Indian Statistical Institute
End-Semester Examination (2019)
M. Tech.(CS) II Year
Cognitive Science

Date: 27.11.2019 | **Full Marks: 75** | **Duration – 3 hours**

Answer as many questions as you like, but you can at most score 75

1. John von Neumann and Alan Turing, whose ideas shaped modern computing technology, both explored network models inspired by the brain. Answer the following questions in the light of the above statement.
   (a) How did Turing's approach to the decidability problem differ from that of Alonzo Church? How did the biological brain inspire Turing to conceive the Finite State Machine?
   (b) In the book "The Computer and the Brain", published posthumously after the death of John von Neumann, he writes a chapter titled "Digital and Analog Parts in the Nervous System". Do digital and analog parts exist in the mechanism of the action potential generation in the individual nerve cell too? Justify your answer explaining each part of the action potential in terms of its underlying ionic mechanism.
   [(4+3) + 8=15]

2. One criticism about using complex and deep neural networks to model information processing in the brain is that, it replaces one black box with another.
   (a) How would you address this above mentioned criticism from the point of view of cognitive science?
   (b) To what extent are deep neural net models (like CNN or RNN) similar to biological brains, as compared to the earlier versions of Artificial Neural Networks (ANN), like say, the Multi-Layer Perceptron? [7+8]

3. Complex systems, especially cognitive ones are best understood in terms of a child's activity of making towers using building blocks by defining some agents and agencies. This is the approach of renowned computer scientist Marvin Minsky to understand the mind. Now answer the following questions.
   (a) How a child's brain or any cognitive system at work, can in general, be thus conceived algorithmically? Explain with a suitable hierarchical diagram.
   (b) Is the system actually hierarchical in Minsky's opinion? Justify your answer.

..........*continued on page 2*

(c) Can you tell a few words about any such attempted implementation at the Artificial Intelligence Lab, Massachusetts Institute of Technology, during the time of Minsky?

(d) How can Machine Intelligence be explained in terms of unintelligent components from Minsky's approach? [4+3+4+4=15]

4. (a) What is the basic unit of the central nervous system? Who would you give credit for identifying the same, Camillo Golgi or Ramon y Cajal? Justify.

(b) How does an ON Center-OFF Surround Ganglion neuron receptive field in the visual network respond when the Center and Surround both are illuminated fully? Justify, if this network can be termed as a Convolutional Neural Network (CNN).

(c) The antagonistic receptive field mentioned in (b) may be considered to be a basic unit of our perceptual experience. Now, explain in the light of the Building Block Model of the Society of the Mind, the necessity of "antagonism" (conflict) along with "compromise" arising in the cognitive experience. [(1+4)+(2+2)+6=15]

5. The studies on the mind of Henry Gustav Molaison, immortalized as H. M. in neuroscience and cognitive science literature, revealed many facets of human memory.

(a) Surgery of which part of H.M.'s brain by neurosurgeon William Scoville, revealed these truths? Choose the correct alternative: "This part is located in the (insect /mammalian) brain".

(b) What happened to H.M.'s IQ and epileptic seizures after he recovered from the surgery?

(c) What were the initial observations on his memory during the post-operative recovery?

(d) Explain the PhD student Brenda Milner's experiments on H.M. and what they revealed about different aspects of memory. [(1+1)+(2+1)+3+7=15]

6. (a) Describe at least two of the experiments of psychologist Jean Piaget who realized that watching children may be the crucial clue to understand the making of the mind.

(b) Explain in the light of the above the Seymour Papert principle.

(c) If one fills up the blank by deciding upon the appropriate alternative in the following sentence: "This is the story of an incident that occurred many, many years ago in a ………… (nearby/far-away) land", what choice do you think most will make? Can you justify such choice in the light of Piaget's experiments?

(d) Suppose an ANN is made to learn in the Piaget-Papert way. Can it be, according to you, a fruitful learning mechanism like, say, the Hebbian learning? [6+3+3+3=15]

INDIAN STATISTICAL INSTITUTE
M. Tech. Computer Science Year II, 2019-20
Pattern Recognition and Image Processing
Semester Examination, Date: 29.11.2019

Maximum Marks: **100** Duration: **3 hours**

Answers should be brief and to the point.
Calculators are allowed.

A. This question is **compulsory**. **[4*5=20]**

a. Define decision boundary between two classes. How do you find the "best-fit" decision boundary for a given dataset having two classes? 2+2

b. Why do the final clusters vary for most clustering algorithms if they are run multiple times? Explain with example. 2+2

c. In which of the following cases will k-means clustering fail to give good results? Why? 4
   i. Data points having clusters and outliers
   ii. Clusters having non-uniform densities
   iii. Clusters having spherical shapes
   iv. Clusters having non-convex shapes

d. What is the meaning of noise in an image? Explain different types of noise models. 1+3

e. State the differences between pixel neighborhood and pixel connectivity. 4

B. Answer **any four** questions [20 marks each] **[20*4=80]**

1. a. The following 1-dimensional points A = 1, B = 2, C = 3, D = 8, E = 9, F = 10 are given. 10
   Show single-link bottom-up hierarchical clustering using d(x, y) = |x – y| as the distance between two points x and y.
   b. Use principal component analysis to reduce the dimensions of the given 2D dataset to 1D. Finally, show the transformed data with respect to the principal component obtained. 10
   Input Dataset: (3, 3), (2, 3), (-1, 0), (2, 2)

2. Write short notes on: 5x4=20
   a) Average linkage algorithm
   b) Boosting
   c) Feature selection
   d) k-fold cross validation

3. a. What are the properties of metrics? Write the expression for Minkowski metric. 3+2
   b. Consider the following balloons dataset: 15

Color	**Size**	**Act**	**Age**	**Inflated**
Yellow	Small	Stretch	Adult	F
Yellow	Small	Stretch	Child	T
Yellow	Small	Dip	Adult	T

Yellow	Small	Dip	Child	T
Yellow	Large	Stretch	Adult	T
Yellow	Large	Stretch	Child	F
Yellow	Large	Dip	Adult	F
Yellow	Large	Dip	Child	F
Purple	Small	Stretch	Adult	T
Purple	Small	Stretch	Child	F
Purple	Small	Dip	Adult	F
Purple	Small	Dip	Child	F
Purple	Large	Stretch	Adult	T
Purple	Large	Stretch	Child	F
Purple	Large	Dip	Adult	F
Purple	Large	Dip	Child	F

Compute the parameters of Naïve Bayes classifier for predicting the *inflated* class and the training set error.

4. a. Explain the following operations with relevant examples:
   i) Contrast stretching 5
   ii) Bit-plane slicing 5

   b. Discuss the limiting effect of repeatedly applying a 3x3 low-pass spatial filter to a digital image. You may ignore border effects. Is this effect different from applying a 5x5 filter? 8+2

5. a. What is meant by image smoothing? Describe any one method. 2+3

   b. How is the entropy of an image calculated? Write the significance of entropy in image processing. 3+2

   c. How many bits are required for encoding the message 'mississippi' in Huffman encoding? Show the corresponding Huffman tree. 10

6. a. Can two images have the same histogram? Justify your answer. Comment on the entropy of these two images. 3+2

   b. Consider two 3 bit images with histograms as follows:

Gray Level	Number of Pixels
0	790
1	1023
2	850
3	656
4	329
5	245
6	122
7	81

Image A

Gray Level	Number of Pixels
0	0
1	0
2	0
3	614
4	819
5	1229
6	819
7	614

Image B

Perform histogram specification on *Image A* with respect to the *Image B*. Show all the steps. 15

INDIAN STATISTICAL INSTITUTE

Semestral Examination : (2019 - 2020)

Course Name : M. Tech. (CS) **Year : 2nd year**

Subject Name : Neural Networks & Applications

Date : November 25, 2019 **Maximum Marks : 100** **Duration : 3 hrs 30 mins**

Answer as many questions as you can, but you can score upto 100 marks.

1. Consider a set of 20×20 RGB images distributed in 5 classes. These images need to be classified using a hypothetical convolutional neural network with two convolution layers and a fully connected layer consisting of 50 nodes, apart from input and output layers. The filter size of each of these convolution layers is 3×3. Each of these convolution layers generates 20×20 convolved images. The number of filters in the first convolution layer is 5 and that for the second convolution layer is 1. Assume appropriate bias and activation function of the nodes.
   (a) Write down the expressions for input and output of the nodes in each layer during forward propagation.
   (b) State/derive and explain the expression for modification in the weight values, in each iteration during training under gradient-descent minimization, of all the links in the network. [10 + 20 = 30]

2. Show how the second principal component of a data can be derived in an artificial neural network framework. Prove that after convergence the network has learned the first and second principal components. Assume appropriate architecture and learning rule for this network.

   [5 + 25 = 30]

3. Consider a Sparse Autoencoder.

   (a) Explain mathematically the reconstruction error along with appropriate regularizer for a data set to be fed to this Autoencoder.

   (b) Write down the expression for the sparsity penalty for this Autoencoder, and explain the expression. [5 + 10 = 15]

4. Consider the following dynamical system:

$$\mathbf{s}(t) = \mathbf{f}(\mathbf{y}(t\text{-}1), \mathbf{x}(t); \boldsymbol{\theta}).$$

Here, $\mathbf{s}(t)$ is an m-dimensional vector representing state of the system at time t, $\mathbf{x}(t)$ is the n-dimensional input signal, $\mathbf{y}(t)$ is l-dimensional output of the system at time t, and $\boldsymbol{\theta}$ is the vector of parameters of the model. In other words, state of the system at time t is obtained through the function $\mathbf{f}$ of the parameters mentioned above. You need to predict the output of the system in an appropriate Recurrent Neural Network (RNN) framework. In order to reduce the number of parameters (weights), they are suitably shared. Assume that the maximum allowable time is T during which the outputs are fed back appropriately under the framework of the RNN.

   (a) Derive the learning rules for modification of all the weight values of the links present in the RNN under Back Propagation Through Time (BPTT).

   (b) Explain mathematically how the problems of vanishing and exploding gradient may arise in such a neural network model. Derive all the expressions that you may need to explain it. [20 + 10 = 30]

INDIAN STATISTICAL INSTITUTE

Semestral Examination (Back Paper): (2019 - 2020)

Course Name: M. Tech. (CS) **Year: 2nd year**

Subject Name: Neural Networks & Applications

Date: 15.01.2020 **Full Marks: 80** **Duration: 2 hours 30 minutes**

Answer all the questions.

1. Describe different functions with appropriate explanation, which are usually used as activation functions of artificial neural networks. [10]

2. Derive the expressions for computing the amount of modification for the weight values of the links, in each iteration during training under backpropagation learning algorithm, in a multilayer perceptron for the purpose of pattern classification. Assume two hidden layers between the input and output layers, and also consider appropriate non-linear activation functions of the computing nodes, and the energy function at the output layer. [20]

3. (a) State the Hadamard's conditions for a problem to be well-posed.
   (b) How is Tikhonov's Regularization Theory used for solving ill-posed problems?
   (c) In the context of artificial neural networks, give an example of a widely used regularizer. Explain how this regularizer is incorporated, according to Tikhonov's Regularization Theory, in the objective function.
   (d) What are the physical significances of the situations when the regularization parameter $\lambda \rightarrow 0$ and $\lambda \rightarrow \infty$? [6 + 5 + 7 + 2 = 20]

4. (a) Explain the advantages of Convolutional Neural Networks over Multilayer Perceptrons for classifying a set of multichannel images.
   (b) Explain with an appropriate example whether convolution operation is equivariant under translation and shifting.
   (c) State and explain the learning rule for training a Convolutional Neural Network.
   [5 + 5 + 20 = 30]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2019-2020

M. Tech. (CS) II year

Data Mining and Knowledge Discovery

Date: 22.11.2019 **Maximum Marks: 100** **Duration: 3 hours**

[Answer as much as you can.]

1. (i) Define Minkowski distance.
   (ii) Define Jaccard coefficient.
   (iii) What are some of the merits and demerits of DBSCAN? State one way of selecting the thresholds.
   (iv) Enumerate some of the limitations of partitive clustering algorithms like k-means, and how they can be overcome.
   (v) Describe algorithm PAM, analysing its swapping process.
   (vi) What are the different ways of computing inter-cluster similarity? Explain.
   [3+3+5+5+6+6=28]

2. (i) What are support and confidence in the context of association rules?
   (ii) How does FP-tree help in rule mining? Explain the tree construction with an example.
   (iii) Outline algorithm CLARANS. How does it handle scalability?
   (iv) Why do we need cluster validation? Define any three cluster validity indices.
   [5+9+8+6=28]

3. Answer any three:
   (i) How is big data characterised? What is Hadoop? How does MapReduce work?
   (ii) What is topic mining? How does text clustering help? What is the utility of opinion analysis?
   (iii) What is an RNN? How is it different from MLP? Enumerate different kinds of RNN.
   (iv) Why do we need distributed data mining? What is ensemble learning? What is "sessionization" in the context of web mining?
   (v) What are the categories of time series movements? How do we estimate a trend curve? What is subsequence matching in time series mining?
   [7+7+7=21]

4.
   (i) What is a Convolutional Neural Network (CNN)? Draw its architecture. Why do we need convolution? What is the advantage of Fully Convolutional Network over CNN?
   (ii) Describe the architecture of DenseNet. [(2+4+4+4)+6=20]

5.

(i) Provide an outline on the different training techniques used in deep learning.

(ii) Given an input image of size 227*227*3 with 55 filters, each of size 7*7:

a. Calculate the total number of trainable parameters.

b. What will be the output image size if zero padding is applied?

(iii) Discuss the different types of activation functions with their advantages and disadvantages. [10+(3+3)+8=24]

Indian Statistical Institute

End-Semester Examination (2019-2020)

M.Tech. (CS) II

Advanced Algorithms for Graph and Combinatorial Optimization Problems

Date: 22.11.2019 Maximum Marks: 100 Time: 3.5 hours

Answer as much as you can. This question paper is of 120 marks but the maximum you can score is 100 marks. Marks allotted to each question are indicated within parentheses near the right margin. This paper has two pages.

1. Let G be a 3-regular plane graph having n vertices, in which every vertex lies on one face of length 4, one face of length 6, and one face of length 8.
   (i) Determine the number of faces of each length in terms of n.
   (ii) How many faces does G have in all?

   [4+2=6]

2. (i) Give an example of a chordal graph which is not an interval graph.
   (ii) Present an efficient algorithm to enumerate all the maximal cliques in a given interval graph. Analyse the worst case time complexity of your algorithm.

   [4+(6+4)=14]

3. Let $G_n = (V, E)$ be a graph with the set of vertices $V(G_n) = \{1, 2, \ldots, n\}$, $n \in \mathbb{N}$ and the set of edges $E(G_n) = \{(a, b) | a, b, \in V(G_n)$ and $(a + b)$ is divisible by 2 or 3 or 5$\}$.
   (i) Draw the graph G_{10}.
   (ii) Prove that the graph G_∞ is perfect.

   [5+10=15]

4. Let G be a graph having n vertices $\{v_1, v_2, \ldots, v_n\}$. Then the Mycielski graph $\mu(G)$ of G has
   - the set of vertices $\mu(V) = \{v_1, v_2, \ldots, v_k, u_1, u_2, \ldots, u_k, w\}$, and
   - the set of edges $\mu(E) = E(G) \cup \{(u_i v_j) | (v_i v_j) \in E(G)\} \cup \{(u_i, w) | i = 1, 2, \ldots n\}$.

   The k^{th} Mycielski graph of G is defined as $\mu^k(G) = \mu(\mu^{k-1}(G))$ where $\mu^1(G) = \mu(G)$.

   (i) If $G = K_2$, the complete graph on two vertices, then draw $\mu(G)$.
   (ii) Show that $\mu^k(G)$ is triangle free for all $k \geq 1$.
   (iii) Prove that the chromatic number $\chi(\mu^k(G)) = k + 2$ for all $k \geq 1$. What is the clique number $\omega(\mu^k(G))$?

   [3+5+(5+2)=15]

P.T.O.

5. (i) Prove that if an undirected graph $G = (V, E)$ and its complement $\overline{G}$ are both comparability graphs, then G is a permutation graph.

(ii) What is the time complexity of constructing a permutation π of $(1, 2, \ldots, n)$ corresponding to the above-mentioned graph G having n vertices?

[10+5=15]

6. Give an example of two non-isomorphic split graphs having the same degree sequence. [6]

7. (i) Define the minimum clique partition problem with constrained bounds in weighted interval graphs.

(ii) Present a linear time approximation algorithm for this problem. Analyse the approximation ratio of your algorithm.

[4+(5+5)=14]

8. (i) What is a basis of a matroid?

(ii) Prove that all the bases of a matroid have the same size.

(iii) Give an example of a co-graphic matroid.

[3+5+2=10]

9. Define the dual $\mathcal{M}^{\mathcal{D}}$ of a matroid $\mathcal{M}$ and show that $\mathcal{M}^{\mathcal{D}}$ is also a matroid. [4+6=10]

10. (i) Define the Unit-time Task Scheduling problem.

(ii) Demonstrate how to apply matroid theory to solve the above problem. What is the worst case time complexity of your method?

[4+(8+3)=15]

INDIAN STATISTICAL INSTITUTE

End Semestral Examination

M. Tech. CS – II Year, 2019-2020 (Semester – III)

Optimization Techniques

28.11.2019

Date: AA.BB.2020 | Maximum Marks: 100 | Duration: 4 hours

Note: The question paper is of 135 marks. Answer as much as you can, but the maximum you can score is 40 from Group-A and 60 from Group-B.

Notations:

For $x \in \mathbb{R}^d$, we write $x \geq 0$ (or $x > 0$) if all the coordinates of x are non-negative (positive).

Transpose of a matrix A would be denoted by A^T and transpose of a vector b would be denoted by b^T.

Whenever we say that, $\mathcal{P}$ is a linear program, we mean $\mathcal{P}$ is of the form

$$\begin{array}{ll} \text{Maximize} & c^T x \\ \text{subject to} & Ax \leq b \end{array}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$.

Let A and B be $n \times n$ matrices, then we write $A \succeq B$ if $A - B$ is positive semi-definite matrix.

The *domain* of a function f will be denoted by $dom(f)$.

Gradient and *Hessian* of a function f at x will be denoted by $\nabla f(x)$ and $\nabla^2 f(x)$, respectively.

A function $f : \mathbb{R}^n \to \mathbb{R}$ is *strongly convex* if there exists $\alpha > 0$ such that the function $f(x) - \alpha \|x\|^2$ is a convex function.

$\mathcal{MP}$ will denote the following mathematical program in $\mathbb{R}^n$:

$$\begin{array}{ll} \text{Minimize} & f_0(x) \\ \text{subject to} & f_i(x) \leq 0, \;\; i \in \{1, \ldots, m\} \\ & h_j(x) = 0, \;\; j \in \{1, \ldots, p\} \end{array}$$

Let $\Omega \subseteq \mathbb{R}^n$, and $f : \Omega \to \mathbb{R}$. Then $g \in \mathbb{R}^n$ is a *subgradient* of f at $x \in \Omega$ if for any $y \in \Omega$ one has

$$f(x) - f(y) \leq g^T(x - y).$$

The set of subgradients of f at x is denoted as $\partial f(x)$.

Group-A

(AQ1) Let $S = \{s_1, s_2, \ldots, s_n\}$ be a set of line segments drawn on a two dimensional grid. For each s_i, the (integer) coordinates of both the end-points are given as input. The problem is to compute the minimum number of axis-parallel (horizontal and vertical) lines that can hit all the members in S. Formulate this problem as an ILP problem.

[10]

(AQ2) Consider the linear program $\mathcal{P}$ as defined earlier, and let $S = \{x \in \mathbb{R}^n : Ax \leq b\}$. Define

$$S' = \{d \in \mathbb{R}^n : \forall x \in S, \forall \lambda \geq 0, x + \lambda d \in S\}.$$

(a) Show that $S' = \{d : Ad \leq 0\}$.

(b) Show that S' is a convex set.

(c) Show that $\mathcal{P}$ is unbounded if and only if there exists a $d \in S'$ such that $c^T d > 0$.

[3 + 2 + 5 = 10]

(AQ3) Let $\mathbf{x}^*$ and (λ^*, μ^*) (where λ^* and μ^* are the Lagrangian multiplier vectors for the inequality and equality constraints, respectively) be any primal and dual optimal solutions to the mathematical program $\mathcal{MP}$ with zero duality gap.

(a) State and prove the necessary KKT optimality conditions in terms of $\mathbf{x}^*$ and (λ^*, μ^*).

(b) State and prove the conditions under which KKT conditions are sufficient for zero duality gap.

[5 + 5 = 10]

(AQ4) State and prove Slater's strong duality condition for $\mathcal{MP}$. [2 + 8 = 10]

(AQ5) Let $C \subseteq \mathbb{R}^n$ be a convex set, and $f : C \to \mathbb{R}$.

(a) Show that if $\forall x \in C, \partial f(x) \neq \emptyset$ then f is a convex function.

(b) Show that if f is a convex function then for any $x \in int(C)$ (interior of C), $\partial f(x) \neq \emptyset$.

(c) Show that if f is differentiable at x then $\nabla f(x) \in \partial f(x)$.

[3 + 5 + 2 = 10]

(AQ6) (a) Consider the following mathematical program in $\mathbb{R}^n$:

$$\begin{aligned} \text{Minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \Omega \end{aligned} \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a differentiable convex function and Ω is a convex set. Then show that $x^* \in \Omega$ is an optimal point for the above mathematical program if and only if for all $x \in \Omega$ we have

$$\nabla f(x^*)^T(x - x^*) \geq 0.$$

(b) Consider the following mathematical program in $\mathbb{R}^n$:

$$\begin{aligned} \text{Minimize} \quad & f(x) \\ \text{subject to} \quad & Ax = b \end{aligned} \tag{2}$$

where f is a differentiable convex function and $A \in \mathbb{R}^{m\times n}$. Show that a point $x^* \in \mathbb{R}^n$ is optimal for the above mathematical program if and only if x^* is feasible and there exists $\mu^* \in \mathbb{R}^m$ such that

$$\nabla f(x^*) = A^T\mu^*.$$

[6 + 4 = 10]

Group-B

(BQ1) (a) Consider a set of intervals $[a_i, b_i], i = 1, 2, \ldots, n$ on a real line $\mathbb{R}$ (i.e., $a_i, b_i \in \mathbb{R}$). Assume that all the intervals are of positive length, and the end-points of the intervals are all distinct. The objective is to find a set $Q = \{q_1, q_2 \ldots\}$ of minimum number of points on the same line $\mathbb{R}$ such that each interval contains at least one point of Q. Formulate this problem as an integer linear programming (ILP) problem.

(b) Show that the LP relaxation of the above ILP produces its optimum solution.

[6+9=15]

(BQ2) (a) Consider a linear programming (LP) problem $\min c'x$ subject to $Ax = b$, $x \geq 0$, where $A \in \mathbb{R}^{m\times n}$ and $m \leq n$. You have solved that LP, and observed that in the optimum solution $x_1, x_2, \ldots, x_m$ are in the basis. You also observed that x_i is fractional valued. Show, how you add a Gomory's constraint for the i-th variable in the basis so that no feasible integer valued solutions becomes infeasible due to that constraint.

(b) Consider the following quadratic programming problem:

$$\begin{aligned} \text{Minimize} \quad & c^Tx + \tfrac{1}{2}x^THx, \\ \text{subject to} \quad & Ax \leq b, \qquad x \geq 0, \end{aligned}$$

where $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, $H \in \mathbb{R}^{n\times n}$ and $A \in \mathbb{R}^{m\times n}$. Describe Wolfe's method to solve this problem.

[7+8=15]

(BQ3) Consider that the following result, in connection with the primal dual central path of a given LP problem, is true:

$$\max f(x) = c^T x \text{ subject to } Ax \leq b,\ x \geq 0,$$

For the given LP problem if $\hat{x}$ is feasible solution, and its dual problem

$$\min b^T y \text{ subject to } A^T y \geq c,\ y \in \mathbb{R}^m$$

has a feasible solution $\hat{y}$ such that the slack vector $\hat{s} = a^T y - c$ satisfies $\hat{s} \geq 0$, then for every $\mu > 0$, the system

$$\begin{aligned} &Ax = b, \\ &a^T y - s = 0, \\ &(s_1 x_1, s_2, x_2, \ldots, s_n x_n) = \mu.\underline{1}, \\ &x, s \geq 0 \end{aligned}$$

has a unique solution $x^* = x^*(\mu)$, $y^* = y^*(\mu)$, $s^* = s^*(\mu)$, and $x^*(\mu)$ is the unique maximizer of the logarithmic barrier function $f_\mu(x) = c^T x + \mu \sum_{j=1}^{n} \log_e x_j$ subject to $Ax = b$, $x \geq 0$.

Here $\{(x^*(\mu), y^*(\mu), s^*(\mu)) \in \mathbb{R}^{2n+m} : \mu > 0\}$ is called the primal dual central path of the given LP problem.

Use this result to formulate an iterative algorithm for the given LP problem. Justify that (i) its each iteration needs polynomial time, and (ii) the number of iterations required is also polynomial. [7+(3+5)=15]

(BQ4) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a twice differentiable strongly convex function. Also, let $x_0 \in dom(f)$, and define $S = \{x \in dom(f) \mid f(x) \leq f(x_0)\}$.

The set S and the function f satisfy the following properties:

- There exist m and M, such that $0 < m < M$ and for all $x \in S$, $m I_n \preceq \nabla^2 f(x) \preceq M I_n$.
- There exists $L > 0$ such that for all $x, y \in S$, $\|\nabla^2 f(x) - \nabla^2 f(y)\|_2 \leq L\|x - y\|_2$.
- There exists $\alpha \in (0, 1/2)$, such that for all $x \in S$, we have

$$\|\nabla f(x)\|_2 < \eta, \quad \text{where} \quad \eta = \min\left\{\frac{m^2}{L},\ 3(1-2\alpha)\frac{m^2}{L}\right\}.$$

We are interested in minimizing $f(x)$ with the starting point $x_0 \in dom(f)$.

(a) Describe Newton's Method for minimizing $f(x)$ with the starting point x_0.

(b) Show that Newton's Method is a valid descent method.

(c) Compute the convergence rate of Newton's Method, with starting point x_0, to minimize $f(x)$. Assume that the Backtracking subroutine, inside the Newton's Method, uses parameters (α, β) where $\beta \in (0, 1)$. [2 + 3 + 10 = 15]

(BQ5) Given a closed convex set $C \subseteq \mathbb{R}^n$ and $x \in \mathbb{R}^n$, the *projection problem* $\Pi_C(x)$ is defined as $\Pi_C(x) = \operatorname{argmin}_{z \in C} \|z - x\|$. Also, assume that there exists an oracle $\mathcal{O}_\Pi$ that can solve the projection problem for any compact convex set, given as a system of convex inequalities, in any dimension.

Consider the following mathematical program $\mathcal{MP}1$ in $\mathbb{R}^n$:

$$\begin{aligned} \text{Minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \Omega \end{aligned} \tag{3}$$

where $f : \Omega \to [0, 1]$ is a convex function and Ω is a compact convex set with diameter R. Note that $x \in \Omega$ if

$$g_i(x) \leq 0, \quad \forall i \in \{1, \ldots, m\}$$

where functions g_i, for all $i \in \{1, \ldots, m\}$, are convex functions. We have access to f via an oracle $\mathcal{O}_f$ that given any $x \in \mathbb{R}^n$ returns the value of f at x.

(a) Design an algorithm for $\mathcal{MP}1$ along the lines of the projected descent method assuming that we are given $x_0 \in \Omega$, the functions g_i, and the value of B as inputs, and we also have access to the oracles $\mathcal{O}_f$ and $\mathcal{O}_\Pi$.

(b) Derive the convergence analysis of the algorithm in terms of number of steps taken, and R.

[9 + 6 = 15]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech. CS – 2 Year, 2019-2020 (Semester – 3)

Information and Coding Theory

Date: 25.11.2019 | Maximum Marks: 100 | Duration: 3 hours

Note: The question paper is of 100 marks.

There are two sections in this paper: Section A and Section B. Section A contains 4 problems (of 20 marks each) and the maximum marks in this section is 40. Section B has 6 problems (of 20 marks each) and the maximum marks in this section is 60.

Section: A

Maximum Marks: 60

(A1) Let $X_1, \ldots, X_n$ be a collection of random variables.

(a) Show that

$$H(X_1, \ldots, X_n) = \sum_{i=1}^{n} H(X_i \mid X_{i-1}, \ldots, X_1).$$

(b) Show that

$$H(X_1, \ldots, X_n) \leq \sum_{i=1}^{n} H(X_i)$$

(c) Let $\mathcal{F}$ be a family of subsets of $[n]$ (possibly with repeats) such that each $i \in [n]$ is included in at least f members of $\mathcal{F}$. Show that

$$H(X_1, \ldots, X_n) \leq \frac{1}{t} \left(\sum_{\sigma \in \mathcal{F}} \right) H(X_\sigma),$$

where X_σ is the set $\{X_1 \mid i \in \sigma\}$.

[20 = 6 + 4 + 10]

(A2) The codeword lengths of any uniquely decodable code for $[n]$ must satisfy the Kraft inequality

$$\sum_{i=1}^{n} 2^{-l_i} \leq 1 \qquad (1)$$

where l_i denotes the length of the code for i. Conversely, given a set of codewords lengths that satisfy Inequality 1, it is possible to construct a uniquely decodable code with these codeword lengths. [20 = 10+10]

(A3) State the Hamming Bound, Gilbert-Varshanov Bound, Plotkin Bound, the Singletom bound and the Johnson Bound. Draw the graph of rate vs error for the different bounds (both for unique decoding and list decoding). What happens when the field sizes increase and where does the Reed-Solomon code sits currently? [20]

(A4) Recall the Reed-Solomon Codes:

(a) Describe Reed-Solomn Codes and Reed-Muller Codes.

(b) What is the rate and distance of Reed-Solomn? Prove your answers.

(c) Give an efficient encoding and decoding algorithm for Reed-Solomn Codes.

[20 = 6 + 6 + 8]

Section: B

Maximum Marks: 40

(B1) Let X_1 and X_2 be discrete random variables with probability distribution $p_1(\cdot)$ and $p_2(\cdot)$ over respective alphabets $\mathcal{X}_1 = \{1, \ldots, m\}$ and $\mathcal{X}_2 = \{m+1, \ldots, n\}$. Let

$$X = \begin{cases} X_1 & \text{with probability } \alpha \\ X_2 & \text{with probability } 1-\alpha \end{cases}$$

(a) Find X in terms of $H(X_1)$, $H(X_2)$, and α.

(b) Show that $2^{H(X)} \leq 2^{H(X_1)} + 2^{H(X_2)}$.

[20 = 10 +10]

(B2) Let $\mathcal{G}$ be a family of subgraphs of K_n (complete graph on n vertices on the vertex set $[n]$ and n is a even number) such that for all $G_1, G_2 \in \mathcal{F}$, the graph $G_1 \cap G_2$ does not contain any isolated vertices. Then, prove that $|\mathcal{G}| \leq 2^{\binom{n}{2} - \frac{n}{2}}$. [20]

(B3) If we use a channel on an alphabet $\Sigma = \{a, b, c\}$ such that when a message in Σ^* through the channel a can get corrupted and be received as a or b, b can get corrupted and be received as b or c and c can get corrupted and be received as c or a.

Show that it is impossible to uniquely decode against such an error. Also show that there is a way of encoding that allows to list-decode against such an error, with list size only 2. [20]

(B4) In an $[n, k, d]$ binary linear code C,

(a) Show that either all the codewords have even weight, or exactly half have even weight and half have odd weight.

(b) If C has a codeword of odd weight, then show that the even weight words of C form an $[n, k-1, d']$ binary linear code. Compute d'.

(c) Show that either all codewords in C begin with a 0, or exactly half begin with a 0 and half with a 1.

(d) Show that the sum of the weights of all the codewords in C is at most $n2^{k-1}$.

[20 = 5+5+5+5]

Indian Statistical Institute
Semestral Examination: 2019
Course Name: M. Tech. in Computer Science
Subject Name: Mobile Computing

Date: 23.11.19 Maximum Marks: 100 Duration: 3 hours 30 minutes

Instructions: You **may** attempt **all** questions which carry a total of **110** marks. The maximum marks you can score is only **100**.

1. Briefly explain the channel assignment problem (CAP) in cellular networks. Consider the 21-node cellular graph as shown in Figure 1, where the label $[x]$ associated with a node represents the demand of that node. The frequency separation matrix $C = (c_{ij})$ is given as: $c_{ii} = 5$ for all i, $c_{ij} = 2$ if nodes i and j are adjacent, $c_{ij} = 1$ if nodes i and j are distance 2 apart, and $c_{ij} = 0$ if nodes i and j are distance 3 or more apart. Consider the homogeneous single-channel assignment of the cellular graph as shown in Figure 2, where the label $[y]$ associated with a node indicates the frequency assigned to that node. Construct the `coalesced CAP` using the given single-channel assignment, demand vector and the frequency separation matrix. [5 + 10 = 15]

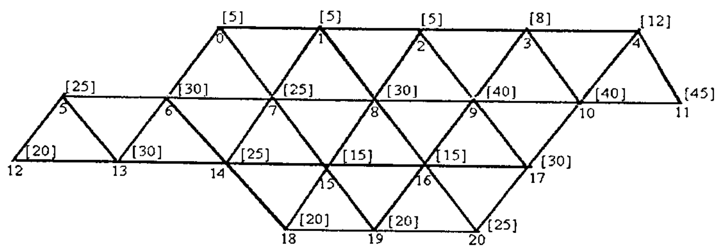

Figure 1: The demand vector.

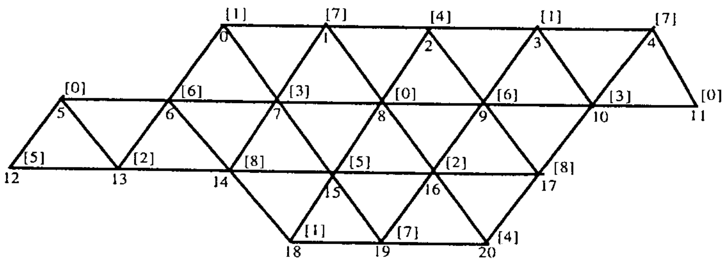

Figure 2: The homogeneous single-channel assignment.

2. What is the perturbation-minimizing frequency assignment problem (`PMFAP`)? Explain with an example the forced assignment with rearrangement (`FAR`) operation used in `PMFAP`. [5+5=10]

3. Describe how randomized rotation of cluster-heads is used to evenly distribute the energy load among the sensors in the Low-Energy Adaptive Clustering Hierarchy (LEACH) protocol in wireless sensor networks. [10]

4. Consider a linear network consisting of n sensor nodes and the base station (BS) as shown in Figure 3 where the distance between two consecutive nodes is r. The distance between the

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last node n and the BS is also r. Circles denote the sensor nodes and square denotes the BS.

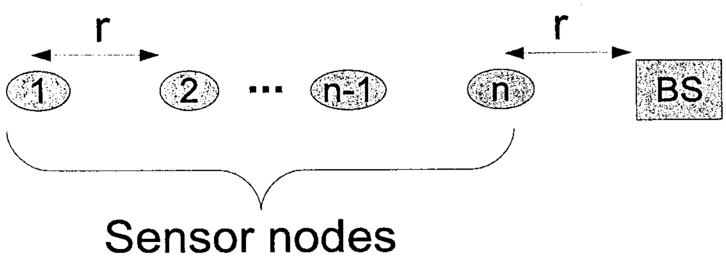


Figure 3: Linear network with n nodes and the base station.

Derive expressions for total energy expended in the system for transmitting a k-bit message from node 1 to the BS using direct communication and minimum transmission energy (MTE) routing protocols. Under what condition does direct communication routing require less energy than MTE routing in this linear network? [5+5=10]

5. State the main difference between self-diagnosis and cooperative diagnosis based fault detection techniques in wireless sensor networks. [5]

6. Present an approximation algorithm for the following minimum relay node placement problem in wireless sensor networks:

   Given a set of sensor nodes S with their locations and an uniform communication radius d of both sensor and relay nodes, the problem is to place a set of relay nodes R such that the whole network G consisting of both sensor and relay nodes is *connected.* The objective of the problem is to minimize $|R|$, where $|R|$ denotes the number of relay nodes in R. Prove the approximation ratio of your algorithm. [8+8=16]

7. What are the different types of cognitive capabilities with which a cognitive radio user should be equipped to support opportunistic and concurrent spectrum access in cognitive radio networks? [6]

8. How energy detection is used for indirect spectrum sensing in cognitive radio networks? [5]

9. State the differences between *open access* and *closed access* mechanisms for security provisioning in 5G cellular networks. [5]

10. Formulate the optimal resource allocation problem in D2D underlaid cellular network as a maximum weight bipartite matching problem. [10]

11. Briefly describe device relaying with device controlled link establishment (DR-DC) model of D2D communication. [6]

12. State three fundamental propagation features of millimeter wave D2D communication in 5G cellular networks. [6]

13. State two different ways to incorporate the impact of blockage in a millimeter wave D2D communication network. [6]

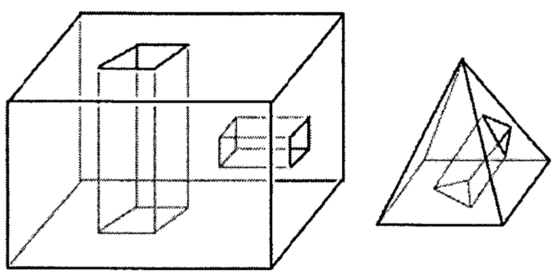
Indian Statistical Institute
First Semestral Examination (2019-20)
M Tech (Computer Science) 2nd Year
Computer Graphics

Date: 03.12.2019

Maximum Marks: 100 Duration: 180 minutes

The answers should be presented point-wise and ***not*** in descriptive style. Clearly specify the input and output in case of an algorithm. Answer not more than **100** marks.

1. How the visibility of a 3D triangular planar patch can be tested with respect to a viewer at (V_x, V_y, V_z)? (5)
2. Propose a visibility test for rendering a scene if the scene contains multiple 3D planar patches. (5)
3. Consider the viewing direction $\boldsymbol{l}(t)=\boldsymbol{s}+\boldsymbol{d}t$, t is a parameter whose value is between 0 and 1, $\boldsymbol{s}$ is the starting point and $\boldsymbol{d}$ is the direction vector. Consider a 3D world containing multiple spheres. How are the visible points on the spherical surfaces estimated? (5)
4. State the differences between the Phong and the Goraud shading. (5)
5. What are the challenges for seed fill algorithm? Suggest techniques to overcome these challenges. (3+2=5)
6. Define bidirectional reflectance distribution function. How transparency can be modeled in rendering a scene? (3+2=5)
7. Give an example showing non-uniqueness of Constructive Solid Geometry based method to model a 3D object. (5)
8. Define simple polyhedron. State Euler's formula for generalized polyhedron. Verify the formula for the following picture. (2+2+1=5)



9. What is meant by Mach band? Which shading method creates maximum Mach band effect and why? (2+3=5)

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10. Write down the Phong's illumination model with the meaning of each term. How the direction of specular reflection is determined using light vector and surface normal? (3+2=5)

11. Given 2D points p_1 and p_2, how a Bezier curve can be drawn starting from p_1 and ending at p_2? Write the algorithmic steps. (5+5=10)

12. Assume a light source and a viewer in a 3D world. Also assume there are two blue planar patches, one close and the other farther away from the viewer. How are the brightness of the blue planar patches generated? How are the brightness vary for the two patches? (7+3=10)

13. State Bresenham's line drawing algorithm. (10)

14. State perspective projection model. State viewing condition responsible for perspective projection. State algorithmic steps to transform an image of parallel lines under perspective projection. (3+3+4=10)

15. Given a light source, a viewer and an illumination model, how the effect of shadow can be rendered in a scene? Using ray tracing algorithm, how is the shadow point detected? (6+4=10)

16. An animation sequence of bouncing ball needs to be created. Assume and state a suitable trajectory of the bouncing ball. Design the steps to implement the animation sequence. Write the pseudocode. (6+4=10)

INDIAN STATISTICAL INSTITUTE

End-Semester Examination (First Semester) : 2019-2020

Course Name : M.TECH. (CS) YEAR II

Subject name : LOGIC FOR COMPUTER SCIENCE

Date : November 28, 2019 Maximum Marks : 50 Duration : 3 hours

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Answer any 10 questions. All questions carry equal marks. Notations/Symbols are as used in the class.

1. Let $\mathcal{L}$ denote the set of formulas in basic modal logic. Is $\mathcal{L}$ countable? Justify your answer.

2. Let Γ be a maximal consistent set of formulas in classical propositional logic. Show the following:

   (a) $\Gamma \vdash \alpha$ iff $\alpha \in \Gamma$.

   (b) $\alpha \vee \beta \in \Gamma$ iff $\alpha \in \Gamma$ or $\beta \in \Gamma$

3. Give conjunctive and disjunctive normal forms of the following formulas:

   (a) $((\alpha \longrightarrow \beta) \longrightarrow \beta) \longrightarrow \beta$

   (b) $(\alpha \longrightarrow (\alpha \wedge \neg\beta)) \wedge (\beta \longrightarrow (\beta \wedge \neg\alpha))$

4. Show that the following formulas are theorems of classical propositional logic:

   (a) $(\alpha \vee \beta) \longleftrightarrow \neg(\neg\alpha \wedge \neg\beta)$

   (b) $((\neg\alpha \longrightarrow \neg\beta) \wedge (\neg\alpha \longrightarrow \beta)) \longrightarrow \alpha$

5. A class of Kripke frames (W, R) with R satisfying a certain property P is said to be defined by a modal formula φ, if for all such frames (W, R), $(W, R) \models \varphi$ iff R satisfies the property P. Here, $(W, R) \models \varphi$ iff $(W, R, V) \models \varphi$ for all valuations V on W. Find the class of frames defined by the following modal operators, with justifications.

   (a) $\Box(\Box\varphi \rightarrow \varphi)$

   (b) $\varphi \rightarrow \Box\Diamond\varphi$

6. Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two Kripke models, such that $(\mathcal{M}_1, w_1)$ is bisimilar to $(\mathcal{M}_2, w_2)$. Show that for all basic modal formulas φ, $\mathcal{M}_1, w_1 \models \varphi$ iff $\mathcal{M}_2, w_2 \models \varphi$.

   Hence, or, otherwise, show that the unary operator $U\varphi$ defined by:

   $$\mathcal{M}, w \models U\varphi \text{ iff } \mathcal{M}, v \models \varphi \text{ for all } v \in W,$$

   is not definable in the basic modal language.

7. Let $\mathcal{M}$ be a Kripke model and w be a world in it. Let φ be a basic modal formula. Consider the modal evaluation game $\mathcal{G}(\mathcal{M}, w, \varphi)$. Let $\mathcal{M}, w \models \varphi$. Show that the player E has a winning strategy in $\mathcal{G}(\mathcal{M}, w, \varphi)$,

8. There exists a finite set of sentences in first order logic whose models are precisely the infinite sets. Prove or disprove.

9. Check whether the following formulas are valid:

   (a) $\forall x \exists y (R(x, y) \longrightarrow R(y, y)) \longrightarrow \exists y (R(y, y) \longrightarrow R(y, y))$.

   (b) $(\exists x.P(x) \rightarrow \exists y \forall z.R(z, f(y))) \rightarrow$
   $((\exists x.P(x) \longrightarrow \forall y \neg \forall z.R(z, f(y))) \longrightarrow \forall x \neg P(x))$.

10. Consider a first order language $\mathcal{L}$ with equality whose vocabulary consists of only a two-place predicate symbol P. Let $\mathfrak{A}$ be a finite structure corresponding to the language $\mathcal{L}$. Let $\mathfrak{B}$ be another structure corresponding to the language $\mathcal{L}$ such that $\mathfrak{A}$ and $\mathfrak{B}$ are elementarily equivalent. Are these two structures isomorphic ? Justify your answer.

11. Consider the relational structure $\mathcal{A} = (\{a, b, c\}, \{(a, b), (a, c)\})$. Is the set $\{b\}$ definable in first order logic whose vocabulory consists of only a two-place predicate symbol P? Justify your answer.

12. Let $\mathcal{M} : (W, R, V)$ be a transitive Kripke model and let Σ be a subformula-closed set of formulas. Let $\mathcal{M}^f : (W^f, R^f, V^f)$ be a filtrated model with respect to Σ, where R^f is defined as follows:

$$R^f|w||v| \text{ iff for all } \varphi, \text{ if } \Diamond\varphi \in \Sigma \text{ and } \mathcal{M}, v \models \varphi \vee \Diamond\varphi \text{ then } \mathcal{M}, w \models \Diamond\varphi$$

Is R^f transitive? Justify your answer.

INDIAN STATISTICAL INSTITUTE

Back Paper Examination (First Semester) : 2019-2020

Course Name : M.TECH. (CS) YEAR II

Subject name : LOGIC FOR COMPUTER SCIENCE

Date : January 15, 2020 Maximum Marks : 100 Duration : 3 hours

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Answer any 8 questions. All questions carry equal marks. 4 marks are reserved for neatness. Notations are as used in the class.

1. Let $X = \{a, b\}$. Let X^* denote the set of all finite strings over X. Is X countable? Justify your answer.

2. Let Γ be a set of formulas and φ be a formula of classical propositional logic. Prove that if $\Gamma \vdash \varphi$ then $\Gamma \models \varphi$, based on the following axiom system:

   Axiom 1: $\varphi \to (\psi \to \varphi)$
   Axiom 2: $(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi))$
   Axiom 3: $(\neg\varphi \to \neg\psi) \to (\psi \to \varphi)$

   and the rule:

   $$\frac{\varphi \qquad \varphi \to \psi}{\psi}$$

3. (a) Give an algorithm to find whether a formula in classical propositional logic is a tautology.

   (b) Show that the following formulas are tautologies of classical propositional logic:

   (a) $\alpha \longrightarrow \neg\neg\alpha$
   (b) $\neg\alpha \longrightarrow (\alpha \longrightarrow \beta)$

4. In each of the following cases, find a suitable first order language and give axioms in it for the given collections of structures:

   (a) Sets of size 3
   (b) Equivalence relation
   (c) Bipartite graphs

5. Check whether the following formulas are valid:

   (a) $\exists x(Px \vee Qx) \to (\exists xPx \vee \exists xQx)$,
   (b) $\forall x(Px \to Qx) \to (\forall xPx \to \forall xQx)$.

6. Give examples to show that the following formulas are not valid.

   (a) $\forall x(\varphi \vee \psi) \to (\forall x\varphi \vee \forall x\psi)$.
   (b) $(\exists x\varphi \wedge \exists x\psi) \to \exists x(\varphi \wedge \psi)$.

7. (a) Prove that the modal formula $\Box\varphi \to \varphi$ characterizes all reflexive Kripke frames.

   (b) Prove that the modal formula $\Box\varphi \to \Box\Box\varphi$ characterizes all transitive Kripke frames.

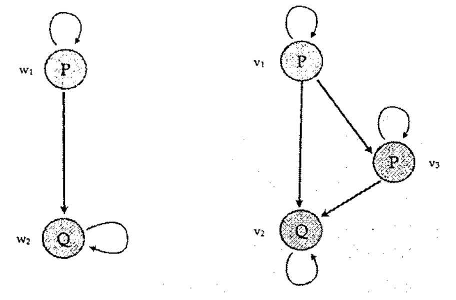
8. Show that the following formulas are valid in basic modal logic.

   (a) $\Box(\varphi \wedge \psi) \to (\Box\varphi \wedge \Box\psi)$,

(b) $\Diamond\varphi \to \neg\Box\neg\varphi$,

(c) $\Diamond(\varphi \vee \psi) \to (\Diamond\varphi \vee \Diamond\psi)$.

9. Are the following two models are bisimilar? Justify your answer.



10. Let Δ be the set of maximal consistent sets in basic modal logic. Let $\Gamma, \Gamma' \in \Delta$. Consider the following two binary relations on Δ.

    (a) $\Gamma R\, \Gamma'$ iff for all formulas φ, $\varphi \in \Gamma'$ implies $\Diamond\varphi \in \Gamma$.

    (b) $\Gamma R' \Gamma'$ iff for all formulas φ, $\Box\varphi \in \Gamma$ implies $\varphi \in \Gamma'$.

    Prove that $\Gamma R\, \Gamma'$ iff $\Gamma R' \Gamma'$.

INDIAN STATISTICAL INSTITUTE
End-Semester Examination: 2019-20 (First Semester)

Course Name: M.Tech. in Computer Science *Subject Name: Distributed Computing Systems*

Date: 26·11·2019 *Maximum Marks: 100* *Duration: 3 hours*

Instructions: *Answer* ***Question 1*** *and* ***any 3 out of the rest.*** *This is an open book examination.*

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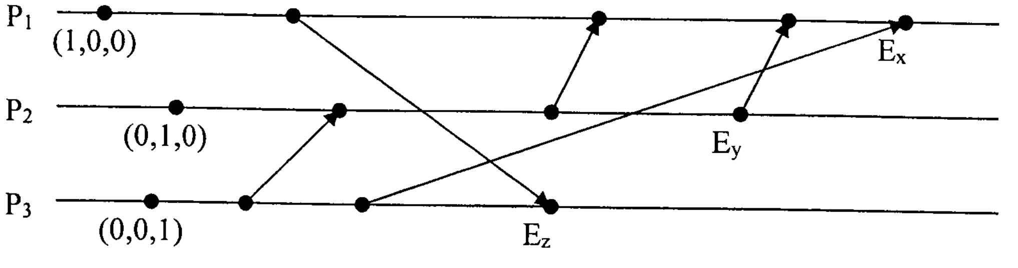
1. Indicate whether each statement below is true or false, with a brief justification / counterexample as necessary. [5 X 5 = 25]
   a) In Lamport's algorithm for distributed mutual exclusion, a process can enter the critical section only when its request is at the top of the request queues of all processes.
   b) The ***2-phase*** algorithm of Ho-Ramamoorthy may detect phantom deadlocks in the ***AND-***request model.
   c) The NetChange algorithm uses more messages in the worst case for stabilizing following a node failure as compared to the number of messages for stabilizing following a node recovery.
   d) Huang's termination detection algorithm is resilient to control message losses.
   e) Vector clocks, like Lamport's clock, define a partial order among events.

2.
   a) In the following space-time diagram, vector clocks are used to timestamp the events. The initial timestamps are given. What are the timestamps of the events E_x, E_y, and E_z?



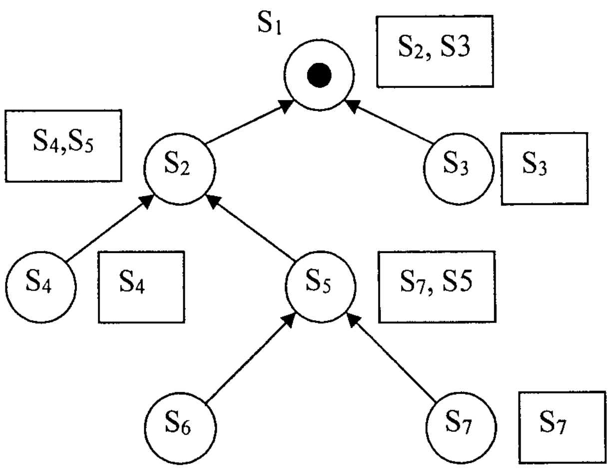
   b) In an implementation of Birman-Schiper-Stephenson's protocol to run on a completely connected network with FIFO links, the checking of the second condition (**$C_j[k] \geq VT_m[k]$ for all $k \neq i$**) got left out by mistake. Show clearly with an example why the protocol will not work correctly in this system.

c) Some of these sequences of records can be present in the log of a site, when the 2-phase commit protocol is in use. Which ones are possible? Justify your answer.

- <no T>, <commit T>
- <ready T>, <abort T>
- <ready T>, <commit T>
- <commit T>, <abort T> [6+7+12]

3.

a) The figure shows a state of Raymond's tree based algorithm for mutual exclusion. S_1 currently holds the token. The request queue at each node is shown beside the node. For example, at S_5, there is a pending request from S_7 followed by one of its own.



- Which sites are requesting?
- In what sequence will the requesting sites enter the critical section?
- Which of these represent possible sequences in which the requests were made? Answer Yes/No for each of the following.
  - (i) S_7, S_3, S_4, S_5
  - (ii) S_4, S_3, S_7, S_5
  - (iii) S_4, S_3, S_5, S_7

b) The *2-exclusion* problem is similar to the mutual exclusion problem, but here at most 2 nodes can be in the critical section at the same time. Design an algorithm to achieve *2-exclusion* in a distributed system. Argue briefly why no more than 2 nodes can be in the critical section at the same time in your algorithm. Analyze the message complexity (per critical section entry) of your algorithm. [(4+5+6)+10]

4. In the lecture we only discussed node failures, but we always assumed that edges (links) never fail. Let us now consider edge failure consensus problem. Assume that all nodes work correctly, but up to f edges may fail. Analogously to node failures, edges may fail at any point during the execution. We say that a failed edge does not forward any message anymore, and remains failed until the algorithm terminates. Assume that an edge always simultaneously fails completely, i.e., no message can be exchanged over that edge anymore in either direction. We assume that the network is initially fully connected, i.e., there is an edge between every pair of nodes and nodes have arbitrary input values in the beginning. Our goal is to solve consensus in such a way, that all nodes know the decision.

(i) What is the smallest ***f*** such that consensus might become impossible? (which edges fail in the worst-case)

(ii) What is the largest ***f*** such that consensus might still be possible? (which edges fail in the best-case)

(iii) Assume that you have a setup which guarantees you that the nodes always remain connected, but possibly many edges might fail. A very simple algorithm for consensus is the following: Every node learns the initial value of all nodes, and then decides locally. How much time might this algorithm require? Assume that a message takes at most 1 time unit from one node to a direct neighbor. [7+8+10]

5.

a) Consider a wait-for graph G = (V,E), where V = {T1, ..., T8} and E={(T1,T2), (T1,T4), (T2,T3), (T2,T6), (T4,T3), (T4,T5), (T5,T6), (T5,T7), (T6,T8), (T7,T8), (T8,T5), (T8,T6)}, where an edge ***(Ta,Tb)*** indicates Ta is waiting for a resource held by Tb.

- If we assume an ***OR***-request model, then which processes are deadlocked?
- If we assume an ***AND***-request model, then which processes are deadlocked?

b) Analyze the truth of this claim: *In the OR-request model, a deadlocked process must belong to one or more directed cycles in the wait-for graph.* Your answer must begin with True/False and then provide the justification/counter-example.

c) What will be the impact of a message loss in Chandy et al.'s diffusion computation based deadlock detection algorithm for the OR-request model. Will it miss a real deadlock? Will it report a phantom deadlock? Justify your answer.

d) Which of the following algorithms assume FIFO channels? For the ones which require FIFO channels, you must indicate what may happen if the channels are not actually FIFO.

- Ricart-Agrawala Algorithm for Mutual Exclusion
- Chandy-Lamport's Algorithm for Global State Recording
- Birman-Schiper-Stephenson protocol for Causal Ordering of messages

[6+7+6+6]

INDIAN STATISTICAL INSTITUTE

Semestral Examination:(2019-2020)

M.TECH (CS) II YEAR

Subject Name: Quantum Information Processing and Quantum Computation

Maximum Marks: 60 Duration: 3.0 hours Date: 26.11.2019

Answer any five of the following six questions

1. (a) Consider two systems A and B situated in two distant laboratories. A_1 and A_2 are two possible observables on system A with possible outcomes $+1$ and -1 and similarly B_1 and B_2 for system B. Prove that statistics collected from measurement of these observables for any preparation of the bipartite system satisfies the following inequality:

$$|\langle A_1 B_1\rangle + \langle A_1 B_1\rangle + \langle A_1 B_1\rangle - \langle A_1 B_1\rangle| \leq 2,$$

if the theory is local and deterministic.

(b) Show that for two-qubit singlet state $|\psi^-\rangle_{AB}$, spin observable can be chosen in such a way that the above quantity can be equal to $2\sqrt{2}$. What is the implication of this violation of the above mentioned inequality?

[8 + 4]

2. Consider the following three qubits state shared between Alice, Bob and Charlie stationed at distant laboratories:

$$|\psi\rangle_{ABC} = \frac{1}{\sqrt{2}}[|000\rangle + |111\rangle],$$

where $|0\rangle$ and $|1\rangle$ form an orthogonal basis in two dimensional Hilbert space.
i) Show that the state can neither be written in the form $|\psi\rangle_A \otimes |\phi\rangle_B \otimes |\chi\rangle_C$ nor in the form $|\eta\rangle_{AB} \otimes |\tau\rangle_C$.
ii) Show that Alice can help to create the Bell state $|\phi^+\rangle$ between Bob and Charlie where all of them are allowed to do local operation and classical

communication.
iii) If Alice shares another state

$$|\phi\rangle_{AD} = a|00\rangle + b|11\rangle, |a|^2 + |b|^2 = 1$$

with Dick then show that this state can be prepared between Bob and Charlie by local operation and classical communication using $|\psi\rangle_{ABC}$ as resource.

[2 + 4 + 6]

3. a) Consider a function $f : \{0,1\}^n \to \{0,1\}$
Let there be a unitary gate U_f which acts in the following way;

$$U_f|x\rangle|y\rangle = |x\rangle|y \oplus f(x)\rangle$$

where $x \in \{0,1\}^n$ and $y \in \{0,1\}$
Show that

$$U_f|x\rangle\frac{1}{\sqrt{2}}[|0\rangle - |1\rangle] = (-1)^{f(x)}|x\rangle\frac{1}{\sqrt{2}}[|0\rangle - |1\rangle]$$

b) Consider a function $f : \{0,1\}^n \to \{0,1\}$, where the function f is either constant or balanced ($f(x) = 0$ for half of the possible input values). Discuss how many queries will be required to learn the nature of the function in classical world. Describe the quantum algorithm by which the function can be shown to be either constant or balanced without calculating the function at various points.

[4 + 2 + 6]

4. Consider a function $f : \{0,1\}^n \to \{0,1\}^n$. The function has a period given by n-bit string a: that is
$f(x) = f(y)$ iff $y = x \oplus a$
a) Discuss how hard it is to find the period a in the classical world.
(b) Show that there is a quantum algorithm by which the period can be found in polynomial time.

[2+10]

5. a) Consider a copy of the following n-qubit state:

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{y\in\{0,1\}^n} (-1)^{x.y}|y\rangle$$

How do you estimate the phase x where $x \in \{0, 1\}^n$?
b) Again consider another copy of the following n-qubit state;

$$|\phi\rangle = \frac{1}{\sqrt{2^n}} \sum_{y=0}^{2^n-1} e^{2\pi i \omega y} |y\rangle,$$

where $\omega \in (0, 1)$ and y is a binary encoding of the integer y taking values from 0 to $2^n - 1$. Obtain a good estimation of the phase ω.
c) Using the Quantum Fourier Transform (QFT) provide a sketch of the quantum algorithm for factoring large numbers.

[2+5+5]

6. The function $f : \{0, 1\}^n \to \{0, 1\}$ is such that:
$f(x) = 1$, for $x = \omega$ and $f(x) = 0$ for $x \neq \omega$.
a) In the classical world, how many queries are required for finding ω?
b) Show how a quantum algorithm provides a quadratic speed up for this search problem.

[2+10]