INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2018-19

Programme Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: Operations Research-I

Date of Exam: 03.09.18 **Total Marks: 60** **Duration: 2 hrs.**

[Assignment: 40 marks]

Answer any six (06) questions. All questions carry equal marks.

1. Consider the following problem of financial planning.
   A bank makes four kinds of loans to its personal customers and these loans yield the following annual interest rates to the bank:

   - First mortgage 14%
   - Second mortgage 20%
   - Home improvement 20%
   - Personal overdraft 10%

   The bank has a maximum foreseeable lending capability of Rs.250 million and is further constrained by the policies:

   a) first mortgages must be at least 55% of all mortgages issued and at least 25% of all loans issued (in Rs. terms)
   b) second mortgages cannot exceed 25% of all loans issued (in Rs. terms)
   c) to avoid public displeasure and the introduction of a new tax the average interest rate on all loans must not exceed 15%.

   Formulate the bank's loan problem as an LP problem so as to maximise interest income whilst satisfying the policy limitations. Please explain while constructing equivalent mathematical constraints based on the above polices of the bank.

   [10]

2. Prove that every basic feasible solution (b.f.s) of a LPP is an extreme point of the convex set of all feasible solutions (f.s.).
Show that the set of vectors $a^1 = (2,-1,0)$, $a^2 = (3,5,1)$ and $a^3 = (1,1,2)$ form a basis in E_3.

[6+4=10]

3. Construct a LP problem with two variables and show graphically that the problem is unbounded. Write down its corresponding dual and interpret its possible solution graphically. State the condition for unboundedness of a primal problem while using Simplex method.

[4+4+2=10]

4. What are the basic assumptions of an EOQ model? Derive optimal parametric values (order qty, ordering interval and cost) of the EOQ model when several items can be procured with the same ordering cost.

[3+7=10]

5. Oranges are grown, picked, and then stored in warehouses in Nagpur, Darjeeling, and Coorg. These warehouses supply oranges to markets in West Bengal, Bihar, U.P., and Andhra Pradesh. The following table shows the relative distance per truckload (in 100s of Km.), along with supply and demand quantities for respective warehouses and markets.

		Markets				
		W.B.	Bihar	U.P.	A.P.	Supply
Warehouse	Nagpur	12	10	9	8	200
	Darjeeling	6	5	10	20	200
	Coorg	21	23	22	7	200
	Demand	140	180	120	160	

(a) Set up the transportation tableau for this problem and determine the initial solution using the minimum cost method.
(b) Test for the optimal solution.

[5+5=10]

6. A construction company has four large tippers located at four different garages. The tippers are to be moved to four different construction sites. The distances in kms. between the tippers and the construction sites are given below.

	Site→	S1	S2	S3	S4
Garage	G1	90	75	75	80
	G2	35	85	55	65
	G3	125	95	90	105
	G4	45	110	95	115

How should the tippers be moved to the construction sites in order to minimize the total distance travelled?

[10]

7. (a) State the basic differences between PERT and CPM techniques under project management.

   (b) Explain the terms for a project network: Critical path, Dummy Activity, Early start and Late finish.

[4+6=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2018-19

Course Name: **M. Tech. (QR&OR) - I Year**

Subject Name: **Quality Management & Systems**

Maximum Marks: 100 Duration: 2 hours

Date: 04 September 2018

Note: Answer all the questions.

1. State the different views of quality with brief explanation. Mention which quality dimensions are generally important for each of these views of quality.

[12 + 8 = 20]

2. Define the customer satisfaction model proposed by Kano.

[10]

3. Write short notes on any three of the following.
   (a) Kaoru Ishikawa
   (b) Philip B. Crosby
   (c) Armand V. Feigenbaum
   (d) EFQM
   (e) MBNQA

[6 + 6 + 6 = 18]

4. Draw a diagram of House of Quality and describe its different components.

[6 + 6 = 12]

5. Assignments.

[40]

Indian Statistical Institute

Mid-Semestral Examination : 2018-19

M-TECH(QR&OR) -- 1st YEAR (E - STREAM)

PROBABILITY-1

{Answer all the questions}

Date: 05.09.18 **Full marks: 60** **Time: 2 hours**

[Symbols have their usual meaning]
Answer all the questions

1. a) State and prove Bonferroni's inequality.
b) State and prove Poincare's theorem.

[12+8=20]

2. a) Let there be 5 letters and 5 envelopes, each letter is having a right pair of envelope. What is the probability that all letters will be placed in wrong envelopes?
b) In a doll manufacturing company there are three machines. 30%, 50% and 20% of the items are produced by machine A, B, and C respectively. Machine A is producing 20% defective items, machine B is producing 30% defective items, machine C is producing 40% defective items. An item is selected randomly and is found to be defective. What is the probability that it is produced by machine B?

[10+10=20]

3. a) Consider a sequence of independent Bernoullii's trials with success probability p. Define a random variable X as # failures preceeding rth success. Find the probability mass function of X. Find its expectation and variance.
b) A set of 8 coins were tossed . The frequency distribution of the number of heads are given below. Fit a binomial distribution to the following data.

# heads	0	1	2	3	4	5	6	7	8
frequency	1	5	25	65	66	48	35	8	2

[10+10=20]

Indian Statistical Institute
M. Tech (QR & OR) I Year (E & S Stream)
Semester-I: 2017-18
Mid-Semester Examination

Sub: Statistical Methods-I

Date: 06/09/2018 Maximum Marks: 60 Time: 2 hours.

Note: Answer all questions.

1. Travel times (in minutes) to work of 20 workers are given below.

$$5, 7, 10, 10, 10, 10, 11, 12, 13, 15, 20, 20, 22, 25, 30, 30, 32, 40, 40, 55, 60.$$

Construct a stem-and-leaf plot of travel times. Comment on the shape of the distribution of travel times.

[7]

2. Write true or false with justification

   (a) Seventy five percent of obervations lie between the first quartile and third quartile.

   (b) The standard deviation is least affected if an extreme outlier is added to the data.

   (c) Suppose the correlation coefficient between x and y is r. If all the values of y are multiplied by a constant $a < 0$, the absolute value of the correlation coefficient remains same.

[2+2+5= 9]

3. Let x be a variable assuming the values $1, 2, \ldots, m$ and $F_1 = n, F_2, \ldots, F_m$ be the corresponding cumulative frequencies of the greater-than type. Show that $\bar{x} = \frac{1}{n}\sum_{i=1}^{m} F_i$. [6]

4. Suppose x_{1j}, $j = 1, \ldots, n_1$ and x_{2j}, $j = 1, 2, \ldots, n_2$ are the values of a variable in the two sets. Then show that the variance of the combined set is

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\bar{x}_1 - \bar{x}_2)^2,$$

where $\bar{x}_i$ and s_i are the mean and standard deviations of ith (i=1, 2) set, respectively.

[7]

5. Show that the correlation coeffiecient (r) between two variables satisfies $-1 \leq r \leq 1$.

[8]

6. Suppose x denotes the sourface roughness of coated interior pipe. The data on surface roughness (in micrometers) of 20 sample sections of coated interior pipe are collected and the following summary statistcs are obtained from the data:
Minimum = 1.060, Maximum = 2.640, $\sum_{i=1}^{20} x_i = 37.62$, $\sum_{i=1}^{20} x_i^2 = 75.9784$, $\sum_{i=1}^{20} x_i^3 = 161.8584$, $\sum_{i=1}^{20} x_i^4 = 358.2773$,, $Q_1 = 1.367$, $Q_2 = 2.040$ and $Q_3 = 2.258$.

P.T.O.

(a) Determine the skewness. Comment on the shape of the distribution of surface roghness based on skewness value.

(b) Provide a rough sketch of box plot. Comment on the shape of the distribution of surface roughness.

(c) Find the correlation coefficient between x and x^2. Comment on the type of relationship between x and x^2 for the given data.

[(5+1)+ (3+1)+ 7 =17]

7. A total of 150 customers of a petrol station were asked about their satisfaction with their car and motor bike insurance. The results are given in the following table.

		Satisfaction level	
		Satisfied	Unsatisfied
Vehicle type	Car (Petrol)	33	25
	Car (Diesel)	29	31
	Motorbike	12	20

(a) Find the marginal frequency distributions of Vehicle type and Satisfaction level.

(b) Find the conditional distribution of Satisfaction level given that Vehicle type is Car (Petrol).

[4 + 2 = 6]

Indian Statistical Institute

Mid-Semester Examination

Course name: M.TECH(QROR)-1st Year (2018)

Subject: Programming Techniques and Data Structures

Date 07.09.2018 Full Marks 50 Time: 2 Hrs

(Answer all questions)

1. Write the output of the following program when executed. [3+5]

```
#include <stdio.h>
int x =9;
int main (void){
        int x=7;
        int i=0;
        printf("Output of Function_1 is: %d\n", function_1(x));
        for (; i<10; i=i+2){
                x = function_2(0);
                 printf(" Output of %d th iteration is %d\n",i,x);
        }
}

int function_1(int counter){
        x++;
        if (!counter){
                 return x;
        }
        x = x+counter--;
        return x;
}

int function_2(int counter){
        static int x = -3;
        x++;
        if (counter==0){
                return x;
        }
        x = counter--;
        return x;
}
```

2. What is a Flow chart? Draw a Flow chart to find the sum of the following series for given values of x and k. [2+8]

$$1 - \frac{x}{1!} + \frac{x^2}{2 \times 2!} - \frac{x^3}{3 \times 3!} + \ldots\ldots\ldots\ldots\ldots + (-1)^k \frac{x^k}{k \times k!}$$

3. Write a C Program to generate first N (take as input) non-prime terms of the Fibonacci sequence which are greater than 100. [14]

4. Write the algorithm for Binary search technique. [8]

5. (a) Convert $(11010101101)_2$ to Hexadecimal. [2+4+2+2]

   (b) Which of the followings are invalid variable names in C and why?

   2018QROR; Float; int x; Mtech@2018

   (c) Write down the equivalent statement of the rewind(fp) using fseek function.

   (d) Write the differences between Malloc and Calloc functions.

Indian Statistical Institute
M. Tech (QR & OR) I Year
Semester-I: 2018-19
Semester Examination

Subject: Statistical Methods-I

Date: 12/11/2018 Maximum Marks: 100 Time: 3 hours.

Note: This paper carries 112 marks. You may answer as much as you can but the maximum you can score is 100.

1. Suppose there are k class intervals in the frequency distribution of a continuous variable x, each of width h. The lower boundary of the first class is x_0 and the frequency of each class is f. Show that the mean of the frequency distribution is $x_0 + \frac{kh}{2}$. Comment on the type of distribution based on the frequency distribution.

[6 + 2 = 8]

2. Show that the mean deviation about the median is least.

[10]

3. Show that Spearman's rank correlation coefficient can be derived as the product moment correlation coefficent between two sets of ranks (Consider the case of no ties).

[10]

4. A medical researcher claimed that smoking can result in wrinkled skin around the eyes. The smoking habit as well as the presence of prominent wrinkles around the eyes are recorded for a random sample of 500 persons. The frequency table is obtained as follows

	Prominent wrinkles	Wrinkles not prominent
Heavy smoker	95	55
Light or nonsmoker	103	247

Compute a measure of association between smoking habit and wrinkle. Comment on the degree of association between smoking habit and wrinkle.

[8+2 =10]

5. (a) What is simple random sampling?

   (b) Let $X_1, \ldots, X_n$ be a SRSWOR sample of size n from a finite population with size N. The population mean and variance are μ and σ^2. Show that the sample mean $\bar{X}$ is an unbiased estimator of μ. Find the variance of $\bar{X}$.

[2+ (3+7)=12]

P.T.O.

6. Suppose a fire insurance company wants to relate the amount of fire damage in major residential fires to the distance between the residence and the nearest fire station. The study is to be conducted on a large suburb of a major city; a sample of 15 recent fires in the suburb is selected. The amount of damage y and the distance between the fire and the nearest fire station x are recorded for each fire. The results are given below.

   x : 3.4, 1.8, 4.6, 2.3, 3.1, 5.5, 0.7, 3.0, 2.6, 4.3, 2.1, 1.1, 6.1, 4.8, 3.8
   y: 26.2, 17.8, 31.3, 23.1, 27.5, 36.0, 14.1, 22.3, 19.6, 31.3, 24.0, 17.3, 43.2, 36.4, 26.1

   Compute the correlation coefficient between x and y. Can you use linear relationship between y and x? - Justify your answer.

   [8+2 =10]

7. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Derive a sufficient statistic for λ. Show that the maximum likelihood estimator (MLE) of λ is a function of the sufficient statistic. Show that the MLE of λ is a consistent estimator of λ.

   [5+ 5 +5 =15]

8. Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with density function

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Derive the maximum likelihood estimator (MLE) of θ.

   (b) Show that the MLE of θ is an unbiased estimator of θ.

   (c) Find the Fisher information for θ. Show that the MLE of θ attain the Cramer-Rao lower bound. Find the efficiency of the estimator.

   [4 + 4 + (3 + 4 + 2) = 17]

9. Highway marking paint is supplied in two colors, white and yellow. The drying time of this paint is of interest, and specially it is suspected that yellow paint dries faster than white paint. Sample measurements for both types of paints are obtained. The drying times (in minutes) are shown below.
   White: 120, 132, 123, 122, 140, 110, 120, 107
   Yellow: 126, 124, 116, 125, 109, 130, 125, 117, 129, 120

   (a) Find a 95% confidence interval on the difference in mean drying times, assuming that the standard deviations of drying times are equal. Assume that drying time is normally distributed.

   (b) Is there any evidence to indicate that yellow paint dries faster than white paint?

   [10+2 = 12]

10. Generate five observations from an exponential distribution with mean 1 using the following observations from an Uniform(0, 1)

$$0.769, 0.443, 0.172, 0.798, 0.216$$

    Hence or otherwise generate an observation from a chi-square distribution with 10 degrees of freedom.

    [5+3 = 8]

Table A.4 Cumulative Normal Distribution

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

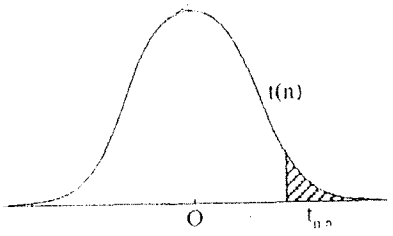
X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.000001

Source: Fisher, R. A. (1953): Statistical Tables for Biological, Agricultural and Medical Research. Oliver and Boyd, London.

Table A.6 Student's t-Distribution

The first column lists the number of degrees of freedom (n). The headings of the other columns give probabilities (α) for t to exceed the entry value, use symmetry for negative t values.



n \ α	.10	.05	0.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1,313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

Source: P. G. Hoel. *Introduction to Mathematical Statistics*, 4th Ed., John Wiley, New York, 1971, page 393.

INDIAN STATISTICAL INSTITUTE

Semestral Examination : 2018-19

M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY

Note : Answer any FIVE questions

[Symbols have their usual meaning]

Date: 14.11.18 **Full marks:100** **Time: 3 hours**

1. a) State and prove weak law of large number due to Khintchine.
   b) State and prove De Moivre's theorem.
   c) Let X ~ gamma distribution with parameter $\alpha = 1$ and p. Show that $P(X \geq 2p) \leq 1/p$. (State the result you have used.)

[6+10+4=20]

2. a) Let $(X, Y) \sim N_2(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$. Find the marginal distribution of Y, conditional expectation and Variance $E(X|Y)$ and $Var(X|Y)$ respectively.
   b) Let $(X, Y) \sim N_2(\mu_1, \mu_2, \sigma_1, \sigma_2, \rho)$ where, $\mu_1 = 0$, $\mu_2 = -1$, $\sigma_1 = 1$, $\sigma_2 = 2$, $\rho = -½$.
   Find i) $P(X+Y > 0)$
   ii) 'a' such that $a*X+Y$ and $X+2*Y$ are independent.

[(5+5)+(5+5)=20]

3. a) Let $(x_1, x_2, \ldots, x_n)$ be a sample of size n drawn from $N(\mu, \sigma^2)$. Let $\bar{x}$ and s^2 be the corresponding sample mean and the sample variance. Find the distribution of $\bar{x}$ and s^2 and show that they are independent.
   b) Let $X_i \sim$ iid exp (λ), $i = 1(1) n$
   Let $X_{(1)} = \text{Min}(X_1, \ldots, X_n)$
   $X_{(n)} = \text{Max}(X_1, \ldots, X_n)$
   Find the distribution of $X_{(1)}$ and $X_{(n)}$.

[12+8=20]

4. a) Two absent minded roommates A and B forget their umbrellas in some way or another. A always takes umbrella when he goes out, while B forgets to take umbrella with probability ½. Probability that each of them forgets his umbrella at a shop is ¼. After visiting 3 shops they return home. Find the probability that they have only one umbrella after their return.
   b) In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices, only one answer being correct. The probability that he makes the guess is 1/3 , the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copies it is 1/8. Find the probability that he knew the answer to the question given that he answered it correctly.
   c) Two athletic teams A and B play a series of independent games until one of them wins 4 games. The probability of each team winning a game is 1/2. Find the probability that the series will end in at most 6 games.

[7+6+7=20]

P.T.O

5. a) Define monotonic sequence of events. Let $\{A_n\}$ be monotonic sequence of events, each belonging to sigma field of events $A \subseteq \Omega$, then prove that,

   $\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)$

   b) Suppose X is a continuous random variable with p.d.f f(x) and c.d.f F(x). Let Y be another random variable such that Y = F(x). Find the p.d.f of Y

   c) Let $X \sim N(\mu, \sigma^2)$ and $Y = X^2$. Find the distribution of Y.

[12+4+4=20]

6. a) Let X follows Normal distribution (with parameters μ and σ) truncated to the left of X = a, and right of X = b . Find the p.d.f and expectation of X.

   b) A student takes a multiple choice test consisting of two problems. The first one has 4 possible answers of which one is correct and the second one has 7 possible answers of which one is correct. The student selects one answer at random for each question and scores 5 if it is correct otherwise gets 0. Let X be the score of the student. Find its expectation and variance.

[13+7=20]

Indian Statistical Institute

Semester Examination

Course name: M.TECH(QROR)-1st Year (2018)

Subject: Programming Techniques and Data Structures

Date 16.11.2018 Full Marks 100 Time: 3 Hrs

(Answer all questions)

1.

(a) What is a pointer in C. Write down the difference between int *p and int **p. [2+2]

(b) Write the output of the following program when executed. [6]

```
#include <stdio.h>
long function(int *a, int N){
    int count, c;
    c = 0;
    count = 1;
    do{
        c += *a;
        a++;
        count ++;
    } while (count<=N);

    return c;
}
int main(void){
    int num[] = {1, 3, 5, 7, 9, 11, 13};
    int *arr = num;
    printf ("The output of function: %ld\n", function(arr, 7));
    return 0;
}
```

2. Write a Flowchart to compute the sum of the series for given values of x and k [10]

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots\ldots\ldots + (-1)^k \frac{x^{2k}}{(2k)!}$$

3. Write the algorithm for the quick sort technique. Also write the complexity of this algorithm. [8+2]

4. An input file contains some sorted integer data. Using the Binary search technique write a C program which detects an input value is present in the file or not. [10]

5. Write a C Program that checks an input number N is prime or not, and generates all prime numbers which are less than or equal to N. [10]

P.T.O.

6. Create a linked-list of N (take as input) nodes where data part of each node contains student name (string), age (integer), class (string) and marks (in Mathematics). Now print the names of students who have received less than 30% marks in Mathematics. Print NONE if no such student is there. [10]

7. A file (take its name as input) contains list of some alphabetic English words. Write a C program that will take each English word from the file and for each word print (on the screen) a respective word as its output where all non-vowel characters will be in the beginning of the output word and all the vowels at the end of the non-vowels. For example, if a word in the file is "Statisical" then the output of this word will be 'Sttsclaiia'. [10]

8. Write the algorithm to convert from Infix to postfix notation of an expression using stack. Write down the postfix expression of the expression A+B^C*D+(E+F)/G-H [8+2]

9. What is a binary tree? Write down the difference between a Binary tree and an AVL tree. Given the following numbers **50, 25, 10, 5, 7, 3, 30,20,8,15** and you need to draw the graphical representation to design an AVL tree using these numbers. Show the graphical representations of the AVL tree after insertion of each number in the tree. [1+1+8]

10. (a) Convert $(10010)_8$ to Hexadecimal. [3]

    (c) What is a B-tree? Give an example of a B-tree of order 4. [1+3]

    (d) Explain the function of free() and fseek() functions in C language. [3]

Indian Statistical Institute

Semester Examination

Course name: M.TECH(QROR)-1st Year (2018)

Subject: Programming Techniques and Data Structures

Date 16.11.2018 Full Marks 100 Time: 3 Hrs

(Answer all questions)

1.

(a) What is a pointer in C. Write down the difference between int *p and int **p. [2+2]

(b) Write the output of the following program when executed. [6]

```
#include <stdio.h>
long function(int *a, int N){
    int count, c;
    c = 0;
    count = 1;
    do{
        c += *a;
        a++;
        count ++;
    } while (count<=N);

    return c;
}
int main(void){
    int num[] = {1, 3, 5, 7, 9, 11, 13};
    int *arr = num;
    printf ("The output of function: %ld\n", function(arr, 7));
    return 0;
}
```

2. Write a Flowchart to compute the sum of the series for given values of x and k [10]

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots\ldots\ldots + (-1)^k \frac{x^{2k}}{(2k)!}$$

3. Write the algorithm for the quick sort technique. Also write the complexity of this algorithm. [8+2]

4. An input file contains some sorted integer data. Using the Binary search technique write a C program which detects an input value is present in the file or not. [10]

5. Write a C Program that checks an input number N is prime or not, and generates all prime numbers which are less than or equal to N. [10]

P.T.O.

6. Create a linked-list of N (take as input) nodes where data part of each node contains student name (string), age (integer), class (string) and marks (in Mathematics). Now print the names of students who have received less than 30% marks in Mathematics. Print NONE if no such student is there. [10]

7. A file (take its name as input) contains list of some alphabetic English words. Write a C program that will take each English word from the file and for each word print (on the screen) a respective word as its output where all non-vowel characters will be in the beginning of the output word and all the vowels at the end of the non-vowels. For example, if a word in the file is "Statisical" then the output of this word will be 'Sttsclaiia'. [10]

8. Write the algorithm to convert from Infix to postfix notation of an expression using stack. Write down the postfix expression of the expression A+B^C*D+(E+F)/G-H [8+2]

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10. (a) Convert $(10010)_8$ to Hexadecimal. [3]

    (c) What is a B-tree? Give an example of a B-tree of order 4. [1+3]

    (d) Explain the function of free() and fseek() functions in C language. [3]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2018-19

Programme Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Course: Operations Research-I

Date of Exam: 19-11-2018 **Max Marks: 100** **Duration: 3** hrs.

Question no. 1 and 8 are compulsory. Use of scientific calculator/ RMMR table is allowed.

Answer as many questions as you can. Maximum you can score is 100.

1. Explain the following terms:
   (a) Project Crashing
   (b) Self Dual Problem
   (c) Inverse Transformation Method in Simulation
   (d) Queue Characteristics
   (e) Safety stock and Stock out cost
   (f) Little's law

[3+3+3+3+3=15]

2. (a) Explain the physical interpretations of a dual problem considering its objective function, dual variables, dual constraints and its coefficients.
   (b) Solve the following problem using Simplex method after converting it to its dual:

Maximize $3X_1 + 2X_2$

Subject to

$$X_1 + X_2 \geq 1$$
$$X_1 + X_2 \leq 7$$
$$X_1 + 2X_2 \leq 10$$
$$X_2 \leq 3$$
$$X_1, X_2 \geq 0.$$

[6+9=15]

P. T. O.

3. As a Project Manager, find the following

   a) Critical Path and expected project completion time;
   b) Least Cost schedule and the corresponding project cost;
   c) 12 days schedule and the corresponding project cost.

   Given: Indirect cost per day = Rs. 40/-

   The information on activities, their durations for completion and corresponding cost figures are tabulated below.

Activity Node	Normal duration (days)	Crash duration (days)	Cost (Rs.)	Crash Cost (Rs.)
(1,2)	8	6	100	160
(1,3)	4	2	150	350
(2,4)	2	1	50	90
(2,5)	10	5	100	400
(3,4)	5	1	100	200
(4,5)	3	1	80	100

   Comment on the solutions obtained in b) and c).

   [5+4+6=15]

4. (a) Derive the optimal order quantity for quantity discount model after stating the most important assumptions and underlying constraints.

   (b) A digital printing press has an annual demand of 1,400 portable hard disk. The cost of a hard disk is Rs. 400. Carrying cost is estimated to be 20% of the unit cost, and the ordering cost is Rs. 25 per order. If the company orders in quantities of 300 or more, it can get a 5% discount on the cost of the hard disks. Should the company take the quantity discount? Assume the demand is constant.

   (c) Mention the basic differences between (t_p, S) policy and (t, S_p) policy. The notations have standard meaning.

   [5+7+3=15]

5. (a) Explain how to estimate safety stock when demand and lead time of items are uncertain and stock out cost is unknown.

   (b) State all the assumptions of a quantity discount inventory model and derive its optimal order quantity for each discount level.

   [7+8=15]

6. Define a Markov process in terms of a stochastic process. In general, what type of Markov process a Queue model is and why? Derive steady-state solution for p_n of a Poisson queue stating all its assumptions. Give one real life example each for encouraging and discouraging queues.

[2+3+7+3=15]

7. State all the assumptions of a (M/M/m/FCFS/∞/∞) model. Derive the expression for the expected number of customers in the queue under steady state conditions.

[4+11=15]

8. SAMSUNG sells and services several brands of home appliances. Past sales for a particular model of Smartphone have resulted in the following probability distribution for demand:

Demand per week	0	1	2	3	4
Probability	0.2	0.35	0.25	0.15	0.05

The lead time, in weeks, is described by the following distribution:

Lead time (week)	1	2	3
Probability	0.10	0.40	0.50

Based on cost considerations as well as storage space, the company has decided to order 10 of these each time an order is placed. The carrying cost is Rs. 50 per week for each unit that is left in the inventory at the end of the week. The stock out cost is set at Rs 4,000 per stock out. The company has decided to place an order whenever there are only 2 smartphones left at the end of the week. Simulate 10 weeks of operation for SAMSUNG with currently 5 units in inventory.

What would be the weekly carrying cost under this situation?

[12+3=15]

Indian Statistical Institute
M. Tech (QR & OR) I Year (E Stream)
Semester-I: 2018-19
Back Paper Examination

Sub: Statistical Methods-I

Date: 07/01/2019 Maximum Marks: 100 Time: 3 hours.

Note: Answer all questions.

1. (a) Describe primary data and secondary data.
   (b) Construct a box plot for the data given below with the summary statistics.

   $$4.2, 4.7, 4.7, 5.0, 3.8, 3.6, 3.0, 5.1, 3.1, 3.8, 4.8, 4.0, 5.2, 4.3, 2.8, 2.0, 2.8, 3.3, 4.8, 5.0$$

   For the above given data set, $Q_1 = 3.25$, $Q_2 = 4.10$ and $Q_3 = 4.80$. What comments can you make about the shape of the distribution?

   [4 + (6+2) =12]

2. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables with mean μ and variance σ^2.
   (a) Consider the following estimators for μ

   $$\hat{\mu}_1 = \frac{X_1}{4} + \frac{X_2 + \cdots + X_{n-1}}{2(n-2)} + \frac{X_n}{4} \text{ and } \hat{\mu}_2 = \bar{X}$$

   Show that both the estimators are unbiased. Find the efficiency $\hat{\mu}_2$ relative to $\hat{\mu}_1$.
   (b) Show that the sample variance $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of σ^2.

   [(5+6) + 7 =18]

3. Let $X_1, \ldots, X_n$ be a random sample from a distribution with the probability density function

   $$f(x) = \frac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}}, \quad x \geq 0, \ \theta > 0$$

   (a) Find a sufficient statistic for θ.
   (b) Find the maximum likelihood etsimator of θ.
   (c) Find the Fisher information for θ.

   [5+5+4=14]

4. Suppose that X is a discrete random variable with

   $$P(X=0) = \frac{2}{3}\theta, \quad P(X=1) = \frac{1}{3}\theta, \quad P(X=2) = \frac{2}{3}(1-\theta), \quad P(X=3) = \frac{1}{3}(1-\theta)$$

   where $0 \leq \theta \leq 1$ is a parameter. The following 10 indepndent observations were taken from such a distribution: (3, 0, 2, 1, 3, 2, 1, 0, 2, 1).

(a) Find the moment estimate of θ.

(b) Construct the likelihood function to obtain maximum likelihood estimate of θ.

[8 + 4 =12]

5. A machine produces metal rods used in an automobile suspension system. A random sample of 15 rods is selected, and the diameter (in mm) is measured. The resulting data are shown below.

$$8.24, 8.21, 8.23, 8.25, 8.26, 8.23, 8.20, 8.26, 8.19, 8.23, 8.20, 8.28, 8.24, 8.25, 8.24$$

(a) Assume that the rod diameter is normally distributed. Derive the maximum likelihood estimates of mean and variance of the rod diameter. Construct a 99% confidence interval on the mean rod diameter.

(b) Describe a graphical method to verify whether normal distribution fits the data.

[(8 + 6) + 6 =20]

6. Generate 10 observations from the Bernoulli distribution with probability of success $p = 0.7$ using the following observations from an Uniform(0, 1)

$$0.769, 0.443, 0.172, 0.799, 0.216, 0.050, 0.495, 0.077, 0.601, 0.319$$

Hence or otherwise generate an observation from a binomial distribution with $n = 10$ and $p = 0.7$.

[6+4 = 10]

7. Two judges rank twelve competitors in a reality show as follows:

Competitor	1	2	3	4	5	6	7	8	9	10	11	12
Judge A	5	1	4	2	7	3	6	8	10	9	11	12
Judge B	10	5	1	2	3	4	7	6	8	11	9	12

Measure the association between the judgements of the two judges by using Kendall's τ. Comment on the degree of association between the judgement of two judges.

[12+2=14]

Indian Statistical Institute
Semester Examination (Back Paper)
Course name: M.TECH(QROR)-1st Year (2018)
Subject: Programming Techniques and Data Structures
Date 7.6.19 Full Marks 100 Time: 3 Hrs

(Answer all questions)

1. (a) What do you mean by a pointer in C? [3]
   (b)Write the output of the following program when executed. [7]

```
#include <stdio.h>

int function_1(int a, int b){
        int c;
        b = a+b;
        c = b;
        return c;
}

int function_2(int *a, int *b){
        int c;
        *b = *a + *b;
        c = *b;
        return c;
}

int main(void){
        int a, b, c;
        a = 5;
        b = 7;
        c = function_1(a, b);
        printf ("The value returned by function_1: %d\n", c);

        c = function_2(&a, &b);
        printf("Current value of b after function_2 executed: %d\n", b);

        return 0;
}
```

2. What is a Flow chart? Draw a Flow chart to find the sum of the following series for given values of x and k. [2+8]

$$1-\frac{x}{1!}+\frac{x^2}{2\times 2!}-\frac{x^3}{3\times 3!}+\ \ldots\ldots\ldots\ldots+(-1)^k\frac{x^k}{k\times k!}$$

3. Write a C Program to generate first N (take as input) Fibonacci numbers which are greater than 100. [10]

P.T.O

4. Write the algorithm for Binary search technique. [10]

5. Given two one-dimensional arrays A and B which are sorted in ascending order. Write a C program to merge them into a single sorted array C (without using sorting algorithm) that contains every item from array A and B in ascending order. [10]

6. Write a C function to multiply a Matrix R of order LxM with a Matrix S of MxN. Now write a C program, using this function to compute the matrix polynomial
$$W = 4XY - 7Y^2 + 5Z$$
where X, Y and Z are square matrices of order 5x5. [10]

7. Write a C program to sort N integer elements using quick sort technique. [10]

8. What is a binary tree? Write down the difference between a Binary tree and B-tree. Given the following numbers **50, 25, 10, 5, 7, 3, 30, 20,8,15** and you need to draw the graphical representation to design a binary search tree using these numbers where 10 is the root of the binary search tree. [1+3+6]

9. What is linked-List? Create a linked-list of N (take as input) nodes where data part of each node contains itemt name (string), and its price (int). Now print the names of the items having price greater than 100. Print NONE if no such item is there. [2+8]

10. (a) Convert $(11010101101)_2$ to Octal. [3+3+4]

(c) Write down the equivalent statement of the rewind(fp) using fseek function.

(d) Write the differences between Malloc and Calloc functions.

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination: 2018-2019 (Second Semester)

M. Tech(QR & OR) I Year (E & S Stream)

Reliability-I

Date: 18 February, 2019 Full Marks: 60 Duration: 2 hours.

Note: Answer all questions.

1. Consider a system with four components. Let x_1, x_2, x_3 and x_4 be the binary state variables of the components and ϕ be the structure function of the system. The states of the components and system are given in the following table.

x_1	x_2	x_3	x_4	$\phi(\underset{\sim}{x})$	x_1	x_2	x_3	x_4	$\phi(\underset{\sim}{x})$
0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	1	1	0	0	0
0	0	1	0	0	1	0	1	0	0
0	0	0	1	0	1	0	0	1	1
0	1	1	0	0	1	1	1	0	1
0	1	0	1	0	1	1	0	1	1
0	0	1	1	0	1	0	1	1	1
0	1	1	1	0	1	1	1	1	1

   (a) Find the min path sets and min cut sets of the system.

   (b) Write down the structure function of the system by using either min path sets or min cut sets.

   (c) Find the structural importance of component 1.

   (d) Give a modular decomposition of the system with two modules.

   (e) Construct a fault tree for the failure of the system and find the min cut sets.

[(4+2) + 3 + 3 + 2 + (4+2) = 20]

2. (a) Consider a coherent system ϕ of n independent components with component reliabilities $p_1, p_2, \ldots, p_n$. Show that the reliability of the system can be expressed as

$$h(\underset{\sim}{p}) = \sum_{\underset{\sim}{x}} \phi(\underset{\sim}{x}) \prod_{i=1}^{n} p_i^{x_i}(1-p_i)^{1-x_i}$$

   where sum is extended over all possible state vectors $\underset{\sim}{x} = (x_1, \ldots, x_n)$.

   (b) Suppose the reliability of each component is $\frac{1}{2}$. Show, by using (a), that the reliability of the system is given by

$$R = \frac{\text{Number of path sets}}{2^n}.$$

   Hence find the reliability of a k-out-of-n system of independent components with each component reliability $\frac{1}{2}$.

[4+(3+3)=10]

3. (a) Show that if $X_1, \ldots, X_n$ are associated binary random variables, then $1 - X_1, \ldots, 1 - X_n$ are also associated binary random variables.

   (b) Let ϕ be a coherent structure of n associated components with component reliabilities $p_1, \ldots, p_n$. Then show that the system reliability satisfy

$$\prod_{i=1}^{n} p_i \leq P[\phi(\underset{\sim}{X}) = 1] \leq \coprod_{i=1}^{n} p_i.$$

[4+11 = 15]

4. Consider a continuous lifetime random variable T with reliability function $R(\cdot)$ and cumulative hazard function $\Lambda(\cdot)$.

   (a) Show that the conditional reliability of a unit of age t is

$$R(x|t) = \exp\left[-\Lambda(t+x) + \Lambda(t)\right].$$

   (b) Show that the mean residual lifetime at age t, $m(t)$, can be expressed as

$$m(t) = \int_0^\infty \exp\left[-\Lambda(t+x) + \Lambda(t)\right] dx$$

   [Note: You can use the expression of $m(t)$ in terms of reliability function to show the above result.]

[6+3 = 9]

5. The lifetime of a household appliance follows exponential distribution with the probability density function

$$f(t) = \frac{1}{\theta} e^{-t/\theta}, \quad t \geq 0, \quad \theta > 0.$$

Find the mean time to failure (MTTF) when it is new. Find the MTTF when a 1-year warranty period ends, assuming it still works.

[3+3=6]

INDIAN STATISTICAL INSTITUTE

Mid -Semester Examination: 2018 – 19

M. Tech (QROR), E-Stream, Semester II

Statistical Methods – II

Date: 19/02/2019 Maximum Marks: 60 Duration: 2 Hrs.

Note: Answer any three (3) questions.
Calculations should be correct to 3 decimal places.

1. a) Define and explain Uniformly Most Powerful Unbiased Test.

   b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. Use likelihood ratio test to define the test statistic and corresponding critical region for the following test hypothesis:

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu \neq \mu_0$$

[5 + 15 =20]

2. a) The mean water temperature downstream from a power plant cooling tower discharge pipe should be no more than 100°F. Past experience has indicated that the standard deviation of temperature is 2°F. The water temperature is measured on nine randomly chosen days, and the average temperature is found to be 100.5°F.

   i) Should the water temperature be judged acceptable with $\alpha = 0.05$?
   ii) What is the P-value of this test?
   iii) What is the probability of failing to reject the null hypothesis at $\alpha = 0.05$ if the true mean temperature is 103°F?

b) Following table gives the number of arrivals per minute in a bank observed in 200 one-minute period.

Arrivals	0	1	2	3	4	5	6	7	8	≥ 9
Frequency	14	31	47	41	29	21	10	5	2	0

Can it be assumed that the number of arrivals per minute follows a Poisson distribution? Perform a goodness-of-fit procedure with $\alpha = 0.05$.

[(5+2+3) + 10 = 20]

3. A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make the cloth for men's shirt. Strength is usually affected by the percentage of cotton used in blend of materials for the fiber. The engineer conducts a completely randomized experiment with five levels of cotton content and replicates the experiment five times. The data are shown in the following table.

Cotton weight Percent	**Observations**				
15	7	7	15	11	9
20	12	17	12	18	18
25	14	19	19	18	18
30	19	25	22	19	23
35	7	10	11	15	11

a) Is there evidence to support the claim the cotton content affects the mean tensile strength? Use $\alpha = 0.05$.

b) Compute a 95% confidence interval estimate of the mean tensile strength value at 30% cotton.

c) Test all pair of means using the Fisher LSD method with $\alpha = 0.05$. What conclusions can you draw?

[9+5+6 = 20]

4. a) Show that for a two factor factorial experiment, the residuals can be expressed as

$$e_{ijk} = y_{ijk} - \bar{y}_{ij\bullet}$$

where terms have their usual meaning.

b) The factors that influence the breaking strength of a synthetic fiber are being studied. A full factorial experiment was carried out involving four production machines and three operators, where machine is a fixed factor and operators are chosen randomly. The results are as follows:

	Machine			
Operator	1	2	3	4
	110	111	108	109
1	110	115	109	108
	111	110	112	114
2	112	111	108	112
	116	112	114	120
3	114	115	119	117

i) Write the corresponding statistical model and hence derive the expression for expected mean square for the model components.

ii) Analyse the data suitably. Present your calculations in the form of ANOVA and revised ANOVA, if required and give your comment. Use $\alpha = 0.05$.

iii) Estimate the variance components of significant effects.

[7+(5+7+1) = 20]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination 2018 - 19

M. Tech. (QR OR); I Year; E-Stream

ELEMENTS OF STOCHASTIC PROCESSES

Date : 21/02/2019 Maximum Marks : 50 Time : 02 hours

Notes:

(i) Unless stated otherwise, M.C. will mean a discrete time parameter, time-homogeneous Markov Chain.

(ii) The symbols carry their usual meanings.

(iii) Use of calculators and other electronic gadgets are strongly prohibited in the examination halls.

(1) Define the following:

(a) Stochastic Process
(b) State Space and Parameter Space.
(c) Transition Probabilities.
(d) Markov Process.
(e) Markov chain. (5 × 4)=[20]

(2) Show that for a M.C.,

$P(X_0 = i_0, X_1 = i_1, \dots, X_n = i_n) = \pi(i_0)p_{i_0 i_1}p_{i_1 i_2} \cdots p_{i_{n-1} i_n}$
for all $n \in \{0,1,2, \dots .\}$ and $i_0, \dots, i_n \in I$. [20]

(3) (a) Show that $P^{(K)} = P^K$ for all $K \in \{0,1,2, \dots .\}$.
(b) State and prove the Chapman Kolmogorov equations. [3 +7]=[10]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2018-19
Course: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Subject: SQC

Date of Exam: 22/02/2019 **Max. Marks: 100**

Duration: 3 hrs.

Answer All Questions.

1) Choose the correct alternative out of a, b, c, d.

A. A process in a state of statistical control means

I. Only common causes are present
II. Only special causes are present
III. The product will always meet its specifications
IV. The process is stable and only random or inherent sources of variation are present in the process

a) I and II above
b) II and III above
c) III and IV above
d) IV and I above

B. The rational subgrouping for the control chart means

a) A subgroup that is free from the assignable causes as much as possible
b) A subgroup that represents the homogeneous conditions as much as possible
c) A small group of consecutively produced parts from a production process
d) All of the above

C. Cause and effect diagram is also known as

a) Ishikawa diagram and affinity diagram
b) Affinity diagram and fish bone diagram
c) Ishikawa diagram and fish bone diagram
d) None of the above

D. Who among the following suggested seven quality tools for controlling quality?

a) Juran
b) Kaoru Ishikawa
c) W. Edward Deming
d) Feigenbaum

E. Specification can be

a) Bilateral with equal value above and below the target value
b) Unilateral
c) Bilateral with unequal value above and below the target value
d) Any one of the above

F. Which type of chart generally uses the point of inflexion for classification of causes?

a) Cause and effect chart

b) Pareto chart
c) Histogram
d) Control chart

G. After the successful process optimization, introduction and maintenance of which check sheets would be beneficial?

a) Defect location check sheet
b) Defective item check sheet
c) Check-up confirmation check sheet
d) Defective cause check sheet

H. A process can have

a) Within-piece variation
b) Time-to-time variation
c) Piece-to-piece variation
d) All of the above

I. In which type of a cause and effect diagram the causes reflecting combined effect of two or more factors belonging to different process steps are difficult to illustrate?

a) Cause enumeration type
b) Dispersion analysis type
c) Process classification type
d) None of the above

J. A control chart is used to

a) Determine if defective parts are being produced
b) Measure process capability
c) Determine sources of process variation
d) Detect occurrence of special causes

K. The SERVQUAL model takes into account

a) 5 gaps
b) 4 gaps
c) 3 gaps
d) 2 gaps

L. The gap between service expectations and service perception gets influenced by

a) Performance, reliability, durability, serviceability, aesthetics
b) Serviceability, aesthetics, features, perceived quality, conformance to standards
c) Reliability, empathy, tangibles, assurance, responsiveness
d) Reliability, serviceability, performance, perceived quality, features

M. Brainstorming rules consist of

a) Non-judgmental, radical, quality, building up, equality
b) Non-judgmental, radical, quantity, building up, equality
c) Non-judgmental, non-radical, quality, building up, equality
d) Judgmental, radical, quantity, building up, equality

N. Process capability is defined as

a) C_P
b) C_{PK}

c) 3σ
d) 6σ

O. Variation in a process exists due to:

a) Chance and assignable causes
b) Chance causes
c) Assignable causes
d) Common causes

[15×2 = 30]

2) Control charts for $\bar{x}$ and R are maintained on the tensile strength of a metal fastener. After 30 samples of size $n = 6$ are analyzed, we find that

$$\sum_{i=1}^{30} \bar{x}_i = 12870 \text{ and } \sum_{i=1}^{30} R_i = 1350$$

a) Compute control limits on the R chart.
b) Assuming that the R chart and $\bar{x}$ chart exhibit control, estimate the parameters μ and σ.
c) If the process output is normally distributed, and if the specifications are 440 ± 40, can the process meet the specifications? Estimate the fraction nonconforming.
d) If the variance remains constant, where should the mean be located to minimize the fraction nonconforming?
e) Recommend control limits on the $\bar{x}$ chart for this relocated mean.

[4+3+(3+4)+2+4=20]

3) A fraction nonconforming control chart with $n = 400$ has the following parameters:

UCL = 0.0962
Centre line = 0.05
LCL = 0.0038

a) Find the width of the control limits in standard deviation units.
b) What sample size would you choose so that the fraction defective chart will have a positive 3σ lower control limit for a center line = 0.05?
c) In a process the in-control fraction defective has been estimated as 0.01. What sample size should be chosen to obtain at least one defective in the sample with a probability = 0.90?
d) In a process the average fraction defective is found to be 0.07. Determine the sample size to have a chance of 0.50 or more of catching on a single sample a shift to a process fraction defective of $p = 0.12$ with 3σ control limits.

[4×5=20]

4) In the context of Acceptance Sampling

a) Discuss the advantages and disadvantages of it.
b) What is the difference between Type A and Type B OC curves?
c) Define AOQ, AOQL, AQL and LTPD.
d) The underlying distribution for an attribute sampling plan is binomial with sample size n and proportion defective p. The lot from which samples are taken, will be accepted if the number of defectives in the sample does not exceed the acceptance number c.

i. Prove that the probability of acceptance is an increasing function of c.
ii. Prove that the probability of acceptance is a decreasing function of n.

[6+4+8+(3+9)=30]

INDIAN STATISTICAL INSTITUTE

Semester Examination: 2018 – 19

M. Tech (QROR), First Year (E-Stream), Semester II

Statistical Methods – II

Date: 16/04/2019 Maximum Marks: 100 Duration: **3** Hrs.

Note: This paper carries 115 marks. Answer as many questions as you can – maximum you can score is 100.
All decimal calculations are to be rounded to 3 decimal places.
Unless other-wise stated, use $\alpha = 0.05$ throughout.

1. An experiment was performed to investigate the capability of a measurement system. Eight parts were randomly selected and two randomly selected operators measured each part three times. The tests were made in random order and the data are shown in Table below.

	Operator 1			**Operator 2**		
	Measurements			**Measurements**		
Part No.	**1**	**2**	**3**	**1**	**2**	**3**
1	49	49	50	50	48	51
2	51	52	51	51	51	51
3	50	51	50	47	50	51
4	50	49	48	49	49	48
5	52	50	50	52	50	50
6	50	51	50	51	48	49
7	48	46	49	46	47	48
8	52	50	51	53	48	50

a) Write appropriate statistical model with proper explanation. Using the rules for expected mean squares, obtain the expected mean squares for the model components.
b) Analyse the data suitably. Present your calculations in the form of ANOVA and revised ANOVA, if required and give your comment.

[(3+7)+10 = 20]

2. Following table gives age (in years) and price (in dollars) for used cars sold in the last year by a car dealership company.

Age	4	5	5	7	7	8	9	10	11	12
Price	6100	5700	4500	4500	4200	4100	3100	2100	2500	2200

a) Fit the simple linear regression using method of least squares.
b) Test the significance of regression.
c) Find the 99% confidence interval on the slope.
d) Find the standardized residuals and hence comment on the normality assumption of errors.

[6+4+3+(6+1)=20]

3. Consider the multiple linear regression model $\boldsymbol{y} = \boldsymbol{Xb} + \boldsymbol{\varepsilon}$, where y is an $(n \times 1)$ vector of responses, $\boldsymbol{X}$ is an $(n \times p)$ matrix of levels of the independent variables, $\boldsymbol{b}$ is a $(p \times 1)$ vector of regression coefficients, and $\boldsymbol{\varepsilon}$ is a $(n \times 1)$ vector of random errors.

a) Show that $SS_E = y^T y - \hat{b}^T X^T y$ and hence derive the ANOVA procedure for testing the significance of regression.
b) Write down the expression of $\boldsymbol{R^2_{adjusted}}$ and explain its superiority over $\boldsymbol{R^2}$.
c) (i) Prove that $\boldsymbol{R^2_{adjusted}} \leq \boldsymbol{R^2}$ and
(ii) Derive the condition under which $\boldsymbol{R^2_{adjusted}}$ may be negative.

[10+4+(3+3)=20]

4. a) Use the general regression significance test to derive the ANOVA procedure for a single factor fixed effect model.

b) Three different formulations of an industrial glue are being tested. The tensile strength of the glue when it is applied to join parts is also related to the application thickness. Four observations on strength (y) in pounds and thickness (x) in 0.01 inches are obtained for each formulation. The data are shown in the following table.

Glue Formulation					
1		2		3	
y	x	y	x	y	x
46.5	13	48.7	12	44.7	16
45.9	14	49.0	10	43.0	15
49.8	12	50.1	11	51.0	10
46.1	12	48.5	12	48.1	12

Does the glue formulation affect the mean tensile strength?

[10+10=20]

5. a) An athletics coach wishes to test the value of an intensive period of weight training to his athletes and selects twelve 400-metre runners from his region and records their times, in seconds, to complete this distance. They then undergo his program of weight training and have their times, in seconds, for 400 meters measured again. The table below summarizes the results

Subject	1	2	3	4	5	6	7	8	9	10	11	12
Before	51.0	49.8	49.5	50.1	51.6	48.9	52.4	50.6	53.1	48.6	52.9	53.4
After	50.6	50.4	48.9	49.1	51.6	47.6	53.5	49.9	51.0	48.5	50.6	51.7

Assuming both populations are continuous, use Wilcoxon Signed Rank Test to test the claim that this particular training program is effective in producing a significant reduction in athletes' times for 400 meters.

b) In a poll 8 subjects (experts) rated 4 paintings on a scale from 0 (don't like at all) 5 (like it very much). Following table shows the data for all subjects and paintings.

Subjects	Paintings			
	1	2	3	4
1	0	5	1	4
2	3	4	2	5
3	1	4	3	4
4	4	2	2	3
5	0	3	5	5
6	3	1	3	4
7	5	3	1	5
8	1	5	2	4

Use Friedman Test to test the differences in median ratings among the four paintings.

[10+10=20]

6. The response time in milliseconds for three different types of circuits, where the circuit types are selected at random from a set of circuit types, in an electronic calculator was determined. For each circuit type, the response times were measured five times and the tests were performed in random order. The results recorded are as under.

Circuit Type	Response Time (in ms)				
1	16	15	18	26	17
2	19	22	20	18	25
3	20	21	33	27	40

Using ANOVA method, test whether the circuit type affect the mean response time and hence estimate the variance components.

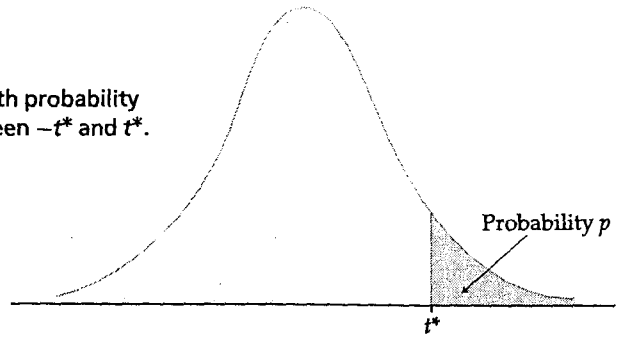
[10+5=15]

ritical values of F for the 0.05 significance level:

Numerator Degrees of freedom

Denominator Degrees of Freedom	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
31	4.16	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15
32	4.15	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
33	4.14	3.29	2.89	2.66	2.50	2.39	2.30	2.24	2.18	2.13
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11
37	4.11	3.25	2/86	2.63	2.47	2.36	2.27	2.20	2.15	2.10
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13	2.08
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2,09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

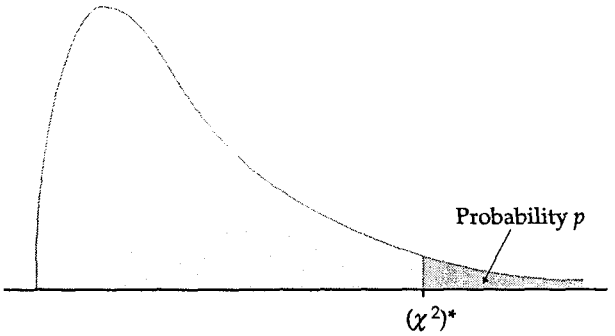
Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.



t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Chi Squared Distribution Table



χ^2 distribution critical values

	Right Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

Critical values for the Wilcoxon Signed-ranks test

n	α (*one-sided test*)			
	0.05	**0.025**	**0.01**	**0.005**
	α (*two-sided test*)			
	0.10	**0.05**	**0.02**	**0.01**
6	2	0	---	---
7	3	2	0	---
8	5	4	2	0
9	8	6	3	2
10	10	8	5	3
11	13	11	7	5
12	17	14	10	7
13	21	17	13	10
14	25	21	16	13
15	30	25	20	16
16	35	30	24	20
17	41	35	28	23
18	47	40	33	28
19	53	46	38	32
20	60	52	43	38
21	67	59	49	43
22	75	66	56	49
23	83	73	62	55
24	91	81	69	61
25	100	89	77	68

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2018 – 19

M. Tech. (QR – OR); I Year; E Stream

Elements of Stochastic Processes

Date: 18.04.2019 Maximum Marks:100 Time: 3 hours

Notes:

1. Unless stated otherwise, "M.C." will mean a discrete time parameter, time-homogeneous Markov Chain.
2. The symbols have their usual meanings.

1. Define the following:
   (a) Class Property
   (b) Essential and Inessential States
   (c) Recurrent and Transient States
   (d) Mean Recurrence Time
   (e) Positive and Null Recurrence (5x4) = [20]

2. Show that Recurrence is a Class Property. [20]

3. Find the relation between the generating functions of the sequences of transition probabilities and first passage probabilities. [20]

4. Consider a M.C. with state space I and transition probability matrix P. Find the probability that exactly m visits are made to state j starting from a state i for various values of $m \geq 0$. [20]

5. Consider a M.C. with state space I = {1, 2, 3, 4, 5, 6} and transition matrix:

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/8 & 7/8 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 4/5 & 0 \\ 0 & 1/4 & 1/4 & 1/2 & 0 & 0 \end{pmatrix}$$

Find the various classes of this chain. [20]

Indian Statistical Institute
Second Semester Examination : 2018-19
M. Tech (QR & OR) I Year (E & S Stream)

Sub: Reliability-I

Date: 22/04/19 Maximum Marks: 100 Time: 3 hours.

Note: The paper carries 112 marks. Answer any part of any question, but the maximum you can score is 100.

1. Write true or false with justification

   (a) The component with lowest reliability is most important to a series system.
   (b) Gamma distribution is always an IFR distribution.
   (c) Type-II censoring scheme is a particular case of hybrid censoring scheme.
   (d) Suppose a unit can fail by any one of the k failure modes. Let T be the failure time of a unit and $\lambda_j(t)$ be the mode-specific hazard rate for mode $j = 1, \ldots, k$. Then the reliability function for T can be expressed as $R(t) = \prod_{j=1}^{k} e^{-\lambda_j(t)}$.

   [3 + 4 + 3 + 5= 15]

2. (a) Let ϕ be a coherent structure of associated components. Let $P_1, \ldots, P_p$ be the min path sets and $K_1, \ldots, K_k$ be the min cut sets corresponding to ϕ. Then show that

$$\max_{1 \le r \le p} \prod_{i \in P_r} p_i \le P[\phi(\underset{\sim}{X}) = 1] \le \min_{1 \le s \le k} \coprod_{i \in K_s} p_i,$$

   where p_i is the reliability of ith component, $i = 1, \ldots, n$.

   (b) Consider a 3-out-of-4 system of associated componnets with component reliabilities $p_1 = 0.6$, $p_2 = 0.8$, $p_3 = 0.8$ and $p_4 = 0.9$. Compute the bounds on system reliability using (a).

   [10 + 10 = 20]

3. Let $h(\underset{\sim}{p})$ be the reliability fundtion of a coherent system ϕ with independent components. Show that $h(\underset{\sim}{p})$ is strictly increasing in each p_i, for $0 < p_i < 1$ for each i.

   [7]

4. Consider a series system of n independent components. Suppose the ith component has reliability function $R_i(t) = \exp(-\alpha_i t^{3/2})$, $i = 1, \ldots, n$. Find the hazard function of the system lifetime. Check whether the system lifetime distribution is IFR.

   [7 + 3 = 10]

5. Find the reliability at 100 h for two design options below (assume exponential distribution for the time to failure of the units and units are independent) (a) Two parallel units with hazard rates $\lambda_1 = 0.0034$ and $\lambda_2 = 0.0105$ and (b) A standby units with two same units having hazard rates $\lambda_1 = 0.0034$ and $\lambda_2 = 0.0105$ (assuming perfect switching and no standby failures). Which system has higher MTTF?

   [(3+6)+6 = 15]

P.T.O.

6. Consider a coherent system with three components. The system functions if component 1 functions and at least one the components 2 and 3 functions. Suppose the system functions under a stress X with the c.d.f. $F_X(x) = 1 - e^{-\theta x}$, $\theta > 0$, $x > 0$. The strengths of the components are i.i.d. with the c.d.f. $F_Y(y) = 1 - e^{-\lambda y}$, $\lambda > 0, y > 0$. Find the reliability of the system.

[10]

7. Twenty items were placed on a life test that lasts 500 h. Ten failures were observed at 50, 62, 75, 115, 190, 254, 312, 389, 412 and 498 h. Suppose the lifetimes of item follow exponential distribution. Derive the maximum likelihood estimates (MLE) of the MTTF and median lifetime along with their variance estimates. Derive an approximate 95% confidence interval for the MTTF. Find the MLE of reliability at 525 h.

[5+5+3+2 =15]

8. The following data represent the failure times in hours of 15 light bulbs, where "+" indicates censored observation.

$$8+, 10, 12, 12+, 16, 31, 35+, 41, 42+, 43, 49+, 75, 81, 109+, 123.$$

   (a) Find the Kaplan-Meier estimate of $R(100)$. Give an approximate 95% confidence interval for $R(125)$.

   (b) Assuming that the underlying life distribution is log-normal with parameters μ and σ, write down the likelihood function to obtain the MLE of μ and σ. Describe, in details, a suitable graphical method to verify the assumption of lognormal distribution for the given data.

[(8+5) + 3 + 4 = 20]

Table A.4 Cumulative Normal Distribution

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt$$

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

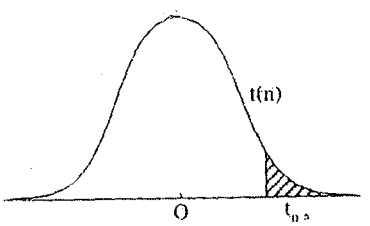
x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.000001

Source: Fisher, R. A. (1953): Statistical Tables for Biological, Agricultural and Medical Research, Oliver and Boyd, London.

P.T.O

Table A.6 Student's t-Distribution

The first column lists the number of degrees of freedom (n). The headings of the other columns give probabilities (α) for t to exceed the entry value, use symmetry for negative t values.



n \ α	.10	.05	0.025	.01	.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

Source: P. G. Hoel, *Introduction to Mathematical Statistics*, 4th Ed., John Wiley, New York, 1971, page 393.

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2018-19

Course Name: M. Tech. (QROR) I Year

Subject Name: Statistical Quality Control

Date: 24/04/2019 Maximum Marks: 100 Duration: 3 hours

[All Symbols Have Usual Meaning.]

Answer All Questions

1. a) A customer has told his supplier that in order to qualify for business with his company, the supplier must demonstrate that C_P exceeds 1.33 for the quality characteristic of concern. If C_P is below 1.33 there will be a probability of 0.90 for detecting this. Whereas if C_P exceeds 1.66 there will be a probability of 0.90 of judging the process capable. To demonstrate capability, what sample size and critical value of C_P the supplier must consider?

Sample Size and Critical Value

Sample Size, n	(a) $\alpha = \beta = 0.10$		(b) $\alpha = \beta = 0.05$	
	$C_P(High)/C_P(Low)$	$C/C_P(Low)$	$C_P(High)/C_P(Low)$	$C/C_P(Low)$
10	1.88	1.27	2.26	1.37
20	1.53	1.20	1.73	1.26
30	1.41	1.16	1.55	1.21
40	1.34	1.14	1.46	1.18
50	1.30	1.13	1.40	1.16
60	1.27	1.11	1.36	1.15
70	1.25	1.10	1.33	1.14
80	1.23	1.10	1.30	1.13
90	1.21	1.10	1.28	1.12
100	1.20	1.09	1.26	1.11

[5]

b) Two parts are linearly assembled so that the second part fits inside the first part. $X_1 \sim N(20, 0.3^2)$ and $X_2 \sim N(19.6, 0.4^2)$. The clearance between the outside wall of the second part and the inside wall of the first part should be 0.5 ± 0.4. Find the probability of nonconformance for clearance.

[8]

c) For a process with an underlying normal distribution of the quality characteristic, prove that $C_{PK} = (1 - K)C_P$ where C_{PK} is the process performance index, K is the process centering index and C_P is the process potential index.

[7]

2. a) If the mean of the quality characteristic of a process operates at the mid-point of the specification limits $\{L, U\}$ then prove that the expected proportion of conforming items is given by $2\Phi(3C_P) - 1$ with an underlying normal distribution.

[6]

b) If the mean of the quality characteristic of a process operates anywhere within the specification limits $\{L, U\}$ then prove that the proportion of nonconformance (NC) is given by the formula $NC = \Phi[-3(2C_P - C_{PK})] + \Phi(-3C_{PK})$ with an underlying normal distribution.

[14]

3. a) Two physicians each examine the knees of 100 patients in an arthritis clinic. After each examination they record whether they believe each patient's knee to be swollen or not. While physician A finds that 30 of the patients have swollen knees, physician B finds 40 of the patients have swollen knees. The physicians agreed that 25 of the patients had swollen knees, and that 55 patients did not. Compute the *kappa* index based on these data and comment about the strength of agreement between the physicians.

[8+2=10]

b) Explain how one can estimate process capability indices for many of the quality characteristics for which it is desirable to have a skewed data.

[5]

4. a) Illustrate graphically the general procedure for double-sampling plans.

[5]

b) What are the advantages and disadvantages of using a double-sampling plan? [4]

c) A double-sampling plan has $n_1 = 30, n_2 = 60, c_1 = 1$ and $c_2 = 3$. For $p = 0.08$, compute the following:

i) P_a^I
ii) P_a^{II}
iii) P_r^I
iv) P_a

[3+9+3+1=16]

5. a) Design a double-sampling plan using the following Grubb's table with $n_2 = 2n_1, \alpha = 0.05$, and $\beta = 0.10$ where $p_1 = 0.01$ and $p_2 = 0.10$. Holding β constant is of primary concern if there are alternatives.

Grubb's Table for $n_2 = 2n_1 (\alpha = 0.05, \beta = 0.10)$

Plan No.	R	c_1	c_2	pn_1	
				$1 - \alpha = 0.95$	$\beta = 0.10$
1	14.50	0	1	0.16	2.32
2	8.07	0	2	0.30	2.42
3	6.48	1	3	0.60	3.89
4	5.39	0	3	0.49	2.64
5	5.09	1	4	0.77	3.92

[5]

b) A single-sampling plan has rectifying inspection that has $AOQL = 0.045, c = 3$ and $y = 1.946$. The lot size is 2000 and the process average is $p = 0.015$.

i) Find n.

ii) Find the ATI.

[5+10=15]

INDIAN STATISTICAL INSTITUTE

Second End-Semester Examination (2018 – 2019)

Course Name	:	**M.Tech (QR & OR)**
Subject	:	**Industrial Engineering and Management**
Date	:	**26/04/2019**
Maximum Marks	:	**60**
Duration	:	**3 Hours**

Question Paper

Attempt ***any three*** questions. All questions carry equal marks.

1. (a) Define 'Financial Management'.
   (b) Explain scopes of financial management.
   (c) How do you improve the efficiency of material handling system?
   (d) Represent a typical balance sheet of an organization. What do you mean by current assets? [2+5+5+ (6+2)]

2. (a) Explain three E's of value for money.
   (b) A company XYZ will start a project, which is expected to have the following cash flows:

Year	Cash Flow
	Rs 000
0	(2500)
1	700
2	600
3	600
4	800
5	500

Find out the expected payback period of the project with cost of capital 10%?

(c) A project requires an initial investment of Rs 800,000 and then earns net cash inflows as follows:

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Cash inflows (Rs000)	100	200	400	400	300	200	150

PTO

In addition, at the end of the seven –year project, the assets, initially purchased, will be sold for Rs 100,000.

Determine the project's ROCE using:

(a) Initial capital costs,

(b) Average capital investment

[7 + 5 + 8]

3. (a) What do you mean by an investment appraisal? Explain one method in relation to investment appraisal.

(b)What is an annuity? How do you estimate annuity?

(c)A payment of Rs 2000 is to be made every year for 4 years, the first payment occurring in one year's time. The interest rate is 12%. What is the PV of the annuity?

[(2+6) + (2+6) + 4]

4. (a) What do you mean by NPV? What are advantages of NPV?

(b)The cash flows of an investment have been estimated as follows:

Year	Rs
0	(35,000)
1	10,000
2	8000
3	6000
4	7000
5	7000
6	2000

The cost of capital is 8%.

Convert these cash flows to a PV.

Calculate the NPV of the project to assess whether it should be undertaken.

(c)What is IRR? Derive the following formula related to IRR. Symbols have their usual meanings.

$$IRR = L + \left[\frac{N_L}{N_L - N_H} \times (H - L) \right]$$

[(2+4) + 6+ (2+6)]

5. (a) How does inflation act on the return on investment?

(b)A company is evaluating a housing project, which requires an initial investment of Rs 125,000. Expected net cash flows are Rs 30,000 pay for 7 years at today's prices. However, these are expected to rise by 4.5 % pa because of inflation. The firms money cost of capital is 15%. Find the NPV by both discounting money cash flows and discounting real cash flows.

(c) An investment of Rs 60,000 today is expected to give rise to annual contribution of Rs 40,000 and annual fixed cost of Rs 15,000 for the next four years. Assume the discount rate is 10%. Calculate

(i) the NPV of this investment and

(ii) the sensitivity of your calculation on the basis of initial investment, contribution, fixed costs, discount rate, and life of the project.

(iii) It is also mentioned that the annual contribution of Rs 40,000 is based on selling one product, with a sales volume of 15,000 units, selling price of Rs 15 and variable costs of Rs12. Calculate sensitivity margin for both the sales volume and the selling price.

[2+8+10]