INDIAN STATISTICAL INSTITUTE

Semester Examination: 2019-20

M. Tech. (QR & OR), 1st Year, 1st Semester

Subject: Statistical Methods I

Date: 18. 11. 2019   **Maximum Marks: 100**   **Duration:** 3 hours

Note: This paper carries 120 marks. Answer as many questions as you can. The maximum you can score is 100.

1. (a) Give an example each of nominal and ordinal variables related to the evaluation of a customer satisfaction measurement process. *[Do not give examples related to the design of the questionnaire].*

(b) What is meaning of the term of 'ratio' in 'ratio scale'?

[4+3=7]

2. Prove the following using Jensen's inequality or otherwise.

(a) Mean absolute deviation (from mean) <= Standard deviation

(b) Arithmetic mean >= Geometric mean

[Note: Jensen's inequality: If p_1, p_2, ..., p_n are positive numbers which sum to 1 and f is a real continuous function that is convex, then

$$f(\sum_{i=1}^{n} p_i x_i) \le \sum_{i=1}^{n} p_i f(x_i).$$

If f is concave, then the inequality reverses.]

[6+6=12]

3. Consider the following ten pairs of observations (x, y).

x	3	7	2	1	8	10	5	4	6	9
y	2	5	1	3	9	8	6	7	4	10

(a)   Estimate the Pearson's correlation coefficient r_{xy}
(b)   Consider that the above values are ranks and obtain an estimate of the Spearman's rank correlation coefficient ρ_{xy}.
(c)   Compare the above results and offer your comments.

[5+5+3=13]

4. A life test is planned with ten units for a maximum duration of 320 hours. The time of failures of the seven units that failed during the test period are noted. Thus the test data we have is as follows:

Time	Frequency
15	1
21	1
75	1
122	1
150	1
182	1
286	1
≥ 320	3

Assume that the distribution of failure time is exponential (with only one parameter). Find the maximum likelihood estimate of the mean of the distribution.

[Hint: The likelihood function for the censored data as above is given by

$$L = L_1 * \prod S(x_r)$$

where L_1 is the likelihood function for the uncensored items and $S(x_r) = P(T_r > x_r)$, x_r is the censored time and $T_r > x_r$ is the real survival time for the rth item.]

[15]

5. Let $\mathbf{Y} = (Y_1, Y_2, ..., Y_n)^T$ be a random sample from the distribution with the pdf given by

$$f(y; \theta) = \begin{cases} \dfrac{2}{\theta^2}(\theta - y), & y \in [0, \theta] \\ 0, & elsewhere. \end{cases}$$

Find an estimator of θ using the method of moments.

[15]

6. Two random samples are taken from two different lots and the following observations on certain characteristic of the items in the two samples are obtained.

Lot 1	5.1, 5.2, 5.7, 9.0, 4.5, 8.2, 6.3, 7.3, 7.2, 4.5, 3.4, 4.3, 11.0, 3.9, 5.9, 7.0, 7.5, 6.5, 10.0, 6.4
Lot 2	4.5, 7.8, 6.0, 5.1, 6.8, 5.7, 3.6, 4.8, 5.9, 3.9, 3.7, 6.0, 3.7, 5.0, 4.1

Construct a Q-Q plot of the two samples and draw your conclusions from the plot. [Use graph paper].

[18]

7. Write an **efficient** algorithm for generating 100 random observations from the following discrete distribution of a random variable X:

Interval of X	Probability
2-4	.1
5-8	.4
9-13	.5

[16]

8. The owner of a small restaurant decides to change the menu. A trade magazine claims that 40% of all diners choose organic foods when eating away from home. A random sample of the 20 diners shows that two of them ordered for organic foods. What can you say about the claim made by the magazine based on the sample results?

[12]

9. The product manager at a local supermarket store would like to estimate the probability that a customer will purchase five or more packets of a certain product. He has collected the following data

No of packets	0	1	2	3	4	5
Frequency	49	35	12	3	1	0

(a) Estimate the probability that is of interest to the product manager

(b) Comment on the estimate

[8+4=12]

INDIAN STATISTICAL INSTITUTE
Semestral Examination : 2019-20
M-TECH(QR&OR) -- 1st YEAR (E-STREAM)

PROBABILITY

Note : Answer any <u>FIVE</u> questions

[Symbols have their usual meaning]

Date: 29.11.19   **Full marks:100**   **Time: 3 hours**

1. a) State and prove Chebyschev's inequality. What does it provide?
   b) State and prove central limit theorem due to Lindeberg and Levy. Explain its application in the field of SQC.

$$[(6+2)+(10+2)=20]$$

2. a) Let $X_i \sim$ Negative binomial distribution with parameters r_i and p respectively, i =1,2 and they are independent. Let $Y = X_1 + X_2$. Find the distribution of Y.
   b) Let X and Y be two random variables such that $Y = \log X$. Let $Y \sim N(\mu, \sigma^2)$. Find out the distribution of X. Find E (X) and Var (X).
   c) Let $X \sim Bin(n, p)$. Prove that $P(X \le k) = \frac{1}{B(n-k,k+1)} \int_0^q z^{n-k-1} (1-z)^k dz$, where q = 1- p.

$$[6+(4+2+2)+6=20]$$

3. a) State and prove Bonferroni's inequality.
   b) There are 3 chests, each having two drawers. Each drawer of chest-1 contains a gold coin, each drawer of chest-2 contains a silver coin. In third chest there is one gold coin in one drawer and one silver coin in another. A chest is opened at random. A drawer is selected. It is found that it contains a gold coin. What is the probability that the other drawer of the selected chest contains another gold coin?

$$[10+10=20]$$

4. a) Define correlation coefficient (ρ_{XY}) between two random variables X and Y. Let U and V be two other random variables such that $U = a + bX$ and $V = c + dX$, where $b, c \ne 0$. Let ρ_{UV} be the correlation coefficient between U and V. Find the relationship between ρ_{UV} and ρ_{XY}. Justify your answer.
   b) The quality assurance manager of a bolt manufacturing company knows that as per contract his client will tolerate a maximum of 2% rejection with respect to diameter of the pipes. He has taken a set of 100 observations on diameter of the selected pipes and estimated the average as 30 cm and the standard deviation as 3 cm. Where should he set his upper specification limit of the diameter of the pipes so that he can satisfy his client (assume that the distribution of the diameter follows Normal distribution and specification for diameter is having only upper specification limit)?

$$[10+10=20]$$

5. a) A person is paying Rs 50 for each participation of the following game:
He is drawing 2 cards from a deck. He gets Rs 20 if he draws 2 red kings. He gets Rs 30 if he draws 2 queens. He gets Rs 30 if he draws 1 red jack and a black ace. Otherwise he gets nothing. What is his expected gain?

b) Five letters are placed at random in five envelopes with addresses written. Find the probability that each letter will be placed in a wrong envelope.

c) Let $X_i \sim \exp(\lambda)$, $i = 1(1)\,n$, X_i's are independent. Let $X_{(1)} = \text{Min}\,(X_1, \ldots, X_n)$. Find the distribution of $X_{(1)}$

$$[8+8+4=20]$$

6. a) Derive the p.d.f of χ^2 distribution with n degree of freedom.

b) Let X and Y be two independent χ^2 variables with degree of freedom n_1 and n_2 respectively. Let $Z = \frac{X}{Y}$. Derive the distribution of Z.

$$[10+10=20]$$

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2019-20

Programme Name: M.Tech (QR & OR) 1st YEAR (E & S Streams)

Course: Operations Research-I

Date of Exam: 25-11-2019   **Max Marks:** 100   **Duration:** 3 hrs.

Question no. 1 and 7 are compulsory. Use of scientific calculator/ RMMR table is allowed.

Answer as many questions as you can. Maximum you can score is 100.

1. Explain the following terms:

    (a) Shortage Cost

    (b) Discrete Event Simulation

    (c) Discouraging Queue Characteristics

    (d) Kendall's notation

    (e) Little's law

    [3+3+3+3+3=15]

2. (a) Explain the physical interpretations of a dual problem considering its objective function, dual variables, dual constraints and its coefficients.

    (b) Solve the following problem using Simplex method after converting it to its dual:

   Maximize $3X_1 + 2X_2$

   Subject to
   $$X_1 + \ X_2 \geq 1$$
   $$X_1 + \ X_2 \leq 7$$
   $$X_1 + 2X_2 \leq 10$$
   $$X_2 \leq 3$$
   $$X_1, X_2 \geq 0.$$

    [6+9=15]

3. As a Project Manager, find the following

   a) Critical Path and expected project completion time;
   b) Least Cost schedule and the corresponding project cost;
   c) 36 days schedule and the corresponding project cost.

   Given: Indirect cost per day = Rs. 175/-

Activity Node	Normal duration (days)	Crash duration (days)	Cost (Rs.)	Crash Cost (Rs.)
(1,2)	8	6	1800	2200
(2,3)	16	11	1500	2200
(1,4)	12	9	2400	3000
(3,5)	14	9	1800	2400
(4,5)	15	14	800	2000
(5,6)	10	8	2000	4000

Comment on the solutions obtained in b) and c)

[5+4+6=15]

4. (a) What are the assumptions of a Production order quantity (POQ) model and how is it different from EOQ model? Derive the conditions for which POQ model approaches to EOQ model. Explain the significance of POQ model parameters towards shortage/stock out situation.

   (b) Mention the basic differences between (t_p, S) policy and (t, S_p) policy. The notations have standard meaning.

   [10+5=15]

5. Define a Markov process in terms of a stochastic process. In general, what type of Markov process a Queue model is and why? Derive steady-state solution for p_n of a Poisson queue stating all its assumptions. Give one real life example each for encouraging and discouraging queues.

   [2+3+7+3=15]

6. Find the expression for p_n and p_0 for $(M/M/m/FCFS/K/\infty)$. Prove that letting $K \to \infty$ and $\lambda/(m\mu) < 1$, the expression for p_n and p_0 yields the results for $(M/M/m/FCFS/\infty/\infty)$.

   [10+5=15]

7. NOKIA sells and services several brands of home appliances. Past sales for a particular model of Smartphone have resulted in the following probability distribution for demand:

Demand per week	0	1	2	3	4
Probability	0.2	0.35	0.25	0.15	0.05

The lead time, in weeks, is described by the following distribution:

Lead time (week)	1	2	3
Probability	0.10	0.60	0.30

Based on cost considerations as well as storage space, the company has decided to order 10 items while placing an order each time. The carrying cost is Rs. 40 per week for each unit that is left in the inventory at the end of the week. The stock out cost is set at Rs 5,000 per stock out. The company has decided to place an order whenever there are only 2 smartphones left at the end of the week. Simulate 12 weeks of operation for NOKIA with currently 3 units in inventory.

What would be the weekly carrying cost under this situation?

[12+3=15]

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2019-20
Course Name: **M. Tech. (QR&OR) - I Year**
Subject Name: **Quality Management & Systems**

Date: **22 November 2019**   Maximum Marks: **100**   Duration: **2 hours**
Note: **Answer all questions.**

1. Define quality function deployment. Describe the structure of House of Quality.

   Or, Define six sigma methodology and explain the various steps of implementation, in brief.

   [10]

2. Write short note on any five of the following terms.

   (a) Seven Quality Management Principles defined by ISO

   (b) Different Views of Quality

   (c) Juran's Trilogy

   (d) Different Dimensions of Service Quality

   (e) Nonconformance Report Structure

   (f) Disposition of Nonconforming Product

   (g) First Party Audit, Second Party Audit and Third Party Audit

   [5 x 6 = 30]

3. Define the customer satisfaction model proposed by Kano.

   Or, State and explain the different dimensions of quality proposed by David A. Garvin.

   [10]

4. Following are the objective evidences of an internal audit conducted according to ISO 9001: 2015 QMS. For any four objective evidences, provide your explanation and justification on being classified as a non-conformance or not.

   (a) Waste records show that waste was removed only once per week on 4 occasions during last three months instead of twice per week as stated in the procedure PR-6.8.

   (b) One batch of final product was dispatched before the completion of final inspection. However, it was found later, that the particular batch passed the final inspection.

   (c) Customer complaints are replied promptly and records are kept. Record shows similar complaints are recurring every month. When asked, the concerned manager tells his duty is only to reply the complaints promptly.

   (d) Testing procedure specifies that the temperature to be controlled within $40 \pm 2^\circ C$. A calibrated thermometer of increment $1^\circ C$ is used for this purpose.

   (e) The company Quality Manual has not been revised since December 2014.

   (f) An Inspector was using a documented instruction detailing how the testing should be carried out, but the testing instruction did not contain any signature.

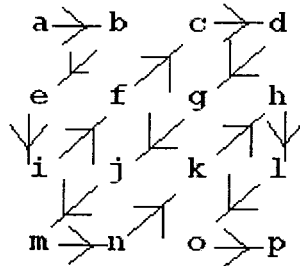   [4 x 5 = 20]

5. Assignments.

   [30]

Date: 20.11.2019   Maximum marks: 100   Duration: 3 hrs.

1. Draw a flow chart (as compact / concise as possible) to print the entities of a given square matrix of order 4 in diagonal pattern, e.g., given the following square matrix

the output should be **a b e i f c d g j m n k h l o p**   [20]

2. Write an algorithm (sequentially list all the major steps) to sort a given set of integers. State its best case and worst case time complexities.   [10 + 2 = 12]

3. Name four of each of built-in and derived data types of C language.   [8]

4. What will take place due to the execution of the following function?   [4]

```
void abc() {
int LA[] = {1,3,5,7,8};  item = 10, k = 3, n = 5, i = 0, j = k;
/*
printf("The original array elements are :\n");
for(i = 0; i<n; i++) {
        printf("LA[%d] = %d \n", i, LA[i]);
}
*/

while( j < n)
    LA[j++] = LA[j-1];

LA[k-1] = item;
```

```
/*
printf("The array elements after insertion :\n");
for(i = 0; i<n; i++)
    printf("LA[%d] = %d \n", i, LA[i]);
*/
}
```

5. Write a C function to search a given integer element in an array of integers both passed as parameters. [4]

6. Write a C code to open a text file containing the roll numbers and names of the students of a class. Print the number of students in the class. [12]

7. What are the library functions available for allocation of memory dynamically? What are the basic differences between them? [2+4=6]

8. What is a pointer variable? Write a function to swap two integers using pointers passed as parameters. If the starting address of an integer array of size 11 is 123456, what will be the address of its last element (state the assumption, if any)? [2+8+2=12]

9. For
   int c, *pc;
   decide the wrong and correct statements.
   a)   pc = c;
   b)   *pc = &c;
   c)   pc = &c;
   d)   *pc = c;   [4 × 2 = 8]

10. (a) Construct a structure for modeling a node in some possible implementation of a binary search tree.

(b) Write a C function for creation of a new node of a binary search tree by allocating memory dynamically and placing a given value into the new node. The prototype of such a function is as follows

                struct node* create_newnode(int)

(c) Write a C function for recursively deletion of all nodes of a binary search tree. The prototype of such a function is as follows

                void delete(struct node*)

                                                    [5+15+10=30]