B II Statistical Methods

Class Test 1 Dated : Feb. 5, 1993

Time : $1\frac{1}{2}$ hrs Full Marks : ~~20~~ ~~100~~ 50

Attempt all questions.

In a 2×2 contingency table, develop an exact test for independence against positive dependence of the attributes. Carry out the test when $n(AB) = 3$, $n(A) = 7$, $n(B) = 5$ and $n = 20$.

[10+10] = [20]

2. In a 2×2 contingency table, suggest a large sample test for independence and justify the nature of its large sample distribution under the null hypothesis of independence.

Carry out the test when $n(AB) = 30$, $n(A) = 70$, $n(B) = 50$, $n = 200$

[15+5] = [20]

3. In a $p \times q$ contingency table with the parameters π_{ij}, $\pi_{i\cdot}$, $\pi_{\cdot j}$ etc in usual notation, suggest tests for

(a) $H_0 : \pi_{i\cdot} = \pi_{j\cdot}$ for a specified pair (i, j)

(b) $H_0 : \pi_{ij} = \pi_{i\cdot}\pi_{\cdot j}$ for a specified pair (i, j)

(c) $H_0 : \pi_{i\cdot} = \pi_{\cdot i}$ for a specified i (when $p = q$)

No derivation is needed.

[3+4+3=10]

B K Sinha
5/2/'93

BStat II Year (1992-93)

Algebraic Structures - Class test

dt 16.2.93

1) If f is a homomorphism of a group G into itself, show that for any element a of G order of $f(a)$ divides the order of a (here order of a is finite) [10]

b) If f is an automorphism of a cyclic group G generated by an element x, show that $f(x)$ also generates G. [10]

a) If G is the additive group of integers, find all its automorphisms [10]

b) If N, M are normal subgroups of a group G, show that
$$NM/M \cong N/N\cap M.$$ [10]

If H is a subgroup of a group G such that the product of any two left cosets of H is also a left coset of H in G show that H is normal in G [10]

If G is an abelian group of order n and m is a +ve integer, relatively prime to n, show that every element g of G can be written as $g = x^m$ for some x in G [10]

For a positive integer n, show that the set of all integers less than n and relatively prime to n forms a group under multiplication modulo n. [15]

G is the group of nonsingular 2×2 matrices of real numbers under matrix multiplication, N is the set of 2×2 matrices with determinant 1. Show that $N \triangleleft G$ and that G/N is isomorphic to the ~~set~~ group of non-zero reals under multiplication [15]

H, K are finite subgroups of a group G and the relation $\sim$ on G is defined by $a\sim b$ if $b = hak$ for $h\in H$, $k\in K$. Show that $\sim$ is an equivalence relation on G. Find the size of the equivalence class containing a in G. [10]

Indian Statistical Institute
B. Stat. II 1992-93

IInd Class Test on Economic Theory : Macroeconomics

Maximum Marks : 20 | Date 3.3.93 | Duration 1 Hour

1. (a) State and derive the result contained in the so-called 'balanced budget multiplier theorem' in the context of a simple economy.

   (b) Consider now an economy in which there are two individuals called the 'poor' and the 'rich' with the following features :

   θ, the proportion of GDP accruing as income to the poor, is less than 1/2 and mpc of the poor (c_p) is higher than that of the rich (c_r).

   The government now adopts the following policy : it raises an additional tax (say of the amount ΔT) from the rich and pays this amount as additional current transfers (ΔTR) to the poor (i.e., $\Delta T = \Delta TR$).

   (i) Will the above policy suffice to raise GDP whatever be the value of θ ?

   (ii) Are there some values of θ for which the above policy will raise GDP by more than the amount of ΔT ?

   (iii) What is the distribution of θ for which the result mentioned in (a) will hold good ?

   [5 + 5] = [10]

2. (a) Describe the economic process underlying the multiplier theory.

   (b) Mention a few cases in which the multiplier theory will not work.

   [5 + 5] = [10]

BSTAT II YEAR 1992-93.

Elements of Algebraic Structures.

Class test III

16-4-93

a) R is a commutative ring. An ideal P of R is said to be a prime ideal in R if $a \cdot b \in P$ implies $a \in P$ or $b \in P$. Show that P is a prime ideal in R iff R/P is a ring with no zero divisors. (15)

b) F is a field and $g(x) \in F[x]$ has degree m. Show that $F[x]/(g(x))$ as a vector space over F has dimension m. (15)

a) Let $\mathfrak{M}$ be the set of all $n \times n$ matrices over a field F such that for $A = (a_{ij}) \in \mathfrak{M}$, $a_{ij} = 0$ for $j < i$.
Let N be the subset of $\mathfrak{M}$ such that for $B = (b_{ij}) \in N$, $b_{ij} = 0$ for $j \leq i$. Show that $\mathfrak{M}$ is a ring and N is an ideal in the ring. Characterise the quotient $\mathfrak{M}/N$. (15)

) F, K are fields, $F \subseteq K$; $a, b \in K$. Let I be the set of all polynomials over F, having a and b as roots. Show that I consists of multiples of a fixed polynomial in I. (15)

a) Show that every maximal ideal in a commutative ring with unity, is a prime ideal. (10)

b) If F is a field of characteristic $p \neq 0$, show that for $f(x) \in F[x]$ with $f'(x) = 0$, $f(x)$ has the form $g(x^p)$ for some $g(x) \in F[x]$. (15)

c) Let $f(x) = x^4 + x^2 + 1 \in Q[x]$. Show that there is a field K containing Q, and containing roots of $f(x)$ such that dimension of K over Q is 2. (15)

Indian Statistical Institute

B.Stat. II year 1992-93

Class Test III on Macroeconomics

Home Assignment : Full marks : 20

(Note : The assignment is to be submitted by 30 April, 1993)

A two-sector economy produces two goods : Sector 1 produces good G_1 and Sector 2, good G_2. It has also two classes of income-earners — workers and capitalists. Money wage rate is the same in both sectors. Let the following notations be used :

W = money wage rate

A = nominal value of autonomous expenditure on G_2

$$\left.\begin{array}{l} Y_i = \text{output of } G_i \\ P_i = \text{price of } G_i \\ L_i = \text{quantity of labour employed in sector } i \end{array}\right\} \text{for } i = 1,2$$

$p = \frac{P_1}{P_2}$, i.e., the relative price of G_1 vis-a-vis G_2

Other features of the economy are given below :

(a) Workers in each sector spend their entire money income on the consumption of G_1 (which is a mass consumption good). Capitalists in each sector consume only G_2 and spend c proportion of their money income on G_2.

(b) All expenditure on G_1 is consumption expenditure. Expenditure on G_2 consists of consumption expenditure (by capitalists) and autonomous expenditure, A (which equals the sum of net export, investment expenditure and government consumption expenditure)

Give also the optimal solution for the dual of the LP problem.

$(10+2) = [12]$

(c) (i) p_2 is fixed by a constant mark-up over unit wage cost :

$$p_2 = \frac{1}{\theta} \, w \frac{L_2}{Y_2} \qquad (0 < \theta < 1)$$

(ii) Per unit requirement of labour in <u>each</u> sector is a constant (say, α_i for sector i) :

$$\frac{L_i}{Y_i} = \alpha_i , \qquad (i = 1, 2)$$

(d) Money wage rate is determined exogenously and hence P_2 is also determined by exogenous factors, α_2 and θ ,(in view of (c) above).

<u>Answer</u> now the following questions for this economy :

1. Show that the real wage rate (measured in terms of P_2) is a constant (say $\bar{w}$),

2. Find the money income of capitalists in each sector and write down the condition for equilibrium for <u>each good</u> , i.e., the relation showing the equality between the nominal value of demand and the nominal value of supply.

3. By using the result: obtained in (2) show that the equilibrium value of Y_2 is a <u>multiple</u> of $\frac{A}{p_2}$. What is the value of the multiplier ? Can you give an intuitive explanation for this result ?

4. By using the earlier results and assumptions of the model show that given A/p_2 , the equilibrium condition for C_1 yields a rectangular hyperbola in p and Y_1 with two asymptotes : $p = \bar{w}\alpha_1$ and $Y_1 = 0$. Draw this

curve on the $(p - Y_1)$ plane. Will this curve shift upward, if θ increases, but A remains unchanged ?

5. Let the (relative) rate of growth of variable x be denoted by $\hat{x}$:

$$\hat{x} = \frac{1}{x} \frac{dx}{dt}$$

Find the rates of growth of Y_2 and p when (A/p_2 grows at an exogenous rate ($\widehat{A/p_2}$). Will $\hat{p}$ be positive necessarily ? (You may approximate the rate of growth of ($p - \bar{w}\alpha_1$) by that of p).

6. Suppose one wants to use the above model to explain the rates of growth of Y_2 and p in the Indian economy during the period 1970-89 (assuming rate of growth of Y_1 to be determined exogenously). Sector I may be represented by <u>foodgrains</u> sector while Y_2 may be represented by gross domestic product at factor cost (GDP) at constant prices originating in the non-agricultural sector. Further, p_2 may be approximated by the wholesale price index of manufactures while a proxy variable for A may be given by the sum total of total fixed investment expenditures and govt. consumption expenditures.

The table below gives data on A, p_1, p_2, Y_1 and Y_2 for the Indian economy for the period 1970-89. Using these data <u>compute</u> annual rates of growth of each variable (i.e., $\hat{x} \equiv (x - x_{-1})/x_{-1}$) over this period. Further, plot observations on each of the following pairs in seperate diagrams :

give also the optimal solution for the dual of the LP problem.

(10+2) = [12]

(i) $\hat{Y}_2$ against $(\widehat{A/P_2})$;

(ii) $\hat{P}$ against $\hat{Y}_1$;

(iii) $\hat{P}$ against $\hat{Z}$; [$\hat{Z} = (\widehat{A/P_2}) - \hat{Y}_1$]

Draw a free-hand best fitting curve for each case. Do the curves corresponding to (i) and (iii) support the two-sector model developed earlier ?

(The table is given in the next page)

Table : Data on Selected Variables of the Indian Economy during 1970-71 to 1979-80.

financial year	Net production of foodgrains (million tonnes) Y_1	GDP at 1980-81 prices (Rs. crs.) originating in non-agricultural sector Y_2	Wholesale price index numbers (base 1981-82 = 100)		Autonomous Expenditure at current prices (Rs crores) A
			foodgrains P_1	manufactures P_2	
1970-71	87.1	49041	42.1	37.0	10106
1971-72	94.9	50678	43.6	40.4	11532
1972-73	92.0	52296	50.3	45.0	12811
1973-74	84.9	53724	59.8	51.6	14129
1974-75	91.6	55378	82.5	62.4	17073
1975-76	87.4	58785	73.3	63.3	20599
1976-77	105.9	62624	64.3	64.7	23509
1977-78	97.3	66290	71.8	66.2	25886
1978-79	110.6	71465	72.7	66.3	28500
1979-80	115.4	71231	78.1	79.7	32332
1980-81	96.0	73891	91.3	95.1	39360
1981-82	113.4	78342	100.0	100.0	46810
1982-83	116.6	82725	109.2	103.5	54041
1983-84	113.3	88334	119.4	109.8	61132
1984-85	133.3	93886	117.1	117.5	69920
1985-86	127.4	99725	124.5	124.4	83429
1986-87	131.6	106398	129.4	129.2	96677
1987-88	125.5	113414	141.3	138.5	113037
1988-89	122.8	122597	161.8	151.6	130433
1989-90	148.7	135389	165.0	168.6	150727

Give also the optimal solution for the dual of the LP problem.

(10+2) = [12]

INDIAN STATISTICAL INSTITUTE

M Stat II Year (SQC - OR) : 1992-93

Class Test I

Optimisation Techniques II

Date 06-08-1992 Maximum Marks: 30 Time: 1 hr.

Answer all the questions.

1. Show that in the simplex method, if a variable x_j leaves the basis, it cannot enter the basis in the next iteration.

[6]

2. Prove: If the slack variable of the ith primal constraint is positive in an optimal solution, there exists an optimal dual ~~solution~~ solution with the ith dual variable equal to zero and conversely; and if the surplus variable of the jth dual constraint is positive in an optimal solution, there exists an optimal solution (primal) with the jth primal variable equal to zero and conversely.

[12]

3. Solve the following LP problem by the dual simplex algorithm:

$$\min \quad x_0 = 2x_1 + 3x_2 + 4x_3$$

$$x_1 + 2x_2 + x_3 \geq 3$$

$$2x_1 - x_2 + 3x_3 \geq 4$$

$$x_1, x_2, x_3 \geq 0$$

Give also the optimal solution for the dual of the LP problem.

(10+2) = [12]

INDIAN STATISTICAL INSTITUTE

B-II (1992-93) Class Test-1

Probability-3

Time 90 mts. Max. marks 50.

1. $X_0, X_1, X_2, \cdots$ are *iid* random variables, where X_0 follows exponential distribution with mean $\frac{1}{\alpha}$, $\alpha > 0$. Let N be an integer-valued random variable defined as follows:

$N = k$ if, and only if $X_1 > X_2, X_2 > X_3, \cdots, X_{k-1} > X_0$, and $X_k < X_0$ for $k = 1,2,3,\cdots$

(a) Find $P(N=k)$ $k=1,2,3,\ldots$

(b) Find $P\left[N=k, X_N \le x\right]$ for $x > 0$. [15]

2. Suppose that (X_1,X_2,X_3) has a joint density given by

$$f(x_1,x_2,x_3) = C\, exp\left[-\tfrac{1}{2}x'Qx \right] \text{ where } x' = (x_1,x_2,x_3)$$

and

$$Q = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

(a) Find C.

(b) Find the conditional density of $X_2 | X_1 = a, X_3 = b$. [15]

3. (a) Write down the Dirichlet $D_k(r_1, r_2, \cdots, r_k; r_{k+1})$ - density.

(b) Let $U_1 < U_2 < U_3$ be the order statistics on the basis of 3 independent observations from $U(0, 1)$ - distribution. Show that (U_1, U_2-U_1, U_3-U_2) follows $D_3(1, 1, 1; 1)$. Find $\text{cov}(U_2, U_3)$. [20]

4. X, Y are *iid* $U(0,1)$ random variables. Let $U = \min(X, Y)$. Find the conditional distribution of $X \mid U = a$, $0 < a < 1$. [10]

B-stat II

Class Test I. Economics

Time. 14-15 hrs to 16-00 hrs

Maximum marks – ~~80~~ 55.

20/8/92

Answer question no. 1 and any two from the rest.

. Prove or disprove

(a) When price elasticity of demand is elastic, then an increase in the price leads to an increase in the amount spent on purchase of the good.

~~b) Perfect competition in the output market in the long run is incompatible with increasing returns to scale and competitive input markets.~~

(b) A production function $f(K,L)$ is homothetic iff it is linear homogeneous.

(c) The elasticity of substitution of the production function

$$f(K,L) = A\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha) L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$ is given by σ.

10+15+10= [35]

State clearly the axioms of consumer's choice. Briefly discuss the relevance of these axioms in the context of the consumer's utility maximisation problem.

[25]

Derive a firm's short run average variable cost, marginal cost and average total cost curves. What are their relationships with each other? Draw a diagram to show these curves.

[25]

P.T.O.

4. Define 'Giffen goods' and 'inferior goods'. What is the relationship between these two types of goods?

Suppose there are two goods x and y. draw diagrams to show the income and substitution effects when

(i) x and y are normal

(ii) x is Giffen, y is normal

(iii) x is inferior, y is normal.

Why is the compensated demand curve always negative sloped?

[25]