79 - 80 | 311

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS
Economics-3: Macroeconomics-II

Date: 12.11.79 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions.

1. Use the production possibilities curve and the consumption indifference map to derive the offer curve of a country. Apply this technique to determine the terms of trade between two countries and also to measure their total production, consumption, imports and exports. Give a diagrammatic exposition the terms of trade. [10+10+5=25]

2. Assuming prices and the rate of exchange to remain constant, discuss how national incomes and balance of payments change in a two-country model when

i) there is an increase in autonomous exports in one of the countries, and

ii) there occurs an increase in investment in one of the countries. [$12\frac{1}{2}+12\frac{1}{2}$]=[25]

3.a) How did the classical economists come to the conclusion that international trade does not require offsetting absolute advantage but is possible where a comparative advantage exists ?

b) The labour theory of value on which their analysis rested was subsequently rejected. What led to this rejection ? How was the law of comparative advantage shown to be valid after the labour theory of value was discarded as invalid. [10+5+10]=[25]

4.a) Explain the following statements:

i) Under conditions of constant opportunity costs at least one country must be fully specialised, and both may be.

ii) With identical tastes and identical comparative costs trade between two countries is impossible.

b) What makes for differences in comparative costs in various countries ?

c) 'Trade tends to bring about equalization of factor prices' - Explain. [(6+6)+6+7]=[25]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III : 1979-80

MID-YEAR EXAMINATIONS

Economics-3: Macroeconomics - II

Date: 24.12.79 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions.
Marks are given in brackets.

1.a) Using a two-country model examine the effects of a devaluation of the domestic currency on the price, quantity and value of the home country's exports and imports in terms of the domestic currency.

b) Work out the conditions under which devaluation of the domestic currency will improve the home country's balance of payments. [16+9]=[25]

2.a) Explain with the help of a diagram the three different forms of the sacrifice principle in taxation. State any assumptions you may have to make.

b) Suppose the utility of an income y is given by the function $u(y)$ and t is rate of taxation so that ty is the tax on y.

Let $u(y) = C - \frac{1}{y}$

where C is a constant.

Discuss whether the tax system would be progressive or otherwise in the following two cases:

i) the sacrifice is absolute for all incomes,
and ii) the sacrifice is proportional for all incomes. [15+10]=[25]

3.a) Examine how the terms of trade are affected by a tariff. What happens when the imposition of a tariff is followed by retaliations and counter-retaliations?

b) 'There is one considerable difference between a tariff and a quota.' Discuss the statement.

c) Discuss the (i) protective, (ii) consumption and (iii) revenue effects of a tariff. [13+6+6]=[25]

4.a) Work out the multiplier effect of an increase in government expenditure when net taxes are an increasing function of national income (net taxes = all taxes - transfer payments).

b) What would be the effect of an increase in resource-using government expenditure (that is, expenditure on goods and services) on the budget ?

c) Briefly discuss how government borrowing may affect national income. [12+5+8]=[25]

p.t.o.

5.a) On what occasions is protection economically justified? Give reasons for your answer.

b) Work out the case when an increase in government expenditure (on goods and services) which is solely financed by taxation has a multiplier effect of unity. [16-9]=7

79 - 80: 313

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

PERIODICAL EXAMINATIONS

Economics-3: Economic Development

Date: 7.4.80 Maximum Marks: 100 Time: 2 hours

Note: Answer Q.No.5 and any two of the remaining questions. Full marks are indicated in brackets [ ].

1. How would you define a society without surplus? Carefully trace the alternative consequences that are possible on an improvement of its techniques of production, given from outside. In what way does this help one understand the process of economic development, in particular the significance of inequalities in the material conditions of life of the people for the process? [30]

2. What are the different forms in which a 'surplus' is appropriated by the landlord in a feudal system of agriculture? What role, if any, does the 'market' play in it? Is the peasants' (serfs') participation in the market necessarily a voluntary one? Would you say that the peasant is 'employed' by the landlord because he works on the latter's farm (demense)? Comment briefly on the potentialities for the growth of production in such a system. [30]

3. Consider an agricultural system with private property rights in land but without wage labour. What would be the basic production relation in this system? Would the entire agricultural population be necessarily involved in this relation on one side or other? Hence point out the basic determinant of the domain of this relation. Would either party in the relation have a motivation for carrying out land improvements on his own? Comment in this context on the significance of alternative systems of the payment arising out of the relation. [30]

4. What are the alternatives for livelihood open to landless persons and marginal landholders in an agricultural system with well defined employment and tenancy relations? What do you think are the major distinguishing characteristics of the two alternative systems possible within these relations, one in which the employment relation is the dominant one and the other in which the tenancy relation is the dominant one? [30]

5. Write short notes on any two of the following:

   a) The concept of economic development.

   b) The concept of agriculture as a broad sector of production.

   c) Capital.

   d) Wage labour.

   e) Marketed surplus.

   f) The general mode of capitalist development. [20 x 2]=[40]

---

79 - 80: 314

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
ANNUAL EXAMINATIONS
Economics-3: Economic Development

Date. 29.5.80 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions.
All questions carry equal marks.

1. Give a general outline of the stages of the development of human society in terms of the growth of productive forces and relations of production. [25]

2. What is the basic difference between the economic development under the capitalist system and that under previous social systems? [25]

3. Give a brief historial account of the First Industrial Revolution in England. [25]

4. What were the effects of the application of machinery on labour during the Industrial Revolution in England ? Discuss in this context the significance of the capitalist motivation for such applications. [25]

5. Write short notes on:

i) Relations of production;

ii) Transport revolution in England. [25]

79- 80 | 321 |

INDIAN STATISTICAL INSTITUTE
B Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS
General Science-4: Physics

Date:29.10.79 Maximum Marks: 100 Time: 3 hours

Note: Attempt all the questions.
All questions carry equal marks.

1. Write down the expressions for the distribution functions in MB, BE and FD statistics, and deduce the one of MB-statistics. [3 x 9+14]=[20]

2. Compare and contrast the three statistics
Establish Planck's radiation law using Bose's statistics.
[6+14]=[20]

3. Starting from Plank's radiation law, deduce
   a) Wien's law
   b) Rayleigh-Jeans law, and
   c) Wien's displacement law.

Suppose in MB-statistics there are three cells in phase space: 1, 2 and 3. Let $N = 30$, $N_1 = N_2 = N_3 = 10$, and $w_1 = 2$ joules, $w_2 = 4$ joules, $w_3 = 6$ joules If $\delta N_3 = -2$, find δN_1 and δN_2. [3+3+4+10]=[20]

4. Explain the following: (a) Microstates and macrostates,
(b) Thermodynamic probability.

Establish Starling's approximation involving the factorial of a large number.

Write down the relation between entropy and probability.
[7+4+6+3]=[20]

5. 'Once the partition function Z is evaluated, all the thermodynamic properties of the system can be calculated'.
- Establish the validity of the statement by calculation using MB-statistics. [20]

INDIAN STATISTICAL INSTITUTE
B.Stat.)Hons.) Part III: 1979-80

MID-YEAR EXAMINATIONS

General Science-4: Physics

Date: 12.12.79 Maximum Marks: 100 Time: 3 hours

Note: Answer any three from Group A.
Answer any two from Group B
Marks are given in brackets.

Group A

1. Write down the distribution function and partition function in MB-statistics. Hence show that the internal energy U of a system is given by

$$U = NkT^2 \frac{d(\ln Z)}{dT}$$

Consider a system of N particles and a phase space of n cells. Suppose that the energy of a particle has the same value w in all cells, so that $w_1 = w_2 = \ldots = w_n = w$ Show that the particles are distributed uniformly among the cells. [2+2+6+6]=[16]

2. What is meant by an equation of state ?
Establish, by the application of MB-statistics the ideal gas equation of state. [4+12]=[16]

3. Establish the law of atmosphere by applying the MB-statistics. Indicate, in outlines, how the procedure could be utilised for the determination of the Avogadro number. [12+4]=[16]

4. Show that the Bose's statistics leads to the same distribution function as the MB-statistics provided N_i/n is very much less than unity.
Show, from Planck's law of radiation, that the wavelength corresponding to the maximum of the curve bears an inverse relation with the absolute temperature. [10+6]=[16]

Group B

5. State the postulates of Bohr's theory of hydrogen atom. Find the expression for the total energy of the electron in the nth orbit. Hence explain the origin of Balmer series. What is the meaning of the total energy of an electron is negative ? [6+11+6+3]=[26]

6.a) Define the radioactive constants and find their inter-relation.

b) Compute the radius of the first Bohr orbit for hydrogen.

c) Draw a neat diagram of a linear accelerator and properly level the diagram. [6+10+5+5]=[26]

7.a) Describe the construction and the principle of action of either a GM-counter or a Wilson cloud chamber.

b) Indicate how the following particles are detected with the help of a cloud chamber.
(i) proton (ii) electron (iii) α-particle.

c) What is the first man-made nuclear reaction? Represent the reaction by a nuclear equation. [6+8+6+6]=[26]

79 - 001 324

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
ANNUAL EXAMINATIONS
General Science - 4: Psychology

Date: 17.3.80 Maximum Marks : 100 Time: 2 hours

Roll No. .....

Instructions:

1. There are 50 multiple-choice questions in this booklet.
2. Answers are to be recorded in this booklet only.
3. For each question, choose the correct (or the best) alternative.
4. To indicate your answer, put a cross mark (X) in the parentheses appearing before your chosen alternative.
5. Do not put more than one cross mark.
6. Attempt all questions.
7. After answering, hand over the booklet to the invigilator.

Question

1. Psychology is best defined as a scientific study of
   ( ) mind.
   ( ) behaviour.
   ( ) consciousness.
   ( ) all the above three.

2. Who founded the first laboratory of experimental psychology?
   ( ) Pavlov
   ( ) Watson
   ( ) Wundt
   ( ) Skinner

3. Which of the following is not a behavioural science?
   ( ) physiology
   ( ) anthropology
   ( ) economics
   ( ) sociology

4. 'Introspection' as the basic method of psychology was used by
   ( ) functionalists.
   ( ) behaviourists.
   ( ) Gestalt psychologists.
   ( ) structuralists.

5. The field of psychology which is solely concerned with the understanding of the fundamental causes of behaviour is

( ) clinical psychology.
( ) developmental psychology.
( ) experimental psychology.
( ) psycho-analysis.

6. The school of psychology most influenced by the field theory in physics was

( ) functional psychology.
( ) Gestalt psychology.
( ) behaviourism.
( ) psycho-analysis.

7. What is the definition of learning?

( ) Learning is a relatively permanent change in behaviour that comes as a result of practice or experience.
( ) Learning is an innate behaviour which is modified by practice and past experience.
( ) Learning is [illegible] practice and experience.
( ) Learning is a change in behaviour which comes primarily through growth and maturation.

8. The main point of difference between the approaches of a social psychologist and a sociologist lies in this that

( ) the former is concerned with the study of the structure and function of social groups and the latter is interested in various various characteristics of the individuals forming such groups.
( ) while the former studies the behaviour of individuals in modern societies, the latter is busy in understanding primitive cultures.
( ) the former uses the experimental method in his his study, while the latter uses only the observational method.
( ) while the former is primarily interested in how individual behaviour is influenced by group-membership, the latter is more concerned with the groups as such.

9. 'Human engineering' is

( ) a branch of engineering which concerns the man-machine relationship in industries.
( ) a specialised application of psychology aimed towards training personnel to operate machines under difficult conditions.
( ) a field of psychology which concerns the design of equipment and the tasks involved in the operation of such equipment.
( ) is a field of psychology specially concerned with the study of problems arising in engineering industry.

10. In which of the following are statistical techniques least applied.

( ) Educational psychology
( ) Psychoanalysis
( ) Industrial psychology
( ) Experimental psychology

11. In a Skinner box two similarly trained animals were placed on an extinction schedule. One of the animals was punished during extinction trials. If the punished animals made as many extinction responses as the unpunished one, it proves that

( ) the punishment administered was a mild one.
( ) the unpunished animal was not highly motivated.
( ) the unpunished animal had other means available for getting reinforcement.
( ) punishment acted as a stimulus for an increased rate of responding.

12. The search for mental elements and rules for combining them were the characteristics of a school of psychology known as

( ) functionalism.
( ) behaviourism.
( ) Gestalt psychology.
( ) structuralism.

13. Which is not true of classical conditioning?

( ) It is possible in the case of animals only.
( ) It is a simple response learning.
( ) It is a kind of associative learning.
( ) It involves pairing of two stimuli.

14. Repeated presentation of the conditioned stimulus without its being followed by the unconditioned stimulus results in

( ) inhibition of the response.
( ) extinction of the response.
( ) suppression of the response.
( ) deconditioning of the response.

15. An active avoidance conditioning requires prior learning of

( ) emotional fear response only.
( ) escape response only.
( ) both emotional fear and escape responses.
( ) neither emotional fear nor escape response.

16. The statement that 'almost any infant, through proper training can be made into a saint, a politician, a thief or a beggar', best characterises the school known as

( ) functionalism.
( ) introspective psychology.
( ) [illegible] psychology.
( ) behaviourism.

17. Which of the following is true ?

( ) What is termed reinforcement in operant conditioning is called the conditioned stimulus in classical conditioning.

( ) In operant conditioning a particular response is sought to be elicited by the experimental design.

( ) In operant conditioning reinforcement increases the frequency of a particular response.

( ) In classical conditioning reinforcement is contingent upon the response.

19. An 'acquisition curve' obtained in a conditioning experiment is

( ) positively accelerated.

( ) negatively accelerated.

( ) sometimes positively and sometimes negatively accelerated.

( ) J-shaped.

19. Which of the following statements is not correct ?

( ) In the initial phases of conditioning, the subject responds to a range of stimuli similar to the conditioned stimulus.

( ) Many of our irrational fears are due to a pairing of a neutral stimulus with an unpleasant experience.

( ) The strength of conditioning is independent of the time-interval between the presentation of the conditioned and unconditioned stimuli.

( ) As learning proceeds, the range of stimuli which elicits the conditioned response becomes progressively reduced.

20. The resistance to extinction of avoidance response is great; because,

( ) avoidance conditioning involves conditioned fear response.

( ) in avoidance conditioning the subject knows when the reinforcement is not present .

( ) avoidance conditioning involves negative rather than positive reinforcement.

( ) in avoidance conditioning there is no chance for the subject to experience the pairing of the conditioned stimulus with the absence of the painful event.

21. Which of the following statements is not true about 'punishment'?

( ) Punishment is less effective the farther it is removed in time and space from the behaviour being punished.

( ) The effectiveness of punishment lies solely in teaching active avoidance of undesirable behaviour.

( ) The more consistently punishment is administered, the more effective it is.

( ) The best way to use punishment is to couple it with a rewarded alternative.

22. Which of the following schedules of reinforcement produces the highest rate of responding?

( ) Fixed interval

( ) Variable interval

( ) Fixed ratio

( ) Variable ratio

23. An 'experimental neurosis' is produced in the subject during conditioning when

( ) too many stimuli are used to elicit the conditioned response.

( ) painful stimuli are used to develop conditioned fear response.

( ) discrimination is pressed for too much resulting in stimulus differentiation being broken down.

( ) an already learned behaviour is severely punished resulting in a basic drive being thwarted.

24. The asymptote of the learning curve in classical conditioning represents

( ) the highest level of conditioned response.

( ) the level of response at the beginning of conditioning.

( ) the level of response at the middle of conditioning.

( ) the lowest level of conditioned response.

25. Which of the following statements is true?

( ) Learning is slower under schedules of continuous reinforcement.

( ) Learning is equally fast under both continuous and partial reinforcement schedules.

( ) There is greater resistance to extinction when responses are learned with partial reinforcement.

( ) Learning with partial reinforcement is not as fast in men as it is in animals.

p.t.o.

26. T.A.T. stands for

( ) Technical Ability Test.
( ) Technical Aptitude Test.
( ) Thematic Apperception Test.
( ) Thematic Apprehension Test.

27. Development of personality is more flexible during

( ) childhood.
( ) adolescence.
( ) adulthood.
( ) old age.

28. How many personality traits were found out by Cattell ?

( ) 7
( ) 12
( ) 18
( ) 18000

29. Sentence-completion test is a kind of

( ) personality inventory.
( ) questionnaire.
( ) rating scale.
( ) projective test.

30. The statement: 'Poverty and want bring out the best efforts of a child, and a strong and self-reliant personality results' , is

( ) always true.
( ) always false.
( ) not necessarily true.

31. In order to standardize a test, it has first to be administered on

( ) a randomized sample.
( ) a sample comprising of individuals of all age groups.
( ) a large representative sample of individuals for whom the test is constructed.
( ) a sample comprising of individuals of both the sexes.

32. The first attempt of listing all the personality traits was undertaken by

( ) Cattell.
( ) Guilford.
( ) Thurstone.
( ) None of these.

33. The stimulus presented in a projective test is often

( ) structured.
( ) nonstructured.
( ) [illegible].

34. The statement: 'Betrayal of love and affection, specially in the early childhood, gives rise to adjustment problems to such individuals when they grow up', is

( ) always true.
( ) always false.
( ) most probably false.
( ) most probably true.

35. The psychologist who extracted only seven primary traits of personality, after analysing his data, was

( ) Guilford.
( ) Cattell.
( ) Thurstone.
( ) Eysenck.

36. Which of the following statements is true about development of personality?

( ) The same condition often has same consequences in different persons.
( ) The same condition often has different consequences in different persons.
( ) The same condition always has same consequences in different persons.
( ) The same condition always has different consequences in different persons.

37. The child is more carefully looked after, tenderly reared up, much hugged and caressed and thus gets more contact comfort in the

( ) Arapesh.
( ) Mundagumor.
( ) Eskimos.
( ) All of the above.

38. The degree of consistency with which a test measures is called its

( ) dependability.
( ) reliability.
( ) objectivity.
( ) validity.

39. When the judgment of the scorer is not a factor in determining the value of the score, the test is

( ) dependable.
( ) reliable.
( ) objective.
( ) valid.

40. Validity of a test implies that the test is

( ) dependable.
( ) yielding nearly the same score for a group of individuals on its repeated use on them.
( ) measuring that very trait for which it is devised.
( ) giving the same score whosoever scores it.

41. Criteria of a good test are

( ) objectivity and reliability.
( ) objectivity, reliability, and validity.
( ) reliability and validity.
( ) objectivity, reliability, validity, and standardization.

42. Which of the following glands does not secret hormones?

( ) Bile
( ) Parathyroid
( ) Adrenal cortex
( ) Pituitary.

43. Rater-bias is expected to be least in

( ) Numerical Rating Scale.
( ) Rank-order Rating Scale.
( ) Graphic Rating Scale.
( ) Forced-Choice Rating Scale.

44. The body-type called Athletic by Kretschmer is similar to Sheldon's concept of

( ) Mesomorphy.
( ) Ectomorphy.
( ) Endomorphy.
( ) None of these.

45. The belief that a man's personality consists of faculties, each located in a specific area of the brain, and evidenced through bumps on the skull, is held by

( ) Physiognomy.
( ) Anatomy.
( ) Graphology.
( ) Phrenology.

46. Rank-order rating scales are in

( ) nominal scale.
( ) ordinal scale.
( ) interval scale.
( ) ratio scale.

47. In a Stress Interview the candidate is asked

( ) questions continuously for hours together to make him exhausted.
( ) questions which will make him perplexed, irritated, and annoyed.
( ) to perform laborious task.
( ) All of these.

p.t.o.

48. Which of the following is not an Absolute Type of rating scale?

( ) Man-to-man rating scale
( ) Numerical rating scale.
( ) Forced choice rating scale
( ) Graphic rating scale

49. The rater, instead of considering the ratee on each trait on its own merit, is affected by an overall impression about him and rates him accordingly. This is called

( ) Error of leniency.
( ) Stereotype.
( ) Contrast Error.
( ) Halo Effect.

50. The Picture-Frustration Test was constructed by

( ) Rosenzweigh.
( ) Murray.
( ) Rorschach.
( ) Kahn.

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
ANNUAL EXAMINATIONS
General Science-4: Anthropology

Date: 29.4.80 Maximum Marks: 100 Time: 2 hours

Note. Answer any five questions.
Each question carries 20 marks.

1. The individuals represented by solid squares and circles in the accompanying pedigree show a certain trait.

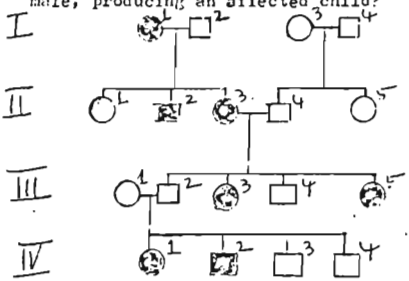
   a) Could the trait be inherited on the basis of a dominant allele ?

   b) Could the trait be inherited on the basis of a recessive allele ?

   Now, ignore generation IV, and assume that the trait is inherited on the basis of a dominant allele.

   c) What are the genotypes of individuals No. I_2, I_4, II_2 and II_4?

   d) What is the probability of II_1, married to an unaffected male, producing an affected child?



2. In the gene pool of a population, the frequency q of the recessive allele b is 0.001.

   a) If a man having genotype Bb marries his mother's step-brother's daughter (i.e., III_1 marries III_2 in the accompanying pedigree), what is the probability that their first child will have the genotype bb ?

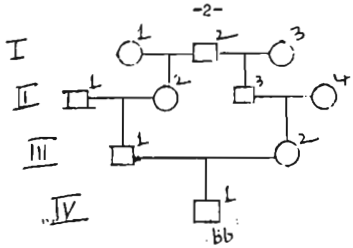
   b) If the same man marries an unrelated woman, what is the probability that their first child will have the genotype bb ?

   c) How many times more likely is production of a bb child if the man marries his mother's step-brother's daughter than it is if he marries an unrelated woman ?

   d) How many times more likely is production of a bb child if the man marries his mother's brother's daughter than it is if he marries his mother's step-brother's daughter?

   [Ignore the small probability that the man and/or his wife inherited allele b from unrelated parents except where such assumption is necessary].

p.t.o.



3. Members of a pair of identical twins have the same genotype. A pair of identical twin brothers married a pair of identical twin sisters. The brothers were albinos, the sisters normally pigmented. One couple produced a normally pigmented daughter and the other couple a normally pigmented son.

   a) If these first cousins marry each other, what is the probability that their first child will be an albino?

   b) Demonstrate your answer to problem (a) diagrammatically.

   c) What is the probability that their second child will also be an albino ?

4. Consider a trait inherited on the basis of a common, dominant, autosomal allele.

   a) Give all possible genotypes of spouses in case each of the following types of marriages:

      i) affected x affected,
      ii) affected x normal, and
      iii) normal x normal.

   b) Give the proportions of affected and unaffected children in case of each marriage type.

5.a) In a random mating population 350 persons were typed for MN blood group system. The phenotypic frequencies of MM, MN and NN were 197, 131 and 22, respectively. Which one of the following gene frequency estimates is correct ?

   a) M = 0.25, N = 0.75

   b) M = 0.50, N = 0.50

   c) M = 0.75, N = 0.25

   d) M = 0.00, N = 1.00.

   b) Assume a population in which the blood group genes O, A, and B are in the proportions 0.6, 0.3, and 0.1. Assuming random mating the frequency of persons with the four blood groups were estimated as follows. Which one of the estimates is correct ?

      i) O = 0.36, A = 0.13, B = 0.45, AB = 0.06

      ii) O = 0.36, A = 0.45, B = 0.13, AB = 0.06

      iii) O = 0.06, A = 0.36, B = 0.13, AB = 0.45.

6.a) The inbreeding coefficient (F) of a child

   a) Whose parents were first cousins is - - - - - - -

   b) whose parents were second cousins is - - - - - - -

   c) whose parents were uncle-neice is - - - - - - -

   Fill in the blanks.

P.T.O.

6.b) In a population survey parental consanguineity was ascertained for 1000 couples. Of these 909 were unrelated, 3 uncle-neice, 30 first cousins and 58 second cousin marriages. Which one of the following F values is correct ?

(a) 0.00294 (b) 0.03156 (c) 0.003156.

7. Which one of the following statements is correct ?

a) the frequency of any particular homozygote is the square-root of the frequency of the gene

b) the frequency of any particular heterozygote is a square of the product of the frequency of the two genes making the heterozygote.

c) The people of Maharashtra constitute a Mendelian population.

d) Assuming no inbreeding then the number of direct lineal ancestors of an individual is given by 2^n, where n = number of generations.

e) The values of coefficient of inbreeding (F) vary from 0 - 100.

f) $F = \Sigma [(1/4)^n]$, where n is the number of ancestors in the path.

8. Albinism, a recessive triat, occurs about 1 in 10,000 persons. Estimate the frequency of both the recessive and dominant alleles as well as the frequency of heterozygote carriers of albinism.

---

79 - 80: 325

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

ANNUAL EXAMINATIONS

General Science-4: Sociology

Date. 31.3.80 Maximum Marks: 100 Time: 3 hours

Note: Answer five questions.
All questions carry equal marks.

GROUP A

Answer any two questions.

1. What is sociology about? How is it related to the other social sciences?

2. Write notes on any 3 (three) of the following, with suitable illustrations.

i) Social positions
ii) Norms; institutionalisation of norms
iii) Role-set
iv) Role conflict, role deviation, role reinforcement.

3. EITHER

Discuss the main approaches to the study of stratification.

OR

Discuss caste and class in their changing context.

GROUP B

4. EITHER

Elucidate the concept of family. How does family differ from lineage and clan? What chief interest of society is served by the social function of family?

OR

What are monogamy and polygamy? Enumerate the different types of preferential marriage found in Indian society? What are the primary causes of polygyny?

5. EITHER

Define kinship. How can you differentiate between the classificatory and descriptive systems? Describe briefly the importance of studying kinship in complex societies.

OR

Write notes on (any four)

(a) Family of orientation
(b) Joint family
(c) Cecisbeism
(d) Cognate
(e) Neolocal
(f) Household.

p.t.o.

GROUP C

6. Point out the broad findings of the following tables, and discuss their respective sociological significance. What type of statistical tests do you propose to strengthen your arguments?

a) Table 1: Occupational distribution of rural households by caste-religion group: 1937 and 1977.

Occupation	Caste/Group: Hindu high		Hindu middle		Hindu low		Muslims		Tribals		All	
	1937	1977	1937	1977	1937	1977	1937	1977	1937	1977	1937	19
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(1
Farmer	-	29	6	46	14	10	—	2	4	--	24	8
Land rentier	11	1	22	5	13	4	--	--	3	3	49	1
Cultivator	--	3	11	29	9	25	--	--	--	8	20	6
Share cropper	--	2	8	8	55	37	2	--	49	31	114	7
Daily labourer	--	2	8	7	53	97	7	23	42	169	110	29
Artisan	6	4	6	3	4	3	--	--	--	1	16	1
Trader	--	5	4	4	4	2	--	1	--	1	8	1
Service-holder	5	16	5	3	4	2	--	1	--	10	14	3
Others	--	--	--	--	8	3	--	1	1	8	9	1
Total	22	62	70	104	164	183	9	28	99	231	364	60

[Source: Socio-economic survey in the surrounding villages of Sriniketan Birbhum: 1979.]

b) Table 2: Year-wise distribution of rural households by family types: 1951 to 1965-66.

Year	Estimated percentage to total ± s.e. in rural West Bengal				
	Non-familial unit	Nuclear family	Extended family	Total	Sample
1951	16.81 ± 3.02	38.13 ± 4.83	45.06 ± 3.18	100.00	(900)
1955-56	11.88 ± 1.19	47.19 ± 2.38	40.93 ± 2.12	100.00	(618)
1960-61	9.19 ± 1.66	58.73 ± 3.52	32.08 ± 2.09	100.00	(2903)
1965-66	9.70 ± 0.53	52.11 ± 0.70	38.19 ± 0.78	100.00	(7822)

[Source. Study of family structures, West Bengal, 1977.]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS
Mathematics-5: Real Analysis

Date: 17.9.79 Maximum Marks: 100 Time: 3 hours

Note: All questions may be attempted.
$\bar{A}$ means the closure of A.
Sum of the marks is 115.

1. Determine the closure, interior, boundary and derivative of the set of real numbers

$$S = E\left\{x : x < \frac{1}{x}\right\}$$ [11]

2.a) What is a topological space? What is an accumulation point of a set (in a topological space)? [11]

b) Give an example to show that the following statement is FALSE. In a topological space, let B_n be a closed set for $n \geq 1$. Then $\bigcup_1^\infty B_n$ is a closed set. [12]

3.a) In a topological space, a set is closed if and only if it contains all its accumulation points (prove). [13]

b) S is a set in a T_1-space and a is an accumulation point of S. Prove that every open neighbourhood of a contains infinitely many points of S. [13]

4. Below, A, B and G are sets in a topological space X.

a) Prove that if G is an open set that does not intersect B, then G does not intersect $\bar{B}$. [11]

b) $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$ (prove) [11]

c) $\bar{A} \cap (\bar{B})^c \leq \overline{A \cap (B^c)}$. [11]

d) G is an open set. Prove that $(G \cap \bar{B}) \leq (\overline{G \cap B})$ [11]

e) Give a concrete example to show that the above statement (in (d)) is false if G is not required to be an open set. [11]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III. 1979-80

MID-YEAR EXAMINATIONS

Mathematics-6: Real Analysis

Date. 18.12.79 Maximum Marks: 100 Time: 3 hours

Note. All questions may be attempted. However you will not be given more than 100 marks. $\bar{A}$ means the closure of A. X and Y are always topological spaces.

1.a) What is a T_2-space ? [4]

b) Y is a T_2-space. f and g are continuous mappings of X to Y. Prove that $E\{x: f(x) \neq g(x)\}$ is an open set (in X). [11]

c) Give an example which shows that the above statement (in (b)) becomes false if Y is not required to be a T_2-space. [5]

2.a) What is a connected set (in a topological space)? [4]

b) S is a connected set in X. Prove that $\bar{S}$ is also a connected set. [11]

c) Show that the finite closed interval $[a,b]$ in the topological space of real numbers is a connected set. [5]

3.a) What is a compact set (in a topological space)? [4]

b) Prove that in a T_2-space, a compact set is a closed set. [5]

c) Prove that in a T_2-space, a compact set and a point not in it can be enclosed in disjoint open sets. In other words, let X be a T_2-space and C a compact set in X. Let $y, \in X$, be such that $y \notin C$. Then there are disjoint open sets P and Q such that $C \subseteq P$ and $y \in Q$. [10]

d) State the Heine-Borel theorem (on the real line). [4]

4.a) X is a topological space. $f(x)$ is a continuous, real-valued function defined on X. That is, f is a continuous mapping of X to the topological space of real numbers. Prove that

$$g(x) = \{f(x)\}^2$$

is also a continuous function. [12]

5.a) Y is a set in X. How is the relative topology on Y defined ? Proofs are not needed. [5]

b) X is a T_2-space and Y is a subset of X. Show that the topological space Y (with the relative topology it gets from X) is also a T_2-space. [9]

6.a) A and B are closed sets in X. Prove that if $(A \cup B)$ is a compact set, then A and B are compact sets. [11]

b) Give an example of a bounded continuous function on $(0,\infty)$ which attains its supremum but does not attain its infimum. [5]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS
Mathematics-6: Real Analysis

Date: 25.2.80 Maximum Marks: 100 Time: 3 hours

Note: All questions may be attempted.
Marks are given in brackets.

1.a) Explain the concept of uniform convergence of a sequence of functions. [7]

b) On a set E, $f_n \to f$ and $g_n \to g$, both uniformly. Show that $(f_n + 2g_n) \to (f + 2g)$ uniformly. [13]

c) On a set E, $|f_n(x)| \leq 1$ and $|g_n(x)| \leq 1$ always.

$f_n(x) \to p(x)$ and $g_n(x) \to t(x)$, both uniformly. Show that $f_n g_n \to pt$ uniformly. [13]

d) Give an example to show that the above statement, in (c), becomes false if the conditions " $|f_n(x)| \leq 1$ and $|g_n(x)| \leq 1$ always." are dropped. [13]

2. State and prove Dini's theorem on the convergence of a monotone sequence of continuous functions on [a, b] to a continuous limit function. [13]

3.a) What are the upper and lower limits of a bounded sequence of real numbers? [7]

No proofs are needed.

b) $\{a_n\}$ and $\{b_n\}$ are bounded sequences of positive real numbers. Prove that $\overline{\lim_n}\,(a_n b_n) \leq \overline{\lim_n}\, a_n \cdot \overline{\lim_n}\, b_n$. [13]

c) Give a numerical example to show that the above statement, in (b), becomes false if $\leq$ is replaced by $=$. [13]

d) $\{a_n\}$ is a bounded sequence.

$$b_n = \frac{a_1 + 2a_2 + a_3 + 2a_4 + \cdots + a_{2n-1} + 2a_{2n}}{3n}.$$

Prove that $\overline{\lim_n}\, b_n \leq \overline{\lim_n}\, a_n$. [13]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
ANNUAL EXAMINATIONS
Mathematics - 4: Real Analysis

Date: 23.5.80 Maximum Marks: 100 Time: 3 hours

Note: All questions may be attempted.
Marks are given in brackets.

1.a) On a set E, $f_n(x) \to g(x)$ uniformly. Show that $f_n^2 \to g^2$ uniformly in case $g(x)$ is a bounded function. [8]

b) Give an example to show that the above statement is not true if $g(x)$ is not required to be bounded. [8]

c) Show that $f(x) = \frac{1}{1^x} + \frac{1}{2^x} + \frac{1}{3^x} + \ldots + \frac{1}{n^x} + \ldots$ is a continuous function on the open half-line $1 < x < \infty$. [9]
(You may assume the fact that the series is convergent when $x > 1$).

2.a) In a compact topological space, every infinite sequence has at least one limit point (prove). [6]

b) $\{a_n\}$ is a bounded sequence of real numbers. What is the definition of $\overline{\lim}\ a_n$? [8]
Also define $\overline{\lim_n}\ a_n$ in terms of the concept of supremum.
Show that the two definitions are equivalent.

c) S is the set of positive integers having 2 as the first digit (in the usual decimal representation). Thus $2 \in S$, $24 \in S$, $42 \notin S$. [6]
Find the upper and lower densities of S. Give reasons. [9]

d) A is a set (of positive integers) having a density. $(S \cup T) = A$ and $(S \cap T)$ is empty. $\bar{\delta}(S) + \bar{\delta}(T) \leq \bar{\delta}(A)$, Show that S has a density.
$\bar{\delta}$ means upper density. [8]

3. Show that the radius of convergence of $a_0 + a_1 x + \ldots + a_n x^n + \ldots$ is 0 if and only if $\overline{\lim_n} \frac{1}{\left|a_n\right|^{\frac{1}{n}}} = 0$. [8]

4.a) What is an absolutely continuous function $f(x)$ on $[a,b]$? [5]

b) Show that the product of two absolutely continuous functions on $[a,b]$ is absolutely continuous. [7]

c) $f(x)$ is > 0 and is absolutely continuous on $[a,b]$. Show that $\frac{1}{f(x)}$ is also absolutely continuous on $[a,b]$. [9]

p.t.o.

5.a) What is a probability distribution on $(-\infty, \infty)$? [5]

b) When do we say that $P_n \Rightarrow P$? The P's are probability distributions on $(-\infty, \infty)$. [5]

c) $P_n \Rightarrow P$. $F(x)$ is continuous on $[a,b]$. Show that the convergence of $F_n(x)$ to $F(x)$ is uniform on $[a,b]$. [7]
$F_n(x)$ is the distribution function of P_n.

d) $P_n \Rightarrow P$ and $F(x)$, the distribution function of P, is continuous throughout $(-\infty, \infty)$. Show that
$F_n(x) \to F(x)$ uniformly on $-\infty < x < \infty$. [9]

INDIAN STATISTICAL INSTITUTE

B.Stat.(Hons.) Part III: 1979-80

PERIODICAL EXAMINATIONS

Statistics- 6: Inference

Date: 24.9.79 Maximum Marks: 100 Time: 3 hours

Note: Attempt all the questions. Marks for each question and specified in the brackets after that question.

1.a) Define 'Sufficient statistic.'

b) State clearly the factorisation criterion for obtaining a sufficient statistic.

c) Let $X \sim U(\theta_1, \theta_2)$, $-\infty < \theta_1 < \theta_2 < \infty$. Let $x_1, \ldots, x_n$ be a random sample from X. Show that (a, b) is a sufficient statistic for (θ_1, θ_2) where $a = \min_i x_i$ and $b = \max x_i$.

d) Let t be a sufficient statistic for a family $\{ f(\cdot, \theta), \theta \in \Omega \}$ of distributions on a sample space X. Let the family of distributions $\{ g(\cdot, \theta), \theta \in \Omega \}$ - t be complete. Show that no function of t which generates a coarser partition of the sample space than that generated by t is sufficient.

e) Let $N = (N_1, \ldots, N_k)$ have a distribution with parameters $\theta_1, \ldots, \theta_k$ such that $\theta_i > 0$ for all i and $\sum_1^k \theta_i = 1$ with the probability function

$$P\{N_1 = n_1, \ldots, N_k = n_k\} = \begin{cases} \dfrac{n!}{n_1! \ldots n_k!} & \text{if } n_1, \ldots, n_k \text{ are nonnegative integers such that } \sum_1^k n_i = n \\ 0 & \text{otherwise} \end{cases}$$

where n is a given fixed positive integer. Show that N is a complete sufficient statistic for $(\theta_1, \ldots, \theta_k)$. [2+5+15+13+15]=[50]

2.a) Let X be a random variable the distribution of which is indexed by θ, $\theta \in \Omega$. Let $x_1, \ldots, x_n$ be a random sample from X with the joint density $f(x, \theta)$. Let $g(x_1, \ldots, x_n)$ be an unbiased estimator of $\psi(\theta)$. Show that, under certain conditions to be specified by you,

$$V_\theta(g) \geq \frac{\{\psi'(\theta)\}^2}{E_\theta\left(\frac{\partial^2 \log f}{\partial \theta^2}\right)}$$

p.t.o.

2.b) Let X have a distribution with the density

$$f(x, \theta) = \frac{\theta^k}{\Gamma(k)} e^{-\theta x} x^{k-1} \qquad 0 < x < \infty \;,\; \theta > 0 \;.$$

Let $x_1, \ldots, x_n$ be a random sample from X. Obtain a function $\gamma(\theta)$ of θ, and its unbiased estimator the variance of which attains the lower bound in (a). Does there exist another parametric function which is not a linear function of $\gamma(\theta)$ which has an unbiased estimator the variance of which attains the lower bound in (a)? If yes, exhibit one such. If no, give reasons for your answer. [10+10+5]=[25]

3.a) Let X have a distribution indexed by a parameter θ, $\theta \in \Omega$. Let $x_1, \ldots, x_n$ be a random sample from X. Let $G(\neq \emptyset)$ be the class of all unbiased estimators with finite variance of $\gamma(\theta)$. Let Z be the class of all zero functions. Show that $g(x) \in G$ has minimum variance among all members of G if and only if it is uncorrelated with every member of Z having finite variance.

b) Let $Y \sim N_n(X\beta, \sigma^2 I)$ where $X_{n \times m}$ is known and β is unknown, $\beta \in R^m$. Show that BLUE of every estimable linear parametric function is in fact the minimum variance unbiased estimator in the class of all unbiased estimators of that parametric function. [10+15]=[25].

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

MID-YEAR EXAMINATIONS

Statistics - 6: Inference

Date: 9.12.79 Maximum Marks: 100 Time: 3 hours

Note: Attempt all the questions. The marks for each question are given in the brackets at the end of the question.

1. Let X be a random variable with a distribution indexed by a parameter θ. Let X possess finite first and second moments. Let x_1, x_2, x_3 be a random sample from X. Let $g_1 = 2x_1 + 3x_2 + 4x_3$ be an unbiased estimator of a function $\psi(\theta)$ of θ. Consider another estimator $g_2 = 3(x_1 + x_2 + x_3)$. Is it an unbiased estimator of $\psi(\theta)$? Suppose you have to choose between the two estimators g_1 and g_2. Which one do you choose and why ? Let x_4 be another observation on X choosen independently of x_1, x_2 and x_3. Let $g_3 = 2.25(x_1+x_2+x_3+x_4)$ and $g_4 = x_1 x_2 x_3 x_4$. Suppose you are told that g_4 is unbiased for $\psi(\theta)$. Compare g_1, g_2, g_3 and g_4. [25]

2.a) Show that the sample k^{th} raw moment is consistent for the k^{th} population raw moment $(k \geq 1)$.

b) Hence or otherwise show that sample k^{th} central moment is consistent for population k^{th} central moment $(k \geq 2)$.

c) Is a consistent estimator necessarily unbiased ? Give reasons for your answer. [10+4+6]=[20]

3.a) Explain clearly the method of moments and the method of maximum likelihood for estimating parameters.

b) Let X have a distribution indexed by a parameter θ. Let $x_1, \ldots, x_n$ be a random sample from X. Let there exist an unbiased estimator $g(x_1, \ldots, x_n)$ of $\psi(\theta)$ the variance of which attains Cramer-Rao lower bound. Show that $g(x_1, x_2, \ldots, x_n)$ satisfies the likelihood equation.

c) Let $X \sim N(\mu, 4)$ where $\mu \in [-4, 8]$. Let $x_1, \ldots, x_n$ be a random sample from X. Obtain the maximum likelihood estimator of μ based on the random sample.

d) Give an example where the method of moments does not give a unique estimator of a parametric function. [6+5+13+6]=[30]

4. Consider a power series distribution with probability

$$P\{X = r\} = \frac{a_r}{f(\theta)}\theta^r \quad \text{for } r = c, c+1, \ldots, \infty$$

where c is a known integer. Let $x_1, \ldots, x_n$ be a random sample from X and let $T = \sum_{1}^{n} x_i$.

a) Show that T is a complete sufficient statistic for θ and the distribution of T is of the form

$$P\{T = t\} = b_t \theta^t \div [f(\theta)]^n \quad \text{for } t = nc, nc+1, \ldots, \infty$$

b) Show that the minimum variance unbiased estimator of θ^r is $\begin{cases} 0 & \text{if } t < r \\ b_{t-r} \div b_t & \end{cases}$

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III. 1979-80
PERIODICAL EXAMINATIONS
Statistics-6: Inference

Date. 13.2.80 Maximum Marks. 100 Time. $2\frac{1}{2}$ hours

Note. Attempt all questions.
Marks are given in brackets.

1. Explain the following.
(i) Simple hypothesis (ii) composite hypothesis,
(iii) Non-randomised test (iv) randomised test
(v) level of a test (vi) uniformly most powerful test. [6 x 3]=[18]

2. Let $x_1,\cdots,x_m$ and $y_1\cdots y_n$ be independent samples from $N(\xi, 1)$ and $N(\eta, 1)$ respectively.

a) Show that the most powerful level α test ϕ^* for testing the simple hypothesis

$$H: \xi = \eta = \frac{m\xi_1 + n\eta_1}{m+n}$$

against the simple alternative $K: \xi = \xi_1, \eta = \eta_1$ (ξ_1 and η_1 are known and $\xi_1 < \eta_1$) rejects H iff $\bar{y} - \bar{x} > c$ where c is so chosen that ϕ^* is of size α.

b) Show that the power of ϕ^* is an increasing function of $\eta - \xi$.

c) Deduce from the above results that ϕ^* is the UMP level α test for testing $H^*: \eta \leq \xi$ against $\eta > \xi$. [20+6+14]=[40]

3. Let $x_1,\cdots,x_n$ be a random sample from $N(\theta, 1)$. Does there exist a uniformly most powerful level α test for testing $\theta = 0$ against $\theta \neq 0$? If yes, obtain the test function. If no, give reasons? [12]

4.a) When do you say that a family of distributions indexed by a single parameter θ has monotone likelihood ratio?

b) Let θ be a real parameter and let X have a probability density with monotone likelihood ratio in $T(x)$. Obtain the uniformly most powerful test of level α for testing $H: \theta \leq \theta_0$ against $K: \theta > \theta_0$. [4+26]=[30]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
ANNUAL EXAMINATIONS

Statistics - 6: Inference

Date: 26.5.80 Maximum Marks: 100 Time: $3\frac{1}{2}$ hours

Note: The paper consists of 110 marks. You may attempt any part of any question. The maximum you can score is 100.

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution which has probability density function

$$f_\theta(x) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $0 < \theta < \infty$. Show that the UMP test of level α for testing $H: \theta = \theta_0$ against $K: \theta > \theta_0$ is given by the the critical region

$$\left\{(x_1, \ldots, x_n): \prod_{i=1}^{n} x_i > \exp\left[-\chi^2_{\alpha,2n} / 2\theta_0\right]\right\}$$

where $\chi^2_{\alpha,2n}$ is the lower α-point of the χ^2 distribution with $2n$ degrees of freedom. [20]

2. Let X_{ij}, $j = 1, \ldots, n_i$ be a random sample from $N(\mu_i, \sigma^2)$ $i = 1, \ldots, k$. Further let the random samples be independent and [illegible] be unknown. Derive the likelihood ratio test for testing $H: \mu_1 = \ldots = \mu_k$ against $K: \mu_i$'s are not all equal. [15]

3. Let θ be a real parameter and let $X = (X_1, \ldots, X_n)$ be a random vector with probability density

$$f_\theta(x) = c(\theta) e^{\theta T(x)} h(x)$$

where $c(\theta)$ is a function only of θ (not involving x) and $T(x)$ and $h(x)$ are functions of only x (not involving θ). Obtain the UMP unbiased level α test for testing $H: \theta = \theta_0$ against $K: \theta \neq \theta_0$. [15]

4. Let X have a distribution F belonging to $\tilde{\mathcal{F}}$. Let $T(x)$ be sufficient for a family of distributions $\mathcal{F}_0 \subset \mathcal{F}$. Show that a test ϕ similar for testing $F \in \mathcal{F}_0$ is of Neyman's structure with respect to $T(X)$ if and only if $T(X)$ is boundedly complete when $F \in \mathcal{F}_0$ [10]

Let $X \sim N(\mu, \sigma^2)$ where σ^2 is unknown. Obtain the UMP unbiased level α test for testing $H: \mu = 0$ against $K: \mu \neq 0$. [12]

p.t.o.

6. Let X have a discrete distribution. Let ϕ be a randomised test based on X. Obtain a nonrandomised test (involving some additional random variables, if required) which has the same power function as ϕ. [10]

7. For each $\theta_0 \in \Omega$, let $A(\theta_0)$ be the acceptance region of a nonrandomised test of level α for testing $H(\theta_0) : \theta = \theta_0$ and for each sample point x let $S(x)$ denote the set of parameter values

$$S(x) = \{\theta;\ x \in A(\theta),\ \theta \in \Omega\}.$$

a) Show that $S(x)$ is a family of confidence sets for θ at confidence level $1-\alpha$.

b) Show that if $A(\theta_0)$ is UMP for testing $H(\theta_0)$ against alternatives $K(\theta_0)$ then $S(x)$ minimises the probability

$$P_\theta\ \theta' \in S(x) \quad \text{for all } \theta \in K(\theta').$$

c) Let the family of densities $f_\theta(x)$, $\theta \in \Omega$ have monotone likelihood ratio in $T(x)$ and suppose that c.d.f. $F_\theta(t)$ of $T(x)$ is continuous in t for each fixed θ. Show that there exists a uniformly most accurate lower bound $\underline{\theta}$ for θ at confidence level $(1-\alpha)$. [2+7+7]=[16]

8. Let X follow $N(\mu, \sigma^2)$ where σ^2 is unknown. Obtain the level $1-\alpha$ confidence interval of shortest expected length based on a random sample of size n from X. [12]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1978-79
PERIODICAL EXAMINATIONS

Statistics- 7: Linear Estimation and Analysis of Variance

Date: 3.9.79 Maximum Marks: 100 Time: 3 hours

Note: Do as many questions as you can. The paper carries 110 marks but the maximum you can score is 100 marks
Marks are given in brackets

1 Prove or disprove the following statements in case of the model $(Y, X\beta, \sigma^2 I)$.

i) Sum of squares of deviations of observed values of y from their expected values is minimum at the points $\hat{\beta} = \beta$ where $\hat{\beta}$ is a solution of $X'X\beta = X'Y$.

ii) If $P'\hat{\beta}$ is a l, u, e of a linear parametric function $P'\beta$, then it is unique and vice-versa.

iii) If the class of linear unbiased estimators for $P'\beta$ is non-empty then $P'\hat{\beta}$ is blue.

iv) All linear parametric functions are estimable.

v) The l.u.e $P'\hat{\beta}$ of $P'\beta$ is uncorrelated with all other linear estimators $L'Y$ if $L'X = \underline{0}$.

vi) An unbiased estimator of σ^2 is $(Y - X\hat{\beta})'(Y - X\hat{\beta})$

vii) $X\bar{S}S = X$ where $S = X'X$ and $\bar{S}$ is a g-inverse of S.

[5+10+5+5+3+5+2]=[35]

2 Obtain best linear unbiased estimator for an estimable parametric function $P'\beta$ in case r linear constraints $H'\beta = \alpha$ (given) are imposed on β in $(Y, X\beta, \sigma^2 I)$. [10]

3 Provide 95% confidence interval for a linear parametric function $P'\beta$ on the basis of a random sample $Y = (y_1, \ldots, y_{15})'$ of size 15 from a normal population with $EY = X\beta$ and $D(Y) = \sigma^2 I$ where β is 5×1 vector and rank of X is 5. [10]

4 A variate y is expected to be related with x_1, x_2 and x_3 by

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

and has variance σ^2.

Thirty independent measurements on y and x's yield

$\bar{y} = 40$, $\bar{x}_1 = 30$, $\bar{x}_2 = 25$, $\bar{x}_3 = 20$

$$S = \begin{pmatrix} 20 & 9 & 7 \\ 9 & 30 & 10 \\ 7 & 10 & 30 \end{pmatrix}$$

$Q_1 = 35$, $Q_2 = 45$, $Q_3 = 40$, $Q_0 = 30$

p.t.o

where $S = (S_{ij})$, $S_{ij} = \sum_k (x_{ik} - \bar{x}_i)(x_{jk} - \bar{x}_j)$

$Q_i = \sum_k (y_k - \bar{y})(x_{ik} - \bar{x}_i)$; $i = 1,2,3$

and $Q_o = \sum_k (y_k - \bar{y})^2$

i) Is $\theta = \lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ estimable whatever be λ's ?

ii) Obtain b,l,u,e for θ.

iii) Obtain an unbiased estimator for σ^2

iv) Obtain 95% confidence interval for $\lambda = 2\beta_1 + \beta_2 - 3\beta_3$

v) Is $\lambda = 2$ at 5% level of significance?

vi) Is $\beta_1 = \beta_2 = \beta_3 = 1$ true at 5% level of significance? [5+10+5+5+10]=[40]

5 Let $\bar{A}$ denote a g-inverse of a matrix A. Then show that

i) For a given A, a g-inverse of A always exists.

ii) $r(A) = r(\bar{A}A) = r(A\bar{A}) = tr(\bar{A}A)$

$= tr(A\bar{A}) \leq r(\bar{A})$

iii) If for a given G; GA is idempotent and r(GA) = r(A) then G is a g-inverse of A. [5+5+5]=[15]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

MID-YEAR EXAMINATIONS

Statistics-7: Linear Estimation and Analysis of Variance

Date: 6.12.79 Maximum Marks: 100 Time: 3 hours

Note: Do as many questions as you can The paper carries 110 marks. The maximum you can score is 100. Marks are given in brackets.

1. Prove or disprove the following statements.

i) In the model $(Y, X\beta, \Sigma)$, β is a random variable with mean μ (known) and dispersion matrix T. Then the estimators $d = a + L'Y$ are BLUE for $P'\beta$ when $L = (\Sigma + XTX')^{-1} XTP$ and $a = \mu'P - \mu'X'L$.

ii) In the GGM model $(Y, X\beta, \sigma^2 G)$; $M(G) \subset M(T)$ and $M(X) \subset M(T)$ where $T = G + XUX'$, U being symmetric Then rank $(X') =$ rank $(X'TX)$.

iii) A set of least squares estimator for $\beta = (\beta_1, \cdots, \beta_p)'$ in the model $(Y, X\beta, \sigma^2 G)$ with G-non singular, is given by $\hat{\beta} = SQ$ where $S = X'G^{-1}X$ and $Q = X'G^{-1}Y$.

iv) The least square estimator $P'\hat{\beta}$ for $P'\beta$ is BLUE, in the model $(Y, X\beta, \sigma^2 I)$. [10+5+10+5]=[30]

2. What do you mean by confidence sets ? Obtain $100(1-\alpha)\%$ confidence ellipsoids for $H\beta$ based on a random sample $(y_1, \cdots, y_n)$ from a normal population $N(X\beta, \sigma^2 I)$ where $H\beta = (P_1'\beta, \cdots, P_k'\beta)'$, the $P_i'\beta$ $(i = 1, \cdots, k)$ being k-linearly independent estimable functions. [15]

3. What do you mean by analysis of variance ? Give estimates for treatment effects and block effects, under an additive model, in a two-way classification with one observation per cell. Give ANOVA for testing the hypotheses that all treatment effects are same and all block-effects are same. [20]

4. Twelve cows, in a cheese plant, were devided into three batches according to race, age etc., each batch consisting of 4 cows. Four types of ration were supplied randomly, to every batch separately, such that a cow in a batch got one type of ration. The data for growth in the milk yield (in gms.) was found as below,

Types of ration	Batches of cows		
	1	2	3
1	500	1000	1500
2	800	600	800
3	1200	500	800
4	1500	1600	1000

Assume the data to come from a normal population with $y_{ij} = \mu + t_i + b_j$ $i = 1, \cdots, 4$ $j = 1, \cdots, 3$.

i) Is there any difference in the effects of various rations ?

ii) Is there any difference in the effects of batches of cows on the milk yield increase ?

iii) Is there any intraction between the types of rations and the types of cows ? [10+10+5]=[25]

5. Let $X_{(i)}$ be the i^{th} column vector of the design matrix X in $(Y, X\beta, \sigma^2 I)$. Let $X'_{(i)} X_{(i)} = C_i^2$ (a given constant) and $\hat{\beta}_i$ be the least squares estimator of β_i. Then show that $V(\hat{\beta}_i) \geq 1/C_i^2$ and that $V(\hat{\beta}_i)$ is minimum when the columns of X are orthogonal to each other. [10]

6. A random sample $(y_1, \ldots, y_n)$ is drawn from a normal population $N(X\beta, \sigma^2 I)$. Give test-statistics for testing the hypotheses

$$\beta_1 = \ldots = \beta_p = \beta_o, \text{ where } \beta_o \text{ is not specified.}$$

Give ANOVA for the same. [10]

[79-80] 361 [1]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS
Statistics-8: Sample Surveys

Date: 10.9.79 Maximum Marks: 100 Time: 3 hours

Note: Four questions are to be answered. Each question carry equal marks. Maximum score of each subdivision of a question is indicated against each question.

1.a) Define and illustrate the concept of

i) population
ii) sample
iii) sample space
iv) sampling design
v) sampling strategy
vi) unbiased estimation

in the context of survey sampling.

b) Give some situations under which a sampling strategy may be said to be better than another sampling strategy. [20+5]=[25]

2.a) Define and illustrate the concept of simple random sampling with replacement (srswr).

b) Taking an artificial population,

U :	1	2	3	4
Y :	2	4	8	6

and taking a srswr- design of n= 2 draws, show that sample mean is an unbiased estimator of population.

c) For srswr, find an expression for variance of sample mean in the general case (arbitrary n) and an unbiased estimator of the variance of sample mean. [5+10+10]=[25]

3.a) Define and illustrate the concept of simple random sampling without replacement (srswor).

b) Show that in srswor, sample mean is an unbiased estimator of population mean. Find an expression for the variance of the sample mean together with an unbiased estimator of this variance.

c) Hence or otherwise show that the sample mean based on a simple random sample (srs) of size n drawn without replacement is preferable to the sample mean based on a srs selected by n draws made with replacement as an estimator of population mean in some sense. [6+15+4]=[25]

4.a) To estimate the proportion P of individuals in the population having a certain characteristic, you are required to draw a simple random sample (srs) of n individuals without replacement from a population of size N. Give an unbiased estimator of P together with its variance and an unbiased estimator of this variance.

b) To estimate the total number of persons living in an area consisting of two zones A and B, a srswor of n households is drawn from the list of N households living in the area and the number of members of each sampled household is noted. From these data, find an unbiased estimator of the total number of persons living in zone A and indicate an expression for the unbiased estimator of the variance of this estimator. Assume that at least two sampled households belonged to zone A.

4.c) A simple random sample of size 3 is drawn from a population of size N with replacement. Show that the probabilities that the sample contains 1,2 and 3 different units (for example, aaa, aab, abc respectively) are,

$$P_1 = \frac{1}{N^2}, \quad P_2 = \frac{3(N-1)}{N^2}, \quad P_3 = \frac{(N-1)(N-2)}{N^2}.$$

[(4+4+4)+(4+3)+6]=[25]

5.a) Define and illustrate the concept of stratified random sampling. List some situations under which stratified random sampling should be used.

b) On the basis of a srs of n units drawn without replacement from a population of size N, find an unbiased estimate of

$$\bar{Z} = \sum_{i \neq j=1}^{N} Y_i^2 Y_j ,$$

where Y_i is the value of y, the variable of interest, on the unit labelled i (i = 1,. , N).

c) In order to estimate the proportion of persons (P) belonging to blood group O, living in a certain island, an anthropologist wants to make a sample survey taking a simple random sample of 'n' persons from the total number of 3200 individuals in the island. He desires to take a sample large enough to ensure that chance is .95 that the unknown true value of the population proportion will be within a range of .05 from the sample estimate on either side. Assuring that a very rough stimate of P is .5, find the desired sample size.. [10+6+9]=[25]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
MID-YEAR EXAMINATIONS

Statistics-8: Sample Surveys

Date: 15.12.79 Maximum Marks: 100 Time: 3 hours

Note: This paper contains 125 marks. Answer as many questions as you can. Maximum you can score is 90. 10 marks have been allotted for submission of note books on practical problems done in your class.

1.a) In a stratified simple random sampling (without replacement) scheme obtain the optimum allocation of sample sizes n_h's, $h = 1,\ldots,L$, to the different strata minimising the variance of the estimate of the population mean subject to a fixed total cost of the survey. Assume a cost function of the type

$$C = C_0 + \sum_{h=1}^{L} C_h n_h$$

where C = total cost, C_0 an overhead cost and C_h = cost of sampling one unit from the h^{th} stratum.

b) With 2 strata a sampler, using a stratified simple random sampling (with replacement) scheme, would like to have $n_1 = n_2$ for administrative convenience. If V_{eq} and V_{opt} denote the variances of the usual unbiased estimate of the population mean given by (i) $n_1 = n_2$ and (ii) Neyman's allocation respectively, show that the proportional increase in variance with respect to V_{opt} is given by

$$\left(\frac{r-1}{r+1}\right)^2$$

where $r = \frac{\hat{n}_1}{\hat{n}_2}$, $\hat{n}_1$, $\hat{n}_2$ being the values of n_1 and n_2 as given by Neyman's allocation.

c) Derive an expression for the estimate of the gain in precision due to stratification. [10+4+6]=[20]

2. What is cluster sampling and why is it resorted to ? Assuming that the clusters are of equal sizes, suggest an estimate which is unbiased under simple random sampling. Obtain its variance in terms of intra-class correlation coefficient and examine the efficiency of cluster sampling with respect to single unit sampling. [6+3+6+5]=[20]

3.a) Describe the ratio estimate of the population mean and state a situation where you think you would use it. Obtain the bias in it and examine its magnitude. Obtain also an approximate expression for the variance of the ratio estimate of population mean.

b) Can you give a ratio-type estimator which is unbiased under simple random sampling without replacement ? [(2+4)+4+5+5]=[20]

4.a) Describe systimatic sampling scheme with an illustration. Give unbiased estimate of population mean and obtain its variance under systematic sampling scheme.

b) A sample of size n is drawn with equal probability and without replacement from a population of size N. Let

P.T.O.

$$\hat{\bar{Y}} = \sum_{r=1}^{n} a_r y'_r$$

be any linear estimate of the population mean $\bar{Y}$, where the a_r's are some constants and y'_r denotes the value of the unit included in the sample at the r^{th} draw. Show that $\hat{\bar{Y}}$ is an unbiased estimate of $\bar{Y}$ if and only if

$$\sum_{r=1}^{n} a_r = 1.$$

c) Either

Compare the efficiency of regression estimate of population mean with respect to mean per unit estimate under simple random sampling without replacement.

OR

Write a short note on use of interpenetrating subsamples in estimating the variance of an estimator. [(3+3+4)+7+3]=[20]

5. Consider two-stage sampling with simple random sampling without replacement at both the stages. Suppose n first stage units (fsu's) are selected from N fsu's and m_i second stage units (ssu's) are selected from the M_i ssu's in the i^{th} selected fsu $(i = 1, \ldots, n)$.

Obtain an unbiased estimator of population total. Find its sampling variance together with an unbiased estimate of this variance. [4+8+8]=[20]

6. In a sample survey for estimating the number of standards of pepper in a tehsil having 72 villages, a sample of 5 villages was selected with simple random sampling without replacement and from each sample village 5 clusters of 20 fields each were drawn wit simple random sampling without replacement. Data on number of clusters in the sample villages and on number ~~number~~ of standards in the sample clusters are given below.

sample village	no. of clusters	number of standards in sample villages				
		1	2	3	4	5
1	27	43	40	36	98	39
2	24	59	123	100	37	34
3	14	116	55	306	172	127
4	116	69	22	84	120	58
5	25	19	27	205	213	24

Estimate unbiasedly the total number of standards in the tehsil along with its standard error. [25]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

PERIODICAL EXAMINATIONS
Statistics - 8: Design of Experiments

Date: 10.3.80 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions.
Marks are indicated in brackets.

1. Consider the following block design with 15 treatments in 6 blocks and the observations.

	Design					Observations				
Block 1	1	2	3	4	5	10	11	12	13	14
2	1	6	7	8	9	10	15	16	17	18
3	2	6	10	11	12	11	15	19	20	21
4	3	7	10	13	14	12	16	19	22	23
5	4	8	11	13	15	13	17	20	22	24
6	5	9	12	14	15	14	18	21	23	24

a) Identify the above design completely. [State values of different parameters.]

b) Is the difference between the effects of treatments 1 and 15 estimable? If the answer is yes , find an estimate of it.

c) Suppose the observations (13, 14 and 17, 18) corresponding to treatments 4, 5 in block 1 and 8,9 in block 2 are missing. In this situation, answer the question (b).

[3x10]=[30]

2.a) State and prove the inequalities for the numbers of blocks and treatments in a BIBD and a PBIBD.

b) For a PBIBD (m) with parameters b, v, r, k, λ_1, n_i, p^i_{jk}, (i,j,k = 0,1,...,m), and association matrices B_i(i = 0,1,...,m), prove the following:

i) $n_i\ p^i_{jk} = n_j\ p^j_{ik}$

ii) $B_j\ B_j = p^0_{jk}\ B_0 + p^1_{jk}\ B_1 + \ldots + p^m_{jk}\ B_m$. [10+2x10]=[30]

3.a) What are the advantages and disadvantages of the principle 'replication' in an experiment ?

b) Explain the terms: Randomization, Heterogeneity.

c) When and where would one use designs with (i) Complete blocks and (ii) incomplete blocks.

d) Suppose you are interested in comparing 3 treatments A, B, and C. There are two available designs to use, namely

Block 1	A	A	C
2	A	B	C

Block 1	A	B	C
2	B	A	C

which design would you prefer ? Give reason. [4x10]=[40]

---

79 - 80: 364

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
ANNUAL EXAMINATIONS
Statistics-8: Design of Experiments

Date: 12.5.80 Maximum Marks: 100 Time: 3 hours

Note: Answer any four questions.
Marks are indicated in brackets.

1.a) When would you use the following designs?

i) F-square, (ii) Latin square,
iii) Lattice and (iv) Youden Square.

b) Construct two orthogonal latin squares of order 11. Where would you use such design? [15+10]=[25]

2.a) Prove that, for a connected block design set-up the adjusted treatment totals are orthogonal to the block totals. Discuss the usefulness of this result.

b) In (a), find the sum of squares for contrasts between treatments (adjusted). [15+10]=[25]

3. Consider a connected block design set up with v treatments. Let $\underset{\sim}{\tau}$ be a vector of treatment effects. Suppose you are interested in making inferences about $\underset{\sim}{\eta} = L\underset{\sim}{\tau}$ where L is a s x v matrix with rank s. Discuss (with proofs) the effects of minimising trace, determinant and maximum characteristic root of $V(\hat{\underset{\sim}{\eta}})$. [You may just state without proofs pertinent results of matrix theory.] [25]

4. Suppose you are interested in comparing the effects of v treatments in a two-way elimination of heterogeneity set up. Suppose further that the null hypothesis of equality of effects of v treatments is rejected. What would you conclude? Discuss in detail Multiple t confidence intervals in this connection. [25]

5. Consider an experiment using a PBIBD (2) with 100 treatments. Discuss how the problem of inverting a 100 x 100 matrix C can be reduced to one of inverting a 2 x 2 matrix A in the analysis. [25]

INDIAN STATISTICAL INSTITUTE
B Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS
Statistics-9: Sampling Distributions

Date: 5.11.79 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can
Total marks allotted in the margin is 115 but maximum marks that you can score is 100.

a) Let x_1 and x_2 be independent observations from a rectangular distribution in the interval (0, 1)
Work out the joint distribution of

$$u = x_1 x_2 , \quad v = (1 - x_1)(1 - x_2).$$

Hence determine the marginal distributions of u and v.

b) Let X_1, X_2, X_3 denote a random sample from Normal population with zero mean and unit variance. Let the random variables Y_1, Y_2, Y_3 be defined by

$$\begin{aligned} X_1 &= Y_1 \cos Y_2 \sin Y_3 \\ X_2 &= Y_1 \sin Y_2 \sin Y_3 \\ X_3 &= Y_1 \cos Y_3 \end{aligned}$$

where $0 < Y_1 < \infty$, $0 < Y_2 \leq 2\pi$, $0 \leq Y_3 \leq \pi$ Find the joint distribution of Y_1, Y_2, Y_3. Hence find the marginal distribution of Y_1. [10+15]=[25]

a) Show that mean and variance of a sample from $N(\mu, \sigma^2)$ are independently distributed. Hence find the mean and variance of the distribution of sample variance.

b) Let $x_1, \ldots, x_n$ be a random sample from a rectantular distribution in the interval (0,1). Show that the density of the geometric mean y of the sample is

$$f(y) = \frac{n^n y^{n-1}}{\lfloor n} (-\log y)^{n-1}, \quad 0 \leq y \leq 1. \qquad [15+10]=[25]$$

a) Suppose $x_1, \ldots, x_n$ are independent random variables each of which is distributed normally with mean zero and variance unity. Show that the conditional distribution of

$$\sum_{i=1}^{n} x_i^2$$

subject to the conditions

$$\sum_{j=1}^{n} a_{ij} x_j = 0, \text{ for } i = 1, \ldots m (< n)$$

where $\sum_{j=1}^{n} a_{ij} a_{i'j} = \delta_{ii'}$, the Kronecker's delta, is a central chi-square with n-m d.f

p.t.o

3.b) Prove that the power of a chi-square test is non-decreasing function of the non-centrality parameter for fixed d.f. [10+10]=[20]

4.a) Derive the distribution of the reciprocal of the sample coefficient of variation from a sample from

$N(\mu, \sigma^2)$, $\mu > 0$.

b) Let y_1 and y_2 be independent chi-square variables each with n d f.

Show that

$$\frac{\sqrt{n}\,(y_1 - y_2)}{2\sqrt{y_1 y_2}}$$

is distributed as student's t with n d.f. [15+15]=[30]

5. Define Fisher's F-statistic. Hence derive its distribution. Also show that under certain limiting conditions it is distributed as a central chi-square [15]

INDIAN STATISTICAL INSTITUTE
B.Stat. (Hons.) Part III:1979-80

MID-YEAR EXAMINATIONS

Statistics-9: Sampling Distributions

Date: 21.12.79 Maximum Marks: 100 Time: 3 hours

Note: Answer as many questions as you can.
Total marks in the margin is 116 but maximum marks that you can score is 100.

1.a) Let x be distributed normally with mean $\mu(\neq 0)$ and variance unity. Work out the distribution of x^2.

b) Let X_1 and X_2 be two random variables with joint p.d.f.

$$f(x_1, x_2) = \begin{cases} 1 & \text{if } 0 \leq x_1, x_2 \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Show that random variables $Y_1 = \sqrt{-2 \log_e X_1} \cos 2\pi X_2$ and $Y_2 = \sqrt{-2 \log_e X_1} \sin 2\pi X_2$ are independent. Hence find the distribution of Y_1. [15+10]=[25]

2.a) State and prove Fisher-Cochran's theorem.

b) Let (x_i, y_i), $i = 1 \ldots n$ be a random sample of size n from a bivariate normal population with parameters $(\mu_1, \mu_2, 1, 1, \rho)$. Show that

$$\sum_{i=1}^{n} \left(\frac{x_i^2 - 2\rho x_i y_i + y_i^2}{1 - \rho^2} \right)$$

follows a non-central Chi-square distribution.

[15+10]=[25]

3.a) Show that t-distribution tends to normal distribution as the d.f. tends to infinity.

b) If $x_1 \leq x_2$ be two ordered observations from a normal distribution with (μ, σ^2), find the distribution of $u = x_2 - x_1$. Further if s^2 is an estimate of σ^2 with ν d.f. and distributed independently of u, show that the statistic $u^2/2s^2$ is distributed as F with d.f. $(1, \nu)$.

c) A random sample of $(2\lambda + 2)$ observation of a uniformly distributed random variables X in the range $0 \leq X \leq 2a$ is ordered, the $(\lambda + 1)$th and $(\lambda + 2)$th observations being x_1 and x_2. Obtain the joint distribution of

$$y = \frac{x_1 + x_2}{2} \quad \text{and} \quad z = \frac{x_2 - x_1}{2}. \qquad [12 \times 3]=[36]$$

4.a) Let x and y be jointly follow a bivariate normal distribution with parameters $(0, 0, 1, 1, \rho)$. Show that $u = x + y$ and $v = x - y$ are independently distributed.

Let r be the sample correlation coefficient between u and v, based on a random sample of size n from the joint distribution of (u, v). Show that r^2 follows a beta distribution with parameters

$$\left(\frac{1}{2}, \frac{n-2}{2}\right).$$

....

4.b) Derive the sampling distribution of the regression coefficient b in the regression equation

$$Y_x = \bar{y} + b(x - \bar{x})$$

Calculated from a sample of size n, assuming the dependent variable y is stochastic and the independent variable x non-stochastic.

Show that some function of b is distributed as student's t with n-2 d.f. What happens to the distribution of this function when both x and y are random variables having a joint probability distribution ? [15+15]=[30]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

PERIODICAL EXAMINATIONS

Statistics-9: Statistical Methods

Date: 24.3.80 Maximum Marks. 100 Time: 3 hours

Note: Answer as many questions as possible.
Total marks in the margin is 110, but
maximum marks that you can score is 100.

1. On the basis of random samples from two normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu^2, \sigma_2^2)$, propose some exact test procedures for the hypothesis $H_0\ [\mu_1 = \mu_2]$ against $H[\mu_1 \neq \mu_2]$, when σ_1^2 and σ_2^2 are different and unknown.
Also work out 95 % confidence limits for the ratio of variances, σ_1^2 / σ_1^2. [12+8]=[20]

2.a) Let X have a binomial distribution with parameter n = 10 and p. The null hypothesis $H_0\ [p = 1/2]$ is rejected and the alternative hypothesis $H[p = 1/4]$ is accepted if the observed value X_1, a random sample of size one, is less than or equal to 3. Obtain the size and power of the test.

b) An engineer wishes to estimate the mean setting time of a new gypsum cement mix used in highway spot repairs. Prior to sampling, the engineer wishes to determine the sample size (i.e. the number of observations of the setting time with the new mix) required to attain a desired precision in estimating the mean. From experience with other cement mixes, the engineer expects the standard deviation of the measurements to be approximately 5. How many observations of the setting times with the new mix should the engineer collect to be 95 % certain that the error of estimation of the true mean does not exceed one minute? [10+10]=[20]

3. A medical researcher wishes to determine if a pill has the undesirable side effect of reducing the blood pressure of the user. The study involves recording the initial blood pressures of 11 college-age women. After they use the pill regularly for six months, their blood pressures are again recorded. Data are given below:

Subject

	1	2	3	4	5	6	7	8	9	10	11
Before using the pill	70	80	72	76	76	76	78	72	82	64	74
After using the pill	68	72	62	70	58	66	52	68	64	72	74

Do the data substantiate the claim that use of the pill reduces blood pressure ? [15]

Obtain the generalized likelihood ratio test for the hypothesis of parallelism of two regression lines (under the usual assumptions). Hence show that this test is equivalent to the classical t.test, [20]

p.t.o.

5. In an experiment designed to determine the relationship between the dose of a compost fertilizer (x) and the yield of a crop (y), the following summary statistics are recorded.

$$n = 15 \qquad \bar{x} = 10.8 \qquad \bar{y} = 122.7$$

Corrected sum of squares and sum of products are

$$S_x^2 = 70.6\ ,\ S_y^2 = 98.5,\quad S_{xy} = 68.3.$$

Do the data contradict the experimenter's conjecture that over the range of x values covered in the study, the average increase in the yield per unit increase in the compost dose is less than 1.5 ? (Assume linear relationship). [15]

6. Explain how you will obtain an F-test for testing linearity of regression. [10]

7. Practical note book. [10]

INDIAN STATISTICAL INSTITUTE
B.Stat(Hons.) Part III: 1979-80

ANNUAL EXAMINATIONS

Statistics-9: Statistical Methods

Date: 16.5.80 Maximum Marks: 100 Time: 4 hours

Note: Answer as many questions as you can. Total marks in the margin is 110 but maximum that you can score is 100.

1. A sample of size n is drawn from each of k normal populations with the same variance. Derive the likelihood ratio criterion for testing the hypothesis that the means are all zero. Show that the criterion is a function of a ratio which has F-distribution. [15]

2. Let $x_1, \ldots, x_n$ denote a random sample from a normal distribution with mean θ and variance 16. Find the sample size n and a UMP test of $H_0[\theta = 25]$ against $H[\theta < 25]$ with power function $K(\theta)$ so that $K(25) = .10$ and $K(23) = .90$. [10]

3. The following table pertains to Carotene content determined by two different methods for 10 wheat varieties. By one method Carotene was determined on the whole wheat and by the other method on flour. The measurements for carotene in wheat are lower than for carotene in flour, contrary to the true situation. Make an appropriate analysis of the data to test the null hypothesis that there is no difference in the experimental error variance between the two methods for determination of carotene contents.

Variety No.:	1	2	3	4	5	6	7	8	9	10
Carotene in flour	2.3	3.11	2.15	1.96	2.02	1.76	2.10	2.12	2.28	1.86
Carotene in wheat	1.18	2.13	1.41	1.42	1.50	1.25	1.65	1.24	1.48	1.35

[15]

4. a) Work out the standard error of the sample standard deviation based on a sample from a normal population with parameters μ and σ^2

b) Let $(\bar{X}_{n_1}, S_1^2)$ and $(\bar{Y}_{n_2}, S_2^2)$ be the means and variances in samples of sizes n_1 and n_2 drawn from normal populations $N_1(\mu_1, \sigma_1^2)$ and $N_2(\mu_2, \sigma_2^2)$ respectively, where σ_1^2 and σ_2^2 are different and unknown.
Show that the asymptotic distribution of the test statistic.

$$Z = \frac{\bar{X}_{n_1} - \bar{Y}_{n_2}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

4.b) (contd.)

assuming H_0 $[\mu_1 = \mu_2]$ to be true, is N(0,1). (Relevant convergence theorems are to be stated only). [10+10]=[20]

5.a) Derive the z-transformation for correlation coefficient. Suppose k ($\geq$ 2) bivariate normal populations have a common correlation coefficient ($\neq$ 0). Describe a procedure for estimating the common unknown correlation coefficient.

b) The correlation coefficient between temperature of rice and breakage percentage calculated from a sample of size 28 was found to be 0.7. Is this value consistent with the assumption that the correlation in the population is 0.5 ? Obtain 95 % confidence limits for the population correlation coefficient. [15+10]=[25]

6. The following table records the observed number of births at a hospital in four consecutive quarters:

Quarter :	Jan.-Math	Apr.-Jun.	July-Sept.	Oct.-Dec.
Nd. of births:	110	57	53	80

It is conjectured that twice as many babies are born during the Jan.-Mar. quarter than are born in any of the other three quarters.

Test if these data strongly contradict the stated conjecture. [15]

7. Practical note-books [10]

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
PERIODICAL EXAMINATIONS

Statistics-10: Demography

Date: 15.10.79 Maximum Marks: 100 Time: 3 hours

Note: Separate answerscripts should be used for groups A and B.

Group A

1. EITHER

Define Demography as a subject of population science. Give the basic concepts one must take care of in measurement and analysis of mortality data.

What special considerations lead to a refined measure of I.M.R. Distinguish between the generation approach from period approach in measuring vital rates. (25)

OR

Why and how a L.T. is considered to be an indispensible tool in demographic analysis. Define L.T.D.R. as a superior index to crude death rate (C.D.R.) as a current measure of mortality. How is it related to e_0^0?

Establish the relationship,

$$\mu_{x+1/2} = m_x \simeq q_{x+1/2}$$

Give different interpretations for L_x, putting reasons.

Why is formula $L_x = \frac{l_x + l_{x+1}}{2}$ considered unsuitable for early years of life, say $x = 0$ and 1 (25)

2. Below is given an extract from a hypothetical fem. table. Complete the table and calculate the following values from it.

1) L.T.D.R. 2) Average age at death of female dying between ages 30 to 50. 3) Of the 1000 fem. aged 20 years, how many would die between ages 50 x 51 ?

age in years	Age d.r $_nm_x$	No.surv. at age x l_x	No.living bet.ages x to x+n $_nL_x$	No.living above age x T_x	Complete expectation of life at age x e_x^0
0	.21356	100000	82622	5690309	
1- 4	.00400	82355	*	5364452	
5- 9	.00182	81052	401720	*	
10-14	.00019	*	401490	*	
15-19	.00066	80245	400960	4478619	
20-24	.00139	79982	398860	4077655	
25-29	.00201	79428	395420	3678799	
30-34	.00260	*	390930	3283379	
35-39	.00336	*	385240	2892449	
40.44	.00442	76323	377990	*	
45-49	.00617	74653	*	*	
50-54	.00910	72378	*	1760679	
55-59	.01384	69147	336230	1405439	
60-64	.02158	64494	308710	1069209	
65-69	.03408	57831	269606	760499	
70-74	.05400	48642	216820	490899	
75-79	.06245	36933	*	274079	
80+	.22545	*	*	*	

N.B.: Given $L_{1-4} = 1.352\, l_1 + 2.648\, l_5$. (25)

Group B

Note: Answer any three questions.
Each question carry equal marks.
20 marks are reserved for class assignments.

3.a) What made the Government of India to organise the Mass Vasectomy Camps?

b) What are the categories of staff employed in primary health centers ?

c) Why the scheme did not work well? (3+3+4)=(10)

4a) What sort of age grouping will you recommend to minimise reporting errors ? Give reasons for your answer.

b) How will you arrive at the usual age groups used in census reports from the grouped data in (a)? (6+4)=(10)

5. What are the indicators which could be used to measure progress of education growth over time period ? Give illustrations with reference to published figures (10)

6. You are given age specific general fertility rates of 131.8, 266.2, 245.7, 154.1, 79.3, 26.6 and 22.1 for age groups 15-19, 20-24, 25-29, 30-34, 35-39, 40-44 and 45-49 respectively for West Bengal (rural). Calculate the net reproduction rate and interpret the result. (7+3)=(10)

7. What are the household membership concepts that could be used in demographic surveys ? State their relative advantages and disadvantages. (4+6)=(10)

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80
MID-YEAR EXAMINATIONS
Statistics-10 : Demography

Date: 27.12.79 Maximum Marks: 100 Time: 3 hours

Note: Answer Groups A and B in separate answerscripts Marks are given in brackets.

Group A:

Max. Marks: 50

1. EITHER

Give details of methodological principles involved in constructing Life Tables based on consecutive age-distributions. [25]

OR

a) Mention the types of sources for obtaining estimates of net migration for fixed intervals, and give estimating procedure for one of them. Express streams of migration as transition probabilities.

b) What is the difference between a current estimate and a population projection ? Mention the methods available for estimating total population at future dates. [25

2. Table below gives age-distribution for two successive censuses and a clm. for 10-year L.T.S R. for a hypothetical population 1961-71.

Table

Age group	Census 1961 population	10-year L.T.S R.	Age group	Census 1971 population
0 - 4	77135	.9087	10 - 14	132870
5 - 9	85134	.9573	15 - 19	170227
10 - 14	79185	.9471	20 - 24	263971
15 - 19	82603	.9308	25 - 29	253964
20 - 24	126247	.9223	30 - 34	195373
25 - 29	155344	.9161	35 - 39	151259
30 - 34	138843	.9047	40 - 44	118383
35 - 39	109356	.8850	45 - 49	76421
40 - 44	81626	.8548	50 - 54	65897
45 - 49	47062	.8122	55 - 59	32265
50 - 54	36908	.7535	60 - 64	22248
55 - 59	15134	.6726	65 - 69	9655
60 +	25059	.3866	70 +	10100
All ages	1059911		All ages	1502633

Use procedure for estimating net intercensal migration by age according to the Forward L.T.S R. method. [25]

p.t.o

Group B.

Max. Marks: 50

Note: Answer any three questions.
11 marks are reserved for class assignments.

3. What is crude birth rate ? How is it obtained through sample Registration System (SRS)? How does this system differ from the data collection system of National Sample Survey (NSS)? [4+5+4]=[13]

4. Write notes on the followings:

a) Occupation mobility ? [5]
b) Recall bias in reporting live births in households ? [5]
c) Infant mortality rate ? [3]

5.a) What are the indicators used to measure the economically active population ? [5]

b) Differentiate between total fertility rate and gross reproduction rate ? [4]

c) What made the Government of India to organise mass Vasectomy Camps ? [4]

6.a) What sort of age grouping will you recommend to minimise reporting errors ? Give reasons for your answer ? [6]

b) You are given age specific general fertility rates of 131.8, 266.2, 245.7, 154.1, 79.3, 26.6 and 22.1 for age groups 15-19, 20-24, 25-29, 30-34, 35-39, 40-44 and 45-49 respectively for Rural West Bengal.
Calculate the net reproduction rate and interpret the result ? [7]

---

79 - 80: 333

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

PERIODICAL EXAMINATIONS

Statistics-10. Index Numbers and Time Series

Date: 14.4.80 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions.
Marks are given in brackets.

1.a) What preliminary adjustments are to be made before time series data can be properly analysed?

b) Indicate the factors giving rise to seasonality. Why is it important to measure these fluctuations?

c) List some important factors that contribute to long-term upward trend and long-term downward trend. [9+8+4]=[20]

2. Describe the method of moving averages. Examine how it distorts the genuine oscillatory part of a time series. [6+14]=[20]

3. What are asymptotic growth curves? Discuss the characteristics of the logistic curve. [5+10]=[15]

4. Write a note on the method of link relatives. [10]

5. Fit a logistic curve to the census populations of England and Wales given below:

Year	Population (in millions)
1831	13.9
1841	15.9
1851	17.9
1861	20.1
1871	22.7
1881	26.0
1891	29.0
1901	32.5
1911	36.1

[35]

---

79 - 80: 384

INDIAN STATISTICAL INSTITUTE
B.Stat.(Hons.) Part III: 1979-80

ANNUAL EXAMINATIONS

Statistics-10: Index Numbers and Time Series

Date: 19.5.80 Maximum Marks: 100 Time: 3 hours

Note: Answer all questions.
Marks are given in brackets.

1. How are chain index numbers (of prices) constructed? Compare chain index numbers with fixed base index numbers and argue a case in favour of the former. [6+14]=[20]

2. Explain what is meant by a stationary time series and mention the different models that account for oscillations in such a series. Discuss how correlograms discriminate among these models. [4+4+12]=[20]

3. Write short notes on any two of the following:

   a) The method of periodogram analysis.

   b) The ratio-to-moving-average method in the estimation of constant seasonal indices.

   c) Weighting diagram in the construction of 'cost of living' index numbers. [10+10]=[20]

4. The table below gives the receipts of State Governments in India for the quarters of the year ending in March (M), June (J), September (S) and December (D). Calculate the indices of seasonal variation, assuming that the seasonal pattern is constant over years.

Year	Quarter			
	M	J	S	D
1952	144	51	49	60
1953	156	62	67	60
1954	161	64	65	75
1955	173	71	74	81

[Hint: Take the total of seasonal indices for the four quarters of the year to be 400]. [40]
