INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1991-92
SEMESTRAL II BACKPAPER EXAMINATION
STATISTICAL METHODS IV

Date: 29.6.92 Maximum Marks: 100 Time: 4 Hours

1. Let $(X_1, \dots, X_r) \sim N_r$. Show the following relations:

For $k \leq r$, the distribution of X_k given $X_1, \dots, X_{k-1}$ is the same as the distribution of X_k given X_{k-1}

$(\Leftrightarrow)$ For $k \leq r$, $E(X_k \mid X_1, \dots, X_{k-1}) = E(X_k \mid X_{k-1})$

$(\Leftrightarrow)$ For $j \leq \nu < k \leq r$, $\rho_{jk} = \rho_{j\nu}\rho_{\nu k}$. [10]

2.(a) State and prove the Slepian's inequality for N_2. [8]

(b) Suppose (X, Y) has a p.d.f which has the TP_2 property. Prove that (X, Y) is associated. [12]

3.(a) Let $X \sim N_n(\underset{\sim}{O}, I_n)$. Show that $U = 1'X$ and $Q = X'CX$ are independently distributed when $C1 = 0$ (C is symmetric); further show that $Q \sim \chi^2$ when C is idempotent. [3]

(b) Use the above result to derive the distribution of the sample correlation coefficient based on a random sample of size N from a bivariate normal distribution with $\rho = 0$. [12]

4.(a) Let F be the c.d.f of $X_1, \dots, X_p$ and F_i be the cdf of X_i. Prove that F is a continuous function of $(X_1, \dots, X_p)$ when each F_i is continuous. [6]

(b) Let (X, Y) be distributed as $N_2\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right]$.

Prove that the correlation between X^2 and Y^2 is ρ^2. [5]

(c) Let r_n be the sample correlation coefficient based on a random sample of size n from N_2 with $\rho = 0$. Obtain the limiting distribution of $(r_n)\sqrt{n}$ as $n \to \infty$. [9]

p.t.o.

5. Let $A \sim W_p(m,\Sigma)$, with Σ p.d. and $m \geq p$.

Prove that $E(A^{-1}) = \left(\frac{1}{m-p-2}\right)\Sigma^{-1}$ when $m-p-2 > 0$. [10]

6. Let X_1 denote the increase in the number of hours of sleep due to sedative A, and X_2 denote the corresponding increase due to sedative B. The following mean vector and the co-variance matrix are obtained for observation of (X_1,X_2) on 10 patients chosen randomly (you may assume N_2 distribution for X_1,X_2).

$$\bar{X}_1 = 2.33 \qquad \bar{X}_2 = 0.75 \qquad S = \begin{pmatrix} 4.01 & 2.85 \\ 2.85 & 3.20 \end{pmatrix}$$

(a) Test the hypothesis H_0 : Var (X_1) = Var (X_2) against H_1: not H_0 at 5% level. [8]

(b) Test H_0: $E(X_1) = 3\,E(X_2)$ against H_1: not H_0 at 5% level. [6

7. The distributions in four blood-group classes O, A, B and AB of 140 Christions who are army cadets and 295 other Christion are given below:

	O	A	B	AB	Total
army cadets	56	60	18	6	140
others	120	122	42	11	295

Test the hypothesis that the blood group distributions in two groups are the same. [6

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1991-92
SEMESTRAL-II BACKPAPER EXAMINATION
ELEMENTS OF ALGEBRAIC STRUCTURE

Date: 22.6.92 Maximum Marks: 100 Time: 3 Hours

Note: You may answer all questions. Maximum score is 100.

I.(a) If G is a finite group such that the equation $x^m = e$ (e = identity in G) has atmost m solutions for any positive integer m, show that G is abelian. [10]

(b) If R is a commutative ring with unity, and I is a maximal ideal, show that every nonzero element of R/I is invertible under multiplication. [10]

II.(a) If G is a group of order 385, prove that its 7-sylow subgroup is in the centre of G. [10]

(b) In a finite group G, show that any two p-sylow subgroups in G are conjugate, where p is a prime dividing the order of G. [10]

III.(a) In a principal ideal domain D, given any increasing sequence $C_1 \quad C_2 \quad \ldots\ldots \quad C_k \quad \ldots\ldots$ of ideals of D, show that there is some l such that $C_m = C_l$ for all $m > l$. [10]

(b) Determine the number of abelian groups of order 720. Justify your answer. [10]

IV.(a) If F is a field of q elements, show that $q = p^r$ for some prime p and integer r. [10]

(b) If F is a field and $g(x) \in F[x]$, show that $F[x]/(g(x))$ is a principal ideal domain. [10]

(c) F is a field of p^m elements and K is a field of p^n elements. If F is a sub-field of K, show that m divides n. [10]

V. Construct a field of 16 elements. Find a primitive element r of the field. Find the minimum polynomial of r^3 over GF(2). [25]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1991-92
SEMESTRAL-II BACKPAPER EXAMINATION
DIFFERENTIAL EQUATIONS

Date: 22.6.92 | Maximum Marks: 100 | Time: 3 Hours

Note: Answer all the questions.

1. Let $f(x,y)$ be continuous on the closed strip $[a,b] \times (-\infty,\infty)$ and satisfy the following Lipschitz condition:

$\exists$ a constant $K > 0$ such that

$|f(x,y_1)-f(x,y_2)| \leq K|y_1-y_2| \quad \forall x \in [a,b]$ and $y_1, y_2 \in (-\infty,\infty)$.

If (x_0,y_0) is any point in $[a,b] \times (-\infty,\infty)$, show that the initial value problem $y' = f(x,y)$, $y(x_0)=y_0$ has a unique solution defined on $[a,b]$. [20]

2.(a) Find the general solution of

$$\frac{d^4y}{dx^4} - y = x^2 e^{-x}.$$

(b) Find a solution of the homogeneous equation by trial, and then find the general solution of

$$xy'' - 2(x+1)y' + (x+2)y = x^3 e^{2x}.$$

[10+10=20]

3. Show that the equation

$$4x^2y'' - 8x^2y' + (4x^2+1)y = 0$$

has only one Frobenius series solution. Find it. Use this solution to find the general solution. [20]

4.(a) Let $A \equiv (x_1,y_1)$, $B \equiv (x_2,y_2)$ be two points lying above the x-axis (ie. $y_1, y_2 > 0$).
Find the curve C (having equation $y = y(x)$) such that

(i) C joins A and B;
(ii) C lies entirely above the x-axis;
(iii) the length of C is some fixed number L
and the area bounded by C, the x-axis and the lines $x = x_1$ and $x = x_2$ is a maximum.

(b) A and B are as above. Find the equation of the curve which satisfies (i) and (ii) of the above part and for which the area of the surface of revolution obtained by revolving the curve about the x-axis is a minimum. [10+10=20]

p.t.o.

5. Let $P_n(x)$ be the n^{th} Legendre polynomial.

Show that

(a) $P_n(x) = C_n \frac{d^n}{dx^n} \left((x^2-1)^n \right)$, where C_n is a constant.

(b) $\int_{-1}^{1} P_n(x)\, Q(x)\, dx = 0$ whenever $Q(x)$ is a polynomial of degree $< n$.

(c) $P_n(x)$ has n simple zeros in $(-1, 1)$. [7+5+8=20]

INDIAN STATISTICAL INSTITUTE
B.STAT. II AND III YEAR:1991-92
SEMESTRAL II EXAMINATION
DIFFERENTIAL EQUATIONS

Date:8.5.92 Maximum Marks:100 Time: 3 Hours

Note: This paper carries 110 marks. You may answer all the questions. But the maximum you can score is 100.

1.(a) Let $f(x,y)$ be continuous on the closed strip $[a,b] \times (-\infty,\infty)$ and satisfy the following Lipschitz condition:

a constant $K > 0$ such that

$$|f(x,y_1)-f(x,y_2)| \leq K |y_1-y_2| \quad x \in [a,b], \text{ and}$$
$$y_1, y_2 \in (-\infty,\infty).$$

If (x_0,y_0) is any point in $[a,b] \times (-\infty,\infty)$, show that the initial value problem $y' = f(x,y)$, $y(x_0)=y_0$ has a unique solution defined on $[a,b]$.

(b) Let $f(x,y)$ be a continuous function on the open strip $(a,b) \times (-\infty,\infty)$ such that for any closed strip $[c,d]\times(-\infty,\infty)$ contained in $(a,b)\times(-\infty,\infty)$, and for any $(x_0,y_0) \in [c,d]\times(-\infty,\infty)$, the initial value problem $y'= f(x,y)$, $y(x_0)=y_0$ has a unique solution defined on $[c,d]$. Show that for any $(x_0,y_0) \in (a,b)\times(-\infty,\infty)$, the initial value problem $y'=f(x,y)$, $y(x_0)=y_0$ has a unique solution defined on (a,b). [15+8=23]

2.(a) Find a solution of the homogeneous equation by trial, and then find the general solution of the equation

$$x(1-x)y'' - (1-2x)y' + (x^2-3x+1)y = (1-x)^3.$$

(b) Let $u(x)$ and $v(x)$ be solution of $y''-4y'+29y=0$ and $y''+4y'+13y=0$ respectively, such that the curves $y=u(x)$ and $y=v(x)$ intersect at the origin and have the same slope at the origin. Determine $u(x)$ and $v(x)$ if $u'(\pi/2) = 1$. [10+6=16]

p.t.o.

3.(a) Show that the boundary value problem

$$y'' + \pi^2 y = f(x), \quad y(0) = y(1) = 0$$

has a solution if and only if $\int_0^1 f(x) \sin \pi x \, dx = 0$.

In this case, find the solution of the above boundary value problem.

(b) Consider the eigen value problem

$$y'' + \alpha^2 y = 0, \quad y(0) + y'(0) = 0, \quad y(1) = 0, \ (\alpha \text{ real})$$

show: (1) any $\alpha < 0$ is not an eigen value.

(2) $\alpha = 0$ is an eigen value.

(3) If $\alpha > 0$ is an eigen value, then $\tan \alpha = \alpha$.

Also, find the eigen functions, corresponding to each eigen value. [10+10=2

4. Solve the initial value problem $y'' + xy' + 2y = 0$, $y(0) = 1$, $y'(0) = 2$ by the power series method. Find the recurrence formula and use it to prove the convergence of the series for all x. [1

5.(a) For n=0,1,2,... let $J_n(x)$ be the Bessel function of the first kind of order n. Show that

$$J_n(x) = c_n \int_0^\pi \cos(n\theta - x \sin \theta) \, d\theta,$$ where c_n is some constant.

(b) Show that $x=\infty$ is a regular singular point of the equation

$$x^3 y'' + 3x^2 y' + (x+4) y = 0$$

Find its indicial equation and its exponents at $x = \infty$. [15+8=2

6.(a) Find the extremals of the functional

$$v(y(x)) = \int_0^4 [x\, y'(x) - (y'(x))^2] \, dx$$

which is determined by the boundary conditions

$y(0) = 0$, $y(4) = 3$.

contd. ....

6.(b) Let A and B be points on the sphere $x^2+y^2+z^2=c^2$. Show that the shortest curve joining A and B and lying on the sphere is an arc of a great circle. [6+10=16]

INDIAN STATISTICAL INSTITUTE
B.STAT. II YEAR: 1991-92
SEMESTRAL-II EXAMINATION
ECONOMICS II

Date: 8.5.92 Maximum Marks: 100 Time: 3 Hours

Note: Answer ANY FOUR questions. All questions carry equal marks. Marks allotted to the different parts of a question are shown in brackets [ ].

1.(A) Consider a simple model of income determination in a closed economy as follows:

$$Y = C + \bar{I} + \bar{G}$$

C = Consumption expenditure

$$= \bar{C} + c\, Y_d \qquad c = \text{mpc},\ t = \text{income tax rate}$$

$$Y_d = Y + TR - tY \qquad TR = \text{transfer payments},\ Y_d = \text{disposable income}$$

where $\bar{C}$, $\bar{I}$ and $\bar{G}$ are autonomous consumption, investment and govt. expenditures, respectively.

(a) Suppose, transfer payments are also autonomous, being given at $\overline{TR}$. What is the value of the multiplier corresponding to (i) an increase in $\bar{G}$ and (ii) a decrease in $\overline{TR}$? [7]

(b) Suppose transfer payments also depend on income Y. When income is high, transfer payments, such as unemployment benefits, fall and when income is low unemployment and hence unemployment benefits increase:

$$TR = \overline{TR} - bY \qquad b > 0$$

(i) What is the value of the multiplier now corresponding to an increase in $\bar{G}$? (ii) Why is the new multiplier less than the corresponding one in (a)? [8]

(B) Write a short note on the Phillips curve. [10]

2.(a) The economy is at full employment. Now the government wants to change the composition of demand toward investment and away from consumption without, however, allowing aggregate demand to go beyond full employment. What is the required policy mix? [15]

(Use IS - LM diagram to show your policy proposal.)

p.t.o.

2.(b) Consider an economy where the government consider two alternative programs for contraction. One is the removal of an investment subsidy, the other is a rise in income tax rates. Use the IS - LM schedule to discuss the impact of these alternative policies on income, interest rates and investment.

[Hint: A subsidy to investment means that the government essentially pays part of the cost of each firm's investment Thus a subsidy to investment shifts the investment schedule (drawn against the rate of interest) right ward, i.e.., at each interest rate, firms now plan to invest more.] [1

3.(a) Explain the terms : internal rate of return and marginal efficiency of capital. [6

(b) Derive the investment schedule, i.e., the one which shows that the amount of investment expenditure will be inversely related to the market rate of interest. [1

(c) What is meant by the term 'automatic stabilizer'? Illustrate. [4

4.(a) What is meant by 'speculative demand for money? Distinguish between 'risk lovers' and 'risk averters'? [1

(b) Use Tobin's portfolio balance approach to demonstrate the following results:

(i) an investor will put his financial wealth partly in cash and partly in bonds.

(ii) an investor's speculative demand for money will be inversely related to the rate of interest. [1

5.(a) (i) Consider an economy with internatinal trade (but no capital flows). What is meant by internal and external balance?

(ii) Suppose in this economy exports are autonomously given, but imports vary positively with income. What kinds of policy dilemma may arise in such an economy, if the government wants to achieve both external balance and full employment of labour? [1

(b) (i) Allow capital inflows and outflows in the above economy. Suppose the economy is initially in internal and external balance. Draw the IS, LM and BP schedules.

contd. ...

5.(b) (ii) Assume there is an increase in the foreign interest rate. Show the effect on the BP schedule. Suppose the central bank does not undertake any offsetting policy. Describe briefly the adjustment process by which internal and external balance will be restored. [10]

6.(a) Consider the following statement: 'one way of comparing Keynesian and Classical models is to view them as postulating alternative closing rules to a complete macroeconomic model'.

In the light of the above statement compare the structures and implications of the Keynesian and the Classical models. [17]

(b) Consider the Keynesian model you have developed in (a) above. Suppose, investment is interest-inelastic. Show that fiscal policy is effective in raising output in this model. Will monetary policy be also effective? Give reasons for your answer. [8]

INDIAN STATISTICAL INSTITUTE
B.STAT. II YEAR:1991-92
SEMESTRAL-II EXAMINATION
BIOLOGY-II

Date: 8.5.92 Maximum Marks:100 Time: 3 Hours

GROUP A

Maximum Marks: 50

Note: All questions carry equal marks.

1. What are the different factors affecting crop production? Briefly discuss about the various types of agricultural production systems. [5+5]

2. Diagramatically represent Nitrogen cycle in Soil - Plant ecosystem. Briefly describe nitrogen fixation phenomenon by legumes. [6+.

3. Undergoes during its life-cycle under transplanted condition. Briefly discuss about the yield components on which paddy grain yield is determined. [6+.

4. What are the main elements of experimentation in agricultural trials? Describe the principles followed in serial correlation procedure to assess soil heterogeneity. [5+5]

5. (a) State and describe Gooduin's Epigenetic Model.
   (b) Discuss the stability properties.
   (c) When does the system show life-cycle properties? [2+4+4]

GROUP B

Answer Question no.4 and 5 and any two of the following.

1. What do you understand by spontaneous mutation and induced mutation? Discuss the procedure utilized for detection of sex linked lethal mutations utilizing CIB method and Muller-5 method. (2+4+4=10.

2. Explain linkage and crossing over phenomenon with examples. (5+5=10)

3. How do you induce polyploidy? Discuss briefly the importance of polyploidy in agriculture and evolution? (2+8=10)

p.t.o.

4. The colour of Onion bulbs in governed by two pairs of genes. A cross between true breeding red bulb strain and true breeding white bulb strain produced all F_1 plants with red bulbs. In F_2, 110 red, 38 yellow and 46 white bulbs were produced. Use x^2 test and explain the results genetically. [10

5. Write short notes on the following (any four):

i) Important cattle breeds in India.
ii) Modern definition of gene.
iii) Major discoveries in molecular biology in twentieth Century.
iv) The origin of tetraploid cultivated cotton.
v) Extranuclear inheritance.
vi) Structure of DNA.
vii) Cross-breeding in Poultry farming. (4x5=20)

INDIAN STATISTICAL INSTITUTE
B.STAT. II YEAR: 1991-92
SEMESTRAL II EXAMINATION
DEMOGRAPHY

Date: 6.5.92 | Maximum Marks: 100 | Time: 3 Hours

Note: Separate answerpapers to be used for Groups A and B.

GROUP A

Maximum Marks: 50 | Time: $1\frac{1}{2}$ Hours

Note: Answer any three questions. All questions carry equal marks.

1.(a) What do you mean by infant mortality? Describe the different methods for computing infant mortality rate. Comment on their merits and demerits.

(b) For a certain life table $l_x = 20900 - 80x - x^2$

(i) What is the ultimate age in the table.

(ii) Find

$${}_{10}p_{20}$$

2.(a) Describe crude birth rate, general fertility rate and total fertility rate, indicating their merits and limitations.

(b) Describe the net reproduction rate.

3.(a) Explain the uses of population estimates in a country's developmental programme. Discuss the mathematical methods for projecting total population.

4. Write short notes on any three of the following:

(a) Standardised death rates

(b) Sample Registration Scheme

(c) Person-Years Lived by a population during a calendar year

(d) Current life table.

GROUP B

SQC AND OR

Maximum Marks: 50 | Time: $1\frac{1}{2}$ Hours

Group A: SQC. Answer Question (1) and any one from the remaining.

1. Discuss (any three): Causes of variation, Rational subgrouping; AOQL and ATI; Interpretation of runs on a control chart. [3x5=15]

2.(a) How can frequency distributions be used to diagnose situations of assignable variations?

(b) What do you mean by the O.C curve of an acceptance sampling plan? [7+3=10]

p.t.o.

3.(a) Derive a single sampling plan for variables if acceptable quality level is p_1 and rejectable quality level is p_2 with α and β as the corresponding risks, when only a lower specification limit is given and σ is known.

(b) What will be the acceptance samples plan for a quality characteristic whose lower specification limit is 25; σ is 2; p_1=.05 with α = .05; p_2=.010 with $1-\beta$=.90. [7+3=

Group B: OR. Answer question (1) and any one from the rest.

1. Discuss (any three): Inventory Ordering System; Basic feasible solutions; Arrivals in queues; Buffer stock. [3x5=1

2.(a) Write down the cost model for an inventory problem.

(b) Derive the EOQ for a single item static demand problem with no lead time. [5+5=

3.(a) Under Poisson arrival/departure, derive W_q=expected waiting time in queue in terms of the expected number of customers and the rate of departure, in the steady state.

(b) When is the steady state reached in an M/M/1 queue ? Cars arrive at a toll gate on a freeway according to Poisson distribution with mean 90 per hour. Average time for passing through the gate is 38 seconds. Assuming steady state condition what is the expected waiting time in queue for a car? [4+2+4=

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1991-92

SEMESTRAL-II EXAMINATION

Statistical Methods IV

Date: 4.5.1992 Maximum Marks: 100 Time: 4 hours

GROUP - A Max. Marks: 80

1. Consider a random vector $\underset{\sim}{X} : p \times 1$ with the p.d.f.

$$f(\underset{\sim}{x}) = \begin{cases} k, & \text{if } \underset{\sim}{x}'\underset{\sim}{x} \leq r^2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Using induction on p, show that $\frac{1}{k} = \pi^{p/2} r^p / \Gamma(\frac{p}{2} + 1)$. [7]

(b) Derive the p.d.f. of $Z = \underset{\sim}{X}'\underset{\sim}{X}$ and obtain $E(Z)$. [5]

(c) Show that the covariance matrix of $\underset{\sim}{X}$ is $\frac{r^2}{p+2} I_p$. [3]

(d) Use the above results to show that $E\underset{\sim}{Y} = 0$, $\mathcal{D}(Y) = \Sigma$, when $\underset{\sim}{Y}$ has the p.d.f. (Y $p \times 1$) [3]

$$g(\underset{\sim}{y}) = C, \text{ if } \underset{\sim}{y}'\Sigma^{-1}\underset{\sim}{y} \leq p+2$$
$$= 0, \text{ elsewhere.}$$

2. Let (X, Y) be distributed as the bivariate normal distribution with zero means, unit variances, and correlation ρ $(|\rho| < 1)$.

(a) Show that $P(X \leq x, Y \leq y)$ is increasing in ρ. [6]

(b) Show that the p.d.f. of (X,Y) is TP_2 iff $\rho \geq 0$. [3]

(c) Show that (X,Y) is associated iff $\rho \geq 0$. [5]

(d) Prove that $P[|X| \leq c, |Y| \leq d] \geq P[|X| \leq c]P[|Y| \leq d]$. [4]

3. (a) Define (i) MP level α test, (ii) UMP level α test, (iii) Unbiased test. [3]

(b) State and prove the Neyman-Pearson Lemma. [5]

contd... 2/-

(c) Let $(X_1, \ldots, X_n)$ be a random sample from $U(0, \theta)$. Show that the following test is UMP level α for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.

$$\phi^*(x_1, \ldots, x_n) = \begin{cases} 0, & \text{if } \theta_0 \alpha^{\frac{1}{n}} < X_{(n)} < \theta_0 \\ 1, & \text{elsewhere.} \end{cases}$$

[10]

Where $X_{(n)} = \text{Max}(X_1, \ldots, X_n)$.

4. (a) Let F be the c.d.f. of $(X_1, \ldots, X_p)$ and F_i be the c.d.f. of X_i. Prove that F is a continuous function of $(X_1, \ldots, X_p)$ when each F_i is continuous. [6]

(b) Let (X,Y) be distributed as $N_2 \left[\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right]$.
Prove that the correlation between X^2 and Y^2 is ρ^2. [5]

(c) Let r_n be the sample correlation coefficient based on a random sample of size n from a bivariate distribution with finite fourth moments, the population correlation being 0. Derive the limiting distribution of $\sqrt{n}\, r_n$ as $n \to \infty$. [7]

5. Let $A \sim W_p(m, \Sigma)$, with Σ p.d. and $m \geq p$.

(a) Show that $\sigma^{11}/a^{11} \sim \chi^2_{m-p+1}$. [9]

(b) For $\underset{\sim}{\ell} \neq 0$, show that $\underset{\sim}{\ell}'\Sigma^{-1}\underset{\sim}{\ell} / \underset{\sim}{\ell}' A^{-1}\underset{\sim}{\ell} \sim \chi^2_{m-p+1}$. [2]

(c) Prove that $E(A^{-1}) = \frac{1}{m-p-2}\Sigma^{-1}$, when $m-p-2>0$. [4]

(d) Let $\underset{\sim}{\bar{X}}$ and S be the MLE's of μ and Σ, respectively, based on a random sample of size N from $N_p(\underset{\sim}{\mu}, \Sigma)$. Find $E(\underset{\sim}{\bar{X}}'\, S^{-1}\, \underset{\sim}{\bar{X}})$. [3]

6. Let $X_1, \ldots, X_N$ be a random sample from $N_2\left[\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}, \begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{pmatrix}\right]$.

(a) It is known that $\mu_1 = 0$. Compare the variance of the MLE of μ_2 with that of the standard estimate $\bar{X}_2$. [5]

(b) Suppose $\mu_1 = \mu_2 = \mu, \sigma_{11} = \sigma_{22}$. Obtain the LRT for $H_0: \rho = 0$ against $H_1: \rho \neq 0$. [6]

contd.... 3/

(c) Let s_{11} and s_{22} be the MLE's of σ_{11} and σ_{22}, respectively.

Obtain the limiting distribution of $\sqrt{n}\,(\frac{s_{11}}{s_{22}} - \frac{\sigma_{11}}{\sigma_{22}})$

as $n \to \infty$. [12]

GROUP - B

Max. Marks: 20

State the methods and the theoretical results used.

1. The means of three biometrical characters and the matrix of pooled variances and covariances were obtained for two groups of female desert locusts - one in the phase gregaria and the other in the intermediate phase between gregaria and solitaria.

Matrix of pooled variances and covariances based on (20 + 72 - 2) = 90 d.f.

	X_1	X_2	X_3
X_1	4.7350	.5622	1.4685
X_2		.1431	.2171
X_3			.5702

Means	X_1	X_2	X_3
Phase gregaria (n = 20)	25.80	7.81	10.77
Intermediate phase (n = 72)	28.35	7.41	10.75

Test the hypothesis of equality of the mean-vectors of the two groups of locusts. (you may assume normal distribution). [13]

2. The sample correlation coefficients based on (independent) samples from two bivariate normal distributions of sizes 22 and 25, were found to be .1164 and .1125. Test the hypothesis of equality of two population correlation coefficients. [7]

3. Let $\bar{X}_1$ $\bar{X}_2$ be the mean values and s_1^2, s_2^2 be estimated variances in two samples of sizes n_1 and n_2 respectively. If n_1 and n_2 are large, the statistic

$$W = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Can be used as a normal variate N(0,1) when the two population means are equal.

In a feeding experiment with pasteurized (x_1) and unpasteurized (x_2) milk, Elderton found the following values of W for five groups of boys, the variate being the height.

Age group of boys (yrs.)	Observed W
$6\frac{3}{4}$	2.69
$7\frac{3}{4}$	-0.71
$8\frac{3}{4}$	1.24
$9\frac{3}{4}$	1.84
$10\frac{3}{4}$	1.06

It is postulated that the two types of milk have the same effect on the stature as opposed to the hypothesis that the pasteurized milk leads to larger mean stature. Combine the evidences supplied by the different groups to make an inference.

[6]

4. The distributions in four blood-group classes O, A, B, and AB of 140 Christians who are army cadets and 295 other Christians are given below:

	O	A	B	AB	Total
Army Cadets	56	60	18	6	140
Others	120	122	42	11	295

Test the hypothesis that the blood-group distributions in the two groups are the same.

[7]

:bcc:

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INDIAN STATISTICAL INSTITUTE
B.STAT (HONS.) II YEAR: 1991-92
SEMESTRAL-II EXAMINATION
ECONOMIC AND OFFICIAL STATISTICS

Date: 30.4.92 Maximum Marks: 100 Time: $3\frac{1}{2}$ Hours

GROUP: A

Answer any two questions

1. Discuss the properties of the logistic curve as used for modelling growth of population over time.
Describe the iterative method of LS estimation of the parameters of the logistic curve proposed by Pearl and Reed. How does one get the starting values in this iterative method? [20]

2. Outline the estimation of the Engel curve of any item of consumption from househod budget data. State the assumptions and approximations underlying the usual procedure. Mention briefly how the appropriate algebraic form of the Engel curve, based on the " per capita formulation", is chsen for any given body of data. [20]

3. Write short notes on any two of the following:
(a) Bias of Laspeyres' formula when used for constructing a consumer price index.
(b) Problems of identification and least squares bias in the time series approach to statistical demand analysis.
(c) The Cobb-Douglas production function. [2x10=20]

GROUP: B

Answer all questions

4. The shows some results based on an all-India enquiry on househo d budgets conducted during 1955-56:

contd. 2/-

total ph. exp/person (Rs. per 30 days)(PCE)	% of persons	average exp./person (Rs. per 30 days)	
		on all items	on clothing
(1)	(2)	(3)	(4)
0-8	14.0	6.3	0.2
8-11	17.4	9.4	0.5
11-13	11.7	12.0	0.8
13-15	9.1	14.0	1.0
15-18	11.6	16.5	1.4
18-21	8.2	19.5	1.9
21-24	7.0	22.5	2.2
24-28	5.5	25.8	2.6
28-34	5.2	30.7	3.9
34-43	4.8	37.7	4.9
43-55	2.6	47.2	4.3
55-	2.9	83.3	10.2
all	100.0		

Do <u>any two</u> of the following:

(a) Examine graphically whether the size distribution of population by PCE is approximately lognormal, and if so, estimate the parameters of this underlying distribution by any suitable method. [2

(b) Construct the Lorenz curve of the size distribution of population by PCE and compute the Lorenz ratio. [2

(c) Obtain the Engel elasticity for clothings by fitting the double-logarithmic form of Engel curve to the data. [Hint: use the weighted Least Squares method.) [2

5. Practical Record. [2

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INDIAN STATISTICAL INSTITUTE
B.STAT. II YEAR: 1991-92
SEMESTRAL-II EXAMINATION
ELEMENTS OF ALGEBRAIC STRUCTURE

Date: 27.4.92 Maximum Marks: 100 Time: 3 Hours

Note: You may answer all questions. Maximum score is 100.

I. (a) If G is a group of order 231, show that its 11-sylow subgroup is normal in G and lies in the centre of G. [10]

(b) If G is a finite group and p is a prime dividing the order of G such that $(ab)^p=a^p.b^p$ for all a,b in G, show that the p-sylow subgroup of G is normal in G. [10]

II. (a) N is a normal subgroup and S any subgroup of a group G. Show that (NS)/N is isomorphic, as a group, to $S/(S\cap N)$. [10]

(b) G is a finite group and T is an automorphism of G such that T(x) = x holds only for x=e, the identity element of G. Show that every element g of G can be written as $g = x^{-1}(T(x))$ for some x in G. [10]

III. (a) Determine the number of abelian groups of order 288. Justify your answer. [10]

(b) R is a commutative ring; an ideal P in R is called a prime ideal if $ab \in P$, $(a,b \in R)$ implies either $a \in P$ or $b \in P$. Show that P is a prime ideal iff R/P is an integral domain. [10]

(c) If R is a commutative ring with unity, show that every maximal ideal in R is a prime ideal. [10]

IV. F,K are fields with $F \subseteq K$ and $a \in K$. Suppose that a satisfies a polynomial equation over F and that its minimum polynomial g(x) over F has degree n.

(a) Show that every non zero element in $R=F[x]/(g(x))$ is invertible in R. [10]

(b) If F(a) is the smallest field contained in K containing both F and a, show that the dimension of F(a) as a vector space over F is n. [10]

V. Given that x^3+2x+2 is irreducible over GF(3) explain how to construct a field F of 27 elements. Find a primitive element α of F and find the minimum polynomial of α^2 over GF(3). [25]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year: 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Calculus III

Date: 2.1.1992 Maximum Marks: 100 Time: 3 hours

Note: The papers carries 100 marks.
Answer ALL the questions.

1. Show that if $f(x)$ is a periodic function of period 2π given by

$$f(x) = \frac{\pi}{2} + x \quad \text{for } -\pi \le x \le 0$$
$$= \frac{\pi}{2} - x \quad \text{for } 0 \le x \le \pi,$$

then $$f(x) = \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots\right].$$

What is the value of the series for $x = \pm\pi$.

Deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ [11]

2. Show that

$$\int_0^\infty \frac{\log(1 + b^2x^2)}{1 + a^2x^2}\,dx = \frac{\pi}{a}\log\frac{a+b}{a}.$$

Justify all your steps. [11]

3. Maximize $x^a y^b z^c$, where a, b, c are positive constants, subject to the condition $x^k + y^k + z^k = 1$ where x, y, z are nonnegative and $k > 0$.

From the above result derive the following inequality for any six positive numbers u, v, w, a, b, c:

$$\left(\frac{u}{a}\right)^a \left(\frac{v}{b}\right)^b \left(\frac{w}{c}\right)^c \le \left(\frac{u+v+w}{a+b+c}\right)^{a+b+c}.$$ [12]

Evaluate the following line integral:

$$\int_C x^2yz\,dx - y^3\,dy + x^2z\,dz$$

where C is the closed polygonal curve with successive vertices (0,2,3), (1,2,3), (1,2,6), (1,4,3), (0,2,3). [11]

p.t.o.

5. Evaluate the integral

$$\iint \frac{dx\,dy}{(x^2 + y^2 + 1)^2}$$

taken over one loop of the lemniscate $(x^2 + y^2)^2 = (x^2 - y^2)$.

[11]

6. u and v are continuously differentiable scalar fields on an open set containing the circular disk R whose boundary is the circle $x^2 + y^2 = 1$. Define two vector fields F and G as follows:

$$F(x,y) = v(x,y)\bar{i} + u(x,y)\bar{j}, \quad G(x,y) = \left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)\bar{i} + \left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right)\bar{j}$$

Find the value of the double integral $\iint_R F \cdot G \, dx\,dy$ if it is known that on the boundary of R we have

$$u(x,y) = 1 \text{ and } v(x,y) = y.$$ [11]

7. If $r > 0$, let $I(r) = \int_{-r}^{r} e^{-u^2} du$.

(a) Show that $I(r)^2 = \iint_R e^{-(x^2 + y^2)} dx\,dy$, where $R = [-r, r] \times [-r, r]$.

(b) If c_1 and c_2 are the circular disks inscribing and circumscribing R, show that

$$\iint_{C_1} e^{-(x^2 + y^2)} dx\,dy < I(r)^2 < \iint_{C_2} e^{-(x^2 + y^2)} dx\,dy .$$

(c) Show that $\int_0^\infty e^{-u^2} du = \sqrt{\pi}/2$. [11]

8. Use Stoke's theorem to show that

$$\int_C (y^2 - z^2)dx + (z^2 - x^2)dy + (x^2 - y^2)dz = 9a^3/2$$

where C is the curve cut from the boundary of the cube $0 \leq x \leq a$, $0 \leq y \leq a$, $0 \leq z \leq a$ by the plane $x+y+z = 3a/2$. Explain how to traverse C to get the given answer.

[11]

9. Let S be the surface of the unit cube, $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$, and let $\bar{n}$ be the unit outer normal to S. If $\bar{F}(x,y,z) = x^2\,\bar{i} + y^2\,\bar{j} + z^2\,\bar{k}$, use the divergence theorem to evaluate the surface integral $\iint_S \bar{F}\cdot\bar{n}\,dS$.

Verify the result by evaluating the surface integral directly.

[11]

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION
Probability Theory and its Applications III

Date: 30.12.1991 Maximum Marks: 100 Time: 3 hours

Note: This paper carries 100 marks.
Answer ALL the questions.

1. Show that if X,Y,Z are independent Exp(1) random variables, then the random variables $U = X+Y+Z$, $V = (X+Y)/(X+Y+Z)$ and $W = X/(X+Y)$ are independent. [10]

2. (a) What is meant by a Dirichlet distribution with parameters $\alpha_1,\ldots,\alpha_{k+1}$?

   (b) Suppose $(X_1,\ldots,X_k)'$ has Dirichlet distribution with parameters $\alpha_1,\ldots,\alpha_{k+1}$. Show that for $i < k$, $(X_1,\ldots,X_i)'$ also has a Dirichlet distribution. Also show that the regression of X_k on $(X_1,\ldots,X_{k-1})$ is linear. (2+(8+8)) = [18]

3. If n points are chosen at random prom the interval (0,1), find the probability that all these points will be within an interval of length at most $\frac{1}{2}$. [10]

4. If X has an n-dimensional normal density with mean μ and dispersion Σ, find the distribution of the random variable $Y = (X - \mu)' \Sigma^{-1} (X - \mu)$. [8]

5. At a bus station, a bus is supposed to appear every hour on the hour, but is subject to delays. The successive delays are independent U(0,1) random variables. If a person arrives at the bus station at a random time between 1 p.m. and 2 p.m., what is the expected waiting time for him to get a bus ? [10]

6. (a) Define convergence in probability.

   (b) Show that if $X_n \xrightarrow{P} X$, then for any continuous function $f: \mathbb{R} \to \mathbb{R}$, $f(X_n) \xrightarrow{P} f(X)$. (2+10) = [12]

p.t.o.

7. Suppose $X_1, X_2, \ldots$ are i.i.d. random variables with a non-zero finite mean, and, let $S_n = X_1 + \ldots + X_n$. Show that

$$\max_{1 \le i \le n} |X_i|/|S_n| \longrightarrow 0, \text{ almost surely.}$$

[10]

8. Show that if $X_n \xrightarrow{a} X$ and $Y_n \xrightarrow{a} 0$, then $X_n + Y_n \xrightarrow{a} X$.

[10]

9. Let φ be the characteristic function of a distribution having density f.

Show that if $\int_0^\infty |\varphi(t)|^2 \, dt$ converges, then so does $\int_{-\infty}^\infty f^2(x) \, dx$ and in that case

$$\int_{-\infty}^{\infty} f^2(x) \, dx = \frac{1}{\pi} \int_0^\infty |\varphi(t)|^2 \, dt$$

Using this deduce that $\int_0^\infty \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$.

(6+6) = [12]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1991-92
SEMESTRAL-I BACKPAPER EXAMINATION

Statistical Methods III

Date: 27.12.1991 Maximum Marks: 70 Time: 4 hours

Note: Answers must be precise and complete.

1. Let $X_{ij}(i = 1,\dots,p;\ j = 1,\dots,k)$ be independent $N(\mu_i, \sigma^2)$ variables.

(a) Show that the MLE's of $\mu_i (i = 1,\dots,p)$ and σ^2 are

$$\hat{\mu}_i = \frac{1}{k}\sum_{j=1}^{k} X_{ij}, \quad \hat{\sigma}^2 = \frac{1}{kp}\sum_{i=1}^{p}\sum_{j=1}^{k}(X_{ij}-\hat{\mu}_i)^2.$$ [6]

(b) Suppose k is fixed but $p \to \infty$. Show that

$$\hat{\sigma}^2 \xrightarrow{P} \frac{k-1}{k}\sigma^2.$$ [6]

~~(c) Construct a consistent estimate of σ^2.~~ [4]

2. Let $X_1,\dots,X_n$ be a random sample from $U(0,\theta)$, $\theta > 0$. Let M_n be the MLE of θ, and define $T = \frac{n+2}{n+1} M_n$. Compare the MSE's of M_n and T. [12]

3. Let $X_i = \frac{\theta}{2} t_i^2 + \varepsilon_i$ $(i = 1,\dots,n)$, where ε_i's are i.i.d. $N(0, \sigma^2)$, σ^2 being known.

(a) Using a pivot $2\Sigma t_i^2 X_i / \Sigma t_i^4$ find a fixed length $(1-\alpha)$ level confidence interval for θ. [5]

(b) If $0 \le t_i \le 1$ $(i = 1,\dots,n)$, but we may otherwise choose the t_i freely, what values should we use for the t_i so as to make the above confidence interval as short as possible for given α ? [5]

4. Let $X_1,\dots,X_n$ be i.i.d. random variables with a common continuous distribution F. Let m be a median for F. Obtain a (non-trivial) lower confidence bound for m of level $(1-\alpha)$. [8]

p.t.o.

5. To determine how an experimental dose of a dental anesthesia affects male and female patients, random samples of 25 males 36 females patients are selected and their reaction times are recorded in minutes. The data have been summarized as follows

	Male	Female
mean	4.8	4.4
s.d.	0.8	0.9

(a) Test that there is no sex difference in the mean reaction time. State the theoretical results used.

(b) Obtain a 95% confidence interval for the difference of the mean reaction times.

(6+4) = [10]

6. Measurements of the left-hand and right-hand gripping strength of 10 left-handed writers are recorded as given below.

	Person									
	1	2	3	4	5	6	7	8	9	
Left-hand strength	140	90	125	130	95	121	85	97	131	[illegible]
Right-hand strength	1?8	87	110	132	96	120	86	90	129	[illegible]

Do the data provide strong evidence that people who write with the left-hand have a greater gripping strength in the left-hand than they do in the right-hand ?

(assume that the difference in strengths is approximately normally distributed.)

[9]

7. The number of monthly road accidents in a busy intersection decreased from 7 to 3 after implementing some precautionary measures. Is there any strong evidence that the precautionary measures were effective ? Test at size 0.1 (assume that the number of monthly road accidents is distributed as Poisson.)

[8]

8. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu_1, \sigma^2)$ and $Y_1, \ldots, Y_?$ be a random sample (drawn independently) from $N(\mu_2, \sigma^2)$. Obtain the likelihood ratio test for testing $H: \mu_1 = \mu_2$ against $K: \mu_1 \neq \mu_2$, σ being unknown. How do you obtain the cut-off point ?

[12]

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INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II and III Year : 1991-92
Elective : Anthropology
PRACTICAL

Date: 27.11.1991 Maximum Marks: 50 Time: 2 hours

Note: Mention the number of supplied dermatoglyphic prints and the name of the subject with age and sex for Anthropometric measurements.

Answer the following questions:

1. Identify the following variables and their bilateral differences:

   (a) Angle a t d. [5]

   (b) Total no. of triradii on finger. [5]

   (c) Total no. of triradii on palm. [5]

   (d) TFRC [10]

   (e) Finger pattern types. [5]

2. Measure the following measurements with land marks and the Instruments used (any five): (5 x 4) = [20]

   (i) Maximum Head Length

   (ii) Least Frontal Breadth

   (iii) Bigonial Breadth

   (iv) Bizygomatic Breadth

   (v) Morphological Facial Length

   (vi) Nasal Length.

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1991-92
SEMESTRAL - I EXAMINATION

Geology

Date: 28.11.1991 Maximum Marks: 100 Time: 3 hours

Note: Attempt Question No.1 and any FIVE from the rest.

Fill up the blanks (any 13). Only write down the appropriate word in the blank space from those given in the parenthesis.

(i) Petroleum is a/an _____ (rocky/mineral/organic/inorganic) substance.

(ii) _____ (Basalt/Feldspar/Mudstone/Granite) is a fine-grained igneous rock.

(iii) Al is an important element in _____ (magnetite/chalcopyrite/dolomite/feldspar).

(iv) The bottom surface of a sedimentary layer can be determined from _____ (cross-bedding/amorphous mineral/dip/metamorphism).

(v) The _____ (muscles/eyes/teeth/blood cells) of a dinosaur are best preserved in rocks.

(vi) Quartz, the most abundant mineral available, has a chemical composition _____ (SiO_3/SiO_2/Mg_2SiO_4/Fe_3O_4).

(vii) When a sea invades the land, it is termed _____ (regression/transgression/onlap/overlap).

(viii) Sedimentary structures include _____ (grain-size/sorting/facies/stromatolite).

(ix) The fossilized bones of the Lower Jurassic (190 m.y.) dinosaur mount in the Indian Statistical Institute have been obtained from _____ (mixed/continental/turbidite/marine) rocks.

(x) A bed in which the grain size decreases upward is called a _____ (lamineted/cross-/trough/graded) bed.

(xi) A horizontal sequence of beds lying over a tilted sequence of beds has a/an _____ (folded/unconformable/sedimentary/metamorphic) contact relationship.

contd..... 2/-

(xii) The most important element that occurs next to O_2 in the earth's crust is _____ (Fe/Mg/Si/Ca).

(xiii) The sand grains in a sea-beach are generally _____ (well/poorly/moderately) sorted.

(xiv) Large mountain-building activity is closely associated with _____ (syncline/anticline/fault/geosyncline).

(xv) The term 'texture' includes _____ (grain-size/structure/cross-bedding/viscosity).

(xvi) Addition of free Oxygen in the primitive atmosphere was by means of _____ (volcanic activity/water vapour/photo-chemical dissociation/photosynthesis).

$((1\frac{1}{2} \times 13) + \frac{1}{2}) = [20]$

2. Describe the role of Si-O tetrahedron in the formation of silicate minerals. Describe the role of various silicate structures in the formation of different kinds of igneous rock from the crystallization of a basaltic magma.

(8+8) = [16]

OR,

What are the essential characteristics of the silicate structure ? How can pyroxene, amphibole, biotite, muscovite and quartz be differentiated on the basis of silicate structure ?

Describe the role of Al in the structure of Plagioclase.

(4+8+4) = [16]

3. Which of these are clastic sedimentary rocks _____ limestone, grainte-gneiss, chalcopyrite, pegmatite, breccia, mica-schist, calcareous sendstone, gabbro ?

Describe the mode of transportation of the weathered material from source to sedimentary basin. Where does the transported material get deposited and what happens after the material has been deposited ?

(4+6+6) = [16]

4. What is a fold and its axis ? Define the dip and the strike of a bed.

contd..... 3/-

Suppose you are visiting an area where others say that a syncline occurs. What are the criteria that you would look for to establish that it is not a syncline but an anticline.

(4+6+6) = [16]

5. A violent earthquake takes place at a remote place. How would you proceed to determine the time and location of the place of origin (epicentre) of the earthquake ?

In what ways earthquakes are useful to the geoscientists ?

(12+4) = [16]

6. What is meant by 'rocks of Proterozoic Age' ? Do you expect to find coal deposits in such rocks ?

Why is the study of the Cretaceous - Tertiary boundary (65 m.y.) to important to the geoscientists ?

'The Archaean rocks contain graphite and thin layers of limestone'- what does this statement signify ?

(6+2+4+4) = [16]

7. Explain how the entire remains of extinct woolly mammoths are still to be found in Siberia and Alaska.

Describe the alteration processes which the hard parts of an organism undergo in the course of fossilization.

Give a good example to show how fossils can be useful.

(4+8+4) = [16]

8. Describe the basic ideas basing on which the theory of radio-activity has been built up. In how many ways the age of a rock may be determined ?

Describe the K-Ar method (without going into mathematical details) employed in the age determination of a marine sedimentary rock.

(5+5+6) = [16]

9. (a) What is meant by 'sorting' of sediments ? How does the fabric of a sedimentary deposit help in understanding the sedimentary processes ?

contd..... 4/-

(b) How do you think the great Himalayan mountain has formed ?

(8+8) = [16]

10. Write short notes (any four):

Conical volcano; Coal and pearl; Shocked quartz; Principle of superposition; Characteristics of a lava; Primary magma; Discontinuous reaction series.

(4 x 4) = [16]

:bcc:

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) II Year : 1991 - 92
SEMESTRAL - I EXAMINATION

Statistical Methods III

Date: 25.11.1991 Maximum Marks: 80 Time: 4 hours

Note: Total marks assigned = 100. Maximum score = 80. Answers must be precise and complete.

1. A national safety council wishes to estimate the proportion of automobile accidents that involve pedestrians. How large a sample of accident records must be examined to be 98% certain that the estimate does not differ from the true proportion by more than 0.04 ? (The council believes that the true proportion is below 0.25). [4]

2. Let X_1 and X_2 be independently distributed as $N(\theta_1, 1)$ and $N(\theta_2, 1)$, respectively. Obtain the maximum likelihood estimates of θ_1 and θ_2, given that $\theta_1 \leq \theta_2$. [5]

3. Consider the linear model

$$Y_i = \alpha + \beta x_i + \varepsilon_i, \quad (i = 1, \ldots, n)$$

Where ε_i's are uncorrelated with zero mean and variance σ^2. Let $\hat{\alpha}$ and $\hat{\beta}$ be the least-squares estimates of α and β, respectively. Show that (assuming the forms of $\hat{\alpha}$ and $\hat{\beta}$)

$$E(\hat{\alpha}) = \alpha, \quad E(\hat{\beta}) = \beta$$

$$\text{var}(\hat{\alpha}) = \sigma^2\left[\frac{1}{n} + \frac{\bar{x}^2}{\Sigma(x_i-\bar{x})^2}\right], \quad \text{var}(\hat{\beta}) = \sigma^2[\Sigma(x_i-\bar{x})^2]^{-1}$$

$$\text{cov}(\alpha, \beta) = \sigma^2\left[-\frac{\bar{x}}{\Sigma(x_i-\bar{x})^2}\right].$$

and

$$E(\hat{Q}) = (n-2)\sigma^2, \quad \text{where } \hat{Q} = \sum_1^n (Y_i - \hat{\alpha} - \hat{\beta} x_i)^2.$$

[15]

p.t.o.

4. Suppose that the ε_i's in Question 3 are i.i.d. $N(0,\sigma^2)$.

(a) Show that the maximum likelihood estimates of α, β and σ^2 are $\hat{\alpha}$, $\hat{\beta}$, and $\hat{Q}/n$, respectively. [4]

(b) Show that the maximum likelihood estimates of α and σ^2, given $\beta = \beta_0$, are given by

$$\hat{\hat{\alpha}} = \bar{y} - \beta_0\bar{x}, \quad \hat{\hat{\sigma}}^2 = \frac{1}{n}[\hat{Q} + (\hat{\beta} - \beta_0)^2 \Sigma(x_i - \bar{x})^2].$$

[5]

(c) Use (a) and (b) to show that the likelihood-ratio test for testing $H: \beta = \beta_0$ against $K: \beta \neq \beta_0$ rejects H if

$$\frac{|\hat{\beta} - \beta_0|}{\left[\frac{1}{\Sigma(x_i - \bar{x})^2} \cdot \frac{\hat{Q}}{n-2}\right]^{1/2}} \geq C.$$

What is the value of C if the test is of size α ?
Justify your answer by theoretical results (without proof).

[8]

5. As part of a study on the rate of combustion of artificial graphite in humid air flow, an experiment was conducted to investigate oxygen diffusivity through water vapour pressure. Sample mixtures of nitrogen and oxygen were prepared with 0.17 mole fraction of water at nine different temperatures, and the oxygen diffusivity was measured for each. The data are given below.

Temperature x (in 100°F)	Oxygen diffusivity y
10	1.69
11	1.99
12	2.31
13	2,65
14	3.01
15	3.39
16	3.79
17	4.21
18	4.61

contd..... 3/-

(a) Do the data provide sufficient evidence to indicate that a rise in temperature for each $100^\circ F$ (in the domain of this study) increases oxygen diffusivity by 0.33 ? [12]

(b) Find a 95% (non-trivial) confidence interval for the mean oxygen diffusivity when the temperature is $1500^\circ F$. (Use a linear model with errors distributed independently as $N(0,\sigma^2)$) [10]

6. Let X denote the time in days to failure of an equipment and assume that X is distributed with the p.d.f.

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0, \quad \theta > 0.$$

Consider the problem of testing the hypothesis H that the mean life $\theta \le \theta_0$ (given) against $K: \theta > \theta_0$. Let $X_1, \ldots, X_n$ denote the times in days to failure of n similar pieces of equipment, selected randomly from a large lot.

(a) Consider the critical region $\bar{X} \ge C$. Obtain C so that the test is of size α. [3]

(b) Give an expression for the power of this test in terms of chi-square distribution. [4]

(c) Use the central limit theorem to show that

$$\Phi[(\theta_0 z_\alpha/\theta) + \sqrt{n}(\theta - \theta_0)/\theta]$$

is an approximation to the power of the test for large n, where Φ is the cdf of $N(0,1)$, and z_α the αth fractile of $N(0,1)$. [5]

(d) The following are days until failure of air monitors in a nuclear plant:

3	150	40	34	32	37	34	2	31
6	5	14	150	27	4	6	27	10
30	37.							

Assume the model described above. Is H rejected at level $\alpha = 0.05$ when $\theta_0 = 25$? [5]

7. Consider the following data which give miles per gallon of a particular make of car before and after application of a newly developed gasoline additive:

Car	1	2	3	4	5	6	7
Before x	17.3	21.3	19.5	19.1	22.0	18.7	20.3
After y	18.3	20.8	20.9	21.2	22.7	18.6	21.9

Use the sign test (randomized, if necessary) of size 0.1 to test the hypothesis that the gasoline additive is not effective. Comment on the validity of the test based on the given information. [8]

8.(a) Someone explains the point of a test of significance as follows: "If the null hypothesis is rejected, the difference is not trivial. It is bigger than what would occur just by chance". Comment briefly. [3]

(b) True or false: The P-value is the probability that the null hypothesis is true. Support your answer. [2]

(c) Suppose one wants to test $H: X \sim N(0,1)$ against $K: X \sim$ Cauchy (0). What kind of critical region seems to make sense ? [2]

(d) Let X be a r.v. with continuous distribution $F(\cdot\,; \theta)$, where θ is unknown (real) parameter, F being known. Let $X_1, \ldots, X_n$ be n independent observations on X. Show that a (non-trivial) confidence interval for θ at a given level can be obtained based on chi-square distribution, when F is stochastically increasing in θ. Use this result when F is the distribution of $U(0, \theta)$. [5]

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SEMESTRAL - I EXAMINATION

Economics I

Date: 22.11.1991 Maximum Marks: 65 Time: $2\frac{1}{2}$ hours

Note: The paper carries 78 marks. You may attempt any part of any question. The maximum you can score is 65. Marks allotted to each question are given in brackets.

1. Consider an industry consisting of a dominant firm and many small firms. The small firms constitute a competitive fringe. Determine, under usual assumptions, the equilibrium output vector of the industry. Show that the consumers are better off with a dominant firm and a competitive fringe than with a monopolist.

(7+5) = [12]

2. A consumer spends his fixed income on the consumption of two goods x and y. The marginal utility of x(y) is independent of the quantity of y(x) consumed. The marginal utility of x is a positive constant but the marginal utility of y falls as consumption rises. What can you say about

(a) the slope of the indifference curve ?

(b) the elasticity of demand for x ?

(4+7) = [11]

3.(a) Clearly explain the central ideas underlying the two fundamental theorems of Welfare Economics. [4]

(b) Suppose that the expected utility function of an individual A is more concave than that of individual B. That is, A's utility function can be expressed as an increasing, concave transform of B's utility function. Then show that A will pay higher insurance premium than B to avoid a fair gamble.

[6]

(c) What is Walras's law ? Prove this law for a two person exchange economy. [6]

contd..... 2/-

(d) Why should a compensated demand curve be always negatively sloped ?

[3]

4. A monopolist produces a homogeneous good from a single plant with the following annual total cost function:

$$TC = \frac{q^3}{3} - 30q^2 + 1000q .$$

He sells his product in two markets between which price discrimination is possible. In the first market the demand function is

$$P_1 = 1100 - 13q_1 .$$

In his profit maximizing equilibrium his total annual output is 48 and the elasticity of the second market demand function at the equilibrium price is -3. How much is his

(a) Price in the first market ?

(b) Price in the second market ?

(c) Total profit ?

(5 x 3) = [15]

5. Prove or disprove the following :

(a) If an individual's utility function U is strictly quasiconcave, then for each positive price vector there is a unique bundle that maximizes U on his budget set.

(b) In a monopsonistic labor market a profit maximiting monopolist would pay a wage rate equal to the marginal revenue product of labor.

(c) In a pure exchange economy if the initial endowments of the two persons are not on the contract curve, then perfect competition will induce them to move to a point on the contract curve.

(4 x 3) = [12]

6. Clearly distinguish between:

(a) Competitive elements and monopoly elements in a monopolistically competitive industry.

(b) The income-consumption curve and the Engel curve.

(c) Normal Profit and Excess Profit.

(3 x 3) = [9]

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SEMESTRAL - I EXAMINATION

Probability Theory and its Applications III

Date: 20.11.1991 Maximum Marks: 120 Time: 4 hours

Note: This paper carries questions worth a total of 140 marks. Answer as much as you can. The maximum you can score is 120 marks.

1. X,Y are independent random variables with a common density $f(x) = \begin{cases} \frac{1}{x^2} & \text{for } x > 1 \\ 0 & \text{otherwise} \end{cases}$. Find the conditional distribution of V given U, where U = X+Y and V = X-Y. [10]

2. $X_1, \ldots, X_n$ are independent random variables with a common continuous distribution function F and $X = \max(X_1, \ldots, X_n)$. Find $E(X_1/X = x)$. [10]

3.(a) Show that if $U_1, \ldots, U_n$ are the order statistics of a sample of size n from an exponential distribution with parameter λ, then $U_1, U_2-U_1, \ldots, U_n-U_{n-1}$ are independent.

(b) Two light bulbs are installed simultaneously. As soon as one of them burns out, it is replaced by a third bulb. Assume that the lifetime of a bulb is exponentially distributed with parameter λ. Using (a), find the distribution of the total time till the last breakdown.

(10+5) = [15]

4.(a) What is an n-dimensional Normal density ?

(b) $\underset{\sim}{X} = (X_1, \ldots, X_n)'$ has a Normal density. Let $1 \leq k < n$ and denote $\underset{\sim}{X}_1 = (X_1, \ldots, X_k)'$ and $\underset{\sim}{X}_2 = (X_{k+1}, \ldots, X_n)'$. Show that for a suitable matrix M, the random vectors $\underset{\sim}{X}_1 + M\underset{\sim}{X}_2$ and $\underset{\sim}{X}_2$ are independent. Deduce that $\underset{\sim}{X}_2$ has a (n-k) - dimensional Normal density and that the conditional distribution of $\underset{\sim}{X}_1$ given $\underset{\sim}{X}_2$ is given by a k-dimensional Normal density.

(2 + (10+8)) = [20]

p.t.o.

5. If (X_1, X_2, X_3) are jointly uniformly distributed on the surface of the unit sphere in $\mathbb{R}^3$, find the distribution of $\sqrt{X_1^2 + X_2^2}$. Hence derive the conditional distribution of $\sqrt{Y_1^2 + Y_2^2}$ given $\sqrt{Y_1^2 + Y_2^2 + Y_3^2}$ where Y_1, Y_2, Y_3 are independent $N(0,1)$ random variables.

(8+4) = [12]

6. $X_0, X_1, X_2, \ldots$ are independent random variables with a common continuous distribution function F. Let $N = \min\{n \geq 1 : X_n > X_0\}$. Find the probability $P(N = n, X_N \leq x)$. Assuming that the common distribution F is U(0,1), show that X_N has density $f(x) = -\log(1-x)$ for $0 < x < 1$.

(10+10) = [20]

7.(a) Define convergence in probability.

(b) Show that if X_n, $n \geq 1$ and X are random variables such that $E|X_n - X| \to 0$, then $\{X_n\}$ converges in probability to X.

(c) Show that if a sequence $\{X_n\}$ converges in probability, then for every $\epsilon > 0$ there is a constant K such that $P(|X_n| > K) < \epsilon$ for all n.

(3+5+10) = [18]

8.(a) X_n is a sequence of random variables with $P(X_n = 2^n) = P(X_n = -2^n) = \frac{1}{2n^2}$ and $P(X_n = 0) = 1 - \frac{1}{n^2}$. Show that $\sum_{n=1}^{\infty} X_n$ converges with probability 1.

(b) $X_1, X_2, \ldots$ are i.i.d. U(0,1) random variables. Using an appropriate law of large numbers (with statement), show that $\left(\prod_{i=1}^{n} X_i\right)^{\frac{1}{n}}$ converges to e^{-1} almost surely.

(8+8) = [16]

9.(a) Find the characteristic function of the random variable X having density $f(x) = \frac{1}{2} e^{-|x|}$ for $-\infty < x < \infty$.

Hence derive the characteristic function of the standard Cauchy distribution.

contd..... 3/4

(b) X_n, $n \geq 1$ is a sequence of random variables converging in distribution to a non-degenerate random variable X. Show that if X_n is distributed as $N(0, \sigma_n^2)$, then σ_n converges to a limit $\sigma > 0$ and that X is distributed as $N(0, \sigma^2)$.

(8+3+8) = [19]

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SEMESTRAL - I EXAMINATION

Calculus III

Date: 18.11.1991 Maximum Marks: 70 Time: 3 hours

Note: The paper carries 90 marks. Answer as many questions as you can, but the maximum marks you can score is 70.

1. Obtain the Fourier series corresponding to the function

$$f(x) = x - \pi, \quad -\pi < x < 0$$
$$= \pi - x, \quad 0 < x < \pi.$$

Show that the series converges to $f(x)$ in $(-\pi,0) \cup (0,\pi)$. What is the value of the series at $x = \pm \pi$, and at $x = 0$? Also show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}.$$

[10]

2. Show that

$$\int_0^{\frac{\pi}{2}} \log(1 - a^2 \sin^2 \theta)\, d\theta = \pi \log \frac{1 + \sqrt{1-a^2}}{2}, \text{ where } a^2 < 1.$$

Hence find the value of $\int_0^{\frac{\pi}{2}} \log \sin \theta \, d\theta$.

[10]

3. Find the minimum volume bounded by the planes $x = 0$, $y = 0$, $z = 0$ and a plane which is tangent to the ellipsoid

$$\frac{x^2}{2} + \frac{y^2}{3} + \frac{z^2}{4} = 1$$

at a point in the octant $x > 0$, $y > 0$, $z > 0$.

[10]

Find the amount of work done by the force field

$$F(x,y) = (x - y)\vec{i} + (x + y)\vec{j}$$

in moving a particle once round the astroid

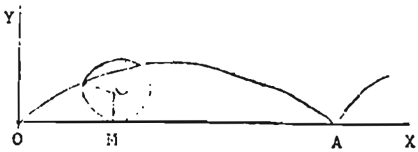
$$x^{2/3} + y^{2/3} = a^{2/3}$$

in the counterclockwise direction.

[10]

p.t.o.

5. A cycloid is a curve traced out by a point P on the circumference of a circle which rolls (without sliding) on a straight line OX called the base of the cycloid.



$x = a(\theta - \sin\theta)$, $y = a(1 - \text{cor}\ \theta)$.

a = radius of the generating circle, c is its centre, $P = (x,y)$.

$\theta = 0$ at O and $\theta = 2\pi$ at A.

Find the volume of the solid generated by revolving one arch of the cycloid about its base.

[10]

6. Let C be the curve of intersection of the hemisphere $x^2 + y^2 + z^2 = 8x$, $z > 0$, and the cylinder $x^2 + y^2 = 4x$

Use Stokes' theorem to show that

$$\int_C (y^2 + z^2)dx + (z^2 + x^2)dy + (x^2 + y^2)dz = 32\pi .$$

Explain how to traverse C to arrive at the given answer.

[10]

7. Compute the area of that portion of the conical surface $x^2 + y^2 = z^2$ which lies between the two planes $z = 0$ and $x + 2z = 3$.

[10]

8. Find the potential of the homogeneous solid bounded by the ellipsoid

$$\frac{x^2 + y^2}{a^2} + \frac{z^2}{b^2} = 1 \quad (a < b)$$

at its centre.

[10]

p.t.o.

9. Evaluate the integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dx\, dy}{(x^2 + y^2 - 2x + 4y + 6)^{3/2}}$$

by representing it as the flat solid angle which the plane $z = 0$ subtends at the point $(1, -2, 1)$.

[10]

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