]

2. a) Formulate the problem of simultaneous linear equations, in matrix form. [4]

b) Show the solution space of a system of linear homogeneous equations in n unknowns has dimension $n - r$, where r is the rank of the matrix of coefficients. [10]

c) Find a value ∂ for which the follows system of equations admits a solution.

$$\begin{aligned} 2x_1 - x_2 + 5x_3 &= 4 \\ 4x_1 \qquad + 6x_3 &= 1 \\ -2x_2 + 4x_3 &= 7 + \partial \end{aligned}$$ [6]

3. a) Define the determinant of a square matrix and show that the determinant is zero if and only if the matrix is not of full rank. [6]

b) If A is a matrix in which all the elements above the main diagonal are zero, show that the determinant of A is equal to the product of the elements in the main diagonal. [8]

c) Explain a method of computing the determinant of a matrix using the pivotal condensation method. [6]

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ANNUAL EXAMINATIONS

Economics-2: Economic Theory

Date: 20.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer the groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Maximum Marks: 50

Attempt any two questions.

1. Give an analytical proof of the proposition that under perfect competition in both the output and factor markets, if Q is the maximum output which can be obtained at the cost of C rupees, then C rupees is the minimum cost at which the output Q can be produced.

   Give a geometrical interpretation of your analysis. [25]

2. a) A monopolist can sell his product in two economically isolated markets. Under what conditions will price discrimination pay?

   b) A monopolist in a market with a demand function $p = 24-q$ can also sell his product in a competitive market where the prevailing price is Rs.10 per unit. The total cost of production is Rs.$0.25\,Q^2$

   (Q = total number of units produced
   q = number of units demanded in the monopoly market at the price Rs. p per unit).

   Show that the monopolist will reap greater profits by selling in both types of market than by selling in the monopoly market alone. [10+15]=[25]

3. a) Explain the concept of marginal revenue product.

   b) Explain how collective bargaining affects wages and employment when there is

   i) perfect competition in output market,

   ii) perfect competition in output market but monopsony in labour market. [10+15]=[25]

4. Under bilateral monopoly, determine the equilibrium positions in the following cases:

   i) both parties behave as quantity-adjusters,

   ii) one of the parties acts as a quantity-adjuster and the other as a monopolist,

   iii) one of the parties is the fixer of an option and the other is the taker. [25]

Group B: Maximum Marks: 50

Attempt any three questions.
The maximum you can score in this group is 50.

1. Discuss the process of income formation under the interaction of the multiplier and acceleration principles. Sketch the different time paths followed by income for different values of the multiplier and the accelerator. [20]

2. Compare the Keynesian and classical theories of employment. [20]

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3\. a) Briefly examine the effects of government borrowing on national income.

b) Given that y_0 is the initial level of national income at market prices, b is the constant proportion of the community's disposable income which is spent on consumption, and t is the ratio of net tax receipts to national income at market prices, prove that, with unchanged private investment and increased government expenditures on goods and services, national income at market prices can be doubled if a budget deficit of the amount $y_0(1-b)(1-t)$ is incurred.

Assume budget to be balanced initially and the economy to be a closed one. [10+15]=[25]

4\. In a two-country model how will incomes in both countries change on account of an increase in autonomous exports in one of them? Give an analysis of the corresponding changes in income if, alternatively, autonomous investment were to rise in the same country. [20]

5\. Do you think that fiscal policy has to be supplemented by monetary policy in order to cope with a depression or an inflationary process? Give reasons for your answer. [20]

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ANNUAL EXAMINATIONS

Economics-2: Indian Economic Problems

Date: 21.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets []. Answer any two questions from each group.

Group A: Maximum Marks: 50

1. Examine the main economic factors behind the present rural unrest in India. Do you agree with the view that the non-implementation of the land reform measures is the basic cause for this unrest? Give reasons for your answer. [$12\frac{1}{2}+12\frac{1}{2}$]=[25]

2. Examine the nature of the financial problems faced by the Indian agriculturists from cultivation to marketing. Do you agree with the view that after the nationalisation of major banks, Commercial banks can play a big role in providing farm credit to the agriculturists? Give reasons for your answer. [$12\frac{1}{2}+12\frac{1}{2}$]=[25]

3. What are the difficulties in the way of marketing of agricultural produce in India? In this context, discuss the case for State Trading in agricultural products. [$12\frac{1}{2}+12\frac{1}{2}$]=[25]

Group B: Maximum Marks: 50

1. Discuss the scope and functions of 'Industrial Finance Corporation' and 'National Industrial Development Corporation'. [25]

2. Discuss the case for using Foreign aid for India's economic development. What are the forms in which Foreign aid may be available to India? [15+10]=[25]

3. Examine the nature of the concentration of economic power in the industrial sector of India. [25]

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ANNUAL EXAMINATIONS

General Science-3: Geology

Date: 22.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Attempt any three questions from Group A and any two from Group B.

Group A.

1. Define the following terms: Rock, Mineral, Crystal. How are the rocks classified into major groups? Give two examples from each group. [6+14]=[20]

2.a) Define folds. What are the different kinds of folds?

b) What is metamorphism? What are the factors of metamorphism? Define metamorphic facies. [10+10]=[20]

3. Define 'weathering' and 'erosion'. What are the major natural agencies that control weathering and erosion? Briefly describe the actions of these natural agencies. [6+3+11]=[20]

4. What is an unconformity? Describe any two types of unconformities found in stratified rocks. [20]

Group B

1. What are fossils? Describe various modes of fossilization. What is the significance of fossils in the geological record? [5+5+10]=[20]

2.a) What are the important rock types that constitute the Gondwana deposits of India? State the geological time span the Gondwana deposits occupy. Where are the important Gondwana coal-fields of India located ?

b) Name the most important economic mineral deposit found in the Tertiary deposits of India. Name the localities from where it is found. [15+5]=[20]

3. How to differentiate between (attempt any five):

(a) Normal fault and thrust fault.
(b) Shale and limestone.
(c) Peat and anthracite.
(d) Cross-stratification and parallel-stratification.
(e) Dykes and sills.
(f) Faults and joints. [4×5]=[20]

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ANNUAL EXAMINATIONS

Statistics-2: Time series and Index Numbers

Date: 23.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ].

Group A: Maximum Marks: 50

Answer all questions.

1. With which characteristic movement of a time series would you mainly associate each of the following:

   (a) a boom in business activity,

   (b) an increase in the employment of agricultural labour during the harvesting of Kharif crops,

   (c) a rising demand for computers in India,

   (d) a Baranagore Bandh.

   Give reasons for your answer. [10]

2. The following table gives the losses due to fire in the U.S.A. in different quarters of the years 1948-51.

year	losses (in million dollars)			
	January-March	April-June	July-September	October-December
1948	209	178	150	171
1949	188	161	149	169
1950	190	178	149	173
1951	209	178	161	183

Assuming seasonal pattern to remain constant, calculate the indices of quarterly variation. [20]

3. Fit a logistic curve to the following U.S. population data obtained at decennial censuses of 1880-1960.

Census Population of U.S.A.

year	census population (in millions)
1880	50.2
1890	62.9
1900	76.0
1910	92.0
1920	105.7
1930	122.8
1940	131.7
1950	150.7
1960	179.3

Plot the observed data and the fitted values on the same graph paper. [20]

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Group B: Maximum Marks: 40

Answer all questions

1. What is a chain index? Discuss its advantages and disadvantages over a fixed base index number. [10]

2. Explain the different types of error arising in connection with the construction of a price index number. How can these errors be measured? [10]

3. Discuss briefly how you will proceed to construct a cost of living index number for workers in jute mills around Calcutta. [10]

4. The following data relate to the group indices and corresponding weights (shown in brackets) for the menial class cost of living index numbers in Calcutta:

year	food (71.28)	clothing (2.89)	fuel and light (9.27)	house-rent (6.69)	miscellaneous (9.87)
1951	396.7	551.4	366.0	116.9	291.8
1952	380.2	504.2	336.8	116.9	283.6

Calculate the general cost of living index for each of the given years.

The total wages and the number of workers employed in jute textiles around Calcutta are given below:

Year	Total wages (Rs. lakhs)	Number of workers (000)
1951	2231	272
1952	2552	275

Calculate the average nominal wages and real wages for the jute textile workers, using the general cost of living indices for the menial class people in Calcutta. [10]

Group C: Maximum Marks: 10

Practical Note-book [10]

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[23[illegible]]

ANNUAL EXAMINATIONS

Statistics-2: Statistics Theory

Date: 25.5.70 | Maximum Marks: 100 | Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

Group A: Maximum Marks: 50

Answer all questions.

1. EITHER

A normal distribution with mean μ and variance σ^2 is truncated below $x = c$. Find the mean and variance of the truncated distribution. [12]

OR

Find the m.g.f. of the gamma distribution with parameter p :

$$f(x) = \frac{1}{\Gamma p} e^{-x} x^{p-1} \quad (p > 0), \quad 0 \leq x < \infty$$

and hence prove the following reproductive property:
If x_1, x_2 are independent gamma variates with parameters p_1, p_2 respectively, then $x_1 + x_2$ is a gamma variate with parameter $p_1 + p_2$. [12]

2. EITHER

Write down the density of the bivariate normal distribution and examine the properties of the conditional distribution of y given x. [12]

OR

The joint density of two r.v.'s x and y is given by:

$$f(x,y) = 1/\pi a^2 \quad \text{for} \quad x^2 + y^2 \leq a^2$$

$$\text{and} \quad = 0 \quad \text{elsewhere.}$$

Find the marginal distribution of y and the conditional distribution of x given y. [12]

3. EITHER

Define the multiple correlation coefficient $R_{1.23...p}$ between x_1, on the one hand, and $x_2, x_3, \ldots, x_p$, on the other, and discuss its significance, quoting necessary formulae. Mention in particular the cases where $R = 0$ and $R = 1$. [14]

OR

Consider the following special case of the Pearsonian differential equation :

$$\frac{dy}{dx} = \frac{y(x+a)}{b_0 + b_1 x}.$$

Integrate this to obtain the equation of the frequency curve and examine the properties of the curve. [14]

Write short notes on any two: (i) conditions for consistency of a correlation matrix; (ii) the multinomial distribution; (iii) Gram-Charlier series type A. [12]

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Group B: Maximum Marks: 50

Answer any three questions.

1.a) Discuss briefly the advantages of the sampling method over complete enumeration and also those of probability sampling over subjective and haphazard selections.

b) Explain the concept of the standard error of a statistic. [12+4]=[16]

2.a) Define random sampling numbers.

b) Show that if x is an observation drawn at random from a continuous theoretical population (like the normal) with distribution function $F(\cdot)$, then $F(x)$ is uniformly distributed over (0,1). How is this result useful in random sampling from continuous populations? [3+7+6]=[16]

3. Let $(x_1, x_2, \ldots, x_n)$ be a random sample drawn without replacement from a finite population of size N. Find the mean and variance of the sampling distribution of

$$\bar{x} = \sum_j x_j/n .$$ [16]

4.a) If $x_1, x_2, \ldots, x_n$ are mutually independent $N(0,1)$ variables, what would be the distribution of

(i) $\sum_1^n x_i$; (ii) $x_1^2 + x_2^2 + x_3^2$; (iii) $\dfrac{\sqrt{n}\,\bar{x}}{\sqrt{\dfrac{\Sigma (x_i-\bar{x})^2}{n-1}}}$

and (iv) $\dfrac{(x_1^2 + x_2^2)/2}{(x_3^2 + \ldots + x_n^2)/(n-2)}$.

(Just state the form of the distribution with values of the parameters, if any.)

b) Given a random sample of size n drawn with replacement from a population, how would you test the null hypothesis $H_0(\mu = \mu_0)$, where μ denotes the population mean and μ_0 some specified value. You may assume that n is large. [8+8]=[16]

Neatness [2]

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ANNUAL EXAMINATIONS

Statistics-2: Statistics Practical

Date: 26.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [].

1. The following shows the frequency distribution of pages in a book according to the number of printing mistakes (x):

x :	0	1	2	3	4	5	Total
No. of pages:	112	120	62	24	9	1	328

Fit a Poisson distribution to the data and test the goodness of fit. [18+7]=[25]

2. The following shows the means, standard deviations and intercorrelations of scores on three tests given to 205 students:

Score	mean	s.d.	inter-correlations		
			y	x_1	x_2
Y	58.9	9.56		0.632	0.758
X_1	49.2	7.23			0.326
X_2	67.5	11.54			

Obtain the multiple regression equation giving y in terms of x_1 and x_2 and examine the effect of dropping x_1 from this equation on the predictive efficiency of the regression equation. [25+10]=[35]

3. Draw a random sample of size 5 from the frequency distribution of heights presented below:

height (inches)	60-62	62-64	64-66	66-68	68-70	70-72
no. of persons	15	88	152	148	59	6

[10]

4. EITHER

A random sample of 562 males (drawn with replacement) showed 213 literates while another random sample (with replacement) of 483 females showed only 102 literates. Examine whether the proportion of literates varies significantly between males and females. [10]

OR

A random sample of 292 persons drawn with replacement showed an average income of Rs.234/- and a s.d. of Rs.68/-. Does this contradict the prevalent idea that the average income in the population is Rs.250/-? [10]

5. Practical Records [10]

6. Viva Voce [10]

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ANNUAL EXAMINATIONS

Statistics-2: Probability

Date: 27.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [ ]. Answer all questions

Group A: Maximum Marks: 57

1.a) A closet contains n pairs of shoes. If $2r$ shoes are chosen at random $(2r < n)$, what is the probability that there will be no complete pair among them?

b) An urn contains a white and b black balls. Balls are drawn one by one until only those of a single colour are left. What is the probability that these are white?

c) There are N tickets numbered $1,2,\ldots, N$, of which n $(< N)$ are taken at random and arranged in increasing order of their numbers: $x_1 < x_2 < \ldots < x_n$. What is the probability that $x_n = M$? [1-]

2. There are s sets of tickets, each set containing n tickets numbered $1,\ldots, n$. From these ns tickets, r $(\geq n)$ tickets are taken at random, without replacement.

i) Show that the probability that all the n numbers are found among the sample of r tickets is

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} \binom{ns-ks}{r} \Big/ \binom{ns}{r} .$$

ii) By computing this probability directly for $r=n$, show that

$$\sum_{k=0}^{n}(-1)^k \binom{n}{k}\binom{ns-ks}{n} = s^n.$$

[14]

3.a) H is an event in a discrete sample space, for which $P(H) \neq 0$. Show that the conditional probability $P(E/H)$ defined as $P(EH)/P(H)$ for all events E of the sample space has the same basic properties as you know $P(E)$ to possess.

b) If $H_1,\ldots,H_n$ are mutually exclusive and exhaustive events, show that for any event E,

$$P(E) = \sum_{j=1}^{n} P(H_j)P(E/H_j).$$

Also derive Bayes' formula.

c) $(N+1)$ urns contain respectively $0,1,\ldots,N$ red balls. One urn is chosen at random and n balls drawn from it with replacement, of which m are found to be red. If now one more ball is drawn, show that the probability that this ball is red is approximately $(m+1)/(n+2)$. [..]

4.a) An urn contains N balls, of which R are red. Let S_n denote the number of red balls in a random sample (without replacement) of n balls drawn from the urn. Show that

$$E(S_n) = \frac{nR}{N}; \quad V(S_n) = \frac{nR(N-R)}{N^2}\left[1-\frac{n-1}{N-1}\right]$$

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[You may introduce a set of variables X_j, $j = 1,\dots,n$, such that $X_j = 1$ if the j-th ball drawn is red and 0 otherwise. Then $S_n = X_1 + \dots + X_n$].

b) State and prove the Chebychev inequality.

c) A random variable X has the distribution

$$P(X = j) = 2^{-j}, \quad j = 1,2,\dots$$

Use the Chebychev inequality to obtain an upper bound for the probability $P\{|X-2| > 2\}$, and compare the value of this upper bound with the actual value of the probability calculated from the distribution. [14]

Group B: Maximum Marks: 43

1.a) Define the generating function of a sequence of numbers, and show how the mean and the variance of a non-negative integer valued random variable can be calculated from the generating function of its probabilities.

b) If $A(s)$ is the probability generating function of X, obtain (i) the probability generating function of $X+1$; (ii) the generating function of the numbers $P(X > n+1)$.

c) Let X_j, $j = 1,2,\dots$ be i.i.d. random variables, each assuming the values $1,\dots,a$ with probabilities $1/a$. Let $S_n = X_1 + \dots + X_n$. Show that the probability generating function of S_n is

$$\left[\frac{s(1-s^a)}{a(1-s)}\right]^n$$

From this, obtain an expression for $P(S_n = j)$. [15]

2.a) If $A(s)$ is the p.g.f., of each of the independent random variables $X_1,\dots,X_N$, where N is itself a random variable with p.g.f. $G(s)$, show that the p.g.f. of $S_N = X_1 + \dots + X_N$ is $G[A(s)]$.

b) Find the p.g.f. of S_N if each X_j is 1 with probability p and 0 with probability $1-p$ $(=q)$, while N has the geometric distribution given by $P(N = n) = (1-b)b^n$, $n = 0,1,\dots$, $0 < b < 1$.

c) X and Y are two random variables such that the conditional probability distribution of Y for any given value of $X = \lambda_j$ is Poisson with parameter λ_j, and X can take the values $\lambda_1, \lambda_2, \dots$, with probabilities $p_1, p_2, \dots$ $(\lambda_j > 0,\ \Sigma p_j = 1)$. Obtain the unconditional distribution of Y and show that

$$E(Y) = E(X) \quad \text{and} \quad V(Y) = V(X) + E(X).$$ [14]

3.a) State the law of large numbers (weak) for i.i.d. random variables with finite mean values.

b) Obtain a sufficient condition for the law of large numbers to hold for a sequence of independent random variables which may not be identically distributed, but have finite variances.

c) Use the above law to deduce the limiting behaviour of the relative number of successes in a sequence of independent trials, when the probabilities of success in the trials are (i) the same, (ii) different. [14]

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ANNUAL EXAMINATIONS

[237]

General Science-2: Physics Theory

Date: 28.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts.
: Marks allotted for each question are given in brackets [].

Group A: Maximum Marks: 50

Answer any three questions.

1. A straight weightless elastic beam of length l and of uniform rectangular cross-section is rigidly clamped at one end and is loaded at the other with a weight W, the bending of the beam from the initial horizontal position remaining within elastic limit. Calculate the displacement of the loaded end.

If the same beam is supported at both ends and the same load is placed at the mid-point what would be the depression of this loaded mid-point compared with the previous displacement of the loaded end? [12+4]=[16]

2. Define the terms: moment of inertia and radius of gyration. What is the physical significance of moment of inertia?

Calculate the moment of inertia of a solid sphere about a diameter. [illegible]

You are given two spheres of same mass and size one being hollow and made of a substance of higher density, while the other is solid but made of a substance of lower density Explain how (no deduction necessary) you will identify the hollow one. [2+2+3+6+3]=[16]

3. What are the characteristics of a simple harmonic motion. The space-time equation for a simple harmonic motion is given by $x = a \sin(wt + \epsilon)$. Show that the velocity v and acceleration f satisfy $w^2v^2 + f^2 = a^2w^4$.

Suppose a smooth straight tunnel is bored through the earth and a body is dropped into it. Assuming the earth to be a uniform homogeneous sphere, show that the body would execute S.H.M. with a time period $T = 2\pi\sqrt{R/g}$ where R= radius of the earth and g = acceleration due to gravity. [3+4+9]=[16]

4. Investigate the growth and decay of a current in a circuit composed of an inductance L, resistance R and a battery of of EMF E all connected in series.

A telephone operates at a current of 120 milliampere and has an inductance 10 henries and resistance 100 ohms. If a 24 volt battery having negligible internal resistance is suddenly applied, calculate the operating time. [6+4+6]=[16]

5. Explain the term 'electrical resonance' with reference to an A.C. circuit consisting of a capacity, inductance and resistance in series.

What do you mean by the term power factor?

A resistance R and a condenser connected in series across a 240 volt a.c. supply of sinusoidal waveform, take a current of 1.6 amp. at a power factor of 0.6. Determine the resistance R and the reactance of the condenser. [6+5+5]=[16]

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Group B: Maximum Marks: 50

Answer Q.5 and any other two of the rest.

1. Explain the Fresnel concept of half-period zones. Show how the principle of half-period zone has been applied to explain the rectilinear propagation of light. [7+12]=[19]

2. Describe with a neat diagram the construction and the principle of action of a Michelson Interferometer. Indicate to what different uses the instrument is put. [4+4+6+5]=[19]

3. Prove that for any substance the ratio of the adiabatic and isothermal elasticities is equal to the ratio of the two specific heats.

Prove the following from thermodynamical considerations

$$\text{(i)}\ C_p - C_v = T(\partial p/\partial T)_v\ (\partial V/\partial T)_p$$

$$\text{(ii)}\ (\partial C_v/\partial V)_T = T(\partial 2p/\partial T)_v^2$$

where the symbols have their usual significance. [7+6+6]=[19]

4. Explain what you mean by reversible and irreversible processes. Show that the efficiency of a reversible engine is the maximum.

A carnot engine whose low temperature reservoir is kept at 12°C has an efficiency 40 per cent. It is desired to increase the efficiency to 60 per cent. By how many degrees centigrade should the temperature of the reservoir at the higher temperature be increased.? [4+4+6+5]=[19]

5. Make a choice of the correct answer from the following:

A. A frictionless heat engine can be 100 per cent efficient only if its exhaust temperature is

a. equal to input temperature c. 0°C
b. less than input temperature d. 0°K.

B. A dynamo is often said to generate electricity. It actually acts as a source of

a. charge, b. emf, c. electrons
d. magnetism

C. Monochromatic green light is used to illuminate ($\lambda = 5 \times 10^{-7}$ m) a pair of narrow slits 1 mm. apart. The separation of bright lines on the interference pattern formed on a screen 2 m. away is

a. 0.1 mm. b. 0.25 mm. c. 0.4 mm.
d. 1.0 mm.

D. Longitudinal waves do not exhibit

a. refraction b. reflection
c. interference d. polarisation. [4×3]=[12]

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General Science-2: Chemistry Theory

Date: 29.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer groups A and B in separate answerscripts. Marks allotted for each question are given in brackets [].

Group A: Maximum Marks: 50

Answer three questions of which question 1 is compulsory.

1.a) Two sets of data from experiments with aqueous solutions of cane-sugar ($C_{12}H_{22}O_{11}$) are reproduced below. Utilise the data to prove that substances in solution do behave like gases.

Table 1

Temp(^oC)	Concentration of cane-sugar solution (gms/litre)	Osmotic pressure (atmosphere)
0	10.0	0.66
0	20.0	1.32
0	45.0	2.97

Table 2

Concentration of cane-sugar solution (moles/litre)	Temperature (^oC)	Osmotic pressure (atmosphere)
0.3	0	7.085
0.3	10	7.334
0.3	20	7.605

b) Calculate the osmotic pressure of a solution containing 5 gms. of urea (mol. wt. 60) per litre of solution at 25^oC. [12+8]=[20]

2.a) Write down the expression correlating the molecular weight of a substance and the depression of the freezing point of the solvent in which the substance has been dissolved.

b) The melting point of phenol is 40^oC. A solution containing 0.172 gm. of acetanilide (C_8H_9ON) in 12.54 gms. of phenol freezes at 39.25^oC. Calculate the freezing point constant and the latent heat of fusion of phenol. [15]

3.a) State Hess's law of constant heat summation and point out its thermodynamic basis. Indicate its use in determining the heat of formation of an organic compound. Define the terms endo-thermic and exothermic as applied to compounds.

b) From the following thermochemical equations, all of which refer to a temperature of 17^oC, calculate the heat evolved in the polymerisation of acetylene to benzene

$$C + O_2 = CO_2 + 96.960 \text{ cals}$$
$$H_2 + \tfrac{1}{2}O_2 = H_2O + 68,360 \text{ cals}$$
$$C_6H_6 + 7\tfrac{1}{2}O_2 = 6CO_2 + 3H_2O + 799,350 \text{ cals}$$
$$C_2H_2 + 2\tfrac{1}{2}O_2 = 2CO_2 + H_2O + 310,050 \text{ cals}$$

[15]

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4. EITHER

What is meant by dynamic equilibrium as applied to chemical system. Find out the expression for the equilibrium constant for any two of the following reactions and from the expression show how pressure influences the equilibria of the systems mentioned:

(a) Synthesis of Ammonia from elements

(b) Dissociation of Nitrogen peroxide (N_2O_4)

(c) Thermal dissociation of hydrogeniodide. [15]

OR

The equilibrium constant for the reaction between acetic acid and alcohol, forming ethyl acetate and water, at 25°C is 4. Explain precisely what is meant by this statement. If 5 gm. molecules of acetic acid react with 1 gm. molecule of alcohol at 25°C as far as practicable, what will be the composition of the equilibrium mixture? [15]

Group B: Maximum Marks: 50

Answer all the questions.

1.a) What is meant by the order of a reaction and what is half-life as applied to a chemical reaction? How do half-lives of different types of reactions depend on the initial concentration of reactants? Give briefly the principles underlying two common methods of verifying the order of a reaction.

b) From the following data show that the decomposition of H_2O_2 in an aqueous solution is a first order reaction.

Time (minutes)	0	10	20
Volume of $KMnO_4$ in ml. which reacts with H_2O_2	22.8	13.8	8.25

[18]

2. What is the difference between a weak acid and a strong acid? What is an amphiprotic solvent? Give an example and illustrate its amphiprotic behaviour. Define pH of a solution? How is the pH of a solution of a weak acid related to its degree of dissociation and its dissociation constant? [16]

OR

(a) Explain why is an aqueous solution of sodium acetate alkaline and an aqueous solution of ammonium chloride acidic.

(b) 'Hydrochloric acid in acetic acid medium appears as a very weak acid'. Give reasons for your arguments.

(c) Calculate the pH of the following solutions:

(i) 1.009 $\frac{N}{10}$ aqueous solution of Hcl

(ii) 0.905 $\frac{N}{100}$ solution of NaoH

(iii) 0.01 N solution of acetic acid (dissociation constant of acetic acid = 1.8×10^{-5}) [16]

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Write short explanatory notes on:-

(a) Le Chatelier principle

(b) Conjungate acids and bases

(c) Ionic product of water. [16]

OR

In a series of six experiments with hydrogen iodide 0.96 gm of the latter in each experiment was entirely vapourise at the given temperatures and constant pressure and then quickly cooled. The amount of iodine liberated in each experiment was determined by titration with 0.1N sodium thiosulphate, and the volumes of the latter for corresponding temperatures were as follows:

Temperature ($0^{o}C$)	250	290	330	360	400	420
Volume of Thiosulphate (c.c.)	13.2	12.4	12.0	12.*	14.6	15.7

Calculate the percentage of hydrogen iodide dissociated at each temperature and express your results in the form of a graph. What conclusions would you arrive at from the nature of the graph. [16]

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ANNUAL EXAMINATIONS

General Science-2: Physics Practical

Date: 30.5.70 Maximum Marks: 100 Time: 3 hours

Note: Marks allotted for each question are given in brackets [ ].

Group A

Perform the experiment as indicated in Card.

Group B

Class work

Group C

Note Book

Group D

Viva Voce

Distribution of marks of Group A

Theory	--	7
Tabulation	--	40
Calculation, graph etc.		7
Accuracy	-- -	6
		60

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[illegible]

ANNUAL EXAMINATIONS

General Science-2: Chemistry Practical

Date: 30.5.70 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions. Marks allotted for each question are given in brackets [ ].

Group A

1. Transfer the solution supplied to a 100 ml. volumetric flask and make up the volume with distilled water. Estimate the total amount of iron present in the sample with the help of $K_2Cr_2O_7$ as the primary standard. [70]

Group B

Viva Voce [10]

Group B

Practical Note Book [10]

Group C

Class-work [10]
