

ABSTRACT

Cops and Robber is a two-player pursuit-evasion game played on graphs. Here, one player controls a set of k cops, and the other player controls a single robber. The game starts with k cops placing themselves on the vertices of a graph. More than one cops can occupy the same vertex. Then the robber enters a vertex of the graph. Then the game occurs in rounds and in each round, first the cops move, and then the robber moves. In a *cop move*, each cop either moves to an adjacent vertex or stays on the same vertex. In a *robber move*, the robber either moves to an adjacent vertex or stays on the same vertex. If at some point in the game, one of the cops occupies the same vertex as the robber, we call it a *capture*. The goal of the cops is to capture the robber, and the robber aims to evade the capture. The *cop number* of a graph G , denoted as $c(G)$, is the minimum number of cops that can ensure the capture against all strategies of the robber. A graph is *cop-win* if its cop number is 1. This game is a *perfect information game*, that is, each player can see other players and their moves.

Many variants of this game have been studied, mainly varying based on the capabilities of the cops and the robber. We begin our study by considering two variants: the *Cops and attacking Robber* and the *lazy Cops and Robber*, on various kinds of grids originating from the Cartesian product of paths and cycles. In particular, we study the *attacking cop number*

of planar grids, cylindrical grids, toroidal grids, high-dimensional grids, and hypercubes. We study the *lazy cop number* of planar, cylindrical, and toroidal grids. We also study the classical Cops and Robber game on solid grids and partial grids.

Next, we study three models of this game on oriented graphs which differ based on the kind of moves the players can make. These models are (i) the *normal cop model*, where both cops and robber can only move along the direction of the arcs; (ii) the *strong cop model*, where the cops can move along or against the direction of the arcs while the robber can only move along them; and (iii) the *weak cop model*, where the robber can move along or against the direction of the arcs while the cops can only move along them. For the normal cop model, we show that there exist strongly connected oriented graphs having high girth, high minimum degree, and high cop number. We also characterize the cop-win graphs in various graph classes like transitive-triangle-free, outerplanar, and subcubic graphs. For the strong cop model, we construct graphs with unbounded cop number, and also study the cop number of grids, outerplanar, and planar graphs. For the weak cop model, we characterize the cop-win graphs.

Next, we study the game of Cops and Robber on string graphs. We give an algorithm to capture the robber on a string graph using at most 14 cops. Thus we prove that the cop number of string graphs is upper bounded by 14.

Let H be a subgraph of G . We say that cops *guard* H if the robber cannot enter the vertices of H without getting captured. Finally, we study the applications of guarding subgraphs to bound the cop number of some graph classes. In particular, we study the cop number of butterfly networks and AT-free graphs. We also study the game of *Cops and fast Robber* for AT-free graphs.