INDIAN STATISTICAL INSTITUTE
Research and Training School
B.Stat. Part III: 1966-67

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PERIODICAL EXAMINATION

Mathematics: 3

Date: 26.9.66. Maximum marks: 100 Time: 3 hours

Note: Each question carried 25 marks. The whole paper carries 125 marks. You may attempt any part of any question.

1.(a) What is an ordered field? Prove that the following relations hold in an ordered field (with the usual notation):

i) $x \leq y \iff -x \geq -y$

ii) $x \leq y,\ z \geq \theta \implies xz \leq yz$

iii) $x \leq \theta,\ y \geq \theta \implies xy \leq \theta$

iv) $x \leq \theta,\ y \leq \theta \implies xy \geq \theta$

v) $e > \theta$

(b) Prove that no ordered field can be finite.

(c) When is an ordered field said to be complete ? If x is a strictly positive number in a complete ordered field, show that there exists a non-negative integer n such that $ne \leq x \leq (n+1)e$. (25)

2.(a) Give a definition of the real number system and identify its rational subfield.

(b) What is the Cantor-Dedekind axiom ?

(c) Show that every open interval on the real line contains a rational number.

3.(a) What is a Metric space? Define open subsets of a Metric space and prove that the class of open sets is closed under finite intersections and arbitrary unions.

(b) Defining a closed subset of a Metric space to be a set which contains all its accumulation points, show that a set is closed if and only if its complement is open.

(c) What is the closure of a set? Show that the closure of any set is closed.

4.(a) When is a subset of a Metric space said to be compact?

(b) Show that every compact set is closed.

(c) Show that a closed and bounded set on the real line is compact.

5.(a) What is a sequence?

(b) When is a sequence of points of a Metric space convergent? Show that a sequence can have at most one limit.

(c) Let A be a subset of a Metric space and x an accumulation point of A. Show that there exists a sequence of points of A which converges to x.

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PERIODICAL EXAMINATION

Statistics 3: Probability

Date:3.10.66. Maximum marks: 100 Time $2\frac{1}{2}$ hours

Note: The paper carries 110 marks. You may attempt any part of any question.

1. Describe very briefly what is meant by a 'discrete probability distribution' and a 'continuous probability distribution' taking one example of each type. Mention as many important differences as you can between these two types of probability distributions. [6+6=12]

2. A gang of enemy infiltrators was known to have been air-dropped in an area which is a square of side 5 miles. Suppose the defending country's army is stationed at one particular corner of this square; what is the probability that the infiltrators are encountered within 6 miles distance from this corner, when a search is conducted?

(Mention if any plausible assumptions are made.) [10]

3. When do you say that a continuous random variable has a 'uniform' or 'rectangular' distribution in the range (a, b)? What are E(X) and Var (X) if X $\frown$ R(a, b)? [4+3+5=12]

4. Show that all the cumulants of $N(\mu, \sigma^2)$ beyond the second are zero. [12]

5.(a) What is meant by saying that a random variable (r.v.) X is distributed as $\Gamma(n, \theta)$. Find the mean and variance of such a r.v. X.

(b) If a r.v. X is distributed as a Beta-distribution of type - I with parameters 1 and 1 (i.e., X $\frown$ $\beta_1(1,1)$), then show that the distribution of $Y = -\log_e X$ is $\Gamma(1, 1)$. [4+3+5 +(6)=18]

6. Given that X $\frown$ $N(\mu, \sigma^2)$, find the distribution of the transformed variate Y, where

(i) $Y = 2X+3$ (ii) $Y = X^2$ (iii) $Y = e^{X^2}$. [4+7+7=18]

7.(a) Show that there exists a suitable transformation, which transforms any absolutely continuous distribution into a rectangular distribution on (0,1).

(b) If X is a non-negative r.v. with median 'm' what is the median of the r.v. $Y = X^2$? [5+5=10]

8. Check if the following function of two variables is eligible to be a joint probability density function for a pair of r.v.'s X and Y:

$$f(x, y) = c.\ \exp\left[-\frac{1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right] \qquad |\rho| < 1.$$

Then what should be the constant c? What are the marginal density functions? [3+7+8= 18]

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PERIODICAL EXAMINATION

Statistics-3: Statistics Practical

Date: 7.10.66. Maximum marks: 50 Time: 3 hours.

1. Three important measurements from which the cranial capacity (C) may be predicted are the glabella-occipital length (L), the maximum parietal breadth (B) and the basio bregmatic height (H). The prediction formula suggested is

$$C = \alpha' L^{\beta_1} B^{\beta_2} H^{\beta_3} .$$

This can be written as

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

where $y = \log_{10} C$, $x_1 = \log_{10} L$, $x_2 = \log_{10} B$, $x_3 = \log_{10} H$.

The mean values and the corrected sum of squares and products of these characteristics computed on the basis of measurements on the 86 male skulls from the Farringdon Street series are given below.

Characters	Mean	Corrected sums of squares and products			
		y	x_1	x_2	x_3
y	3.1685	.12692	.03030	.04410	.03629
x_1	2.2752		.01875	.00848	.00694
x_2	2.1523			.02904	.00878
x_3	2.1128				.02886

i) Find the least-square values of α, β_1, β_2, β_3. [10]

ii) Find the multiple correlation coefficient between y and (x_1, x_2, x_3). [6]

2. The following data give the velocity (V) of the [illegible] Missisipi River in feet per second corresponding to various depths expressed in terms of the ratio D of the measured depth to the depth of the river.

D	0	.1	.2	.3	.4
V	3.195	3.230	3.253	3.261	3.252

D	.5	.6	.7	.8
V	3.228	3.181	3.127	3.059

Fit a suitable polynomial curve to the data by means of 'Orthogonal polynomial fitting method'. [15]

GO ON TO THE NEXT PAGE

3. Table 1 presents the summary of data for complete census of all the 340 villages in Ghaziabad subdivision. The villages were stratified by size of their agricultural area into 4 strata. The population values of the strata means and variances for the area (X) under wheat are given.

Table 1

Stratum number i	Size of village in Bighas	N_i	μ_i	$\frac{N_i}{N_i-1}\sigma_i^2$
1	0 - 500	63	112.1	3170
2	501 - 1500	199	276.7	13550
3	1501 - 2500	53	558.1	34600
4	> 2500	25	960.1	130540

Let μ be the population mean.

i) Find the variance σ^2 of X in the whole population. [5]

ii) Find the variance of $\hat{\mu}_{st}$ for

(a) Proportional allocation, [5]
and (b) Neyman allocation, [5]

based on a sample of size $n = 34$ drawn by stratified random sampling without replacement.

iii) Compare the variances obtained in (a) and (b) with the corresponding variance of $\hat{\mu}$ based on a sample of size 34 drawn by SRS without replacement. [2]

For neatness. [2]

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PERIODICAL EXAMINATION

Statistics-3: Statistics Theory

Date: 10.10.66. Maximum marks: 100 Time: $2\frac{1}{2}$ hours.

1. Let $X_1, X_2, \ldots, X_p$ be p random variables.

(a) Define the multiple correlation coefficient between X_1 and $(X_2, \ldots, X_p)$ and show that it is always non-negative provided it exists. [3]

(b) Assuming that the distribution of $(X_1, \ldots, X_p)$ is non-singular, find a necessary and sufficient condition for the above multiple correlation coefficient to be equal to zero. [3]

(c) Show that the multiple correlation coefficient between X_1 and $(X_2, \ldots, X_p)$ is not less than the multiple correlation coefficient between X_1 and $(X_2, \ldots, X_{p-1})$. [5]

(d) Prove:

$$1 - \rho^2_{1(23\ldots p)}$$

$$=(1-\rho^2_{12})(1-\rho^2_{13.2})(1-\rho^2_{14.23})\ldots(1-\rho^2_{1p.23\ldots(p-1)}).$$

[5]

2.(a) Describe the simple random sampling schemes with and without replacement. [2]

(b) A sample of size n is drawn from a population of N units by simple random sampling without replacement. Two real valued characteristics x and y are observed for each sampled unit. Find an expression for the expectation of the sample covariance between x and y. [5]

(c) Why the simple random sampling scheme without replacement is considered to be better than the simple random sampling with replacement? [7]

3.(a) Describe the stratified random sampling scheme and the principle of Neyman allocation. [6]

(b) Let μ be the mean of a real-valued characteristic x in a population of N units. The population is divided into k strata. For each of the following schemes a sample of size n is drawn with replacement:

i) simple random sampling
ii) stratified random sampling with proportional allocation,
iii) stratified random sampling with Neyman allocation.

State the standard estimators of μ in these three cases and obtain their respective variances V_1, V_2, V_3. Show that $V_1 \geq V_2 \geq V_3$. When are these equalities attained? [14]

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PERIODICAL EXAMINATION

Economics-3

Date: 17.10.66. Maximum marks: 100 Time: 3 hours.

Answer Groups A, B and C in separate booklets.

Group A

Answer any one question. Max.marks:25

1. Review your understanding of Adam Smith's model of economic development.

2. Examine the Ricardian theory of income distribution in a growing economy.

Group B Max.marks:30

(10 marks will be reserved for the class work on credit and financial system in India, and 20 marks for answering any one question in this group).

Answer any one question.

3. Describe the basic characteristics of low-income countries. In this context explain the nature of vicious circle of poverty from both supply and demand sides of capital.

4. Give an exposition of the concept of the low-level equilibrium trap and bring out the broad reasonings behind the theory of the 'big push'.

Group C Max. marks:45

Answer any two questions.

5. 'The pattern of India's foreign trade, on the eve of the First 5-year Plan, reflected the under-developed state of Indian economy'. Illustrate.

6. ~~Received~~ Review the nature of changes in India's foreign trade during the period of the three 5-year Plans (1951-2 — 1965-6).

7. Examine the balance of payment situation of India and bring out the principal factors contributing to the expanding gap between receipts and payments.

8. Out-line the chief measure you would suggest for reducing the margin of India's foreign trade deficit, without adversely affecting the rate of growth of the national economy.

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PERIODICAL EXAMINATION

Statistics-3: Data Processing

Date: ~~19.10~~ 7.11.66. Maximum marks: ~~100~~ 50 Time: 2 hours.

Note: Blank Panel Diagrams are supplied with the answerscript.

1.(a) Write alphabetic codes for the word 'Wage'.

(b) What are the advantages of a fully automatic punching machine over a manual one?

(c) Why one corner of a Hollerith card is cut? [2 + 5 + 4=11]

2. There are numeric as well as 'x' punches on columns 40-41 of a deck of cards of which a copy is to be made according to the following design:

Punches of columns 5 to 39 are to be transferred on columns 3 to 37 of the new deck. Design index number 72 to be punched on columns 1 - 2 of the copy deck. The numeric punches of columns 40 of the mother deck are to be transferred on column 38 of the copy deck. The 'x' punches of column 40 and numeric punches of column 41 are to be transferred on column 39 of the copy deck. x-punches of column 41 of the original deck, should be ignored.

Show the reproducer panel wiring diagram required for this job. [11]

3. How will you set up a X-distributor from column 32 by digit 5. Use the Blank Panel diagram to show the connections required for the job. [8]

4. Draw the Complete Control Panel diagram for the following job in respect of 416 accounting machine.

Card-design:	Card Cols.
cdi	1 - 4
roll number	5 - 8
category code (staff - 1, helper - 2)	9
monthly pay rate (in Rs.)	10 - 13
name (alpha)	14 - 40
department code	41
section code	42

Tabulation required with totals by section (minor), department (inter) and Final in the form given below.

department INTER	section MINOR	number of worker	amount payable
X	X		

[20]

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PERIODICAL EXAMINATION

General Science-5: Sociology- I

Date: 21.11.66. Maximum marks: 100 Time: 3 hours

Answer Question 7 and any four from the rest.
All questions carry equal marks.

1. Define Sociology, Social Group and Social event. 'Sociology is the science of society' - Discuss.

2. What is Institution? Distinguish between customary and contractual institutions. Compare briefly their roles in primitive and modern societies.

3. What are Monogamy and Polygamy? Enumerate different types of preferential marriage found in Indian societies. What are the chief functions of marriage?

4. Define family. How does it differ from a household? What are the chief interests of society served by the social and cultural functions of family?

5. Write short notes on any four of the following:

   Ego, Exogamy, Corporeal property, Dual organisation, Couvade, Parallel cousin.

6. Illustrate the application of statistical methods in the study of social structure.

7.(a) What do you mean by the term 'social norm'?

(b) 'Extended family is the norm of our society'. Discuss this assertion with reference to the following table:

Area	Family type		Total
	Extended family	Non-extended family	
(1)	(2)	(3)	(4)
Rural	686	1956	2642
Urban	357	1483	1840
Total	1043	3439	4482

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PERIODICAL EXAMINATION

General Science-4: Biochemistry Theory

Date:28.11.66. Maximum marks:100 Time:2 hours

1. Describe how glucose is metabolised anaerobically in mammalian system. [20]

2. Describe the properties of different hormones present in pituitary gland. [20]

3.(a) Name the different vitamins present in the group -B complex. [8]

(b) Describe the source and deficiency symptoms of vitamin A. [12]

4. Give examples of each of the following Polysaccharide, carbohydrate, ketoacid, amino acid with - SH group, triglyceride. [10]

5. What is an enzyme? Describe the properties of

1) Amylase
2) Pepsin [15]

6. How can you estimate

1) glucose in blood ?
2) Vitamin C in lemon ? [15]

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PERIODICAL EXAMINATION

General Science-4: Biochemistry Practical

Date: 5.12.66. Maximum marks: 100 Time: 3 hours

Estimate the total amount of glucose in the given sample.

Dilute the unknown glucose solution with distilled water and make up the volume to 100 ml.

Take 5 ml. of Fehling's solution number (1) and 5 ml. of Fehling's solution number (2) in a conical flask and titrate with the unknown glucose solution using methylene blue as an indicator.

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PERIODICAL EXAMINATION

Sociology - II

Date: 12.12.66. Maximum marks: 100 Time: 3 hours

Answer Question 7 and any four from the rest.
All questions carry equal marks

1. EITHER

Define the relations between social change, evolution and progress. Give a general account with illustrations of the various types of social change.

OR

What is social structure? How does it differ from social stratification? How can you ascertain the social structure of a given society?

2. Distinguish between caste and class. Is caste system weakening in India? Give reasons for your answer.

3. EITHER

Define Nationality. Enumerate the essential elements which constitute a group of people into nationality.

OR

Define Community and Association. How do you distinguish between rural and urban community? What factors are responsible for the formation of a community?

4. What are Consanguineal and Affinal relationships? Compare the role of kinship in simple and modern societies.

5. What is Religion? Describe the nature of religion comparing it with magic and animism.

6. Write short notes indicating the social significance attached to the following customs:

(a) Lobola (b) Kula (c) Potlatch

7. Critically examine the need of application of statistical methods in sociological analyses.

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MID-YEAR EXAMINATION

Mathematics-3: Analysis

Date: 21.12.66 Maximum marks: 100 Time: 3 hours

Note: The whole paper carries 120 marks. You may attempt any part of any question.

1.(a) Define the norm of a vector in R^p. [2]

(b) If $||x||$ denotes the norm of x in R^p prove that $d(x,y) = ||x-y||$ is a metric on R^p. (You may assume the Cauchy-Schwartz Inequality.) [6]

(c) What is meant by the interior of a set in a Metric Space? Show that the interior of a set A is the largest open set contained in A. [6]

(d) What is meant by the closure of a set in a Metric Space? Show that the closure of a set A is the smallest closed set containing A. [6]

2.(a) When is a sequence of points in R^p said to be a Cauchy sequence? [2]

(b) Show that a Cauchy sequence in R^p is bounded. Can you conclude that every Cauchy sequence in R^p has at least one limit point? Give reasons. [6]

(c) Show that if a Cauchy sequence has a limit point then it has a limit. [6]

(d) Show that a sequence in R^p is convergent if and only if it is Cauchy. [6]

3.(a) When is a function said to be continuous at a point? (Give your definition in terms of neighbourhoods.) [2]

(b) Prove that a function f is continuous at a point a of its domain if and only if for every sequence $\{x_n\}$ of points in its domain converging to a, $\{f(x_n)\}$ converges to f(a). Hence conclude that any function defined on the set of integers on the real line is necessarily continuous. [10]

(c) A function f on R to R is additive if it satisfies $f(x+y) = f(x) + f(y)$ for all x, y in R. Show that if f is a continuous additive function then it is of the form $f(x) = cx$ for all x in R, where c is a constant. [8]

4.(a) Let f be defined on all of R^p taking value in R^q. Show that the following conditions are equivalent:

i) f is continuous on R^p.

ii) If G is any open set in R^q, then $f^{-1}(G)$ is open in R^p.

iii) If F is any closed set in R^q then, $f^{-1}(F)$ is closed in R^p. [12]

(b) Show that every linear functional on R^p is continuous. [4]

Please Turn Over

4.(c) If f is a real valued function defined and continuous on R^p and if $f(a) > 0$, then prove that f is positive on some neighbourhood of the point a. [4]

5.(a) When is a subset of R^p said to be connected? [4]

(b) If f is defined and continuous on a connected set C in R^p taking values in R^q, show that $f(C)$ is connected in R^q. [10]

(c) State Bolzano's Intermediate Value Theorem. If a real valued function f is defined and continuous on a closed interval $[a, b]$ in R such that $f(a)\,f(b) < 0$, can you conclude that f vanishes at some point of $[a, b]$? Give reasons. [6]

6. If f is defined and continuous on a compact subset K of R^p with range in R^q, prove that $f(K)$ is compact in R^q. Can you conclude that if, in particular, f is real valued then it attains its maximum and minimum? Give reasons. [20]

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MID-YEAR EXAMINATION

Economics-3

Date: 22.12.66. Maximum marks: 100 Time: 3 hours

Answer Groups A, B, and C in different answerscripts

Group A Maximum marks: 40

Answer any two questions

1. Explain when and under what assumptions a state of no accumulation and stationary population turns up in the Ricardian theory of economic development. [20]

2. Discuss the Marxian theory on the tendency of the rate of profit to fall in the course of capitalist development. [20]

3. 'So far as capitalism is concerned we are undoubtedly justified in calling under-consumption a disease of old age'. Examine the statement. [20]

Group B Maximum marks: 35

4. Write short notes on any two of the following in the context of development of low-income countries:-

   (a) Terms of trade
   (b) The Theory of comparative advantage
   (c) Chronic balance of payments problem
   (d) Complementarity [15]

5. EITHER

(a) Give an exposition of the critical minimum effort hypothesis.

OR

(b) Show that a sustained rise in labour productivity is economic development. Discuss the major problems involved in inducing such a rise. [20]

Group C Maximum marks: 25

Answer any one question

6. Examine the Budgetary policy of the Government of India, during the three 5-year plans, in the background of the plan objectives. [25]

7. 'Taxation should be confined to raise funds for meeting non-revenue-yielding activities of the Government and revenue-yielding enterprises of the Public Sector should be financed from Public loans'.

Comment on the above, in the light of Indian experience, specially since 1955-6. [25]

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MID-YEAR EXAMINATION

Statistics-3:Probability

Date: 23.12.66 | Maximum marks: 100 | Time: $2\frac{1}{2}$ hours

The whole paper carries 110 marks. You may attempt any part of any question.

1.(a) Suppose a needle of length ℓ is thrown at random on a board ruled with horizontal lines distance $d(d > \ell)$ apart.

i) What is the probability that the needle intersects one of the lines? [10]

ii) State clearly the assumptions made. [2]

(b) If $X \frown N(\mu, \sigma^2)$, show that $E(|X-\mu|) = \sigma\sqrt{\frac{2}{\pi}}$. [10]

2. Let $X \frown \beta_1(m, n)$. Then show that

(a) $1 - X \frown \beta_1(n, m)$ [4]

(b) $\frac{X}{1-X} \frown \beta_2(m, n)$ [8]

(c) $\frac{1-X}{X} \frown \beta_2(n, m)$ [4]

3.(a) Show that the ratio of 2 independent $N(0,1)$ variables is distributed as $C(0, 1)$. [10]

(b) If $X \frown \Gamma(n_1, \theta)$, $Y \frown \Gamma(n_2, \theta)$ and further they are independent, what is the distribution of the random variable $\frac{X}{X+Y}$? [10]

4.(a) Let $X_1, X_2, \ldots, X_n$ be independent and have the same distribution function F (with density f). Denoting by

$$u = \max.(X_1, \ldots, X_n)$$

and

$$v = \min.(X_1, \ldots, X_n),$$

find the distributions of u, v and $(u-v)$ in terms of F and f. [5 + 5 + 10] =[20]

(b) What would the distribution of the sample range, $(u-v)$, reduce to if F is $R(0, 1)$? [4]

5.(a) Define the convolution of two distribution functions. [6]

(b) To what random variable does this new distribution function correspond? [2]

(c) Show using the above definition that

$$\Gamma(n_1, \theta) * \Gamma(n_2, \theta) = \Gamma(n_1 + n_2, \theta)$$

where * denotes the operation of convolution. [10]

(d) If $X_1, \ldots, X_n$ are independent $R(0, 1)$ variables, show that the distribution of $-\log_e(X_1 X_2 \ldots X_n)$ is $\Gamma(n, 1)$. [10]

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MID-YEAR EXAMINATION

Statistics-3: Statistics Theory

Date: 24.12.66 Maximum marks: 100 Time: 3 hours

1.(a) Define the following terms:

(i) Simple and composite hypotheses.
(ii) Randomized and nonrandomized test.
(iii) Level and size of a test.
(iv) Most powerful and uniformly most powerful test. [10]

(b) State and prove the Neyman-Pearson Lemma. How is it used to obtain a most powerful test for testing a simple hypothesis against a simple alternative? [15]

(c) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, 1)$. Obtain (explicitly) the UMP level-α test for testing $H: \mu = 0$ against the alternative $K: \mu > 0$. Show that there does not exist a UMP test for testing $\mu = 0$ against $\mu \neq 0$. [20]

2. EITHER

(a) Describe the cumulative total method and Lahiri's method for selecting a sample of size 1 under a pps scheme. [12]

(b) On the basis of a sample of size 2 selected with pps without replacement, state the Horvitz-Thompson estimator for the population total, and find the variance of this estimator. [15]

OR

(c) Describe the circular systematic sampling scheme. Show that, under this scheme, the sample mean is an unbiased estimate of the population mean. [12]

(d) Suppose the units in a population of size N are so arranged serially that the value of the characteristic under study corresponding to the ith unit is i $(i = 1, 2, \ldots, N)$. For a systematic sample of size n obtain the variance of the sample mean and compare it with the variance of the sample mean based on n units selected by SRS without replacement. Assume that N is an integral multiple of n. [15]

3.(a) Show that the factor reversal test and time reversal test are not satisfied by Laspeyres' and Paasche's indexes. Further show that these tests are satisfied by Fisher's ideal index. [12]

(b) It is desired to determine a trend line in a time-series by a weighted moving average covering consecutive sets of seven points which shall accurately represent the series if it consists of a quadratic polynomial in time variable. Obtain the formula in the form

$$\frac{1}{21}[-2u_{-3} + 3u_{-2} + 6u_{-1} + 7u_0 + 6u_1 + 3u_2 - 2u_3].$$

Show that this formula is in fact accurate for a cubic polynomial. [16]

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MID-YEAR EXAMINATION

Statistics-3: Statistics Practical

Date: 26.12.66 Maximum marks: 100 Time: 3 hours

1. The numbers of letters posted in a certain city on each day in a period of five consecutive weeks are given below:

Sl. No. of week	Number of letters posted on different days of the week (in 00)						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday.
1	18	161	170	154	143	161	76
2	18	165	179	157	168	195	85
3	21	162	169	153	139	185	82
4	24	171	182	170	162	179	95
5	27	172	196	180	170	202	120

Fit a straight line trend (by least squares) to the weekly averages, and use this to obtain trend values for each day. Find the ratios of the daily figures to the corresponding trend values. Using the median of these ratios as the representative daily figure, calculate indices of variation from day to day within a week. [45]

2. EITHER

(a) For a certain population divided into 8 strata, the following table gives the mean value of a variable under investigation and its standard deviation in each stratum, along with sizes of these strata.

stratum	size	mean	s.d.	stratum	size	mean	s.d.
1	400	5.5	8.5	5	373	13.2	17.6
2	513	6.7	8.8	6	733	15.6	18.3
3	385	9.3	11.1	7	215	19.3	23.0
4	613	10.8	13.5	8	173	24.0	31.7

Compute the standard error of the estimate of population average for each of the following methods of sampling. The sample size in each case is 100.

i) Stratified random sampling with replacement. Proportional allocation of sample size to different strata.

ii) Stratified random sampling with replacement. Neyman's optimum allocation of sample size to the different strata.

iii) Sampling with replacement. SRS. [25]

OR

(b) The results of a public opinion survey in Calcutta revealed from amongst Hindu males, classified in 4 age groups, the following frequencies for the different types of opinions on the question of widow re-marriage. Test (at 5 percent level of significance) whether the age factor influences the type of opinion expressed.

Please Turn Over

2.(b) (contd.)

Opinion	Age groups				Total
	19-25	26-35	36-55	over 55	
Unconditional support	49	79	69	9	206
Conditional support	149	256	297	43	785
Opposition	7	9	14	2	32
Unconditional opposition	18	59	95	23	195
Total	223	443	475	77	1218

[25]

3. EITHER

(a) From some data the retail price index for a certain year was worked out to be 204.6. Percentage increases in prices over the base period were as follows:

Rents and Rates	65
Clothing	220
Fuel and Light	110
Miscellaneous	125

What was the percentage increase in the Food group if the weights of the different items were as follows:

Food 60, Rents and Rates 16, Fuel and Light 8, Clothing 12, Miscellaneous 4.

(All items = 100). [10]

OR

(b) Weldon threw twelve dice 26306 times and counting the occurrence of a 'five or a six' as a success, obtained in all 106602 successes. Is this consistent with the assumption that the probability of success with each die is 1/3 ? [10]

4. Practical Record [10]

5. Viva voce [10]

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MID-YEAR EXAMINATION

General Science-5: Statistical Mechanics

Date: 27.12.66 Maximum marks: 100 Time: $2\frac{1}{2}$ hours

Answer Question 1 and any three of the rest.

1.(a) Show that if x is large $\log(x!) = x \log x - x$. [12]

(b) Suppose there are three cells in phase space: 1, 2, and 3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$, and $W_1 = 2$ joules, $W_2 = 4$ joules, $W_3 = 6$ joules. If $\partial N_3 = -2$, find ∂N_1 and ∂N_2. [9]

(c) Consider a system of 10^6 particles and a phase space of 5×10^5 cells, in which the energy w_1 is the same for all cells. What is the thermodynamic probability of (i) the most probable distribution. (ii) the least probable distribution. [13]

2.(a) Apply the Boltzmann statestics to derive the monoatomic ideal gas equation. [16]

(b) Show that the molar specific heat capacity at constant volume is (3/2)R, where R is the universal gas constant. [6]

3.(a) Enumerate the difficulties with the Maxwell-Boltzman statistics. [6]

(b) Deduce Planck's radiation law by the application of Bose's statistics. [12]

(c) Hence deduce Rayleigh-Jeans and Wien's laws of radiation. [4]

4.(a) Distinguish between Fermic-Dirac, Bose-Einstein and Maxwell-Boltzmann statistics. [8]

(b) Deduce the distribution equation of F.D.Statistics. [9]

(c) What is a boson? Give examples of two boson-particles. [3]

(d) What statistics does an ensemble of β-particles obey? [2]

5.(a) What is Thermodynamic probability? [3]

(b) Distinguish clearly between micro-state and macro-state. [6]

(c) Calculate the thermodynamic probability in the case of Bose statistics. [6]

(d) What is Heisenberg's uncertainty relation? Explain how it leads to the quantum statistics. [2 + 5]=[7]

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PERIODICAL EXAMINATION

Statistics-3: Probability

Date: 20.3.67. Maximum marks: 100 Time: 3 hours

The whole paper carries 120 marks. You may attempt any part of any question, without necessarily restricting the total to 100.

1.(a) Find the mean, median and mode for the distribution with the density

$$f(x) = \sin x, \quad 0 \leq x \leq \pi/2.$$ [6]

(b) Find the inter-quartile range of the Cauchy distribution with the density

$$f(x) = \frac{1}{\pi} \frac{\sigma}{\sigma^2 + (x-\mu)^2}, \quad -\infty < x < \infty .$$ [6]

(c) Find the mean and variance of a random variable X, where X is distributed as a chi-square distribution with n degrees of freedom. [8]

2.(a) How do you get a lower bound for the probability, $P[\,|X - \mu| < t]$, from the Tchebycheff's inequality, on the basis of given expected value μ of X and the 50th central moment μ_{50} of X? [6]

(b) Let X_1 and X_2 be 2 independent random variables distributed with the density

$$f(x) = e^{-x}, \quad 0 < x < \infty$$
$$= 0, \quad \text{otherwise}.$$

Then show that

(i) $Z = \frac{X_1}{X_2}$ has an F-distribution [6]

(ii) $Z = \frac{X_1}{X_2}$ and $S = X_1 + X_2$ are independently distributed. [4]

3.(a) Why are χ^2, t, F distributions called the 'derived distributions'? What relations are there between these three distributions among themselves and with the Normal distribution? (No proofs needed.) [10]

(b) Suppose we have a sample of size n from a standard normal population. Then find the distributions of the sample mean and sample variance and show that they are independent. [15]

4.(a) Define the characteristic function of a distribution function. What is the advantage of characteristic functions (c.f.) over the moment generating functions? [5]

(b) Show that the c.f. of any distribution function is uniformly continuous over the entire real line. [8]

4.(c) If $\varphi(t)$ is a c.f., show that its complex conjugate $\bar{\varphi}(t)$ is also a c.f. To what distribution function does this $\bar{\varphi}(t)$ correspond? What can you say about the distribution function if $\varphi(t)$ is real-valued? [8]

5.(a) Define a continuity point of a distribution function and show that the set of continuity points of any distribution function is dense in the real line. [8]

(b) If $\{X_n\}$ is a sequence of random variables converging in probability to a constant c, then show that the sequence of random variables $\{Y_n\} = \{f(X_n)\}$ converges to $f(c)$ in probability, if f is continuous function on the real line. [10]

(c) If the sequence of random variables $\{X_n\}$ converges to another random variable X in probability, then does $f(X_n) \rightarrow f(X)$ in probability for any continuous function f? Justify your answer. [4]

6.(a) If $\{X_n\}$ is a sequence of random variables converging to X in probability, then do the corresponding expectations, $\{E(X_n)\}$ converge to $E(X)$? Justify your answer. [4]

(b) If $x_1, x_2, \dots, x_n$ is a sample of independent observations from $R(0, \theta)$, find the distribution of the sample maximum u. Show that

$$T = \frac{n+1}{n} u$$

is an unbiased estimator of the parameter θ. What is the variance of the estimator T? [12]

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PERIODICAL EXAMINATION

Statistics-3: Statistics Theory

Date: 27.3.67. Maximum marks: 100 Time: 3 hours

1.(a) Discuss briefly the different problems of point estimation introducing the concepts of estimator and estimate.

(b) What are the different criteria for defining a 'good' estimator? [10]

2.(a) Let $\{T_n\}$ be a consistent sequence of estimators for estimating θ (real-valued). Let g be a real-valued continuous function. Show that $\{g(T_n)\}$ is consistent for estimating $g(\theta)$. [12]

(b) Let $\{X_n\}$ be a sequence of i.i.d. random variables with the common p.d.f. $f(.;\theta)$. Consider the following three forms of f:

(A) $f(x;\theta) = \frac{1}{\sqrt{2\pi}} e^{-(x-\theta)^2/2}$, $-\infty < x < \infty$.

(B) $f(x;\theta) = \frac{1}{\pi}\cdot\frac{1}{1+(x-\theta)^2}$, $-\infty < x < \infty$.

(C) $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}$, $x \geq 0$

$= 0$, otherwise.

For each of the following estimators state whether it will be consistent for estimating θ for each of the above cases (A), (B) and (C).

(α) $T_n(X_1,\ldots,X_n) = (X_1 + \ldots + X_n)/n$

(β) $T_n(X_1,\ldots,X_n) =$ median of $X_1,\ldots,X_n$.

(γ) $T_n(X_1,\ldots,X_n) = X_1$

(δ) $T_n(X_1,\ldots,X_n) = \frac{n+2}{n^2}(X_1 + \ldots + X_n)$. [12 X1]=[12]

3.(a) State and prove the Cramér-Rao Inequality mentioning the the regularity conditions involved. [12]

(b) Let $X_1,\ldots,X_n$ be a random sample from $N(\theta, 1)$. Show that the sample mean is the best unbiased estimator of θ. [8]

4.(a) Define the likelihood function and explain the method of estimation by maximum likelihood principle. [4]

(b) Let $X_1, X_2,\ldots,X_n$ be i.i.d. random variables with the common p.d.f (or probability function) $g(.;\theta)$. Consider the following different forms of g:

(A) $g(x;\theta) = \frac{e^{-x^2/2\theta}}{\sqrt{2\pi\theta}}$, $-\infty < x < \infty$, $\theta > 0$

(B) $g(x;\theta) = \theta^x(1-\theta)^{1-x}$, $x = 0, 1$, $0 \leq \theta \leq 1$

(C) $g(x;\theta) = e^{\theta - x}$, if $\theta \leq x$

$= 0$, otherwise.

Please Turn Over

For each of the above cases find the maximum likelihood estimate of θ. Do you get the same estimate of θ by the method of moments? [12]

5. Consider n uncorrelated random variables $Y_1, \ldots, Y_n$ having the common variance σ^2. Assume that

$$E(Y) = \begin{pmatrix} EY_1 \\ \vdots \\ EY_n \end{pmatrix} = A\,\underset{\sim}{\theta}$$

where A is a known $n \times k$ matrix of rank r and $\underset{\sim}{\theta}' = (\theta_1, \ldots, \theta_k)$ is a vector of unknown parameters.

a) Obtain a necessary and sufficient condition for a linear parametric function to be linearly estimable. Show that all the components $\underset{\sim}{\theta}$ are linearly estimable iff $r = k$ which is equivalent to the condition that all linear parametric functions are linearly estimable. [8]

b) Show that the set of equations $A'A\,\underset{\sim}{\theta} = A'\underset{\sim}{Y}$ is consistent and every solution $\hat{\underset{\sim}{\theta}}$ of the above equations is a least-square estimate of $\underset{\sim}{\theta}$ and vice-versa. [12]

c) Show that, if $\underset{\sim}{\ell}'\underset{\sim}{\theta}$ is linearly estimable then it has a unique best linear unbiased estimate given by $\underset{\sim}{\ell}'\hat{\underset{\sim}{\theta}}$ where $\hat{\underset{\sim}{\theta}}$ is a solution of $A'A\,\underset{\sim}{\theta} = A'\,\underset{\sim}{Y}$. [10]

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PERIODICAL EXAMINATION

Statistics-3: Statistics Practical

Date:27.3.67. Maximum marks: 100 Time: 3 hours

1. EITHER

(a) Suppose that $X_1, X_2, \ldots, X_{12}$ are 12 independent random samples drawn from a $N(\mu, 1)$ population.

i) How will you test the hypothesis $H : \mu = 2$ against the alternative $K : \mu > 2$? [5]

ii) Compute the power function of this test for $\mu = 2.0(0.2)4.0$. [15]

(b) Suppose $X_1, X_2, \ldots, X_{15}$ are 15 independent observations from a $N(\mu, \sigma)$ population.

i) How will you test the hypothesis $H : \sigma = 1.5$ against the alternative $K : \sigma > 1.5$? [5]

ii) Compute the power function of this test for $\sigma = 1.5(0.1)2.5$ [10]

OR

(c) The following table gives the stature in c.m.(X), and the weight in lbs.(Y) of 30 school-boys from a certain school.

i) Obtain the linear regression of Y on X. [10]

ii) Test the following hypotheses about the regression equation $Y = \alpha + \beta X$.

A. $\alpha = 1.2$. [10]

B. $\alpha = 2.0$ and $\beta = 0.5$. [15]

sl.no.of individuals	stature (cm.)	weight (lbs.)	sl.no.of individuals	stature (cm.)	weight (lbs.)
1	146	83	16	153	78
2	152	76	17	159	76
3	143	70	18	146	63
4	157	82	19	142	98
5	155	88	20	157	64
6	159	92	21	154	80
7	138	58	22	154	88
8	138	58	23	154	95
9	148	74	24	162	98
10	146	70	25	161	98
11	145	62	26	154	105
12	138	62	27	171	100
13	146	70	28	164	100
14	158	106	29	158	94
15	168	88	30	163	108

2. In a University Examination the frequency distribution of marks in Statistics and Mathematics was found to be Normal with the following parameters.

	mean	s.d.
Statistics	55	10
Mathematics	60	15

with a correlation coefficient of 0.6.

Please Turn Over

a) Amongst those that score 70 in Statistics what proportion score more than 70 in Mathematics? [7]

b) Amongst those that score 70 in Mathematics what proportion score more than 70 in Statistics? [7]

c) What proportion of students score more than 150 in the aggregate? [6]

3.(a) The following gives the heights in inches of 15 individuals taken at random from a certain population. Examine whether the average height in that population differs significantly from 64 inches.

Heights (in inches) of individuals:

66, 56, 59, 55, 63, 62, 70, 72, 69, 49, 49, 53, 60, 64, 66. [10]

(b) The standard deviation of stature (in inches) of 2 groups of boys, one of ages 11-13 and the other of ages 14-16, estimated from 2 samples of sizes 10 and 15 respectively are 1.15 and 2.36. Does this prove that in respect of stature, adolescent boys (age group: 14-16) are more heterogeneous? [10]

4. The correlation coefficient between the scores in two halves of a psychological test applied on a group of 25 students was 0.48. Examine whether the correlation coefficient between the two halves of the test is

a) significantly different from zero. [8]

b) significantly different from 0.5. [7]

5. Practical Record. [10]

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PERIODICAL EXAMINATION

Mathematics-3: Analysis

Date: 3.4.67. Maximum marks: 100 Time: 3 hours

The whole paper carries 120 marks. You may attempt any part of any question, without necessarily restricting the total to 100.

1. Let f be a real-valued function on a subset D of the real line.

a) Define the derivative $f'(c)$ of f at a point $c \in D \cap D'$ and prove that $f'(c)$, if it exists, is unique. [4]

b) Prove that the existence of $f'(c)$ implies the continuity of f at c. Show by example that the converse is not true. [6]

c) Starting with the definition of $f'(c)$ show that for an interior point c of D at which f has a relative maximum, if $f'(c)$ exists then $f'(c) = 0$. Show by example that this need not be true if c is not an interior point. [10]

d) State Taylor's Theorem. [4]

2. Let (f_n) be a sequence of real-valued functions on a closed interval $J = [a, b]$ of the real line and let f_n' exist on J for every n.

a) If (f_n) converges uniformly on J to a function f then does it follow that f' exists on J? If so give a proof, if not give a counter-example. [5]

b) Suppose that there exists a point $x_0 \in J$ at which the sequence $(f_n(x_0))$ converges and that the sequence (f_n') is uniformly convergent to a function g on J. Prove that the sequence (f_n) converges uniformly on J to a function f which has a derivative at every point and that $f' = g$. [15]

c) Give an example to show that the hypothesis of the existence of x_0 in (b) above cannot be dropped. [4]

3. Let f and g be bounded real-valued functions on a closed interval $J = [a, b]$ of the real line.

a) Define the Riemann-Stieltjes integral of f w.r.t. g on J and prove that, if it exists, it is unique. [4]

b) State and prove the Cauchy criterion for Riemann-Stieltjes integrability. [10]

c) Suppose that $a < c < b$ and f is integrable w.r.t. g over both the sub-intervals $[a, c]$ and $[c, b]$. Prove that f is integrable w.r.t. g on the interval $[a, b]$ and

$$\int_a^b f\,dg = \int_a^c f\,dg + \int_c^b f\,dg. \quad [10]$$

4.(a) What are the axioms which make R^2 a field ? [2]

(b) Explain how the real numbers may be identified with a subset of the complex numbers. [4]

(c) Show that a sequence (z_n) of complex numbers converges to a complex number z_0 if and only if $(\bar{z}_n)$ converges to $\bar{z}_0$. [4]

(d) Show that a continuous function on a compact subset of the complex plane is uniformly continuous. [14]

5. Let f be a complex-valued function on a subset D of the complex plane and let u and v be its real and imaginary parts respectively. Let $c \in D \cap D'$.

a) If $f'(c)$ exists, prove that the partial derivatives of u and v exist at c. What are the equations connecting u_x, v_x, u_y and v_y? [12]

b) If $f'(c)$ exists and $v = 0$ on D, then show that $f'(c) = 0$. [6]

c) Prove that the function f defined everywhere by $f(z) = \bar{z}$ does not have a derivative at 0. [6]

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PERIODICAL EXAMINATION

Economics-3: Indian Planning

Date: 10.4.67. Maximum marks: 100 Time: $1\frac{1}{2}$ hours

Answer any three questions.
All questions carry equal marks.

1. Discuss planning as a means of economic development.

2. Compare the approach and objectives of the Bombay Plan and the second five-year plan.

3. Discuss the view that the first five year plan was a success despite many weaknesses in the plan.

4. Write short notes on the estimates of financial resources of the second and third five year plans.

5. Critically evaluate the results of the second five year plan.

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ANNUAL EXAMINATION

Mathematics-3: Analysis

Date: 23.5.67. Maximum marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer Groups A and B in separate answerscripts. The whole paper carries 120 marks. You may attempt any part of any question, without necessarily restricting the total to 100.

Group A

1.(a) When is a subset of a Euclidean space R^p said to be connected? [2]

(b) Prove that an open subset of R^p is connected if and only if it cannot be expressed as the disjoint union of two non-empty open sets. [3]

(c) Let f be defined and continuous on R^p with range in R^q. If C is a connected set in R^p, prove that f(C) is connected in R^q. [10]

(d) State Bolzano's Intermediate Value Theorem. If f is defined and continuous on a closed interval of the real line and assumes only rational values then prove that $f \equiv$ a constant. [5]

2.(a) Let f be a real valued function on a subset D of R^p.

i) When is f said to be uniformly continuous on D? [2]

ii) State a necessary and sufficient condition for f to be not uniformly continuous on D. Give an example of a function which is continuous but not uniformly continuous on its domain of definition. [4]

(b) Let (f_n) be a sequence of real valued functions defined on a subset D of R^p.

i) When is (f_n) said to be uniformly convergent to a function f on D? [2]

ii) State a necessary and sufficient condition for (f_n) to be not uniformly convergent to a function f on D. [2]

iii) Let (f_n) be uniformly convergent to a function f on D. If each f_n is continuous on D, prove that f is also continuous on D. Show by example that f need not be differentiable on D even if each f_n is differentiable on D. [10]

3. Let f be a complex valued function on a subset D of the complex plane and let u and v be its real and imaginary parts respectively. Let c be an interior point of D.

(a) If f'(c) exists, prove that the partial derivatives of u and v exist at c. What are the equations connecting u_x, v_x, u_y and v_y ? [10]

Go on to the next page

3. (contd.)

(b) If $f'(c)$ exists and v is a constant in a neighbourhood of c, then show that $f'(c) = 0$. [4]

(c) Prove that the function defined by $g(z) = \bar{z}$ for all complex numbers z is continuous everywhere but differentiable nowhere. [6]

Group B

4. Let f and g be bounded real valued functions on a closed interval $J = [a, b]$ of the real line.

(a) Define the Riemann-Stieltjes integral of f w.r.t. g over J. [2]

(b) If f is continuous on J and g is monotone increasing on J, then show that f is integrable w.r.t. g over J. [12]

(c) If f and g are as in (b) above then prove that there exists a point $c \in J$ such that

$$\int_a^b f \, dg = f(c)[g(b)-g(a)].$$

Give an example to show that this result need not be true if f is not assumed to be continuous. [6]

5.(a) If f is Riemann integrable over $J = [a, b]$ and F is a function on J such that $F' = f$ on J, then prove that

$$F(b) - F(a) = \int_a^b f.$$ [10]

(b) Let D be the rectangle in $R \times R$ given by $D = \{(x,\theta): a \leq x \leq b, c \leq \theta \leq d\}$ and suppose that f and its partial derivative f_θ are continuous on D to R. Prove that the function ϕ defined by

$$\phi(\theta) = \int_a^b f(x,\theta)dx$$

has a derivative on $[c, d]$ and

$$\phi'(\theta) = \int_a^b f_\theta(x, \theta)d x.$$ [10]

6. Let f be a real-valued function on $I = \{x: x \in R, x \geq a\}$. [2]

a) Define the infinite integral of f over I.

b) State and prove the Cauchy Criterion for the integrability of f over I. [7]

c) Suppose that f and g are integrable over $[a,c]$ for all $c \geq a$ and that $0 \leq f(x) \leq g(x)$ for all $x \geq a$. If the infinite integral of g exists then prove that the infinite integral of f exists and that

$$0 \leq \int_a^{+\infty} f \leq \int_a^{+\infty} g.$$ [6]

d) State the Dominated Convergence Theorem. Formulate an example and verify the theorem. [5]

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ANNUAL EXAMINATION

Economics-3

Date: 24.5.67. Maximum marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer Groups A and B in separate answerscripts.

Group A

Answer any two questions.

1. 'Important developments in the economy at large had to take place before a realistic plan could be produced'. Discuss these developments in the field of industry and agriculture of USSR since 1917. [25]

2. Why despite the problems associated with the 'success' indicators, was there rapid economic development in the USSR? What is the 'synthetic indicator'? How is it expected to solve the problems arising out of the earlier success indicators? [25]

3. Critically discuss the domestic price policy of the U.S.S.R. [25]

4. Discuss in broad outlines the assumptions and approach of the first five year plan of U.S.S.R. and the course of development of the plan during this period. [25]

Group B

Answer Question 5 and any one of the rest.

5. Describe the main elements of mathematical growth models and explain the conditions of their practical applicability. How can these models be used as the hard core of planning? [25]

6. An economy is said to be in equilibrium when its productive capacity (i.e., the total output when its labour force is fully employed) equals its national income. Explain how its investment should grow so that the economy is always in equilibrium. What happens to the economy if investment grows in a different manner? State any simplifying and even heroic assumptions you may have to make and explain the parameters you may use. [25]

7. 'If sufficient investment is not forthcoming today, unemployment will be here today, but if enough is invested today, still more will be needed tomorrow in order to increase demand so that the expanded capacity can be fully utilized and excessive capital accumulation avoided tomorrow'. Elucidate the statement and explain the conditions necessary for a steady growth of an economy. [25]

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ANNUAL EXAMINATION

Statistics-3: Probability

Date: 25.5.67. Maximum marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer Groups A and B in separate answerscripts.

The whole paper carries 120 marks. You may attempt any part of any question without necessarily restricting the total to 100.

Group A

1. A line of given length is divided into 3 parts at random by choosing two points randomly on it. Find the probability that these 3 parts form the sides of a triangle. [6]

2. If the random variable, X has the Cauchy distribution with density

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2} \qquad -\infty < x < \infty$$

and if x_1 and x_2 are two independent observations from it,

(a) then show that the density of their mean, $\bar{x} = \frac{x_1 + x_2}{2}$ is given by

$$g(\bar{x}) = \frac{1}{\pi} \frac{1}{(1+\bar{x}^2)} \qquad -\infty < \bar{x} < \infty .$$ [6]

(b) What can you conclude from this?

Compare it (without proof) with the situation where X is distributed as $N(0, 1)$. [4]

3.(a) Define 'Statistical Tolerance Limits'. [3]

(b) Let $x_1, x_2 \ldots, x_n$ be n independent observations from the density $f(x)$. Denoting by

$$u = \max. (x_1, \ldots, x_n),$$
$$v = \min. (x_1, \ldots, x_n),$$

derive the distribution of the random variable

$$Y = \int_v^u f(x)dx .$$ [8]

(c) Describe how you utilise the distribution of Y to obtain statistical tolerance limits for the population having the density $f(x)$. [4]

4.(a) Describe briefly how the χ^2, t and F distributions are related to standard normal distribution. [4]

(b) Show that if a random variable X has a t-distribution with n d.f., then X^2, has an F distribution. [3]

(c) If X is distributed as F_{n_1,n_2} then find the distribution of X^{-1}. [3]

5.(a) Define the characteristic function (c.f.) of a distribution function (d.f.) defined on the real line. [2]

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5.(b) If $\varphi_X(t)$, $\varphi_Y(t)$ and $\varphi_{X+Y}(t)$ denote the c.f.'s of the random variables X, Y and X+Y respectively and X and Y are independent, show that $\varphi_{X+Y}(t) = \varphi_X(t)\ \varphi_Y(t)$ for any t. [6]

(c) Show conversely that if $\varphi_{X+Y}(t) = \varphi_X(t).\varphi_Y(t)$ for all t, that does not imply the independence of X and Y, by considering the joint density function for X and Y to be

$$f(x,y) = \frac{1}{4}\left[1 + xy(x^2 - y^2)\right] \quad \text{for} \quad -1 \leq x \leq 1,\ -1 \leq y \leq 1$$

$$= 0 \quad \text{elsewhere.}$$ [8]

(d) State the uniqueness theorem for c.f.'s. [3]

Group B

Assume that all the random variables considered here are defined on some basic probability space.

6.(a) When do you say that a sequence of random variables $\{X_n\}$ converges to a random variables X, in probability? [2]

(b) Show that the probability limit of a sequence of random variables is unique almost surely. [8]

7.(a) If $X_1, X_2, \ldots$ is a sequence of independent identically distributed random variables with finite variance, then show that the random variable

$$\bar{X}_n = \frac{X_1 + \ldots + X_n}{n} \to E(X)$$

in probability. [6]

(b) Can a stronger statement be made regarding the convergence of $\bar{X}_n$ to the expectation, under the above assumptions? [2]

8.(a) Define convergence in mean of a sequence of random variables, and prove that convergence in mean implies convergence in probability. [2+4]=[6]

(b) If $\{X_n\}$ is a sequence of bounded random variables converging to a random variable X in probability, then show that $E(X_n)$ converges to E(X). [6]

9.(a) Define weak convergence of a sequence of distribution functions. [3]

(b) If a sequence of distribution functions $\{F_n(x), n \geq 1\}$ converges weakly to a distribution function F(x), then show that

$$\int_{-\infty}^{\infty} g(x)dF_n(x) \to \int_{-\infty}^{\infty} g(x)dF(x) \quad \text{as} \quad n \to \infty$$

for every bounded continuous function g(x) defined on R^1. [10]

(c) Use this Helly-Bray theorem to show that the c.f.'s $\{\varphi_n(t), n \geq 1\}$ of $\{F_n(x), n \geq 1\}$ converge to the c.f. $\varphi(t)$ of F(x), if $F_n(x)$ converges to F(x) weakly. [3]

10.(a) State and prove Kolmogorov's inequality. [10]

(b) State the central limit theorem of Lindeberg and Lévy. [5]

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ANNUAL EXAMINATION

Statistics-3: Statistics Theory

Date:26.5.67. Maximum marks: 100 Time:3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer Groups A and B in separate answerscripts.

Answer any five questions.

Group A

1.(a) State and prove Neyman-Pearson Lemma and indicate how it is used for obtaining a 'Most Powerful' test for testing a simple hypothesis against a simple alternative. [10]

(b) On the basis of a random sample of size n from $N(\theta,1)$ obtain the UMP level α test for testing $\theta = 0$ against $\theta > 0$. Show that this test is unbiased for alternatives $\theta > 0$ but not for alternatives $\theta < 0$. [10]

2. Give examples (one for each) to illustrate the following:

(a) A consistent estimator may be biased.

(b) An unbiased estimator may not be consistent.

(c) The maximum-likelihood estimator may be biased.

(d) The maximum-likelihood estimator may be different from the corresponding estimator obtained by the method of moments.

(e) The maximum-likelihood estimator is the same as the least-squares estimator. [5 X4]=[20]

3.(a) i) On the basis of a random sample of size n from $N(\theta, \sigma^2)$ obtain (explicitly) the likelihood-ratio test of size α for testing $\theta = 0$ against $\theta \neq 0$, σ^2 being unknown. [10]

ii) Is this test unbiased? [2]

iii) What is the distribution of the test statistic when $\theta = 0$. [2]

(b) On the basis of a random sample of size n from $N(\theta, \sigma^2)$ obtain a confidence interval for θ with confidence coefficient $1-\alpha$, σ^2 being unknown. [6]

4.(a) Let $X_1, \ldots, X_m$ be a random sample from $N(\theta, \sigma^2)$ and $Y_1, \ldots, Y_n$ be a random sample from $N(\theta, \tau^2)$.

i) Obtain the maximum likelihood estimate $\hat{\theta}$ of θ assuming that σ^2 and τ^2 are known. Find the variance of $\hat{\theta}$ and show that it is a function of m, n and σ^2/τ^2. [8]

ii) Obtain Fisher's information contained in the combined sample assuming σ^2 and τ^2 to be known and show that $\hat{\theta}$ is the best unbiased estimate of θ. [4]

(b) Let $X_1, \ldots, X_n$ be a random sample from the Poisson distribution with parameter λ. In order to estimate $e^{-\lambda}$ the following estimator is suggested:

$$g(X_1, \ldots, X_n) = 1, \quad \text{if } X_1 = 0$$
$$= 0, \quad \text{if } X_1 > 0$$

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4.(b)(contd.)

i) Show that ε is unbiased for estimating $e^{-\lambda}$. [2]

ii) Obtain a 'better' unbiased estimator of $e^{-\lambda}$ based on the sufficient statistic $T = X_1 + \ldots + X_n$. [6]

(Hint: Show that the conditional distribution of X_1 given $T = t$ is binomial $B(t, \frac{1}{n})$.)

Group B

5. Let $Y_1, \ldots, Y_n$ be n uncorrelated random variables with the common variance σ^2. Suppose

$$E(Y_i) = \sum_{j=1}^{m} a_{ij}\,\theta_j, \quad i = 1, \ldots, n.$$

where $A = [a_{ij}]$ is a known $n \times m$ matrix of rank r and $\theta' = (\theta_1, \ldots, \theta_m)$ are unknown parameters.

(a) Obtain a necessary and sufficient condition for $\lambda_1\theta_1 + \ldots + \lambda_m\theta_m$ to be 'linearly estimable'. [4]

(b) i) If $\lambda_1\theta_1 + \ldots + \lambda_m\theta_m$ is linearly estimable then show that it has a unique best linear unbiased estimate. [4]

ii) How do you obtain such an estimate? [2]

(c) Show that $E(Q^*) = (n-r)\sigma^2$, where

$$Q^* = \min_{\theta} \sum_{i=1}^{n} [\, Y_i - \sum_{j=1}^{m} a_{ij}\theta_j]^2$$ [10]

6. From each of k normal distributions $N(\theta_1, \sigma^2), N(\theta_2, \sigma^2), \ldots, N(\theta_k, \sigma^2)$ a random sample of size n is drawn. Obtain (explicitly) a size α test for testing the hypothesis

$$H_0 : \theta_i = \alpha + \beta w_i, \quad i = 1, \ldots, k$$

where α, β are unknown and $w_1, \ldots, w_k$ are k different known numbers; assume the alternative to be the complement of H_0 and consider σ^2 to be unknown. [2]

7.(a) i) Let r be the sample correlation coefficient based on a random sample of size n from a bivariate normal distribution with correlation coefficient ρ. Obtain the distribution of r when $\rho = 0$. [10]

ii) How do you test the hypothesis $\rho = 0$ against $\rho \neq 0$? (Statement is enough.) [4]

(b) Consider k normal distributions $N(\theta_1, \sigma^2), \ldots, N(\theta_k, \sigma^2)$. It is desired to test the hypothesis $\theta_1 = \ldots = \theta_k$, σ^2 being unknown.

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i) State a test procedure based on random samples of size $n_1, n_2, \ldots, n_k$ from the k distributions, respectively. [4]

ii) State the distribution of the test statistic under the hypothesis. [2]

8.(a) Discuss the basic principles of design of experiments, explaining, in particular, the principles of local control, replication and randomization. [10]

(b) Explain the 'Latin-square' design. Illustrate the use of such a design. Given an outline of the analysis of such a design specifying the assumptions used. [10]

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ANNUAL EXAMINATION

Statistics-3: Statistics Practical

Date: 26.5.67. Maximum marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

EITHER

1. The observed frequency and the probabilities of different combinations of colour and pollen shape in sweet pea are given below:

Probability			Observed frequency		
pollen shape	colour: purple	colour: red	pollen shape	colour: purple	colour: red
long	$\frac{1}{4}(2+\theta)$	$\frac{1}{4}(1-\theta)$	long	296	27
round	$\frac{1}{4}(1-\theta)$	$\frac{1}{4}\theta$	round	19	85

$0 \leq \theta \leq 1.$

Estimate θ from the above data by the method of maximum likelihood and find out the asymptotic variance of this estimate. [20]

OR.

2. Independent measurements on the three internal angles A, B, C of a triangle are given below in degrees.

angle	no. of times measured	measurements in degrees
A	4	58.6, 59.1, 59.5, 57.9
B	4	60.4, 61.3, 60.2, 61.3
C	4	60.1, 59.2, 59.3, 58.5

Making use of the geometrical properties of a triangle, estimate A, B, C by the method of least squares and compute their standard errors. [20]

3. In a dairy farm an experiment was conducted to compare the effects of 4 types of feeds 1, 2, 3 and 4 on yield of milk. The average yield of milk in litres, the standard deviation and number of cows are given for each of the batches treated by different food types separately. Is there any significant difference between the four types of feed with regard to the effects on average milk yield?

type of feed	number of cows	coverage yield in litres	standard deviation*
1	12	2180	147
2	14	1978	158
3	15	3246	165
4	13	1936	142

[* divisor used is the corresponding sample size] [20]

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4. The table below shows measurements on height made on 10 pairs of twins of opposite sex, all of ten years of age.

sl. no. of twin	height (cm.) male	height (cm.) female	sl. no. of twin	height (cm.) male	height (cm.) female
1	136	132	6	137	140
2	141	133	7	130	133
3	137	135	8	139	140
4	134	131	9	128	123
5	134	137	10	132	136

On the basis of the above data test the following hypotheses.

(a) Among the pairs of twins of opposite sex, all of ten years of age, the boys are taller than their twin sisters. [6]

(b) The standard deviation of the height among the twin brothers is the same as that among the twin sisters. [12]

5. Answer any two from the following:

(a) The following gives the weight in kilogram of 15 boys in a school. Obtain a 95 °/° confidence interval for the average weight of a boy in that school.

Weight (in Kg.) of 15 boys.

30.4, 39.6, 40.2, 37.3, 29.6, 24.9, 44.3, 26.0, 33.0, 32.9, 35.3, 36.2, 41.2, 22.9, 29.3.

(b) Estimates of the percentage of employed persons in a certain city obtained from 2 independent sample surveys are given below. Is the difference between the two estimates due to fluctuations of sampling?

Survey	Sample size	Percent employed
I	2350	39.66
II	1675	38.68

(c) Consider the data given in Question 4 and test the hypothesis given in 4(a) by the sign test of size 0.1.

(d) To study the effect of group pressures for conformity upon an individual in a situation involving monetary risk, a measure of authoritarianism and a measure of social status striving were found for each of 12 college students and the data are given below.

Student	score authoritarianism	score Social status striving
1	82	42
2	98	46
3	87	39
4	40	37
5	116	65
6	113	88
7	111	86
8	83	56
9	85	62
10	126	92
11	106	54
12	117	81

Compute Spearman's rank correlation coefficient between the two scores and test for their independence. [2 X10]=[20]

6. Viva Voce [10]

7. Practical Record [10]

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ANNUAL EXAMINATION

General Science-4: Biology Theory

Date: 27.5.67. Maximum marks: 100 Time: 3 hours

The number of marks allotted to each question is given in bracket [ ].

Answer Groups A and B in separate answerscripts.

Group A

Answer Question No.1 and any two of the rest.

1.(a) Give the statistics on the area under cultivation and production of Grain Sorghums in the various agricultural regions of the world. [9]

(b) Mention separately the respective figures relating to India and Pakistan. [6]

(c) Write the names of 5 important species of Sorghum. [5]

2.(a) What is a legume? [5]

(b) Mention the names of 10 important pulse crops. [5]

(c) Give a botanical account of any one of the pulses. [5]

3. Write brief accounts on any three of the following:

a) Genialogy of the sugarcane, Co.312.

b) Clonal propagation in the coconut.

c) Plants yielding essential oil.

d) Spikelet of rice. [3 ×5]=[15]

4.(a) Mention the world production figures for potato tubers. [5]

(b) Name 5 important starch storage crop plants. [5]

(c) Give a brief morphological account on tapica plant (*Manihot utilissima*). [5]

Group B

Answer Question No.5 and any two of the rest.

5.(a) What is meant by inbreeding? [4]

(b) Why selection for vigour in inbred lines may delay the attainment of homozygosity? [6]

(c) Three inbred lines (Line 1-6, Line 1-7, and Line 1-9) of maize were isolated from thesamme cross-pollinated variety and were propagated by selfing for 30 generations. The parental variety yielded 81 bushels/acre. Average yield (bushels/acre) for 5 successive inbred generations in these inbred lines are given in the following table:

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Generation	Line 1-6	Line 1-7	Line 1-9
0	81	81	81
1 - 5	64	51	41
6 - 10	45	36	34
11 - 15	38	34	26
16 - 20	22	24	14
21 - 25	20	21	13
26 - 30	24	18	9

Comment on the results. [10]

6.(a) What is meant by polyploidy? [5]

(b) How are polyploids induced artificially? [5]

(c) Discuss briefly the role of polyploidy in plant breeding. [5]

7. Give an outline scheme of the methods of breeding self-fertilizing crops. [15]

8. Write short notes on any three of the following :

a) Induced mutation.

b) Meiosis.

c) Chromosomal abberations.

d) Heterosis. [3×5]=[15]

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ANNUAL EXAMINATION

General Science-5: Psychology Theory

Date: 29.5.67.

The number of marks allotted to each question is given in brackets [ ].

Part II.

Answer all questions

1. Explain Ockham's razor, indicating its relevance for (i) science in general, (ii) statistics, and (iii) the concept memory. [10]

2. Compare a computer 'memory' with the memory of a human being. [10]

3. Discuss the differentiation of abilities in statistical terms and comment on the statement:
"Research ability is inborn, and hence training or knowledge does not affect it." [10]

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ANNUAL EXAMINATION

General Science-5: Engineering

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Date: 30.5.67. Maximum marks: 100 Time: 3 hours

The number of marks allotted to each question is given in brackets [ ].

Answer any five questions.

1. Find the ratio of the maximum to the mean intensity of vertical shear in cross-section of a hollow beam of square section with a square hollow therein, the outside width being twice the inner. The cross-section is symmetrical about it horizontal axis. Compare this ratio with that for a beam of rectangular cross-section. [20]

2. Determine the central bending moment and deflection for the beam loaded with a concentrated load W at a point distant L/4 from the left hand support in a simply supported beam of uniform cross-section, whose span is L. Derive the formula which you may use. [20]

3. The external and internal diameter of a hollow steel shaft are 15" and 9" respectively. Determine what horse power it will transmit when the speed is 90 revolutions per minute. The maximum intensity of shear stress is not to exceed 8000 p.s.i. Derive the formula which you use. [20]

4. Define 'Centre of Pressure'

An equilateral triangle (whose side is 6 feet long) is placed in water vertically with its apex up and its base horizontal, so that the apex is 4 feet below the surface. Find the depth of the centre of pressure and the total pressure on the plate. [20]

5. Write a short note on 'Venturi-meter'

The difference of head registered in two limbs of a mercury gauge, with water above the mercury, connected to a venturimeter was 6". The diameters of the pipe and the throat of the meter were 12" and 6" respectively. The coefficient of the meter was 0.97. Find the discharge through the meter. [20]

6. State clearly Bernoulli's Theorem.

A cast iron pipe 9" in diameter and 1500 feet long connects two reservoirs. If the difference of water level in the two reservoirs is 100 feet, find the discharge through the pipe. f = 0.01. Ignore all losses other than friction. [20]
