INDIAN STATISTICAL INSTITUTE

First Semester Examination : (2019-2020)

B.Stat. 3rd Year

PARAMETRIC INFERENCE

Date: 21 November, 2019 Maximum Marks: 100 Duration: 3 Hours

Answer all questions.

1. Let $X_1, \ldots, X_n$ be a random sample from an exponential distribution with mean θ.

(a) Find the UMP level α test for testing $H_0 : \theta \leq \theta_0$ against the alternative $H_1 : \theta > \theta_0$ where $\theta_0 > 0$ is some specified value. Give a direct proof of your result without using the general theorem on UMP test for MLR families of distributions.

(b) Consider now the problem of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. Show that for any $0 < \alpha < 1$, there does not exist a UMP level α test for this problem.

Derive the usual optimum test using the general result (to be stated by you) for an exponential family of distributions.

[14+(7+9)=30]

2. Let $\boldsymbol{X}$ have distribution P_θ with a density $f(\boldsymbol{x}|\theta)$, $\theta \in \Theta$, an open interval of R, so that the family $\{f(\cdot|\theta), \theta \in \Theta\}$ has monotone likelihood ratio in some statistic $T(\mathbf{x})$. Consider a test of the form

$$\phi(\boldsymbol{x}) = \begin{cases} 1, & \text{if } T(\boldsymbol{x}) > c \\ 0, & \text{if } T(\boldsymbol{x}) < c. \end{cases}$$

Show that the power function of this test is nondecreasing in θ. [10]

3. Let $X_1, \ldots, X_n$ be i.i.d., each following discrete uniform on $\{1, 2, \ldots, \theta\}$ where θ is an unknown positive integer. Consider the problem of testing $H_0 : \theta = \theta_0$ against the alternative $H_1 : \theta = \theta_1$ where $\theta_1 > \theta_0$. The sample space here is $\mathcal{X} = \{(x_1, \ldots, x_n) : x_i \in \{1, 2, \ldots, \theta_1\} \text{ for i} = 1, \ldots, n\}$.

PTO

(a) Find the most powerful test of level α $(0 < \alpha < 1)$ for the above problem.

(b) Let $\mathcal{S} = \{(x_1, \ldots, x_n) \in \mathcal{X} : \max(x_1, \ldots, x_n) > \theta_0\}$. Show that for a test with rejection region $\mathcal{R}$ to be most powerful of its size, it is necessary that $\mathcal{S} \subset \mathcal{R}$. [7+9=16]

4. (a) Derive a condition for equality in Cramer-Rao Inequality. Show that a minimum variance bound (MVB) estimator of a parametric function $g(\theta)$ must be a sufficient statistic.

(b) Let $X_1, \ldots, X_n$ be i.i.d. with common p.d.f.

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta > 0$. Find the Cramer-Rao lower bound for the variance of an unbiased estimator of $1/\theta$ and find an estimator which attains this lower bound. [6+6=12]

5. Let $(X_1, \ldots, X_n)$ be a random sample from a $U(\theta - 1/2, \theta + 1/2)$ distribution where $\theta \in R$ is unknown. Let $X_{(j)}$ be the j-th order statistic. Show that $(X_{(1)} + X_{(n)})/2$ is a consistent estimator of θ. [10]

6. Describe how the posterior distribution can be used for estimation of a real parameter θ. How do you measure the accuracy of an estimate?

Given a loss function $L(\theta, a)$, how do you find an optimum estimate in the Bayesian paradigm? [6]

~~5~~ 7. Let $X_1, \ldots, X_n$ be i.i.d $N(\theta, 1)$. Consider the uniform prior $\pi(\theta) \equiv 1$.

(a) Consider the problem of testing $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$. Suppose we reject H_0 for large values of the test statistic $T = \sqrt{n}(\bar{X} - \theta_0)$. Let t be the observed value of the statistic T. Show that the P-value of this test is equal to the posterior probability of H_0.

(b) Find a $100(1 - \alpha)\%$ HPD credible set for θ and compare it with the classical confidence interval.

[9+7=16]

INDIAN STATISTICAL INSTITUTE

Backpaper Examination : (2019-2020)

B.Stat. 3rd Year

PARAMETRIC INFERENCE

Date: ~~?? December, 2019~~ 7 January, 2020 Maximum Marks: 100 Duration: 3 Hours

Answer all questions.

1. Describe the notions of sufficiency, minimal sufficiency and completeness of a statistic. Illustrate with examples (no derivation is needed). [13]

2. Show that if a minimal sufficient statistic exists then a complete sufficient statistic is also minimal sufficient. [7]

3. Let $X_1, \ldots, X_m$ be a random sample from $N(\mu_1, \sigma^2)$ and let $Y_1, \ldots, Y_n$ be a random sample from $N(\mu_2, \sigma^2)$ where $\mu_1 \in R$, $\mu_2 \in R$ and $\sigma^2 > 0$ are all unknown.

(a) Find a complete sufficient statistic for (μ_1, μ_2, σ^2).

(b) Find the UMVUE of $1/\sigma$.

(c) Find the UMVUE of $(\mu_1 - \mu_2)/\sigma$ [8+8+8=24]

4. Let $X_1, \ldots, X_n$ be a random sample from an $N(\mu, \sigma^2)$ population where μ is known.

(a) Find a UMP level α test for testing $H_0 : \sigma^2 \leq \sigma_0^2$ against the alternative $H_1 : \sigma^2 > \sigma_0^2$ where $\sigma_0^2 > 0$ is some specified value. Give a direct proof of your result without using the general theorem on UMP test for MLR families of distributions. [12]

(b) Consider the problem of testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$. Show that for any $0 < \alpha < 1$, there does not exist a UMP level α test for this problem.

Derive the usual optimum test using a suitable general result (to be stated by you) for an exponential family of distributions. [6+11=17]

(c) Show that there exists a minimum variance bound (MVB) estimator of σ^2. Does there exist an MVB estimator of σ? Justify your answer. [9]

5. Let $X_1, \ldots, X_n$ be i.i.d $N(0, \sigma^2)$, $\sigma^2 > 0$. Consider an inverse-gamma (α, β) prior distribution having density

$$g(\sigma^2) = \frac{\beta^\alpha}{\Gamma(\alpha)}(\sigma^2)^{-(\alpha+1)}e^{-\beta/\sigma^2}, \ \sigma^2 > 0$$

where $\alpha > 0, \beta > 0$ are known. Find the Bayes estimate of σ^2 for the loss function $L(\sigma^2, a) = (a - \sigma^2)^2/\sigma^4$. [It is given that the mean of an inverse-gamma (α, β) distribution is $\beta/(\alpha - 1)$.] [10]

6. Let X be a random variable with probability distributions under a simple hypothesis H_0 and a simple alternative H_1 given in the following table.

Values of X	1	2	3	4	5	6
Probability under H_1	0.15	0.15	0.15	0.15	0.15	0.25
Probability under H_0	0.15	0.20	0.15	0.10	0.10	0.30

Suppose that we want to find a most powerful (MP) test of level 0.3. Is the MP test of level 0.3 unique ? Justify your answer. [8]

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2019–20
Linear Statistical Models
Maximum marks: 50

Date: Nov 25, 2019 Duration: 3 hrs

This paper carries 55 marks. Attempt all questions. The maximum you can score is 50. This is a closed book, closed note exam. You may use your own calculator.

1. We have a machine that halves tablets of a fixed size into two equal halves. Due to mechanical errors the halves may not be exactly equal. We have taken n identical tablets and halved them with the machine, and measured the masses of both the halves to get the data $(u_1, v_1), ..., (u_n, v_n)$. We want to test

   H_0 : the machine is doing its job satisfactorily.

   The staff statistician has suggested using paired t-test for equality of means. But the director of the firm thinks that this method fails to take into account the fact that the masses of the two parts should be highly negatively correlated. In view of this, suggest how you may use a linear mixed effects model to improve upon the paired t-test method. Give the R commands to implement your method. [10+5]

2. We had a data set consisting of 10 pairs (x_i, y_i) in a file data.txt. The following analysis was carried out:

```
dat = read.table('data.txt',head=T)
fit = lm(y~x+I(x^2)+I(x^3),dat=dat)
```

   Unfortunately, the data file was lost after this, and only the $(x_i, \hat{y}_i)$ pairs remain. It is now required to fit a straight line

$$y_i = \alpha + \beta x_i + \epsilon_i$$

   to the original data, where ϵ_i's are iid $N(0, \sigma^2)$, for some unknown σ^2. A statistician has used the $(x_i, \hat{y}_i)$ values in place of the (x_i, y_i)'s, and computed $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\sigma}^2$. Will these values be the same as (or more or less than) the estimates based on the original data? Justify your answer mathematically.

   [20]

3. 3 sewing machines are chosen randomly from a large number of similar machines. Independently of this, 5 operators are chosen randomly from a large number of available operators. Each selected operator has worked on each machine to make a shirt of the same fixed size. The data consist of y_{ij}, which is the time required by the i-th operator to finish the shirt using the j-th machine. Assuming that all the operators are equally comfortable with all the machines, we use the following 2-way ANOVA mixed effects model:

$$y_{ij} = \mu + a_i + b_j + \epsilon_{ij},$$

   where

$$a_i \sim N(0, \sigma_a^2), \quad b_j \sim N(0, \sigma_b^2), \text{ and } \epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ (all independent).}$$

   Write down the "machine MS" (No need to derive it). Find its expectation. [2+8]

4. Submit the project on "Shape Analysis using LME". [10]

INDIAN STATISTICAL INSTITUTE
First Backpaper Examination: 2019–20
Linear Statistical Models

Date: 10.1.2020   Maximum marks: 100   Duration: 3 hrs

This paper carries 100 marks. Attempt all questions. The maximum you can score is 100. This is a closed book, closed note exam. You may use your own calculator.

1. We have a machine that is supposed to cut a fixed length of cloth into three equal parts. We have used this machine to cut n identical pieces of cloth of this length. The measured lengths of the parts are $(x_1, y_1, z_1), ..., (x_1, y_1, z_1)$. We want to test

   H_0 : the machine is doing its job satisfactorily.

   Suggest how you may use a linear mixed effect model to do this. Give the R commands to implement your method. [10+5]

2. We had a data set consisting of 10 pairs (x_i, y_i) in a file `data.txt`. The following analysis was carried out:

```
dat = read.table('data.txt',head=T)
fit = lm(y~x,dat=dat)
```

   Unfortunately, the data file was lost after this, and only the $(x_i, \hat{y}_i)$ pairs remain. It is now required to fit a straight line

   $$y_i = \alpha + \beta x_i + \gamma x_i^2 + \epsilon_i$$

   to the original data, where ϵ_i's are iid $N(0, \sigma^2)$, for some unknown σ^2. A statistician has use the $(x_i, \hat{y}_i)$ values in place of the (x_i, y_i)'s, and computed $\hat{\alpha}$ $\hat{\beta}$ and $\hat{\sigma}^2$. Will these values be the same as the estimates based on the original data? Justify your answer.

   [20]

3. Consider the model $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$, where $i = 1, ..., n$ and $j = 1, ..., m$ and $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$ for some unknown $\sigma^2 > 0$. Will the ML estimator of σ_ϵ^2 increase (or decrease) if α_i's are considered random (iid $N(0, \sigma_a^2)$)? [10]

4. Obtain the REML estimator of σ^2 based on the data y_{ij}, $(i = 1, ...10$ and $j = 1, ..., 5)$ where

   $$y_{ij} = \mu + \alpha_i + \epsilon_{ij},$$

   where $e_{ij} \sim N(0, \sigma_e^2)$. [10]

5. We want to write a program to apply MLE to a linear model involving a 6×6 correlation matrix with all off-diagonal entries equal to ρ. The software needs to know the allowable range values for ρ such that the correlation matrix is positive definite. Find this range. [10]

6. Consider the model

$$y_i = \mu + \epsilon_i, \quad i = 1, ..., 6$$

where $\vec{\epsilon} = \epsilon_i$'s has normal distribution with mean zero and dispersion matrix $\sigma^2 diag(R, R, R)$; where $R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, and $\sigma^2 > 0$ is unknown. Obtain MLE of μ. [10]

7. Consider the model $y_i = \alpha + \beta x_i + \epsilon_{ij}$, for $i = 1, ..., n$, with $\epsilon_i \sim N(0, \sigma^2)$ for some unknown $\sigma^2 > 0$. Is it possible to have a data set $\{(x_i, y_i)\}_1^n$, for some $n \geq 3$, such that in the least squares fit we have $|\hat{\alpha}| < |\hat{\beta}|$, but $H_0 : \alpha = 0$ is rejected while $H_0 : \beta = 0$ is accepted at 5% level of significance? Justify your answer. [15]

8. We want to assess the performance of 3 types of sewing machines. 5 operators are chosen randomly from a large number of available operators. Each selected operator has worked on each type of machine to make a shirt of the same fixed size. The data consist of y_{ij}, which is the time required by the i-th operator to finish the shirt using the j-th machine. Assuming that all the operators are equally comfortable with all the machines, we use the following 2-way ANOVA mixed effects model:

$$y_{ij} = \mu + a_i + \beta_j + \epsilon_{ij},$$

where

$$a_i \sim N(0, \sigma_a^2), \text{ and } \epsilon_{ij} \sim N(0, \sigma_\epsilon^2) \text{ (all independent).}$$

Write down the "operator MS" (No need to derive it). Find its expectation. [2+8]

INDIAN STATISTICAL INSTITUTE

First Semester Examination : 2019-20

Course Name : B.Stat. 3rd Year

Subject Name : Sample Surveys

Date : 27.11.19, 2019 Total Marks : 60 Duration : 3 hrs.

Answer Q.4 and any 2 from Q.1 to Q.3.

Symbols and notations are as usual.

1. (a) Let Y be the population total of a variable of interest y. Obtain an unbiased estimator of Y based on a two-stage cluster sampling (SRSWOR-SRSWOR) scheme.

   (b) Derive the variance of the estimator in 1(a). Also derive an unbiased estimator of that variance. [20]

2. (a) Prove that the Horvitz and Thompson's (HT) estimator is an unbiased estimator for the population total Y of a variable of interest y.

   (b) Derive the variance of the estimator in 2(a) and an unbiased estimator of this variance.

   Clearly state the condition which should be satisfied for using HT estimator and variance estimator.

   (c) For PPSWR scheme, show how the HT estimator can be used to estimate Y.

   [20]

3. (a) For estimating the population ratio $R = \frac{Y}{X}$ by an SRSWOR, consider an estimator as $\hat{R} = \frac{\bar{y}}{\bar{x}}$, where $\bar{y}$ and $\bar{x}$ are the sample means of the variables y and x respectively. Examine whether $\hat{R}$ is unbiased. Also derive the mean squared error of $\hat{R}$.

   (b) Explain how Hansen and Hurwitz's double sampling approach can be utilized in the non-response situation to estimate the population mean $\bar{Y}$ by SRSWOR scheme. Obtain the variance of this estimator of $\bar{Y}$. [20]

4. The following figures relate to a group of 15 households.

Serial No.	hh-size	Expenditure in last month (Rs.)
1	8	5470.35
2	6	2716.80
3	5	1873.75
4	4	1693.20
5	3	1393.55
6	6	2398.74
7	2	3153.35
8	5	2708.75
9	7	2873.60
10	6	3775.80
11	8	5027.25
12	3	1175.28
13	4	2952.15
14	2	1032.27
15	2	2075.41

Draw a sample of size 6 by SRSWOR from these 15 households.

Based on the sample drawn by you, give (i) an estimate of average household size, (ii) an estimate of the total population of this area, (iii) an estimate of average household expenditure, and (iv) an estimate of the average per-head expenditure.

With each of the above estimates, give an estimate of the standard error, and the coefficient of variation expressed as a percentage. [20]

Part of a Table of Random Numbers

61424	20419	86546	00517
90222	27993	04952	66762
50349	71146	97668	86523
85676	10005	08216	25906
02429	19761	15370	43882
90519	61988	40164	15815
20631	88967	19660	89624
89990	78733	16447	27932

INDIAN STATISTICAL INSTITUTE

Semestral Examination

Bachelor of Statistics(Hons.) 3rd year 2019-20 (Semester-I)

Design and Analysis of Algorithms

Date: **November 18, 2019** Maximum Marks: **100** Duration: **3 hours**

Note: The question paper carries a total of **120** marks. You can answer as much as you can, but the maximum you can score is 100.

You need to prove the correctness of your algorithms.

1. (a) Given an array of intergers $A[1..n]$, such that, for all i, $1 \leq i < n$, we have $|A[i]-A[i-1]| \leq 1$. Let $A[1] = x$ and $A[n] = y$, such that $x < y$. Design an efficient algorithm to find j such that $A[j] = z$ for some given integer z, $x \leq z \leq y$. Derive the maximum number of comparisons used by your algorithm.

   (b) Prove that a comparison-based algorithm for sorting requires $\Omega(n \log n)$ comparisons.

   [(8+2)+10=20]

2. Consider an undirected graph $G = (V, E)$ with a weight function w providing nonnegative real-valued weights, such that the weights of all the edges are different.

   (a) Prove that, under the given uniqueness assumption, G has a unique Minimum Spanning Tree.

   (b) Algorithm A operates in phases. In each phase, it first finds a simple cycle in the graph. Then it identifies the heaviest edge on each cycle, and removes this edge. Phases continue until we have an acyclic graph. Is this a correct MST algorithm? If so, give a proof. If not, give a specific counterexample.

   (c) Algorithm B uses a simple Divide-and-Conquer strategy. It divides the set V of vertices arbitrarily into disjoint sets V_1 and V_2, each of size roughly $\frac{V}{2}$. Define graph $G_1 = (V_1, E_1)$, where E_1 is the subset of E for which both endpoints are in V_1. Define $G_2 = (V_2, E_2)$ analogously. Recursively find (unique) MSTs for both G_1 and G_2; call them T_1 and T_2. Then find the (unique) lightest edge that crosses the cut between the two sets of vertices V_1 and V_2, and add that to form the final spanning tree T.
   Is this a correct MST algorithm? If so, give a proof. If not, give a specific counterexample.

   [7+10+8=25]

3. Describe the Ford-Fulkerson method for finding the maximum flow in a network. Give an example where it may not work.

   [10+10=20]

4. Write an $O(n \log n)$ time algorithm for finding the convex hull of n given points on the plane.

   [15]

5. Let $A_1, A_2, \ldots, A_n$ be n matrices such that the dimension of A_i is $p_i \times p_{i+1}$. Write an efficient algorithm to find the minimum number of multiplications required to find the product $A_1 \times A_2 \times \ldots \times A_n$.

   [12]

6. Write an efficient algorithm to find a hamiltonian cycle in a given a connected undirected graph $G = (V, E)$ with $n \geq 3$ vertices such that each pair of non-adjacent vertices v and w satisfies $degree(v) + degree(w) \geq n$.

[15]

7. (a) Define the terms NP-hard and NP-complete.

   (b) Let MULTI-SAT4 denote the following decision problem: given a Boolean formula ϕ, decide whether ϕ has at least four distinct satisfying assignments. Prove that MULTI-SAT4 is NP-complete.

[3+10=13]

B.STAT. THIRD YEAR (2019-20)
End-Semester Examination
ECONOMIC AND OFFICIAL STATISTICS
Total Marks = 50. Time allotted = 1hour 30 minutes
29th November, 2019

Question papers consist of two parts.
Part A (30 marks) and Part B. Part A questions are given separately.

Part B (20 marks): It contains questions of total 30 marks.
Answer as many as you can. Maximum one can score is 20.

Q.1 Let p^0 and p be price vectors of length n with all positive entries. A scalar function $P(p^0, p)$ taking positive values is called a price index depending only on prices, if it satisfies the following properties:

(i). Monotonicity: P is strictly increasing w.r.t. p and strictly decreasing w.r.t. p^0
(ii). Linear homogeneity: $P(p^0, \lambda p) = \lambda P(p^0, p)$ for $\lambda > 0$.
(iii). Identity: $P(p^0, p^0) = 1$
(iv). Invariance to unit of measurement: $P(\lambda p^0, \lambda p) = P(p^0, p)$ for $\lambda > 0$

1.1 Prove that the following functions satisfy the above properties (i) to (iv):

(a) $P(p^0, p) = c.p/c.p^0$, where c is a constant vector of length n with all positive entries. (.) stands for dot product. [2]
(b) $P(p^0, p) = \prod(p_i/p_i^0)^{a_i}$, where $a_i > 0$ for all i and $\sum a_i = 1$ [3]

1.2 Give examples to show that:
(a) Properties (ii), (iii) and (iv) together do not imply property (i). [2]
(b) Properties (i), (iii) and (iv) together do not imply property (ii). [3]

Q.2 Consider a production function q = f(k, l), with one output q and two inputs k and l denoting capital and labour, respectively. For such a production function:

2.1 Define returns to scale, marginal rate of technical substitution (MRTS) and elasticity of substitution (ES). [3]

2.2. For Cobb-Douglas production function, find MRTS. [3]

2.3 For CES production function, find the value of ES. [4]

3.1 Banks performs many functions such as accepting deposits from public, give loans to producers and consumers, invest money in government bonds. Banks have large number of branches in different places of the country for carrying out these functions. Banks offer different rates of interest for various types of deposits. Also banks charge different rates of interest on different types of loans, different duration of loans. Loan interest rates can also vary for different borrowers. Deposit and loan interest rates and interest on bond investments also vary over time. Banks collect and aggregate various types of data for monitoring the performance of these functions. Give a list of various types of data that should be collected by banks for this purpose. [5]

3.2 Explain briefly two important functions of money. Name three main components of money supply and describe the sources of data for compilation of each of these components. [2+3]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination 2019-20

Course Name: B.Stat. (Hons.) 3rd Year

Subject: Economic and Official Statistics and Demography

Group: C (Demography)

Date: November 29, 2019 Maximum Marks: 50 Time: 2 hours

(Answer any five questions.)

*(**Instruction**: Standard notation are followed.)*

(Use separate answer booklet for Group C. Use of calculator is allowed.)

1) Given that the quality of age data in most developing nations is poor, suggest a method of adjustment for the age distribution of a developing nation, which is done by comparing with the age distribution of a suitable stable population, assuming it as a standard. State the necessary assumption(s). [10]

2) a) Express ${}_nL_x$ in terms of l_x, ${}_na_x$ and ${}_nd_x$.
   b) For age groups with width of n years, derive an equation that relates q-type mortality rate with m-type mortality rate.
   c) Write a short note on Greville's method with full explanation of all used symbols.
   [2.5+2.5+5=10]

3) a) There are two methods for allowing precise comparison of two or more crude death rates by eliminating the effect of the differences in age structure. What are they? Describe the method where age specific death rates are known only for the standard population.
   b) Define three mortality rates which are concerned with infant deaths. [0.5+5+4.5=10]

4) a) The table below gives estimates of the life expectation e_x at various ages x for females in Nicaragua 1990-95 and the United States 1989. Use them to estimate q_0, ${}_4q_1$, ${}_5q_5$ and ${}_5q_{10}$ for these two countries.

Age x	Life expectation	
	Nicaragua, 1990-95	United States, 1989
0	67.7	78.6
1	70.0	78.3
5	67.7	74.4
10	63.2	69.5
15	58.5	64.6

b) Define age parity specific fertility rate with full explanation of all used symbols. Establish relation between birth order and parity. [6+1.5+2.5=10]

5) a) Derive the Chandrasekhar-Deming formula and show that it estimates the completeness of the coverage of the R system as the match rate of the S system and also estimates the completeness of the coverage of the S system as the match rate of the R system.

b) Define total fertility rate and mention one of its deficiencies. If the crude birth rate in a country remains constant over a number of years, but the general fertility rate increases steadily, what does this tell you about the country's population? [7+1.5+1.5=10]

6) a) Define Gross Reproduction Rate (GRR). In what way is it different from Total Fertility Rate (TFR)? Define and explain Net Reproduction Rate (NRR).

b) Define parity progression ratio. Derive an expression of completed fertility rate of a cohort by using the ratios. [1.5+1+2+1.5+4=10]

= END =