INDIAN STATISTICAL INSTITUTE

B I Semester I

Statistical Methods I

Midsemestral Examination 2005

Total marks 25

Date: 7.9.2005 Duration : 2 Hours

1. Consider 5 attributes A, B, C, D and E having 2, 2, 3, 7 and 9 levels respectively.

   (a) List all possible associations between the attributes A and B. [2]

   (b) Now consider the attributes D and E, *i.e.* the 7×9 contingency table. Define the odds of D_{i1} versus D_{i2}, $1 \leq i1, i2 \leq 7, i1 \neq i2$ at any fixed level, say j, of E as

   $$\frac{(D_{i1}\,E_j)}{(D_{i2}\,E_j)}.$$

   Note that there may be more than one odds at each level of E or at each level of D.

   i. Define an odds ratio (any one) for the 7×9 contingency table. [1]

   ii. What is the total number of possible odds ratios for the above table? Justify. [2]

   iii. How many of them will be algebraically independent? Justify. [2]

2. The following is a $2 \times 2 \times 2$ contingency table from an article that studied effects of racial characteristics on whether persons convicted of homicide received death penalty.

Victim's Race	Defendant's Race	Death Penalty Yes	Death Penalty No
White	White	53	414
	Black	11	37
Black	White	0	16
	Black	4	139

White activists argue that death penalty is imposed more often on white defendants than on black defendants. Are they justified in that argument? [3]

n the following table the samle mean, sample standard deviation and sample frequencies from 4 groups re given. Compute the overall sample variance. [5]

	G1	G2	G3	G4
$\overline{x}$	10	15	-5	20
s	4	3	2	6
f	12	10	30	6

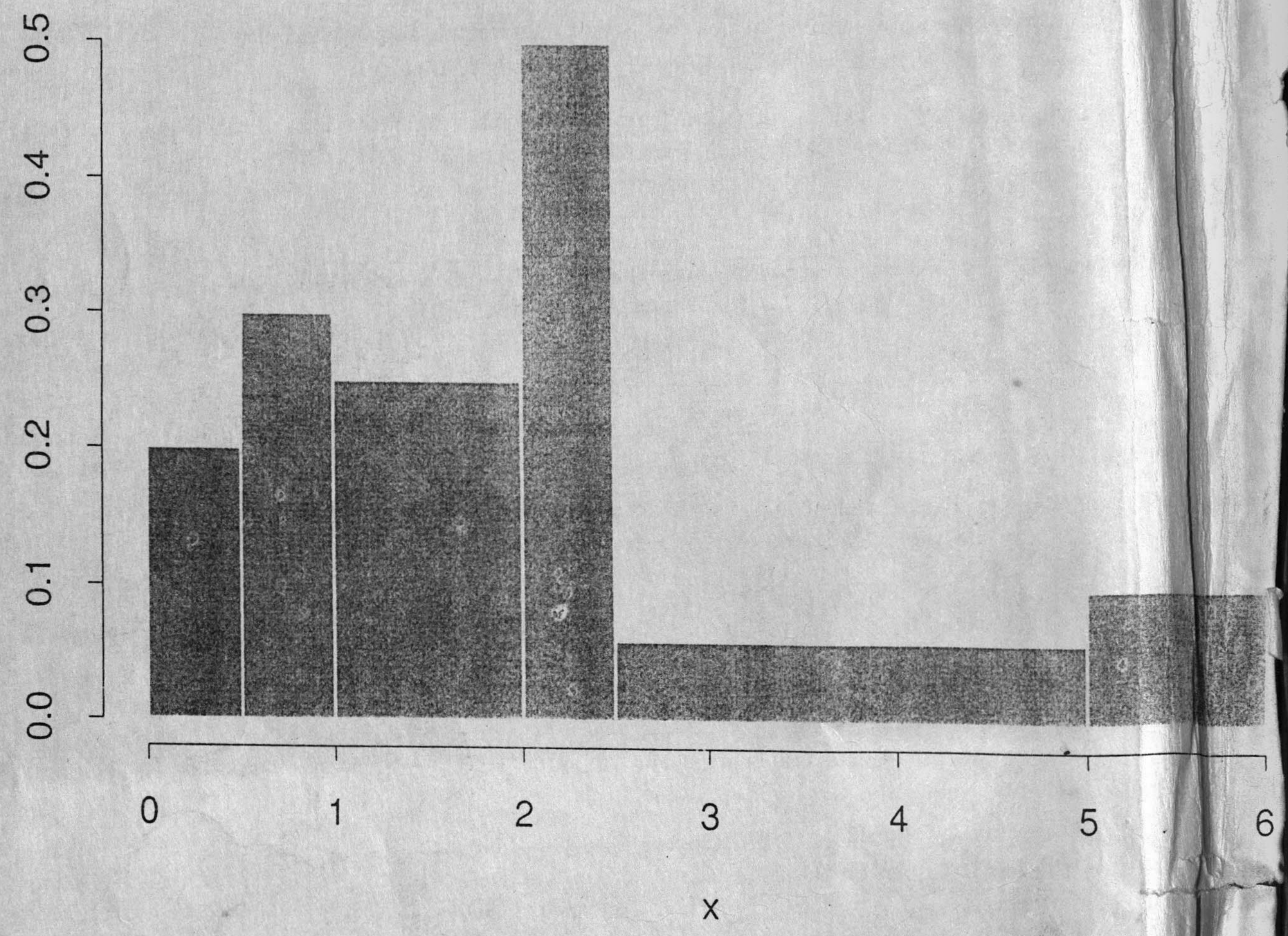
The first four moments of a distribution about the value 4 (i.e. $\frac{1}{n}\sum_{i=1}^{n}(X_i - 4)^r, r = 1, \ldots, 4$) are $-1.5, 17, -30$ and 108, respectively. Find the kurtosis. [5]

onsider the histogram on the back constructed for a sample of size $n = 100$. The class boundaries are osed on the left but open on the right except for the last class which is closed on both sides. Define set of new classes as $C_1 = [0,1), C_2 = [1,2), C_3 = [2,3), C_4 = [3,4), C_5 = [4,5)$ and $C_6 = [5,6]$. ssume further that no observation falls on any boundary. Following questions pertain to the new set f classes. Answer them as accurately as possible. If any approximation has to be made state them ery clearly.

(a) Which class has the highest frequency and what is it? [2]

(b) Define and find the interquartile range for the sample. [3]

PTO



INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : (2005 - 2006)

B.Stat First Year : Probability I

Date : 09-09-05. Maximum Marks : 40. Duration : Two hours.

Note: Justify your answers.
Quote precisely if you are using any result proved in class.
This paper is set for 45 marks. Maximum you can score is 40 marks.

1. (a). A number x is selected at random from $\{1, 2, \cdots, N\}$. Find the probability that $x^2 - 1$ is divisible by 10. If this probability is denoted by p_N, find $\lim_{n \to \infty} p_N$.

   (b). One number is selected from $\{1, 2, \cdots, 20\}$ in such a way that chance of selecting number k is proportional to k. Given that the selected number is divisible by 2, find the conditional probability that it is divisible by 3. You should simplify your answer.

[3+4]

2. n balls are placed in n boxes. All arrangements are equally likely.

   (a). Find the probability that exactly one box is empty.

   (b). Given that box I is empty, find the conditional probability that exactly one box is empty.

   (c). Given that exactly one box is empty, find the conditional probability that box I is empty.

[2+3+3]

3. Let $a > c > 0$ and $b > 0$ be integers. Show the following: the number of paths which touch or cross the line $x = -b$ and then lead to (n, c) without having touched the line $x = a$ equals $N_{n,2b+c} - N_{n,2a+2b-c}$. Note that paths that touch the line $x = a$ before touching $x = -b$ are included.

[7]

P.T.O

4. There are three sets of palying cards. One deck is spread out in a row. I have one set and you have another. We both shuffle and spread them against the first deck. Say that there is a double match at place i if both our cards agree with the card of the first deck at that place. Show that the probability of no double match is

$$\frac{1}{52!}\sum_{k=0}^{52}(-1)^k\frac{(52-k)!}{k!}$$

[7]

5. In a sample space we have two events A and B. We know that

$$P(A) = 1/4; \quad P(B|A) = 1/2; \quad P(A|B) = 1/4.$$

For each of the four statements below, state whether it is true or false, and justify your answer.

(i) $A \subset B$. (ii) $P(B) = 1/4$.

(iii) $(A \cup B)^c = \emptyset$. (iv) $P(B|A^c) = 1/2$.

[2 × 4]

6. Consider the Bose-Einstein Statistics with n cells ($n > 2$) and r balls. Show that the most probable number of balls in box I is zero.

[8]

G O O D L U C K

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2005-2006)

B.Stat I yr.

Vectors and matrices I

Date: 13 Sept. 2005. Maximum marks 30 Duration: 3hrs.

Note: Class room notation is used.

1. a) Define rank factorization of a matrix. Show that every nonnull matrix admits rank factorization.

b) Show that r(AB) = r(B) − d{ C(B) ∩ N(A)}.

[5 + 5 = 10]

2. Prove or disprove the following:

a) (-1).α = -α, where α is a vector and 1 is the unity of the field.

b) The set of all 2-tuples of complex numbers form a vector space over real field and its dimension is 4.

c) Let S and T are subsets of a vector space V. Then $S^{\perp}$ is a subspace of $T^{\perp}$ implies T is a subset of S.

d) Rank of the matrix product AZB is less than or equal to the rank of AB if Z is nonsingular.

e) A is an idempotent matrix implies rank(A) = trace(A).

[5 x 2 = 10]

3. Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 1 \end{bmatrix}$$

Find the rank, a rank factorization, a basis of column space, a basis of row space, a basis of null space and a generalized inverse of A. Also find the set of all solutions of the system of linear equations Ax = b, for b = (0 -1 0 1)′ and for b = (1 1 0 -1)′.

[10]

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INDIAN STATISTICAL INSTITUTE

B.STAT I YEAR

PERIODICAL EXAMINATION

Computing Techniques and Programming I

Time : 3 hours. Date : 16. 09. 05

Remarks : Answer as many parts as you like. Maximum you can score is 100.

1.a) Convert the given numbers to the number systems required

$(83)_{10}=(\ )_3$ $(45)_7=(\ )_8$ $(AF)_{16}=(\ )_5$ $(124.3)_6=(\ )_{10}$ $(110.11)_2=(\ )_4$ $(22.12)_3=(\ )_9$

(3X2+3X3=

b) Two arithmetic expressions are given below.

-187 + 59 **187 - 59**

Convert the decimal integers to internal binary integer representations assuming that integers are stored internally in two bytes and negative numbers are represented in 2's complement form. Then perform the additions in binary 2's complement form and convert the results to decimal. **(5+5=**

2 The following two hexadecimal numbers are representations of two single-precision floating point numbers in **IEEE exponent and significand format.** Convert to binary numbers in exponent and mantissa form, perform addition of these two numbers and then convert the result to the same format after normalization.

3 D D 0 0 0 0 0 Hexa
3 C 8 0 0 0 0 0 Hexa **(2X5=**

3. A computer has **a word** length of **24 binary digits**. Each instruction that it has is stored in one word. An instruction has **three fields** in the following order, **first field** is for **op-code, second and third fields** are for **memory addresses.**

a) If the memory has **1600 words** then how many instructions can it have ?
b) If it has **64 instructions** then what can be the maximum size of the memory that it can have?
c) If the **first bit** (sign bit) of an instruction word is **not to be used** and the number of instructions are **64** then what problem will occur during allocating space to the **two address fields**? **(3+3+4=**

4. Give a flowchart for finding the roots of the quadratic equation **$ax^2+bx+c=0$** where **a,b,** and **c** are to be re from the keyboard.

5. What would be the outcome of the following programs

a)
```
#include<stdio.h>
void swap(int a, int b)
{ int temp;
  temp=a;
     a=b;
     b=temp;
}
main()
{ int a=5,b=6;
   swap(a,b);
   printf(" a= %d, b=%d", a, b);
}
```

b)
```
#include<stdio.h>
main()
{ int x=1, y=2;
   int *ip;
   ip=&x;
   *ip=0;
  printf("r = %d \n", x);
}
```

c)
```
#include<stdio.h>
main()
{ float sum;
   int i;
   for(i=0;i<20;i++)
  {  sum = sum +i;
  }
  printf(" %f ", sum);
}
```

P.T.O

d)
```
#include<stdio.h>
#include<math.h>

struct point{ float x;
              float y;
            };
struct rect { struct point pt1;
              struct point pt2;
            };
float diagonal( struct rect r)
  {  float d;
     d = ( r.pt1.x – r.pt2.x );
     d =  d*d + ( r.pt1.y –r.pt2.y) * ( r.pt1.y – r.pt2.y );
     d =  sqrt ( (double) d);
     return d;
  }
main()
  {  struct rect r;
       r.pt1.x = 2.0; r.pt1.y=3.0;
       r.pt2.x = 3.0; r.pt2.y=2.0;
     printf( " %f \n", diagonal ( r ) );
  }
```

e)
```
#include<stdio.h>
main()
{ printf(" %d ", printf ( " INDIAN STATISTICAL INSTITUTE "));
}
```

f)
```
#include<stdio.h>
main()
{ unsigned int a,b;
   a = 5; b = 6;
   b = b∧a;
   a = b∧a;
   b = b∧a;
  printf ( " a = %d,  b= %d ", a, b ) ;
}
```

(3X6=18)

6. a) Write a function **my_alloc** which takes as argument two integers **m, n** and allocates memory for a two dimensional array of size **m X n**. It returns the **pointer** to the array if successful and returns **NULL** if not successful.
   b) Write a function **my_sort** which sorts elements of an integer array. The function my-sort takes as argument an integer pointer **p** and an integer **n.** Assume that **p** points to the first element of an integer array of **n** elements. **(5+5=10)**

7. Write a function which takes as input two integers **m** and **n (m > n).** The program lists all **composite** numbers between **n and m.** **(10)**

8. Write a program to **add two polynomials.** Use **arrays of structures** to represent the polynomials. **(10)**

9. Let (a_{ij}) be a **5 X 5** matrix. Write a program to construct a **5 X 5** matrix (b_{ij}) such that

$$b_{ij} = \sum_{k=1}^{5} a_{kj} + \sum_{k=1}^{5} a_{ik} - 2a_{ij}$$

Find the **number of additions** required by your program. **(10+5=15)**

Indian Statistical Institute

First semsetral examination : (2005-2006)

B. Stat. I year

Analysis I

Date : 28.11.05 Maximum marks : 60 Duration : $3\frac{1}{2}$ hours.

Answer **ANY SIX** questions. Marks are indicated in brackets.

(1) (a) Determine, with justification, which of the following functions from $\mathbb{R} \times \mathbb{R}$ to $[0, \infty)$ is a metric on $\mathbb{R}$.

$$d_1(x, y) = |x - y|^{\frac{1}{3}}.$$

$$d_2(x, y) = |x^2 - y^2|.$$

(b) For a subset A of a metric space (X, d), recall that the closure of A, denoted by $\overline{A}$, is defined as follows :

$$\overline{A} := \{a \in X \ : \ \forall \epsilon > 0 \ \exists b \in A \text{ s.t. } d(a, b) < \epsilon\}.$$

Prove that $\overline{A}$ is closed. [4+6=10]

(2) Determine, clearly mentioning the results you are using, which of the following series converges.

(i)

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}.$$

(ii)

$$\sum_{n=3}^{\infty} \frac{1}{n \log(\log n)}.$$

(iii)

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n}.$$

[3+3+4=10]

(3) Let $a_n > 0$ for $n = 1, 2, \ldots$. Prove that the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the series $\sum_{n=1}^{\infty} \frac{a_n}{1+a_n}$ is so. [10]

P.T.O

(4) Let $f : (0,\infty) \to \mathbb{R}$ be a continuous function satisfying

$$f(xy) = f(x) + f(y).$$

Prove that f must be of the form $f(x) = c \log x$ for some constant $c \in \mathbb{R}$. [10]

(5) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(U)$ is open for every open subset U of $\mathbb{R}$. Prove that f must be monotone. [10]

(6) Find out all the functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the following :

$$|f(x) - f(y)| = |x - y| \quad \forall x, y \in \mathbb{R}.$$

[10]

(7) Let $f : \mathbb{R} \to \mathbb{R}$ be a three times differentiable function such that $f'(0) = f(0) = 0$. Let $a = f(1)$, $b = f(-1)$. Prove that there exists some $x \in (-1,1)$ such that $f^{(3)}(x) = 3(a-b)$. [10]

INDIAN STATISTICAL INSTITUTE
First Semester Examination: 2005-06
B. Stat. I Year
Probability Theory I

Date: 1.12.05 Maximum Marks: 60 Duration: 3 Hours

1. **State any theorem/formula if you are using it.**
2. **Answers should be simplified as for as possible.**
3. **The paper carries 70 marks. Maximum you can score is 60.**

1.a) From a box containing m white balls and n black balls $(m > n)$, balls are drawn one after another without replacement. Calculate the probability that at some stage of this process, the number of white balls drawn equals the number of black balls drawn.

b) There are n tickets numbered $1,2,\ldots n$ in a box. The tickets are drawn one after another without replacement. Find the probability that at some stage the number on the ticket drawn coincides with the number of the draw.

[5+5]

2. An urn contains 3 Black balls and 2 White balls. I draw 3 balls without replacement from the urn. I now add a ball to the urn as follows : if there are more white balls in my sample then I add a white ball, otherwise I add a black ball. Now you select a ball at random from the urn. For $i = 0,1,2,$ find the conditional probability that there were i white balls in my sample given that you selected white ball.

[6]

3. I keep tossing a coin, whose chance of heads is p in a toss. For $i \geq 1, T_i$ denotes the number of tosses upto and including the ith Head. For $n \geq 1$, let N_n denote the number of Heads in the first n tosses.

a) For $1 \leq x \leq n$, show that $P(T_1 = x | N_n = 1) = \frac{1}{n}$

b) For $1 \leq x_1 < x_2 < \cdots\cdots < x_r \leq n$, show that

$$P(T_1 = x_1, T_2 = x_2, \cdots\cdots, T_r = x_r | N_n = r) = \frac{1}{\binom{n}{r}}$$

[5+5]

P.T.O.

4. X, Y are independent random variables each having geometric distribution with parameter p.

a) Calculate $P(X \geq 2Y)$

b) If $U = Min(X,Y)$ and $V = X - Y$, then show that U, V are independent. [5+10]

5. There are N tickets numbered $1, 2, \cdots\cdots N$ in an Urn. A sample of size n without replacement is taken from the Urn. Let S be the sum of the numbers on the tickets chosen. Calcutta $E(S)$ and $Var(S)$. [5+10]

6. X, Y are two independent random variables each having the p.g.f $Q(t) = e^{4(t^2-1)}$. Calculate

a) $P(X + Y = 3)$ b) $P(X + Y = 4)$. [4]

7. Let X_n be the number of heads obtained by tossing n coins, out of which one is fair coin and each of the others has chance of heads $\frac{1}{n}$. For $k = 0, 1, 2, \cdots\cdots$, let $p_n(k) = P(X_n = k)$. Calculate $\underset{n \to \infty}{Lt}\, p_n(k)$. [10]

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First Semester Examinations (2005-2006)
B. Stat – I yr.
Remedial English
100 marks
1½ hours

Date: 2.12.05

1) Write an essay *on any one* of the following topics. Five paragraphs are expected.
   a) Our duty to the backward communities in our country.
   b) Television – a blessing or a curse.
   c) Attitude or aptitude – which is more important in today's world?

(60 marks)

2) Fill in the blanks with appropriate prepositions:

a) The stories in that book are full ___ interest.
b) Do not indulge ___ strong language.
c) The head-dress of the Cossacks is similar ___ that of the ancient Persians.
d) He is grateful ___ his master ___many favours.
e) This discussion is hardly relevant ___ the subject.
f) Even the enemies admit that he is endowed ___rare talents.
g) He is very different ___ his brother.
h) His views do not accord ___ mine.
i) He is incapable ___ doing good work.
j) He is dependant ___ his parents.
k) He was born ___ humble parents in Deulipur.
l) He has been very much indulgent ___ his children.
m) He did not profit ___ experience.
n) He abstains ___ liquor.
o) He is true ___ his king.
p) He is devoid ___ sense.
q) I am obliged ___ you ___ your kindness.
r) I prefer tea ___ coffee.

(20 marks)

3) Fill in the blanks with appropriate words:

I stood _____ out at the sea. The gray-blue _____ came hurtling onto the wet sand and I felt the cold water gather around my bare _____. The _____ was near the western horizon, a dull red glare almost hidden behind the white _____. Seagulls soared through the darkening blue _____, struggling hard against the strong _____ that blew in from the _____. Their shrill _____ rented the air.

I _____ someone call my name and _____ around to face the beach.

(Turn over)

It was a little boy dressed in black, his face somewhat familiar. He stood a little away _____ me. a tattered cloth bag hanging from his _____ and a large sea shell held up high in his right _____. It was the latter that he was waving at me, a lopsided grin on his _____.

"Very good shell, sir, very good," he _____ in broken English. "I _____ it up today morning from this beach. Will you _____ it, sir? Not expensive, not expensive, only forty _____."

I _____ my head, urging him to go away.

(20 marks)

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: (2005-2006)

B.Stat I yr.

Vectors and matrices I

Date: 5 Dec. 2005. Maximum marks 60 Duration: 3hrs.

Note: Class room notation is used. All the matrices and vectors considered here are over real field unless otherwise stated.

1. **Show that $r(A+B) \leq r(A)+r(B)$ and equality occurs if and only if $C(A) \cap C(B) = \{\varphi\}$ and $R(A) \cap R(B) = \{\varphi\}$.** [5]

2. **Find the inverse of the matrix**

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}.$$ [5]

3. **Let A be a square matrix of order n and rank r. Then prove the equivalence of the following:**
   a) $r(A) = r(A^2)$.
   b) $C(A) \cap N(A) = \{\varphi\}$.
   c) $|CB| \neq 0$ where $A = BC$ is any rank factorization of A.
   d) $A = P \begin{bmatrix} D & O \\ O & O \end{bmatrix} P^{-1}$, for some nonsingular matrix P, where D is a nonsingular matrix of order r.

[10]

4. **Let A and B be matrices such that $C(A) \oplus C(B) = R^n$. Then**
   a) show that $(AA^* + BB^*)$ is nonsingular and its inverse is a generalized inverse of AA^*.

b) **State a set of necessary and sufficient conditions for the matrix P_{AB} to be the projection operator onto C(A) along C(B).**

c) **Obtain P_{AB} in terms of A and B.**

[5+2+3=10]

5 **Show that A is skew symmetric matrix $\iff$ $x'Ax = 0$ for all x.** [5]

6 **Let A be a square matrix of order n and rank r, and x and y be vectors such that $x'Ay \neq 0$. Then show that the rank of the matrix $A - Ay\,x'A / (x'Ay)$ is r-1.**

[5]

7 **Given that the matrix A is nonsingular show that**

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = |A|\,|D - CA^{-1}B|.$$

[5]

8 **Show that commuting generalized inverse of a matrix A exists if and only if $r(A) = r(A^2)$.**

[5]

9 **Prove or disprove the following:**

a) $r(A^k) = r(A^{k+1}) \Rightarrow r(A^k) = r(A^p)$ for all $p > k$.

b) Let A be a square matrix of order n and rank r. Then A has a nonzero principal minor of order r $\Rightarrow r(A) = r(A^2)$.

c) For any matrix A, $A_m^- A$ is unique.

d) $C(A) = N(I - A) \Rightarrow A = A^2$.

e) Let A and B be the matrices of same order. Then $AB = O = BA$ and $r(A) = r(A^2) \Rightarrow r(A+B) = r(A) + r(B)$.

[5X3=15]

INDIAN STATISTICAL INSTITUTE

B I Semester I

Statistical Methods I

First Semestral Examination 2005

Total marks 60

Date: 7.12.2005 Duration : 3 Hours

1. Consider the joint distribution of several attributes, each at 2 levels (present and absent). Let the capital letters $A, B, C, \ldots$, denote the presence of the attributes, and the Greek letters $\alpha, \beta, \gamma, \ldots$ denote their absence. The class frequencies are denoted by putting the corresponding symbols within "( )" brackets. Thus $(AB\gamma)$ represents the frequency of the class where A and B are present, but C is absent. Let n be the total number of observations.

   (a) For three attributes A, B and C show that [3]

   $$(AB) - (ABC) \leq (A) - (AC).$$

   (b) For two attributes A and B define $\delta = (\mathrm{AB}) - \frac{(\mathrm{A})(\mathrm{B})}{n}$ and $V^2 = \frac{n^2\delta^2}{(\mathrm{A})(\mathrm{B})(\alpha)(\beta)}$. Write down the goodness of fit χ^2 statistic under independence of A and B and show that $\chi^2 = nV^2$. [1 + 4]

2. Suppose that X and Y are two random variables, which may be dependent with finite variances. Moreover, assume $Var(X) = Var(Y)$. Show that the random variables $X+Y$ and $X-Y$ are uncorrelated. [5]

3. Consider X to be the number of successes in a series of n independent Bernoulli trials in which the probability of success of i-th trial is p_i. What is the maximum value of $Var(X)$ for fixed n? [5]

4. Let $X_1, \ldots, X_n$ be independent random variables. Suppose X_i has moment generating function (mgf)

   $$M_i(t) = e^{t^2+it}.$$

   Define $W = \sum \frac{X_i}{i}$. Find the mgf of W. Hence find $Var(W)$. [6 + 6]

5. Suppose the demand for petrol in a petrol pump averages 1500 litres with a standard deviation of 200 litres per day. The pump opens everyday with a store of 1800 litres of petrol. Let the proportion (over a large number of days, say a year) of days the pump cannot meet the demand be p. If demand follows a normal distribution find p. [6]

[P. T. O.]

6. The following table is abstracted from a report of army pension fund where certain number of widows receive pensions. It shows the distribution of number of children of those widows.

No of children	0	1	2	3	4	5	6
No of widows	3062	587	284	103	33	4	2

(a) If you are to fit a Poisson distribution to this data what will be the parameter of that distribution? [5]

(b) Notice that the number of widows in category 0 is too large. If you disregard that category completely and fit a truncated Poisson to this data what will be the expected number of children? [4]

7. Data are collected for $n = 10$ eighteen-year old girls on X = height in centimeters and Y = weight in kilograms. Following are the summary statistics related to that data

$$\bar{x} = 165.52, \quad \bar{y} = 59.47$$

$$S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = 472.076, \quad S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = 731.961$$

$$S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 274.786.$$

(a) Find the numerical value of the correlation coefficient between X and Y? [4]

(b) If an eighteen-year old girl is 160 cm tall what would you expect her weight to be? [3]

(c) Assuming the normal error model of simple linear regression construct a 90% confidence interval for $E(Y|X = 160)$. Take MSE to be true value of σ^2. [8]

INDIAN STATISTICAL INSTITUTE

Semestral Examination : (2005-2006)

B.Stat. First Year ✓

Computational Techniques and Programming I

Date : 9.12.05 Maximum Marks : 100 Duration : 3 hours

Note : You can attempt any part of any question.

1. a) Briefly explain the four methods of representing negative numbers that have been used in digital computers. (6)

b) A company has decided to construct machines with 16 bit floating point numbers. The model has 1 sign bit, 5 bit excess 16 exponent, and 10 bit fraction. It uses radix 2 exponentiation and for the normalized fraction part does not have an implied 1 bit to the left of the binary point. What are the smallest and largest positive normalized numbers on the model in decimals ? (10)

c) A computer has a bus with a 200 nsec cycle time. During one cycle time it can either i) read a 32-bit word from hard disk buffer into memory,
or, ii) transfer a 32-bit word from memory to CPU,
The CPU cannot read directly from the hard disk. It can read a 32-bit word from memory after it has been written from hard disk buffer to memory.
The computer has a hard disk that writes 512 bytes (one sector) at a time to its buffer at a speed of 4 Megabytes/sec.
Assume that the CPU (in a single user system) requires a 32-bit word from hard disk and sends a request to the I/O controller with the block address, then after how much time does it get it if the read/write head is already in position, and the required word is the first 32-bits of the block ? (1 nsec = 10^{-9} sec) (10)

d) Assume that it is possible to access a file game1 using the path /user/own/bin/game. The following are also valid paths
/user/doc/notes
/user/own/fast/file1
/user/doc/lecture/maths
While current directory is /user/own
the path bin/new/play3 is also valid.
Draw a diagram showing the above directory structure. (4)

e) A computer system allocates space to files in its hard disk in fixed size blocks of 1 sector of 512 bytes. For a file of size 12146 bytes
i) how many of the bytes allocated to it would remain unused in case of linked allocation if 3-bytes are required to store a hard disk address ?
ii) how many of the bytes allocated would remain unused in case of contiguous allocation ?
iii) if each logical record in the file consists of 132 bytes then in case of contiguous allocation what is the disk address of logical record number 21 in sectors and bytes if space allocation for the file starts from byte zero of sector zero. (3+3+4=10)

f) Information is inserted into a buffer at a rate of k bytes per second and is is deleted at a rate of m bytes per second. The capacity of the buffer is n bytes where n>m, n>k.
i) how long does it take to empty the buffer when it is initially full and k<m ?
ii) how long does it take to fill an empty buffer when k>m ? (3+2)

P.T.O

2. What would be the output of the following program segments (assume they are compiled and run in sun-solaris system which you use (5 X 3 = 15)

i)
```
#include <stdio.h>
main()
{ union { char uchar;
          int  uint;
          long ulong;
        }u;
  printf("%d", sizeof(u));
}
```

ii)
```
#include<stdio.h>
int fun(unsigned int x)
{ int b;
  for(b=0; x!=0; x=x>>1)
      if(x&1==1)
          b++;
  return b;
}
main()
{ int i = 255;
  printf("%d", fun(i);
}
```

iii)
```
#include<stdio.h>
printd(int i)
  { if (n<0)
      { putchar('-');
        n = -n;
      }
    if(n/10)
       printd(n/10);
       putchar(n%10 + '0');
  }
main()
 { int i=-5996;
   printd(i);
 }
```

iv)
```
#include<stdio.h>
process(char *s)
 { int n;
   for(n=0; *s!='\0'; s++)
        n++;
   return n;
 }
main()
{ char string[] = "HowAreYou";

  printf("%d\n", process(string));
}
```

v)
```
#include<stdio.h>
main()
{ FILE *fp;
  long offset;

  fp = fopen("file.dat","r");
  fseek(fp,0,SEEK_END);
  offset = ftell(fp);
  printf("%ld \n", offset);
}
```

3. (a)Write a function nalloc which creates a node for a linklist. nalloc takes in four parameters, the roll number of a student and the marks obtained in Mathematics, Physics and Chemistry. nalloc returns a pointer to the created node. (4)

(b) Write a function createList which creates a linked list of n nodes containing the student's roll number and marks (as created by nalloc) sorted according to the students roll numbers. createList returns the head of the created list. (8)

(c)Write a function deleteFailed which deletes the nodes containing student records for students who failed in all the 3 subjects (assume a pass mark of 40). deleteFailed takes in the head of a linklist created by createList and returns the head of the modified list. (8)

4. Assume you have got an image file of which you know the following information:

(i) The file has a 36 byte header.
(ii) The header is followed by the image data written as one byte per pixel
(iii) The height and width of the image stored in the file are equal.

Write a function which reads the image data from the file into a two dimensional array. (10)

5. (a) An array of 2002 integers is given to you. Write a function to find both the maximum and minimum of the integers. Your program should not use more than 3003 comparisons. (6)

(b) An array of 256 integers sorted in the increasing order is given to you. Write a function, which takes as parameter an integer n, and returns 1 if n is present in the array and returns 0 otherwise. Your function should not use more than 16 comparisons. (6)

(c) An array A of n elements is given to you. The elements of A are either 1 or -1. Write a program to sort the elements in A. Your program should use O(n) operations. (8)

Indian Statistical Institute

Backpaper examination : (2005-2006)

B. Stat. I year

Analysis I

Date : 6.1.06 Maximum marks : 100 Duration : 3 hours.

Answer all the questions. Marks are indicated in brackets.

(1) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function satisfying $|f'(x)| < \frac{1}{2}$ for all x.
that there must exist a point x such that $f(x) = x$.
Hint : Choose any point $x_0 \in \mathbb{R}$ and consider a sequence $(x_n)_{n=0}^{\infty}$ given by $x_{n+1} =$
[20]

(2) Prove that any nonempty open subset of $\mathbb{R}$ is a countable union of disjoin
of the form (a, b) where we allow the choices $a = -\infty$ or $b = \infty$. [20]

(3) Let $s_n = 1 + 1 + \frac{1}{2!} + ... + \frac{1}{n!}$, for $n = 1, 2, ...$ Prove that $e - s_n < \frac{1}{n(n!)}$. Hence con
that e is an irrational number. [10+10=20]

(4) Consider the following two series :

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$$

and

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + ...$$

Prove that both the series converge. If s_1 and s_2 denote res
the first and the second series converge, prove that $s_1 \neq s_2$.

(5) Suppose that $f : [0, 1] \to \mathbb{R}$ is continuous and f is
$f(0) = 0$ and $|f'(x)| \leq 2|f(x)|$ $\forall x \in (0, 1)$. Prove that

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination (2005-2006)

BStat I (Probability Theory II)

Date: 22-02-06 Maximum Marks: 40 Duration: two hours

Justify your steps. Simplify your expressions.

1. (*a*) I toss a fair coin twice and count X, the number of heads obtained. Independently, you pick a number Y at random from the interval $(0,1)$. Calculate the density of $Z = X + Y$.

   (*b*) X is a random variable uniform over an interval. Expected value of X is 2 and variance is 3. Find the density of X^2. [5+5]

2. A pair of random variables (X, Y) have the joint density

$$f(x,y) = e^{-y} \quad \text{for} \quad 0 < x < y < \infty.$$

   (*a*) Calculate the marginal densities of X and Y.

   (*b*) Find $P(X > 4 || Y > 2)$. Find $P(Y < X + 2)$. [4+6]

3. Let X be a random variable having the distribution function

$$\begin{aligned} F(x) &= 0 && \text{for } x < 0 \\ &= x/100 && \text{for } 0 \le x < 1 \\ &= x^2/64 && \text{for } 1 \le x < 2 \\ &= (x-1)/16 && \text{for } 2 \le x < 16 \\ &= 1 && \text{for } x \ge 16 \end{aligned}$$

   Calculate the following.

   (*a*) $P(X$ is an integer). (*b*) $P(1 \le X \le 16)$. (*c*) $P(1 < X \le 16)$.
   (*d*) $P(1 < X < 2) || \frac{1}{2} < X < \frac{3}{2})$. [8]

4. When a current of X Amperes flows through a resistance of Y Ohms, the power generated is $W = X^2Y$ Watts. Suppose that both X and Y are random variables. If X has density $f(x) = 6x(1-x)$ for $0 < x < 1$ and Y has density $g(y) = 2y$ for $0 < y < 1$, calculate the density of W. [7]

5. X and Y are independent random variables; X is standard normal and $Y \sim \chi^2_k$. Derive the density of $T = \dfrac{X}{\sqrt{Y/k}}$. [10]

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination : 2005 – 06
B.Stat. (Hons.) I Year
Statistical Methods II

Date :24.02.2006 **Maximum Marks : 100** **Duration : 3 Hours**

Answer all the questions . Marks allotted to each question are given within the parentheses . Standard notations and symbols are used .

1. (a) Show that

$$b_{p2.34...p-1} = b_{p2.13...p-1} + b_{p1.23...p-1} b_{12.34...p-1}$$

(b) Suppose a computer has found , for a given set of values of x_1 , x_2 and x_3

$$r_{12} = 0.91 \text{ , } r_{13} = 0.33 \text{ and } r_{23} = 0.81 \text{ .}$$

Examine whether his computations may be said to be free from errors .

(15 + 10) = [25]

2. Let $T_i = \sum_{\alpha=1}^{n} x_{i\alpha}$, $T_{ij} = \sum_{\alpha=1}^{n} x_{i\alpha} x_{j\alpha}$ and $s_{ij} = nT_{ij} - T_i T_j$, $1 \le i, j \le 3$.

For a given set of data , suppose T_1 = 581 , T_2 = 179 , T_3 = 66 , s_{11} = 63,713 , s_{12} =15,449 s_{13} =4,620 , s_{22} = 6,353 , s_{23} = 1,056 , s_{33} = 648 and n = 18 .

(a) Obtain the multiple regression equation of x_3 on x_1 and x_2 . Hence determine the value of x_3 when x_1 = 19 and x_2 = 8 .
(b) Give a measure of usefulness of the above regression equation as a predicting formula .
(c) Compute the partial correlation coefficient between x_2 and x_3 , eliminating the effect of x_1 .

(10 +5+5+5) =[25]

3. 12 dice were thrown 2,630 times and each time the number of dice which had 5 or 6 on the uppermost faces was recorded . The results are given in the following table :

Number of dice with 5 or 6 uppermost	Frequency
0	18
1	115
2	326
3	548
4	611
5	519
6	307
7	133

P.T.O.

8	40
9	11
10	2

Fit a suitable distribution to the above data and test for goodness of fit . Consider both the case when the value of the underlying population parameter is known and when it is unknown .

(15 +10) = [25]

4. (a) Give an algorithm to simulate observations from a Hypergeometric distribution with parameters N , n and p .
   (b) Use your algorithm to simulate 15 observations from a Hypergeometric distribution with parameters N = 60 , n = 10 and p = 0.15 .
   (c) Use your sample observations to get an unbiased estimate of p .

(8 +12 +5) = [25]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2005-2006)

B.Stat I yr.

Vectors and matrices II

Date: 28 Feb. 2006. Maximum marks 30 Duration: 3hrs.

Note: Class room notation is used. 2 marks for neatness.

1 Let $Q(x) = x'Ax$ be a real quadratic form. Then prove the following:

a) The definiteness of the quadratic form is unaltered by a nonsingular transformation of the variables.

b) If A is not indefinite then $a_{ii} = 0$ implies $a_{ij} = 0$ for all j.

c) If A is nnd then it can be expressed as $A = LL'$, for some lower triangular matrix L.

[2+3+5 = 10]

2.

a) Every real square matrix A can be expressed as A = RTR* for some unitary matrix R and a triangular matrix T.

b) Define and derive Spectral Decomposition of a real symmetric matrix.

c) Define and derive Singular Value Decomposition of real matrix A of order mxn.

[4+3+3 = 10]

3. Prove the following:

a) Let μ be the maximum singular value and λ be any eigenvalue of a square matrix A, then $\mu \geq |\lambda|$.

b) The sets of nonzero eigenvalues of AB and BA are same.

c) Let A be a positive definite matrix and B be any symmetric matrix of same order. Then there exists a nonmsingular matrix R such that RAR and RBR are diagonal.

d) Real part of every eigenvalue of a real skew symmetric matrix is zero.

[4x2=8]

----------x----------

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2005–2006)

B.Stat.(Hons.) I

Computational Techniques and Programming II

Date: 3.3.06 Maximum marks: 20 Duration: 2hrs

1. (a) Write down the $QRflip$ algorithm using Given's transform. You must write every loop and step explicitly. [3]

   (b) Consider the following symmetric tri-diagonal matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \\ 0 & 3 & 2 \end{bmatrix}$$

   Apply your $QRflip$ algorithm to it, without explicitly computing Q. All numbers are to be shown up to exactly 4 decimal places. Show each step clearly. [7]

2. For $i = 0, ..., n$, let $L_i(x)$ denote the i-th Lagrange polynomial based on the points $(x_0, y_0), ..., (x_n, y_n)$, where the x_i's are distinct. Show that

$$\sum_{i=0}^{n}(3x_i^2 - 2x_i + 1)L_i(x) = 3x^2 - 2x + 1.$$

   Assume that $n \geq 3$. [2]

3. Consider the polynomial

$$p(x) = a_0 + a_1 x + \cdots a_n x^n.$$

   To evaluate it at a given point t we can use Horner's method as follows.

$$\begin{aligned} b_n &= a_n \\ b_i &= a_i + b_{i+1}t \text{ for } i = n-1, ..., 0. \end{aligned}$$

   Let $Q(x)$ and R denote, respectively, the quotient and remainder when we divide $p(x)$ by the polynomial $(x - t)$, *i.e.*,

$$p(x) = Q(x)(x - t) + R \quad \forall x.$$

   Suggest (with justification) how you can use the b_i's to compute R and the coefficients of $Q(x)$. [8]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination

Second semester 2005–2006

B. Stat (First year)

Analysis II

Date: 5 May, 2006 Maximum Marks: 60

Duration: 3 hours 30 minutes

Answer all questions.

(1) Let f_0 be the constant function on [0,1] which takes the value 1. Construct a sequence of functions on $[0, 1]$ by the following rule: For $n \geq 1$,

$$f_n(x) = \sqrt{x f_{n-1}(x)} \text{ for all } x \in [0, 1].$$

Show that the sequence $\{f_n\}_{n=1}^{\infty}$ is monotone. Determine the limit function. Hence prove that the sequence converges uniformly on $[0, 1]$. 12

(2) Let $\{f_n\}$ be a sequence of continuously differentiable real valued functions on $[a, b]$ converging uniformly to a function f. Suppose, the sequence $\{f_n'\}$ converges uniformly to g. Show that f is continuously differentiable and $g = f'$. 10

(3) Let f be a continuous real valued function on $[0, 1]$ such that

$$\int_0^1 x^{2n+1} f(x)\, dx = 0 \text{ for } n = 0, 1, 2, \ldots.$$

Show that f is identically zero on $[0, 1]$. 8

(4) Define

$$f(x) = \begin{cases} e^{-1/x^2} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Prove that f is infinitely many times differentiable but f does not have power series expansion about the point 0. 10

P. T. O

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination (Backpaper)

Second semester 2005–2006

B. Stat (First year)

Analysis II

Date 20 July, 2006 — Maximum Marks: 100

Duration: 3 hours

(1) Let C be the cantor set and let f be a bounded real function on $[0,1]$ which is continuous at every point outside C. Prove that f is Riemann integrable on $[0,1]$. 15

(2) Let f be a Riemann integrable function defined on the interval $[a,b]$. Define $\tilde{f} : [a,b] \longrightarrow \mathbb{R}$ by

$$\tilde{f}(x) = \int_a^x f(t)\, dt.$$

(a) Give an example of a function f such that $\tilde{f}$ is not differentiable.

(b) Give an example of a function f which is not continuous but $\tilde{f}$ is differentiable. 15

(3) Let g be a continuous function on $[0,1]$ such that $g(1) = 0$. Define a sequence of functions $\{f_n\}$ on $[0,1]$ by

$$f_n(x) = x^n g(x), \; n = 1, 2, 3, \ldots$$

Prove that f_n converges uniformly on $[0,1]$. Find the limit function of the sequence. 15

(4) Show that $\log(1+x)$ has the following power series expansion in the interval $(-1,1)$:

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n+1}\frac{x^n}{n} \cdots.$$

What is the sum of the series at $x = 1$? Justify your answer. 15

P.T.O

(5) Evaluate the following limit for all possible integer values of m and n:

$$\lim_{x \to 0+} \frac{\sin^m x}{x^n}.$$

Discuss the convergence of $\int_0^1 \frac{\sin^m x}{x^n} dx$ for different values of m and n. 12

(6) Let $f(x) = |x|$ on $[-\pi, \pi]$. Discuss pointwise and uniform convergence of the Fourier series of f.

Find the sum of the series:

$$\sum_{n \text{ odd} \geq 1} \frac{1}{n^4} \quad \text{and} \quad \sum_{n \in \mathbb{Z}} \frac{1}{n^2}$$

7+7

(5) Let $f(x,y)$ be a continuous differentiable real valued function on the set

$$R = \{(x,y) : x \geq a, c \leq y \leq d\}.$$

Suppose that the integrals $\int_a^\infty f(x,y)dx$ and $\int_a^\infty \frac{\partial f}{\partial x}(x,y)dx$ converge uniformly on $[c,d]$. Prove that

$$\frac{d}{dy}\int_a^\infty f(x,y)dx = \int_a^\infty \frac{\partial f}{\partial y}(x,y)dx.$$

Evaluate the integral

$$\int_0^\infty te^{-\alpha t}\cos mt\, dt,\ \alpha > 0.$$

10+10

(6) Assume the following result: *If f is a Riemann integrable function on $[-\pi,\pi]$ then the series*

$$\sum_{n=-\infty}^{\infty} |\hat{f}(n)|^2$$

converges, where $\hat{f}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx$.

Let g be a continuously differentiable 2π-periodic function on $\mathbb{R}$. Show that

$$\sum_{n=-\infty}^{\infty} n^2|\hat{g}(n)|^2 \text{ converges.}$$

Hence prove that $\sum_{n=-\infty}^{\infty} \hat{g}(n)e^{inx}$ converges uniformly on $\mathbb{R}$. 8+12

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2005-2006)

B.Stat. I year

Vectors and Matrices-II

Date: 9 May 2006. Maximum Marks 60. Duration: 3 Hrs.

Note: Class-room notation is used. You may answer any part of any question. 5 marks are allotted for neat and precise answers.

1. Prove the following:

a) A is a normal matrix <=> A is unitarily similar to a diagonal matrix.

b) Every square matrix satisfies its characteristic equation.

c) Eigenvectors corresponding to distinct eigenvalues of a matrix form an independent set.

d) Algebraic multiplicity of any eigenvalue λ of a matrix A is greater than or equal to its geometric multiplicity.

e) ABA = 0 implies B can be expressed as C + D where AC = 0 and DA = 0.

f) Every square matrix A admits a decomposition A = B+C, where (i) rank(B^2) = rank(B), (ii) C is nilpotent, and (iii) BC = 0 = CB.

g) Every singular value of an idempotent matrix is 1 implies A is symmetric.

h) H is idempotent implies that there exists a positive definite matrix M such that HM is hermitian.

i) Every reflexive g-inverse of a matrix A is a Moore-Penrose inverse of A under appropriate inner products.

j) Let A = B + C and rank(A) = rank(B) + rank(C). Then A is nonnegative definite and B is symmetric implies B and C are nonnegative definite.

k) Geometric multiplicity of the eigenvalue 1 of A is equal to rank(A) <=> A^2 = A.

l) A and B are nonnegative definite matrices implies eigenvalues of AB are nonnegative.

[12×5 = 60]

2. Prove or disprove the following:

a) A and B are square matrices of the same order such that (-B′ : A′)′ is a (A : B)$^-$ implies both A and B are null.

b) Matrix A is semisimple implies A is normal.

c) All the leading principal minors of a symmetric matrix are nonnegative implies A is nonnegative definite.

d) The minimum polynomial of a square matrix is the product $(\lambda-\lambda_1)(\lambda-\lambda_2)\cdots(\lambda-\lambda_k)$, where $\lambda_1, \lambda_2, \cdots \lambda_k$, are the distinct eigenvalues of the matrix.

e) λ is a nonzero eigenvalue of A implies $(1/\lambda)$ is an eigenvalue of every reflexive g- inverse of A.

[5X2 = 10]

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INDIAN STATISTICAL INSTITUTE
Second Semestral Examination : 2005 –06
B. Stat(Hons.) I Year
Statistical Methods II

Date : 16.05.2006 **Maximum Marks : 100** **Duration : 3 Hours**

Answer Question No.5 and ANY THREE questions from the rest . Marks allotted to each question are given within the parentheses . Standard notations and symbols are used .

1.(a) Express a partial correlation coefficient of a higher order in terms of partial correlation coefficients of lower order .

(b) Prove the relation

$$1-R^2_{1.23...p} = \left(1-r^2_{12}\right)\left(1-r^2_{13.2}\right).......\left(1-r^2_{1p.23....p-1}\right) .$$

Use this relation to show that the multiple correlation coefficient $R_{1.23...p}$ must be numerically as high as any of the total or partial correlation coefficients of x_1 with the independent variables .

(15 + 10) = [25]

2. Suppose x_1 , x_2 and x_3 satisfy the relation

$$a_1x_1 + a_2x_2 + a_3x_3 = k$$

where k is a constant .

(a) Determine the three total correlation coefficients in terms of the standard deviations of the variables and the constants a_1 , a_2 and a_3 .

(b) State the partial and the multiple correlation coefficients in this context .

(15 + 10) = [25]

3. (a) Starting from the Pearsonian differential equation derive the p.d.f. of the Pearsonian Type IV curve . Discuss in details how you would estimate the unknown parameters of the distribution based on a given frequency distribution for which the Pearsonian Type IV curve is appropriate . Also estimate how you would fit such a probability distribution to the given data .

(b) Show that , for the Pearsonian family of distributions ,

$$\text{Skewness} = \frac{\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2-6\beta_1-9)}$$

(15 + 10) = [25]

P.T.O.

4. Give an algorithm to simulate observations from each of the following probability distributions having the p.d.f.'s :

(a) $f(x) = C\left(1+\frac{x}{a_1}\right)^{m_1}\left(1-\frac{x}{a_2}\right)^{m_2}$, $-a_1 \le x \le a_2$, $a_1, a_2, m_1, m_2 > 0$

(b) $f(x) = Cx^{-p}.\exp\left[-\frac{\gamma}{x}\right]$, $0 < x < \infty$, $p > 0$.

$(15 + 10) = [25]$

5. In an experimental sampling scheme , 342 samples were drawn from a certain population and a statistic T was calculated from each sample . The frequency distribution of T is given below .

Class – interval	Frequency
0 – 2	4
2 – 4	23
4 – 6	51
6 – 8	69
8 – 10	59
10 – 12	57
12 – 14	35
14 – 16	14
16 – 18	18
18 – 20	3
20 – 22	4
22 – 24	3
24 – 26	1
26 – 28	1
Total	342

Fit a suitable Pearsonian curve to the above data and test for goodness of fit .

Given

$$\mu_1' = 9.36758 \ , \ \mu_2 = 19.35396$$
$$\mu_3 = 72.98265 \ , \ \mu_4 = 1518.64 .$$

$(20 + 5) = [25]$

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2005–2006)

B.Stat.(Hons.) I

Computational Techniques and Programming II

Date: 19. 5. 06 Maximum marks: 50 Duration: 3hrs

This is a closed-book and closed-note examination. You may use calculators.

1. The function add adds two complex numbers, and the function sum uses it to find the sum of n complex numbers stored in a CMPLX array a of length n.

```
typedef struct {double re, im;} CMPLX;

CMPLX *add(CMPLX *a, CMPLX *b) {
  CMPLX *ans;
  ans = (CMPLX*) malloc(sizeof(CMPLX));
  ans->re = a->re + b->re;
  ans->im = a->im + b->im;
  return ans;
}

CMPLX *sum(CMPLX a[], int n) {
  CMPLX *result;
  result = (CMPLX*) malloc(sizeof(CMPLX));
  result->re = result->im = 0.0;
  for(i=0;i<n;i++) {
    result = add(result,&a[i]);
  }
  return result;
}
```

Discuss why it is a bad program. Suggest how you can improve it. [5+5]

2. Consider the fixed point iteration $x_k = f(x_{k-1})$, where f is continuously differentiable over $\mathbf{R}$. If $f(\xi) = \xi$, and $|f'(\xi)| < 1$, does this imply that there is an open interval (a, b) containing ξ such that the iteration will converge to ξ for all $x_0 \in (a, b)$? Justify your answer. [6]

3. State and prove the Fundamental Theorem of Gaussian Quadrature using the weight function $w(x) \equiv 1$. [3+7]

4. Perform two iterations of the 4^{th} order Runge-Kutta method for the differential equation:
$$y' = e^{xy}, \quad y(0) = 1.$$
Take $h = 0.1$. Show each step clearly. [8]

P.T.O.

5. Consider the (possibly inconsistent) system $A\mathbf{x} = \mathbf{b}$, where A has linearly independent columns. Let $\hat{\mathbf{x}}$ be the unique least square solution. Suggest how you can use QR-decomposition to compute $||\mathbf{b} - A\hat{\mathbf{x}}||^2$ without explicitly computing $\hat{\mathbf{x}}$. [8]

6. Define the condition number of a nonsingular matrix with respect to a given norm. Obtain upper and lower bounds for the norm of the error of a nonsingular, linear system of equations in terms of the condition number of the coeffcient matrix and the norm of the residual. [2+6]

INDIAN STATISTICAL INSTITUTE
Second Semestral Back Paper Examination: 2005-06
B. Stat. I Year
Probability Theory II

Date: 17.7.06 Maximum Marks: 100

Justify your answers.

1. I select a number X and you select a number Y
We toss a fair coin. If it is heads up, my number
probabilities, whereas your number Y is uniform
my number X is uniform (0, 1) and your numbe
equal probabilities. Find the joint distribution
marginal distribution function of X. Find the marg
Are X and Y identically distributed? Are X and Y i

2. X is a random variable having density f. Assume
Let m be a median of X. Show that for any real numb

$$E|X-b| = E|X-m| + 2\int_m^b (b-x)$$

Deduce that $E|X-b|$ is least when $b = m$.

3. Let $X \sim \Gamma\gamma$ and $Y \sim \Gamma\left(\gamma + \frac{1}{2}\right)$ be independent. Show th

4. I pick a point (X,Y) at random from the region $\{(x,y): 0$
density of Z=Y – X.

5. X, Y are independent standard Normal variables. Show that X
independent. (When Y=0, we take $\frac{X}{Y}$ to be 0).

6. $X \sim Exp(1)$. Given $X = x$, the conditional distributi
joint density of (X,Y). Find the conditional densi

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