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AN EXTENSION OF A MODEL FOR FIRST BIRTH INTERVAL AND SOME SOCIAL FACTORS

By B. N. BHATTACHARYA
Indian Statistical Institute
and
K. K. SINGH, UTTAM SINGH and C. M. PANDEY
Banaras Hindu University, India

SUMMARY. A model for the time of first birth is presented under certain assumptions, which involve biological and socio-cultural factors. The model has been applied to real data.

1. Introduction

Interval from marriage to the first birth to a female in a population provides good data for the study of fecundability. A variety of mathematical models to explain the nature of this interval have been proposed and are applied to real data to estimate unknown parameters (See, Sheps and Menken, 1973 ; Leridon, 1977 ; Mode, 1985). It is usually assumed in most widely used models that—all the females are fecund at marriage, fecundability is constant for a female till the occurrence of first live birth conception and fecundability may vary among women. However, these models often do not describe satisfactorily rural data, especially where age at start of cohabitation is low (Singh, 1964 ; Singh and Singh, 1983 ; Bhattacharya, *et al.*, 1986).

In many Asian countries and some areas of tropical Africa, age at first marriage has always been very low, particularly in rural areas. However, the start of active sex life after such marriages depends on the permitted social and religious practice (Ohadike, 1979 ; Murphy and Dyson, 1985). In most of the rural communities in India, the bride does not start living with husband immediately after marriage. The consummation of marriage occurs following another ceremony known as return marriage (RM) (Jain, 1975) which also in large number of cases occur well before the age of 15 years. Thus a substantial proportion of such females are expected to be either in a pre-menarcheal state or in a state of adolescent subfertility following the onset of menarche at the time of RM.

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Besides biological factors, time of first birth is also affected by several socio-cultural factors. Even in case of early age at consummation cohabitation begins late due to the social custom of stay for a considerable period by the bride at her father's place after RM. Also she makes frequent short visits to her parents during the early phase of married life. The couple lives under the influence of joint family, young bride has to observe taboos, owing to the presence and control of mother-in-law and other elderly women. Recently, Bhattacharya *et al.* (1986) have reviewed the modifications in the existing models proposed by various authors to tackle such situations. It becomes cumbersome to formulate a model accounting for all such factors.

This paper presents a model for the time of first live birth which accounts for the delayed exposure to risk of conception due to first visit of the female to her parents, takes fecundability to be time dependent during early part of marital life thus indirectly incorporates biological as well as socio-cultural factors responsible for lowering the fecundability. It also takes into account of foetal wastages preceding the first live-birth conception, hnence the model becomes more flexible and may be applied to data in societies where sexual relation starts at an early age. Thus model may be considered as an extension of the model proposed in Bhattacharya *et al.* (1986).

2. Model

Let us consider a cohort of women of the same age at RM, same duration of effective married life say, exactly T years, and have given at least one live birth. A probability distribution of the time between RM and first live birth for such women is obtained under the following assumptions :

(i) The duration of first visit of the female to her parents immediately after RM, say Z, is a random variable having distribution function $H(t)$. The female is fecund when she returns to her husband.

(ii) The conditional instantaneous risk of first conception is low during is low during early part of cohabitation which gradually increases till it reahces a normal level say m_0. Given that the duration of first stay is of length z, the conditional probability that the first conception to a female occurs during the interval $(t, t+\Delta t)$ is

$$\begin{cases} m(t/z)\Delta t + 0(\Delta t)\ (m(t/z) > 0) & \text{for } t > z \\ 0 & \text{otherwise.} \end{cases} \qquad \dots (2.1)$$

The assumed pattern of $m(t/z)$ reflects the sociocultural practices associated with early part of conjugal life, which govern the timing and frequency of intercourse as well as subfecundability which slowly decreases with age.

(iii) A conception ends in either a live birth or a foetal loss. Let θ be the probability that a conception results in a foetal loss, $0 \leqslant \theta < 1$.

(iv) The length of non-susceptible period comprising the duration of pregnancy and post-partum amenorrhoea associated with a foetal loss is an exponentially distributed random variable with mean $1/c$, $c > 0$.

(v) Given that the first pregnancy has resulted in a foetal loss, the instantaneous risk of conception thereafter is constant with a normal value m_0 and remains the same until the occurrence of first live birth conception.

The interval between RM and first live birth, say T_0, given that she has n foetal wastages before the first live birth is the sum of the following components :

(a) Z, the duration of first stay of the female with her parents.

(b) $X_0+X_1+X_2+ \dots +X_n$, the total duration of stay in the fecundable state between RM and the first live birth ; where X_0 : the time elapsed between return to husband after first visit to the parents home and first conception and X_j : the time spent in the fecundable state following j-th foetal loss till the next conception, $j \geqslant 1$.

(c) $Y_1+Y_2+\dots+Y_n$; where Y_j is the duration of non-susceptible period associated with the j-th foetal loss.

(d) g, the period of pregnancy associated with the first live birth. Thus

$$T_0/n = Z+X_0+\sum_{i=1}^{n}(X_i+Y_i)+g. \qquad \dots \quad (2.2)$$

Under the assumptions (i) and (ii), the distribution of $(Z+X_0)$ is

$$F_0(t) = \int_0^t \left[1-\exp\left(-\int_z^t m(x/z)dx\right)\right]dH(z). \qquad \dots \quad (2.3)$$

Z and X_0 may not be independent but $(Z+X_0)$, $X_1, X_2, \dots, X_n$, $Y_1, Y_2, \dots, Y_n$ are statistically independent due to assumptions (iv) and (v). Moreover, for each $j \geqslant 1$, X_j and Y_j are exponentially distributed with parameters m_0 and c respectively.

Let us denote the Laplace transformation of the distribution of $(Z+X_0)$ by $F_0(s)$. Following the procedure described in Suchindran and Lachenbruch (1974) in the derivation of Laplace transform of the time of first live birth, it can be shown that the Laplace transform of the unconditional distribution function of T_0 is

$$\phi(s) = \sum_{n=0}^{\infty} (1-\theta)\theta^n F_0(s) \left(\frac{m_0}{m_0+s}\right)^n \left(\frac{c}{c+s}\right)^n \exp(-sg) \quad \ldots \quad (2.4)$$

which can be re-written as

$$\phi(s) = F_0(s) \left[A_0 + A_1\left(\frac{V_1}{V_1+s}\right) + A_2\left(\frac{V_2}{V_2+s}\right)\right] \exp(-sg) \quad \ldots \quad (2$$

where

$$A_0 = (1-\theta),\ A_1 = \frac{m_0c\theta(1-\theta)}{V_1(V_2-V_1)},\ A_2 = \frac{m_0c\theta(1-\theta)}{V_2(V_1-V_2)}.$$

V_1 and V_2 are the additive inverses of the roots of the equation $s^2+(m_0+c)s+m_0c(1-\theta) = 0$ and V_1 and V_2 are non-negative and distinct. The inverse of $\phi(s)$, the complete distribution function of T_0 is

$$K(t) = \sum_{j=0}^{2} A_j K_j(t) \quad \ldots \quad (2.6)$$

where

$$K_0(t) = \int_0^{t-g} \left[1-\exp\left(-\int_z^{t-g} m(x/z)dx\right)\right] dH(z) \quad t > g \quad \ldots \quad (2.7)$$

and

$$K_j(t) = \int_g^t K_0(y) V_j e^{-V_j(t-y)} dy,\ t > g,\ j = 1, 2. \quad \ldots \quad (2.8)$$

Since the distribution is truncated at the T year, the truncated population is governed by the probability law

$$K^*(t) = \frac{\sum_{j=0}^{2} A_j K_j(t)}{K(T)} \quad g \leqslant t \leqslant T. \quad \ldots \quad (2.9)$$

We specify the assumptions (i) and (ii) as follows :

(i) Z is a discrete random variable taking values $\tau_1 < \tau_2 < \tau_3 < \ldots$ with associated probability function $b_i = \text{Prob.}\ (Z = \tau_i),\ i = 1, 2, \ldots$

(ii) $m(t/z)$ is a polynomial of degree r in t for $z < t \leqslant z+T_1$ and is of the form

$$m(t/z) = \sum_{j=0}^{r} q_j(t-z)^r \quad \ldots \quad (2.10)$$

and for $t > z+T_1$

$$m(t/z) = m((T_1+z)/z) = \Sigma q_j T_1^j \quad \ldots \quad (2.11)$$

i.e., between time points $(z, z+T_1)$ the instantaneous risk of conception depends on distance between the start of cohabitation and of t and thereafter becomes constant and the expressions (2.7) and (2.8) reduce to

$$K_0(t) = \sum_{i; h_i < T} b_i \left[1-\exp\left\{-\int_{\tau_i}^{t-g} m(x/\tau_i)\,dx\right\}\right]$$

$$_j(t) = \sum_{i; h_i < T} b_i\left[1-\exp\{-V_j(t-h_i)\}-V_j \int_{h_i}^{t} \exp\left\{-\int_{\tau_i}^{y-g} m(x/\tau_i)dx - V_j(t-y)dy\right\}\right]$$

$$\text{for } i = 1, 2, \ldots \qquad h_i = \tau_i+g$$

where
$$\int_{\tau_i}^{a-g} m(x/\tau_i)\,dx = \sum_{j=0}^{r} q_j \frac{(a-h_i)^{j+1}}{j+1} \qquad h_i < a \leqslant h_i+T_1$$

$$= \sum_{j=0}^{r} \frac{q_j T_1^{j+1}}{j+1} + m_0(a-h_i-T_1) \quad a > h_i+T_1$$

3. Estimation

A procedure to obtain maximum likelihood estimates (MLE) of the parameter in the distribution (2.9) when $m(t/z)$ is of the form given in (2.10 and 2.11) and for $r = 2$, for known values of T_1, 0 and discrete form of the distribution $H(t)$ is outlined below for the grouped data. In this case, distribution involves three parameters q_0, q_1 and q_2.

Let range of first live birth interval be partitioned into k intervals with the end points of interval being t_j, $j = 0, 1, 2, \ldots, k$, $t_0 = 0$, $t_k = T$ and $t_1 > h_1$. Let P_j denote the expected proportion of women with their times of first birth in the j-th interval $(t_{j-1}, t_j]$ where

$$P_1 = \frac{K(t_1)}{K(T)} \text{ and } P_j = \frac{K(t_j)-K(t_{j-1})}{K(T)} \qquad j = 2, 3, \ldots, k$$

In a sample of N women $n_1, n_2, \ldots, n_k$ women are observed to deliver the the first child during intervals $1, 2, \ldots, k$ respectively and $\sum_{i=1}^{k} n_i = n$. It can be seen that the equations

$$\sum_{i=1}^{k} \frac{n_i}{P_i} \frac{\delta P_i}{\delta q_j} = 0 \qquad (j = 0, 1, 2)$$

do not provide the explicit expressions of MLE. Hence MLE of the parameters may be computed by scoring method. A method of obtaining the pilot values of the unknown parameters which are required for scoring is given below.

TABLE 1. DISTRIBUTION OF FEMALES GIVING BIRTH IN FIRST SEVEN YEARS OF R.M. ACCORDING TO TIME OF FIRST LIVE-BIRTH.

interval between R.M. and first live-birth (in years)	number of females having age at R.M.											
	12*		13		14		15		16		17-19	
	O	*E*	*O*	*E*	*O*	*E*	*O*	*E*	*O*	*E*	*O*	*E*
0 —1.75	12	12.4	7	7.0	17	18.0	45	41.8	49	46.0	69	66.4
1.75—2.75	36	33.8	49	49.7	98	94.8	123	130.8	102	108.6	136	143.2
2.75—3.75	39	37.8	96	87.9	143	141.4	163	155.9	113	106.5	138	129.0
3.75—4.75	22	27.2	75	84.1	104	109.2	113	121.0	67	74.7	73	77.3
4.75—5.75	16	16.1	52	54.3	56	56.3	79	69.9	49	41.4	34	34.0
5.75—7.00	13	10.7	37	33.1	29	27.3	37	40.6	21	23.8	15	14.5
Total	138	138.0	316	316.0	447	447.0	560	560.0	401	401.0	465	465.0
χ^2	1.067		2.341		0.548		3.080		3.496		1.202	
Estimates of												
q_0	0.004		0.011		0.013		0.107		0.385		0.496	
q_1	0.281		0.305		0.420		0.240		0.104		0.334	
var. $(\hat{q}_0)\times 10^{-4}$	30		18		19		21		53		105	
var. $(\hat{q}_1)\times 10^{-4}$	41		27		29		22		60		122	
corr. $(\hat{q}_0, \hat{q}_1)$	−0.786		−0.805		−0.817		−0.801		−0.811		−0.857	

1. Variance
2. Correlation

When $m(t/z)$ is constant, the model involves only one parameter q_0. The pilot value of q_0 can be obtained by equating $\bar{X}$—the mean length of first birth interval of those giving birth in $(0, T)$, to its theoretical expression, where

$$\bar{X} = \frac{\sum\limits_{i/h_i<T} b_i\left[h_i + \sum\limits_{j=1}^{2} \frac{B_j}{V_j}\left\{1-(\exp(-V_j(T-h_i)))(1+V_jT)\right\}\right]}{\sum\limits_{i/h_i<T} b_i\left[1-\sum\limits_{j=1}^{2} B_j e^{-V_j(T-h_i)}\right]}$$

where

$$B_j = A_j \frac{m_0}{m_0-V_j} \qquad j = 1, 2.$$

This equation may be solved by Newton-Raphson iteration procedure.

When $m(t/z)$ is a polynomial of degree one, there are two parameters viz. q_0 and q_1. The MLE of q_0 obtained by taking $m(t/z)$ to be constant and zero can be taken as the pilot values of q_0 and q_1 respectively. Similarly the MLE of q_0 and q_1 obtained above and zero may serve as the pilot values of q_0, q_1 and q_2 respectively, when the form of $m(t/z)$ is quadratic.

4. Application

For illustration of the model we use data from the survey described below. The data were compiled from the survey entitled "Rural Development and Population Growth—A Sample Survey, 1978" conducted by Centre of Population Studies, Banaras Hindu University, Varanasi in the year 1978. The survey included all the households numbering 3514 of 19 villages which were slected from Varanasi district and adjoining areas. The details of the survey are given in Bhattacharya and Nath (1984). Besides other information, marriage and birth histories of all eligible couples (both husband and wife alive and wife below 50 years on the reference date : 25th March, 1978, the Holi festival) were obtained. Table 1 provides age at RM wise distributions of women separately by length of interval between RM and the first live birth. Data presented in the Table 1 refers to couple who did not practice any family planning method, both husband and wife were normal residents of the village, female partner was married only once, whose date of RM preceeds the reference date by at least seven years, at least one child was born during the seven years after RM, and the age at RM was 12 years or more. In this connection it is important to note that in case of the couples where age of female at RM is 12 years, the duration is taken at least 8 years instead of 7 keeping in view that the average age at monarche is at least 13 years. Table 2 presents the distribution of duration of first stay of females according to the age at RM. To cope with the truncation bias the women with order of marriage one and effective marital duration 5 or more years were considered in computation of the proportions given in Table 2. Assuming distribution of first stay (given in Table 2), value of T_1 as 3.00 years, the probability of foetal loss, θ, as 0.15 and the mean duration of non-susceptible period associated with a foetal loss, as 0.50 year, the distribution (2.9) taking risk of first conception to be constant, linear and quadratic in t during an early part of cohabitation and constant thereafter is fitted to data presented in Table 1. It is worthwhile to mention that the distribution with constant risk function of first conception does not explain any observed set of data. The distribution seems suitable with linear and quadratic forms of risk function. However, the improvement in fit using quadratic form of $m(t/\tau_k)$ is negligible in comparison with the linear form. Age at RM wise expected distributions of women by age at return marriage according to time of first birth and estimates of paramoters when $m(t/\tau_k)$ is linear in t are presented in Table 1. Variances of the estimates and correlation coefficients between estimators are also obtained.

Here q_0 and q_1 can be interpreted as the risk of first pregnancy just at the time of first exposure to stable sexual relationship, and the rate of increase in risk of first pregnancy during the period from start of stable union till further time T_1. From the Table 1 it can be seen that the value of $\hat{q}_0$ is almost zero for females with ages at R.M. between 12 to 14 years. However $\hat{q}_1$ increases with age. Its values relating to ages 12 and 14 at R.M. are 0.28 and 0.43 respectively. It is well known that the average age of a female at menarche is between 13 and 14 years, several montstrual cycles after menarche are anovulatory, and females visit their parents more frequently at these ages. The increasing values of $\hat{q}_1$ indicates transition from adolescent sterility to sub-fecundity and from it to fecundity. The values of $\hat{p}_0$ for females with ages at RM, 15, 16, 17—19 years are markedly different from zero and sharply increas with age at R.M. The values of $\hat{q}_1$ for these females particularly for those having ages at R.M. 15 and 16 years, are much lower as compared to its value for females with age at R.M. 14 years. Reasons are obious as most of the females enter cohabitation at mature ages and social factors hinder sexual relations also become weaker at those ages. Desire to conceive the first child soon may be the cause of little higher q_1 for women with age at R.M. 17—19 years than those with age at R.M. 15 or 16 years.

For simplicity, risk of the first pregnancy is assumed to be independent of the duration of first visit to parents. Of course, a deviation from the assumption may be considered.

TABLE 2. DISTRIBUTION OF DURATION OF FIRST STAY OF FEMALES BY AGE AT RETURN MARRIAGE

duration of first stay (in months)	i	τ_i (in years)	$h_i = \tau_i + g$ (in years)	proportion of females with $h = h_i$ having age at return marriage					
				12	13	14	15	16	17,18,19
0	1	0	0.750	0.50	0.03	0.05	0.07	0.10	0.14
0 — 5	2	0.208	0.958	0.35	0.15	0.18	0.17	0.14	0.12
5 — 10	3	0.605	1.375	0.04	0.30	0.34	0.32	0.32	0.25
10 — 20	4	1.250	2.000	0.04	0.43	0.35	0.34	0.34	0.36
20 — 30	5	2.083	2.833	0.03	0.05	0.05	0.07	0.07	0.09
more than 30	6	2.917	3.667	0.04	0.03	0.03	0.03	0.03	0.04
total females	—	—	—	179	414	541	676	491	551

Here it is assumed that hazard of first pregnancy, once the woman has returned from her parents, begins to increase and attains a plateau after time T_1. In fact T_1 for a single woman may depend on age at first stable union and also may vary among women of same age at stable union.

When information on duration of first stay of bride with parents after R.M. is available, as in the present case, easier way of dating the exposure to risk of pregnancy is simply to take return to husband on the onset of stable sexual union rather than posing a probability distribution of Z. However, it may be mentioned that data on this duration is rarely collected and date of R.M. is considered as the effective beginning of coitus. Thus estimate of fecundability obtained ignoring the duration of first stay become under-estimated, particularly at the beginning.

On the other hand, if empirical distribution of Z is available which is useful in choosing a suitable theoretical distribution of Z, the model in Section 2 may be modified accordingly in order to explain data in a better fashion and to obtain more reliable estimates as shown in the section.

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