INDIAN STATISTICAL INSTITUTE

Midsemester Examination : (2001 - 2002)

B.Stat II year : Probability III

Date 03 - 09 - 2001 Maximum Marks 100 Duration : 3 hours

1. Let X, Y, Z be i.i.d random variables with a common density f. Let φ be a continuous function on R to R. Show that $\varphi(X+2Y+3Z)$ and $\varphi(3X+Y+2Z)$ have the same distribution.

[10]

2. Find the distribution of the sample mean for a sample of size n from $exp(1)$ population.

[10]

3. For a random sample of size n from standard normal, show that the sample mean and sample variance are independent.

[10]

4. Let $Y_1 < Y_2 < \cdots < Y_n$ be an order statistic of size n from $unif(-1,1)$. Calculate the joint density of Y_1 and Y_n.

   Hence show that the sample range, $W = Y_n - Y_1$ has density

$$f(w) = \frac{n(n-1)}{2^n}\, w^{n-2}\,(2-w) \quad \text{for } 0 < w < 2$$

[9 + 6]

5. I select a number p at random from $(0,1)$. Then I take a coin whose chance of heads in a single toss is p and toss it 100 times. Let X be the number of heads obtained. For each integer k, $0 \le k \le 100$ calculate $P(X = k)$.

[10]

6. $X_1, X_2, \cdots, X_9$ are i.i.d $exp(1)$. Let $S_1, S_2, \cdots, S_9$ be their successive partial sums. Find the conditional distribution of $(S_1, \cdots, S_8)$ given that $S_9 = 17$.

[15]

P. T. O

7. (a) State Borel - Cantelli Lemma.

(b) Suppose that X_n is a sequence of – not necessarily independent – random variables such that

$$P(X_n = +e^{e^n}) = \frac{1}{2n^2}, \quad P(X_n = -e^{e^n}) = \frac{1}{2n^2}; \quad P(X_n = 0) = 1 - \frac{1}{n^2}$$

Show that the SLLN holds for this sequence of random variables.

[5 + 5]

8. Suppose that $X_n \to X_\infty$ in probability. Show that for any $\epsilon > 0$, there is a number A such that,

$$P(-A < X_n < A) > 1 - \epsilon \quad \text{for each } n, 1 \leq n \leq \infty.$$

Deduce that for any given continuous function φ on R to R , $\varphi(X_n) \to \varphi(X_\infty)$ in probability.

[10 + 10]

GOOD LUCK

INDIAN STATISTICAL INSTITUTE

Midsemester Examination: (2001 - 2002)

Course Name: B. Stat. (IInd year)

Subject Name: C & Data Structures

Date: 05.09.2001 **Maximum Marks: 50** **Duration: 2 hrs.**

Question 1. 10 X 1 = 10

1. Say, a queue is implemented with a linked list, keeping track of a front pointer and a rear pointer. Which of these pointers will change during an insertion into an EMPTY queue?

   A. Neither changes B. Only front_ptr changes

   C. Only rear_ptr changes D. Both change

2. What is the worst-case time for serial search finding a single item in an array?

   A. Constant time B. Logarithmic time

   C. Linear time D. Quadratic time

3. What is the worst-case time for binary search finding a single item in an array?

   A. Constant time B. Logarithmic time

   C. Linear time D. Quadratic time

4. What additional requirement is placed on an array, so that binary search may be used to locate an entry?

   A. The array elements must form a heap B. The array must be sorted

   C. The array must have at least 2 entries D. The array's size must be a power of two

5. Suppose that a Selection Sort of 100 items has completed 42 iterations of the main loop. How many items are now guaranteed to be in their final spot (never to be moved again)?

A. 21 B. 41 C. 42 D. 43

6. Suppose we are sorting an array of ten integers using a some quadratic sorting algorithm. After four iterations of the algorithm's main loop, the array elements are ordered as shown here:

1 2 3 4 5 0 6 7 8 9

Which statement is correct?

A. The algorithm might be either Selection Sort or Insertion Sort.

B. The algorithm might be Selection Sort, but could not be Insertion Sort.

C. The algorithm might be Insertion Sort, but could not be Selection Sort.

D. The algorithm is neither Selection Sort nor Insertion Sort.

7. When is Insertion Sort a good choice for sorting an array?

A. Each component of the array requires a large amount of memory.

B. Each component of the array requires a small amount of memory.

C. The array has only a few items out of place.

D. The processor speed is fast.

8. Suppose we are sorting an array of eight integers using Heap Sort, and we have just finished one of the reheapifications downward. The array now looks like this:

6 4 5 1 2 7 8

How many reheapifications downward have been performed so far?

A. 1 B. 2 C. 3 or 4 D. 5 or 6

9. What kind of list is best to answer questions such as "What is the item at position n?"

A. Lists implemented with an array B. Doubly-linked lists.

C. Singly-linked lists D. Doubly-linked or singly-linked lists are equally best

10. Suppose that p is a pointer variable that contains the NULL pointer. What happens if your program tries to read or write *p?

A. A syntax error always occurs at compilation time.

B. A run-time error always occurs when *p is evaluated.

C. A run-time error always occurs when the program finishes.

D. The results are unpredictable.

Question 2.

The 18th century German city of Königsberg was situated on the river Pregel. Within a park built on the banks of the river, there were two islands joined by seven bridges (see figure 1).

The puzzle asks whether it is possible to take a tour through the park, crossing each bridge only once.

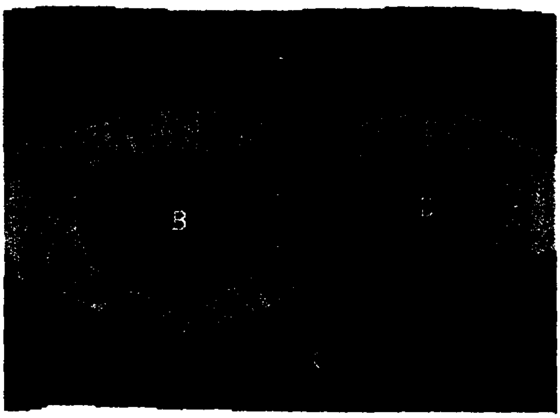

Figure 1.

Transforming the map in fig.1 into a graph in which the vertices represent the "dry land" points (i.e. A, B, C and D) and the edges represent the bridges, write an algorithm that solves the above puzzle. Compute complexity of your algorithm.

8 + 2 = 10

Question 3.

Suppose you are given a list of numbers stored in an array as below:

4	25	90	41	28	3	80	78

Organize the above numbers in a MaxHeap structure (a binary tree is MaxHeap *iff* (i) it is empty *or* (ii) the key in the root is larger than that in either child and both subtrees have the heap property). Show each of the steps separately. Compute the complexity for organizing **n** such elements into a heap.

5 + 2 = 7

Consider the following pseudocode:

```
declare a stack of characters
   while ( there are more characters in the word to read )
   {
         read a character
         push the character on the stack
   }
   while ( the stack is not empty )
   {
         write the stack's top character to the screen
         pop a character off the stack
   }
```

What is written to the screen for the input "carpets"? 3

A. serc B. carpets C. steprac D. ccaarrppeettss

Question 4.

With brief explanation write down the functionality of the following routine named *'func'*? Coupute its complexity.

7 + 3 = 10

```
func( r, lo, up )
      Array r;
      int lo, up;
{
      int d, i, j;
      ArrayEntry tempr;
      for( d = up-lo+1; d > 1; ){
            if(d < 5) d = 1;
            else d = (5 * d -1) / 11;
            for(i = up - d; i >= lo; i--){
                  tempr = r[i];
                  for(j=i+d;j<=up&&(tempr.k>r[j].k;j+=d)
                        r[j-d] = r[j];
                  r[j-d] = tempr;
            }
      }
}
```

Question 5.

1. Here is a small binary tree:

```
     14
    /  \
  2      11
 / \    / \
1   3  10   30
       /   /
      7   40
```

Write the order of the nodes visited in:

A. An in-order traversal,

B. A pre-order traversal,

C. A post-order traversal:

2 + 2 + 2 = 6

2. Suppose that a binary search tree contains the number 42 at a node with two children. Write two or three clear sentences to describe the process required to delete the 42 from the tree.

4

INDIAN STATISTICAL INSTITUTE

MID-SEMESTRAL EXAMINATION:B.STAT (HONS) II-YEAR

(2001 – 2002)

ANALYSIS-III

Max Marks-40 7.9.2001 Time:3 hours

(a) (i) Calculate the directional derivatives of each of the function below.

$$f(x) = x \cdot T(x) \quad \text{where } T : R^n \longrightarrow R^n \text{ is linear}$$
$$f(x) = \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j, \quad a_{ij} = a_{ji}$$

Deduce in each case giving proper justification if the function is differentiable. [4]

(ii) Assume that f is differentiable at a with derivative T_a. Show that all directional derivatives $f'(a;u)$ exist and

$$T_a(u) = f'(a;u).$$

Give an example to show that the existence of all directional derivatives at a point need not imply that the function is differentiable at that point. [6]

(b) (i) State and prove the mean value theorem. Show that if a diffrentiable function on an open convex subset of R^n has zero derivative at each point, then the function is a constant. [4]

(ii) Suppose that $f : U \subseteq R^n \longrightarrow R^m$, U open is a function. Suppose $f = (f_1, \ldots, f_m)$ is such that $D_j f_i(x)$ exists for each $x \in U$ and for all i, j. If each $D_j f_i$ is contimuous at $c \in U$, then show that f is differentiable at c. [6]

(c) (i) Show that the function

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\frac{1}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is differentiable at $(0,0)$. Are $D_1 f$ and $D_2 f$ continuous at $(0,0)$. [3]

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(ii) Find the critical points of the saclar field $f(x, y) = x^4 + y^4 + z^4$. Compute the eigen values of the Hessian matrix at the critical points and hence determine the nature of the critical points. [3]

(iii) Let $f(x, y) = 3x^4 - 4x^2 + y^2$. Show that on every line $y = mx$ the function has a minimum at $(0, 0)$. However show that $(0, 0)$ is a saddle point. [4]

(d) (i) Let F be a continuously differentiable vestor field. If F has a potential function, then show that $curl(F) = 0$. [2]

(ii) State and prove the first fundamental theorem for line integrals. [5]

(iii) A wire has the shape of a circle $x^2 + y^2 = a^2$. Determine ite mass and moment of inertia about a diameter if the density at (x, y) is $|x| + |y|$. [3]

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B.STAT. (HONS.) - II YEAR

MIDSEMESTER EXAMINATION (2001-2002)

PHYSICS - I

te : 10.09.01 Maximum Marks : 30 Duration : 1.5 hrs.

GROUP - A

swer all questions :-

An inextensible string of negligible mass hanging over a smooth peg at the int A connects the mass m_1 on a frictionless inclined plane of inclination θ to the rizontal, to another mass m_2 which is hanging vertically from the peg at A. Use Alembert's principle to show that the massess will be in equilibrium if $m_2 = m_1$ n θ.

(5)

A particle of mass M moves on a plane in the field of force given by (in polar ordinates)

$$\vec{F} = - \hat{n}_r \, kr \cos\theta$$

where k is a constant and $\hat{n}_r$ is the radial unit vector.

Obtain the differential equation of the orbit of the particle using Lagrange's uations.

(5)

Two frames S and S' have a common origin and the frame S' is rotating with spect to the frame S being at rest, with an angular velocity $\vec{w} = \hat{e}_3'\lambda$, where λ is a constant. At time t = 0, the frames are coincident. Let $\vec{A}$ be a vector fined by the equation $\vec{A} = \sin t \, \hat{e}_1' - \cos t \, \hat{e}_2' + \text{expt} \, \hat{e}_3'$. Determine the time rate change of $\vec{A}$ with respect to frame S' and the Coriolis acceleration.

(5)

($\hat{e}_1'$, $\hat{e}_2'$, $\hat{e}_3'$ are unit vectors in the frame S')

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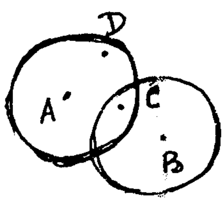
GROUP - B

..: any two of the following questions.

1.



In the above figure, there are two overlapping charge distributions, both of radius 'R' having their centres at A & B. The constant charge densities of A & B are $(+\rho)$ and $(-\rho)$ respectively.

(a) Find the electric field at C. (4.5)

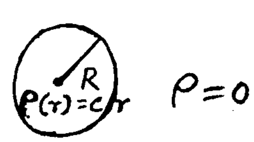
(b) Find the electric field at D. (3.0)

What is the special feature of the answer (a).

2. (a) Find the total electrostatic energy of a spherical charge distribution $\rho(r)$ = cr, of radius R,

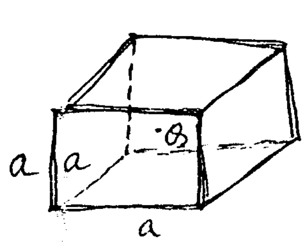
$$\rho(r) = cr, \quad r \le R$$

$$\rho(r) = 0, \quad r > R.$$



(5)

(b) Suppose a point charge Q is placed at the centre of a hollow cubical surface, each square surface of which has area (a^2). Find the flux of electric field linked to any one of the surfaces.



(2.5)

(a) Suppose the electrostatic potential of a charge distribution is given by

$$V(\bar{r}) = p\,\frac{\cos\theta}{r},$$

in spherical polar coordinates where (p) is a constant. Find the magnitude of the electric field $|\bar{E}(\bar{r})|$ at $\bar{r}$.

(Hint : Find out the cartesian components of $\bar{E}$.)

(5)

(b) Show that the following line integral is independent of the path,

$$I = \int_{\bar{A}}^{\bar{B}} (Y\hat{i} + (X+Z)\hat{j} + Y\hat{k}).\overline{d\ell}$$

(2.5)

INDIAN STATISTICAL INSTITUTE

First Semestral Midterm Examination: 2001-2002

B.Stat. (Hons.). 2nd Year

Statistical Methods III

Date: September 14, 2001 **Maximum Marks: 100** **Duration: 3 hours**

- This question-paper carries 105 points. Answer as much as you can, but maximum you can score is 100.
- You must state clearly any result stated and proved in class you may need in order to answer a particular question. Keep the answers brief and to the point.

1(a). Let $X_1, \ldots, X_n$ be iid $N(\theta, \sigma^2)$. Find out the MLE of σ^2. Is it unbiased? Among all estimators of σ^2 of the form $c\sum_1^n (X_i - \bar{X})^2$, find out (if exists) the best choice of c that minimizes the MSE.
(b) Let $X_1, \ldots, X_n$ be iid Bernoulli(p). Find out a minimax estimator of p under the squared error loss function.
(c) Show that if a Bayes estimator is unbiased then its Bayes risk must be 0 under the squared error loss. [(5+10)+10+10= 35]

2(a). Describe a two person zero-sum game where each player has a finite number of actions through a realistic example (not (b) below).
(b) Consider the two person zero-sum game where Player I chooses a number $\theta \in [0,1]$ and Player II guesses it by $a \in [0,1]$. The loss is squared error for Player II. Explicitly derive the optimal solution for this game along with the value of the game.
(c) Let π and δ be two mixed strategies for two players respectively in a two person zero-sum game. Show that if each is Bayes with respect to the other then they constitute an optimal solution of the game. [8+10+12=30]

3. Let $p \in (0,1]$ be the parameter of a statistical model where $f(x|p) = 1/3$ if $x \in \{0, p, 1/p\}$. Suppose X is a single observation of the experiment described by this model.
(a) Find out an unbiased estimator of p.
(b) Find out the Bayes estimator of p under Beta (α, β) prior.
(c) Show that for any unbiased estimator $g(X)$ of p, $\text{Var}_p\{g(X)\} \geq p^2/2$ for all $0 < p \leq 1$. Hence find an unbiased estimator $g^*(X)$ that has minimum variance among all unbiased estimators of p (uniformly in p). [10+15+15=40]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: 2001-2002

B.Stat. (Hons.). 2nd Year

Statistical Methods III

Date: December 03, 2001 **Maximum Marks: 100** **Duration: 3 hours**

- This question-paper carries 110 points. Answer as much as you can, but maximum you can score is 100.
- You must state clearly any result stated and proved in class you may need in order to answer a particular question. Keep the answers brief and to the point.

1(a). Let X_{ij} $(i = 1, \ldots, p; j = 1, \ldots, k)$ be independent, normally distributed with mean μ_i $(i = 1, \ldots, p)$ and variance σ^2. All the parameters are unknown. Find out MLE of the parameters.
(b) Find out the bias of $\hat{\sigma}^2_{MLE}$. Construct an unbiased estimate of σ^2. [10+(6+4)=20]

2(a). Describe briefly the notion of type I error, level of significance and power in a testing problem.
(b) State the Neyman-Pearson lemma.
(c) Suppose X has a Binomial(n, θ) distribution. List all the non-randomized Neyman-Pearson tests for testing $H_0 : \theta = 1/2$ against $H_1 : \theta = \theta_1$ $(\theta_1 > 1/2)$. Hence obtain the most powerful test for an arbitrary level of significance $\alpha \in [0, 1]$.
(d) Show that the most powerful test in (c) is actually uniformly most powerful for testing $H_0 : \theta = 1/2$ against $H_1 : \theta > 1/2$.
(e) Let $X_1, \ldots, X_n$ be a random sample from normal population with mean μ and variance σ^2 (both unknown). We want to test $H_0 : \mu = 0$. Present in a *brief and systematic* manner the statistical arguments leading to the one-sample Student's t test statistic for this problem (**no proofs necessary**).
(f) Show that the sampling distribution of $|t|$ depends on the noncentrality parameter $n\mu^2/\sigma^2$ (derivation of the actual density function is not necessary). [10+6+(6+6)+6+10+6=50]

3. Let $\{X_0, X_1, \ldots\}$ be a time series such that $X_t = \rho X_{t-1} + \epsilon_t$ for $t \geq 1$, where $\epsilon_1, \ldots$ are iid $N(0, \sigma^2)$ (independent of X_0). Find out a distribution for X_0 that makes the time series $\{X_i\}$ stationary for $|\rho| < 1$. Will the time series remain stationary when $\rho = -1$? [10]

4 (a) Suppose we have two sets of observations that have been experimentally obtained as follows. The first set gives Haemoglobin (Hb) counts of a random sample of 10 individuals from a healthy population without any history of a particular disease (D) : 13.2, 12.5, 13.4, 12.2, 11.7, 12.8, 14.0, 13.8, 12.6, 13.6. The second set is a random sample of Hb counts for 6 individuals from the

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population with disease D : 10.6, 11.3, 9.6, 9.8, 12.4, 11.8. Test the hypothesis whether the give disease affects the Hb count negatively at 5% level of significance (your assumptions about th model must be clearly described). **Also**, obtain a 95% confidence interval for the difference i mean Hb counts between the normal and diseased populations.

(b) Suppose the frequency table for a random sample of size 100 is as follows.

Classes	Frequency
$x \leq -1$	12
$-1 < x < 0$	35
$0 \leq x < 1$	42
$x \geq 1$	11

Test if the sample is drawn from a standard normal population at 5% level. [20 + 10 =3(

INDIAN STATISTICAL INSTITUTE
MIDSEMESTER EXAMINATION: 2001 – 2002
COURSE: B.STAT.-II yr.
SUBJECT :– ECONOMICS-I

Date: 10/9/2001 | Maximum Marks: 40 | Duration: 2 Hours

Answer any TWO questions [2x20]

1. There are only two commodities, X and Y, the quantities of each being denoted by x and y respectively. Consider a consumer whose utility function is, $U(x,y)=x^2+y^2;\ x,y\geq 0$. The prices of X and Y are, respectively, $P_x=2$ and $P_y=1$ per unit, and income of the consumer is $M=20$.
   (i) At what commodity bundle is his marginal rate of substitution equal to the price ratio? Call this bundle A. Now consider the commodity bundles where the consumer spends all his income buying only X and only Y. Call these bundles B and C respectively. Compare U(A) with U(B) and U(C). What is the consumer's optimal commodity bundle at these prices and income? [2 + 2 + 2]
   (ii) For the utility function given above, discuss the relation between M and x (that is, the Engel curve for X). [4]
   (iii) For the same utility function derive the relation between P_y and y (that is, the Marshallian demand curve). [5]
   (iv) What can you say about the own price elasticity of demand for Y at different prices? [5]

2. (i) Make clear the concept of consumption set and budget set. Consider a consumer with utility function $U=U(x,y)$ and money income M, where x is the distance (in km) traveled by bus (call it commodity X) and y is pollution generated by bus travel (call it commodity Y). Thus X has positive marginal utility and Y has positive marginal disutility. Assume that every unit consumption of X is associated with α unit consumption of Y, i.e., $y=\alpha x,\ \alpha>0$. Let P_x and P_y denote the unit price of X and Y respectively. Quite obviously, $P_x>o$ and $P_y<0$. Define $P=P_x+\alpha P_y>0$. Explain the consumption set and the budget set in this case? [4 + 6]
   (ii) What can you say about the equilibrium of the consumer? [10]

3. (a) Describe the structure of the cobweb market model. Assume that demand and supply relations are linear. Discuss the nature of the time path of price if the initial price is different from the equilibrium price. [3 + 7]
   (b) Let the market demand for a product be given by $D=a-bP$ and the market supply by $S=c-dP$ where P is the market price, and $a,\ b,\ c,d>0;\ a>c$. Find the equilibrium price for the product. If a tax of $t>0$ per unit of the product is imposed, estimate the loss of welfare. (Welfare = consumers' surplus + producers' surplus.) [3 + 7]

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INDIAN STATISTICAL INSTITUTE
FIRST SEMESTER EXAMINATION: 2001 – 2002
COURSE NAME: B. STAT. II
SUBJECT NAME: ECONOMICS-I

Date: 07/12/2001 Maximum Marks: 60 Duration: 3 Hours

Answer any Six questions [6x10]

1. Consider a firm which takes the product price as given. It has the following cost function:

$$C = F + cq \text{ if } q \le q^*$$
$$= \infty \qquad \text{if } q > q^*$$

where $F > 0$ if $q > 0$ and $F = 0$ if $q = 0$, and q^* is a given level of output. Derive the supply curve of the firm.

2. Consider a revenue-maximizing firm which produces two goods, Q_1 and Q_2, using an input X. Its available input is given and is fixed at a level x^0. The prices of the products are P_1 and P_2, respectively, which also remain unchanged. Explain how this firm will decide its optimal output levels, q_1^* and q_2^*.

3. A labor union first fixes up the wage rate which is taken as constant by the producer, whose objective is to maximize profits with respect to the output level, given its production function $q = f(L)$ which is assumed to be strictly concave (L = employment). The objective of the labor union is to maximize income per labor (including the unemployed labor, if any). How do you solve the problem? Show that in equilibrium wage elasticity of labor demand is unity.

4. State and explain the Weak Axiom of Revealed Preference (WARP). A consumer is observed to purchase $x_1 = 20$ and $x_2 = 10$ at prices $p_1 = 2$, $p_2 = 6$. He is also observed to purchase $x_1 = 18$, $x_2 = 4$ at prices $p_1 = 3$, $p_2 = 5$. Is his behavior consistent with the WARP?

5. Consider the market for a homogeneous good served by two symmetric firms. The cost function is assumed to be linear. The market demand for the product is given by the following expression:

$$P = A - B(q_1 + q_2), \quad A, B > 0$$

where P is the product price and q_i $(i = 1, 2)$ is the supply of the i-th firm.

a) If the firms decide their quantities simultaneously and non-cooperatively, determine their output levels.
b) If firm 1 behaves as a Stackelberg leader and firm 2 as a Stackelberg follower, find the corresponding output levels.

6. What do you mean by Pareto optimal allocation? Derive condition(s) for Pareto optimal allocation in consumption. Is such an allocation unique?

7. Consider a monopoly market for a product. The demand for the product is given by the equation, $X = a - P$, $a > 0$, where X is quantity demand and P is the price of the product. The cost function of the monopolist is $C = bX$, $(0 < b < a)$. Estimate the deadweight loss under uniform monopoly pricing. If the monopolist could successfully practice perfect price discrimination, what would happen to the corresponding estimate?

INDIAN STATISTICAL INSTITUTE
203 B.T. ROAD, KOLKATA – 35

B. STAT. – II YEAR

FIRST SEMESTRAL EXAMINATION (2001-2002)

PHYSICS – I

ate : 07.12.01 | Maximum Marks : 70 | Duration : 3 hrs.

GROUP – A

nswer Q.1 and Q.2 and any one of the rest.

. a) Which components of linear momentum $\vec{P}$ and angular momentum $\vec{l}$ are conserved in motion of the system consisting of a set of mass points ıoving in a potential field generated by fixed sources that are homogenuous ıass distributions

i) in a circular cylinder of infinite length with axis along the z-axis.

ii) in the infinite half plane $x \geq 0$. (3)

b) Two frames S and S′ have a common origin and the frame S′ is otating with respect to S being at rest, with an angular velocity $\vec{w}\,(SS') = \hat{e}_3\,\lambda$, vhere λ is a constant. At time $t = 0$, the frames are coincident. Let $\vec{A}$ be a vector lefined by the equation $\vec{A} = \sin t\,\hat{e}_{1'} - \cos t\,\hat{e}_{2'} + \exp t\,\hat{e}_{3'}$, $\hat{e}_{1'}, \hat{e}_{2'}, \hat{e}_{3'}$ being unit ectors in frame S′, $\hat{e}_3$ is a unit vector in frame S. (8)

Determine $\dot{A}'$ in terms of its components with respect to S.
(dot denotes differentiation with respect to time)

c) Suppose that in a conservative system q_j is a cyclic coordinate such hat dq_j corresponds to a rotation of the system of particles around some axis. hen show that

$$\frac{d}{dt}\left(\sum_i \hat{n}.\vec{r}_i \times m_i \vec{v}_i\right) = \sum_i \hat{n}.\vec{r}_i \times \vec{F}_i$$

vhere $\hat{n}$ is a unit vector along the axis of rotation and the other symbols have heir usual meaning (Diagram is necessary). (4)

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(4)

a) Obtain the equation of motion for a particle falling vertically under the influence of gravity when frictional forces obtainable from a dissipation function ½ kv^2 are present. Integrate the equation to obtain the velocity as a function of time and show that the maximum possible velocity for fall from rest $v = \frac{mg}{k}$. Assume that at t = 0, v = 0 and the particle is at a height h.

(6)

b) Consider the path of a particle of unit mass in a uniform gravitational field. The displacement of the particle in time t is given by

$$x = ut + \tfrac{1}{2} gt^2$$

Constructing a varied path in the following way

$$x = ut + \tfrac{1}{2} gt^2 + \eta(t),$$ with η vanishing at the end points,

show that the action along the actual path is smaller than along any other path.

(4)

3. a) A wedge of mass M is free to move smoothly on a horizontal plane. The wedge has a perfectly smooth surface inclined at an angle α to the horizontal. A particle of mass m is free to move in the xy plane on the inclined surface. Obtain the Hamiltonian and Hamilton's equations of motion. (6)

b) A Hamiltonian of one degree of freedom has the form

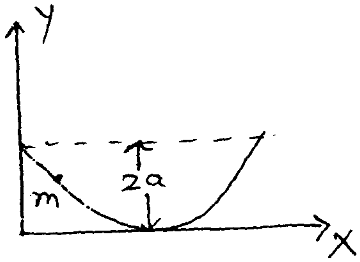
$$H = \frac{p^2}{2a} - bqp\,e^{-\alpha t} + \frac{ba}{2} q^2 e^{-\alpha t}(\alpha + b e^{-\alpha t}) + \frac{kq^2}{2}$$ where a, b, α and k are constants. Find a Lagrangian corresponding to this Hamiltonian. (4)

4. a) A particle moves in a plane under the influence of a force, acting towards a centre of force whose magnitude is

$$F = \frac{1}{r^2}\left(1 - \frac{\dot{r}^2 - 2\ddot{r}r}{c^2}\right)$$

where r is the distance of the particle to the centre of force. Find the velocity dependent potential and from that the Hamiltonian for the motion in a plane. (dot denotes differentiation with respect to time) (5)

b) A bead slides on a wire in the shape of a cycloid (as shown in the gure) described by equations



$$x = a(\theta - \sin\theta)$$

$$y = a(1 + \cos\theta) \quad \text{where } 0 \le \theta \le z\Pi$$

)btain the equation of motion .
[ake the mass of the bead to be m and ıeglect the friction between the bead .nd the wire). (2.5)

c) Show that the generating function for the transformation

$$p = \frac{1}{Q}, \quad q = PQ^2 \text{ is } G = \frac{q}{Q}$$ (2.5)

(Take constants of integration to be zero)

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GROUP – B

Answer No. 5 and any three of the rest of the questions :

1. a) Find the electrostatic potential of a charge distribution, having polarization $\bar{p}(\bar{r})$, in terms of bound charge distributions. What is the physical significance of these bound charges ? (3)

b) Derive the charge distributions for the following cases

i) $\bar{p}(\bar{r}) = \bar{c}$

ii) $\bar{p}(\bar{r}) = c\bar{r}$

In the above, $\bar{c}$ is a constant vector and 'c' is a constant (5)

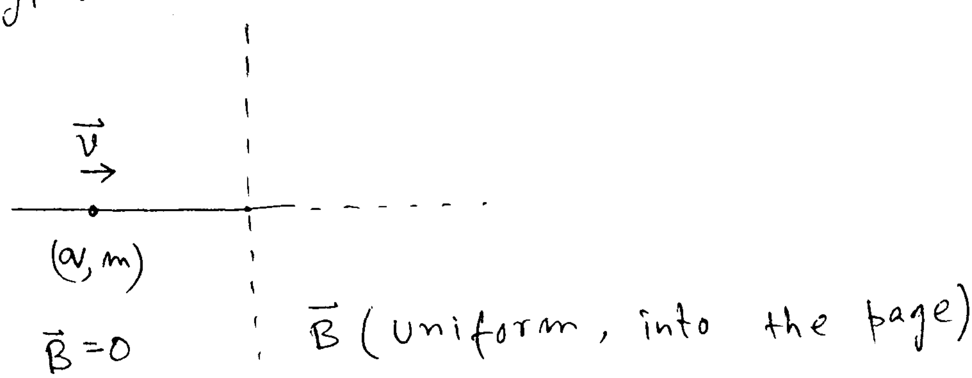
c) For a spherical dielectric body of radius 'R', placed at the origin, fir the electric field for case (ii) $\bar{p}(\bar{r}) = c\hat{r}$, $\hat{r} = \frac{\bar{r}}{r}$. (2)

2. a) Using the Biot-Savart law, find the magnetic field due to a steady current 'I', passing through an infinite, straight wire. (3)

b) Derive Ampere's law $\bar{\nabla} \times \bar{\beta} = \mu_0 \bar{J}$, for steady currents, passing through infinite, straight wires. Hence, find the magnetic field due to an infinit ideal solenoid. (3)

c) Derive the expression for the force per unit length between two parallel, infinite, straight wires, carrying currents I_1 and I_2 separated by a distance 'd'. (4)

3. A particle of charge 'q' of mass 'm' having velocity $\bar{v}$ enters a region of uniform magnetic field $\bar{B}$ (pointing into the page). From the figure below, find the position of the particle time 't'. At $t=0$, the particle is at origin. (10



4. a) Calculate the torque on a magnetic dipole, placed in an external, constant magnetic field $\overline{B}$. Value of the dipole moment is $\overline{m}$. (5)

b) Consider a point charge rotating in a circular path. Let a constant magnetic field be applied, perpendicular to the plane of the particle orbit. Show that the change in the magnetic moment of the rotating charge will always be in a direction, opposite to that of the applied magnetic field. (5)

5. Short note (any two) :

a) Gyromagnetic ratio

b) Capacitor

c) Magnetic field at the origin due to a circular current 'I' of radius 'R'.

(2.5+2.5+2.5)

INDIAN STATISTICAL INSTITUTE

First Semestral Examination: 2001-2002

B.Stat. (Hons.). 2nd Year

Statistical Methods III

Date: December 03, 2001 **Maximum Marks: 100** **Duration: 3 hours**

- This question-paper carries 110 points. Answer as much as you can, but maximum you can score is 100.
- You must state clearly any result stated and proved in class you may need in order to answer a particular question. Keep the answers brief and to the point.

1(a). Let X_{ij} ($i = 1, \ldots, p; j = 1, \ldots, k$) be independent, normally distributed with mean μ_i ($i = 1, \ldots, p$) and variance σ^2. All the parameters are unknown. Find out MLE of the parameters.
(b) Find out the bias of $\hat{\sigma}^2_{MLE}$. Construct an unbiased estimate of σ^2. [10+(6+4)=20]

2(a). Describe briefly the notion of type I error, level of significance and power in a testing problem.
(b) State the Neyman-Pearson lemma.
(c) Suppose X has a Binomial(n, θ) distribution. List all the non-randomized Neyman-Pearson tests for testing $H_0 : \theta = 1/2$ against $H_1 : \theta = \theta_1$ ($\theta_1 > 1/2$). Hence obtain the most powerful test for an arbitrary level of significance $\alpha \in [0, 1]$.
(d) Show that the most powerful test in (c) is actually uniformly most powerful for testing $H_0 : \theta = 1/2$ against $H_1 : \theta > 1/2$.
(e) Let $X_1, \ldots, X_n$ be a random sample from normal population with mean μ and variance σ^2 (both unknown). We want to test $H_0 : \mu = 0$. Present in a *brief and systematic* manner the statistical arguments leading to the one-sample Student's t test statistic for this problem (**no proofs necessary**).
(f) Show that the sampling distribution of $|t|$ depends on the noncentrality parameter $n\mu^2/\sigma^2$ (derivation of the actual density function is not necessary). [10+6+(6+6)+6+10+6=50]

3. Let $\{X_0, X_1, \ldots\}$ be a time series such that $X_t = \rho X_{t-1} + \epsilon_t$ for $t \geq 1$, where $\epsilon_1, \ldots$ are iid $N(0, \sigma^2)$ (independent of X_0). Find out a distribution for X_0 that makes the time series $\{X_i\}$ stationary for $|\rho| < 1$. Will the time series remain stationary when $\rho = -1$? [10]

4 (a) Suppose we have two sets of observations that have been experimentally obtained as follows. The first set gives Haemoglobin (Hb) counts of a random sample of 10 individuals from a healthy population without any history of a particular disease (D) : 13.2, 12.5, 13.4, 12.2, 11.7, 12.8, 14.0, 13.8, 12.6, 13.6. The second set is a random sample of Hb counts for 6 individuals from the

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population with disease D : 10.6, 11.3, 9.6, 9.8, 12.4, 11.8. Test the hypothesis whether the disease affects the Hb count negatively at 5% level of significance (your assumptions abou model must be clearly described). **Also**, obtain a 95% confidence interval for the differen mean Hb counts between the normal and diseased populations.

(b) Suppose the frequency table for a random sample of size 100 is as follows.

Classes	Frequency
$x \leq -1$	12
$-1 < x < 0$	35
$0 \leq x < 1$	42
$x \geq 1$	11

Test if the sample is drawn from a standard normal population at 5% level. [20 + 10

INDIAN STATISTICAL INSTITUTE

Semester Examination: (2001 - 2002)

Course Name: B. Stat. **Year: IInd year**

Subject Name: C & Data Structures

Date: 10.12.01 **Maximum Marks: 50** **Duration: 2 hrs.**

Note: Answer any five.

1. (a) Let n be the number of elements in a sorted list. If n lies in the range $[2^{k-1}, 2^k]$, show that binary search makes at most k element comparisons for a successful search, and either (k-1) or k comparisons for an unsuccessful one. [5]

(b) Consider an array with elements

0 5 17 22 25 31 81

Search the element 30 using binary search. Write down the steps clearly. [5]

2. (a) Construct a B+ tree with the following key values

7 18 22 36 45 50 76 81 92 [4]

(b) Insert 48 in the above B+ tree. Explain the steps. [3]

(c) Delete 92 from the above tree. Explain the steps. [3]

3. (a) Explain the phenomena of primary and secondary clustering in hashing. [5]

(b) What is done to reduce secondary clustering? [5]

4. (a) Why is external sorting used ? [5]

(b) Explain polyphase merge with a suitable example. [5]

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B.STAT. – II YEAR

FIRST SEMESTRAL EXAMINATION (2001-2002)

ANALYSIS – III

Date : 14.12.01 Maximum Marks : 70 Duration : 3 hrs.

Answer all questions.

1. Decide whether the following statements are true or false. Give proper justifications for your answer.

   i) If $f'(a : y)$ exists for all $y \in \mathbb{R}^n$, then f is differentiable at a.

   ii) If f is scalar field with $D_1 f(o,o) = D_2 f(o,o) = 0$ and $f'((o,o): i+j) = 3$, the f cannot be differentiable at (o,o)

   iii) There is no scalar field f with $f'(a: y) > 0$ for a fixed vector a and every non-zero vector y.

   iv) There exists a scalar field f such that $\nabla f = yi - xj$

   v) There exists a vector field F such that curl (F) = xi + yj + zk.

(15)

2. i) State and prove the mean value theorem (5)

   ii) Let $f : u \to \mathbb{R}$, $u \subseteq \mathbb{R}^2$ open, be a function. Assume that the partial derivatives $D_1 f$, $D_2 f$, $D_{1,2} f$ and $D_{2,1} f$ exist and are continuous on U. Then show that $D_{1,2} f = D_{2,1} f$ on u. Show by an example that the above conditions are sufficient but not necessary.

(8)

P. T. O

3. i) Find the critical points of the scalar field $f(x,y,z) = x^4+y^4+z^4$. Compute the eiger values of the Hessian matrix at the critical poi and hence determine the nature of the critical points.

(4)

ii) State and prove the first fundamental theorem for line integrals.

(6)

iii) Compute the following line integrals.

a) $\int_c (x^2 - 2xy)\,dx + (y^2 - 2xy)dy$

where c is the path from (-2,4) to (1,1) along the parabola $y=x^2$.

b) $\int_c \frac{(x+y)dx - (x-y)dy}{x^2+y^2}$

where c is the circle $x^2+y^2 = a^2$ traversed once in the anti-clockwis direction.

(4+4)

4. i) Let f be defined and bounded on a rectangle Q = [a,b] x [c,d]. Assume that f is integrable on Q. For each y Є [c,d] assume that

$$A(y) = \int_a^b f(x,y)\,dx$$

Exists. If $\int_c^d A(y)dy$ exists, then show that

$$\iint_Q f(x,y)\,dx\,dy = \int_c^d \left(\int_a^b f(x,y)\,dx\right) dy$$

(4)

ii) State Greens' theorem in the plane. Use Greens' theorem to evaluate the following integral

$$\int_c y^2dx + xdy$$

a) where C is the square with vertices (0,0), (2,0), (2,2), (0,2)

b) C has the vector equation $\alpha(t) = 2\cos^3 t\, i + 2\sin^3 t\, j;\ 0 \le t \le 2\bar{\Lambda}$

(4)

iii) State and prove Stokes' theorem for solids which are xy, xz, yz projectable. (8)

iv) Use Stokes' theorem to show that the integrals have the given value. Describe explictly how to traverse C.

a) $\int_c ydx + zdy + zdx = \overline{\Lambda} a^2 \sqrt{3}$

where C is the curve of intersection of the plane $x+y+z = 0$ with the sphere $x^2+y^2+z^2 = a$

b) $\int_c (y+z)dx + (x+z)dy + (x+y)dz = 0$

where C is the curve intersection of the plane $y=z$ with $x^2+y^2=2z$.

(4+4)

INDIAN STATISTICAL INSTITUTE

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SEMESTRAL EXAMINATION

PROBABILITY III
B.STAT SECOND YEAR

29 - 11 - 2001
Duration: Three hours

This paper is set for 70 marks. Maximum you can score is 65.
Justify your steps.

1. Suppose that $Y_1, Y_2, \cdots, Y_{k+1}$ are i.i.d standard normal variables. Put

$$X_i = \frac{Y_i^2}{\sum_{j=1}^{k+1} Y_j^2} \quad \text{for } 1 \le i \le k.$$

Show that $(X_1, X_2, \cdots X_k)$ has Dirichlet distribution.

[8]

2. Suppose that $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ is the order statistic of size n from $Exp(\lambda)$. Show that $X_{(1)}$, $X_{(2)} - X_{(1)}$, $X_{(3)} - X_{(2)}$, $\cdots$, are independent.

[8]

3. Suppose that (X, Y) has joint density

$$f(x,y) = \frac{1}{8}\,(x^2 - y^2)\,e^{-x} \quad \text{for } 0 < x < \infty \text{ and } |y| < x.$$

(i) Find the marginal densities of X and Y.

(ii) Find the conditional density of X given Y.

[8+8]

4. Stating precisely any theorem about characterstic functions that you are using, calculate the characterstic function of the standard Cauchy variable. Show that the average of two (independent) standard Cauchy variables is again a standard Cauchy variable.

[6]

P. T. O

5. Suppose that $F_n \Rightarrow F$, where F is a continuous distribution function. Show that

$$Sup_{x \in R} |F_n(x) - F(x)| \to 0 \qquad \text{as } n \to \infty.$$

[8]

6. Suppose that $\{X_n\}$ is a sequence of independent random variables such that

$$P(X_n = 2^n) = \frac{1}{2} = P(X_n = -2^n)$$

for $n \geq 1$. Show that the WLLN fails for this sequence.

[8]

7. (i) Suppose that $X_n \to X$ in probability. Let $\epsilon > 0$. show that there is a number $A > 0$ such that for each $n \geq 1$, $P(-A < X_n < A) > 1 - \epsilon$.

(ii) If $X_n \to X$ in probability and $Y_n \to Y$ in probability, show that $X_n Y_n \to XY$ in probability.

[8+8]

GOOD LUCK

INDIAN STATISTICAL INSTITUTE
B.STAT-I (2001-02)
Theory of Probability and its Applications - I
Semestral-I examinations
Maximum marks: 65. Time: 3 hours.

Date : 18 Dec, 2001.

Note: ***Answer as many questions as you wish. The whole question paper carries 75 marks. The maximum you can score is 65.***

1. (a) Define limsup of a sequence of subsets A_n, $n = 1, 2, \cdots$. Show that
$$\limsup_n (A_n \cup B_n) = (\limsup_n A_n) \cup (\limsup_n B_n).$$
(b) When do you say that $lim\, A_n$ exists where A_n is a sequence of subsets in a sample space. Show that if $lim\, A_n$ exists, then $P(lim\, A_n) = \lim_{n\to\infty} P(A_n)$.
(c) Show that $$(\bigcup_1^\infty A_n)\Delta(\bigcup_1^\infty B_n) \subset \bigcup_1^\infty (A_n \Delta B_n).$$

[3+5+2]

2.(a) An airport shuttle bus with 9 passengers makes 7 scheduled stops. Assuming that all possible distributions of 9 passengers in the 7 stops are equally likely find the probability that someone (at least one) gets down at each stop.
(b) Consider 2 urns U_i containing r_i red and b_i black balls , $i = 1, 2$. A ball is drawn at random from U_1 , put into U_2 and then a ball is drawn from U_2 and put into U_1. After all this what is the probability that a ball drawn from U_1 now is red ? Verify that if $b_1 = r_1$ and $b_2 = r_2$, then this probability is the same as the probability when no transfer takes place.
(c) Let $A_1, A_2, \cdots, A_n$ be independent events each having the same probability p, $(0 < p < 1)$. Let B denote the event that ***exactly one*** among the $A_i's$ occurs and C denote the event that ***exactly two*** among the $A_i's$ occur. If $P(B) = P(C)$, find p.
(d) A class has 12 boys and 10 girls. I roll a pair dice, note the sum of the 2 faces that show up and pick that many students at random from the class. Given that my sample has 5 girls what is the (conditional) probability that I picked no more than 2 boys.

[6+6+2+6]

3.(a) Let X be a non-negative integer valued random variable with probability generating function (p.g.f) $P(s)$. Let $Y = 3X$. Express the p.g.f $Q(s)$ of Y in terms of $P(s)$.
(b) Find the p.m.f of the random variable whose p.g.f is $P(s) = e^{\lambda(s^3-1)}$, $(\lambda > 0)$.
(c) Let $X_1, X_2, \cdots$ be independent random variables with common distribution given by::
$$P(X_i = 1) = p \text{ and } P(X_i = -1) = 1 - p,\ i = 1, 2, \cdots, (0 < p < 1).$$
Let $S_0 = 0$, and $S_k = X_1 + X_2 + \cdots + X_k$, $k = 1, 2, \cdots$. Let
$$u_n = P(S_n = 0),\ n = 0, 1, 2, \cdots \text{ and}$$
$$f_0 = 0,\ f_n = P(X_1 \neq 0, X_2 \neq 0, \cdots, X_{n-1} \neq 0, X_n = 0),\ n = 1, 2, \cdots.$$
(i) Show that $u_{2n} = \binom{2n}{n} p^n (1-p)^n$ and $u_{2n-1} = 0$, for $n = 1, 2,$

Please turn overleaf

(ii) Show that $u_n = f_1u_{n-1} + f_2u_{n-2} + \cdots + f_nu_0, n = 1, 2, \cdots$ and $u_0 = 1$.

(iii) Find the generating function $U(s)$ and $F(s)$ of the sequences $\{u_n\}$ and $\{f_n\}$ respectively. Find f_n explicitly using $F(s)$.

[2+2+(2+3+6)]

4. The number of accidents N in a given interval of time has Poisson(λ) distribution. The number of persons injured in each accident is a random variable which is distrbuted independent of N and also independent of of other accidents with p_k being the probability of k injured, $k = 0, 1, 2, \cdots$. Let N_i to be the number of accidents in which exactly i persons are injured $i = 0, 1, 2, \cdots$.
(a) Find $P(N_1 = r, N_2 = s \mid N = k)$, $k = 0, 1, 2, \cdots$.
(b) Find $P(N_1 = r, N_2 = s)$, for $r = 0, 1, 2, \cdots$, $s = 0, 1, 2, \cdots$. Are N_1 and N_2 independent?
(c) Let M be the total number of persons injured. Find the probability distribution of M and $E(M)$.

[3+3+4]

5. An urn contains α_1 balls numbered 1, α_2 balls numbered 2, $\cdots$, α_r balls numbered r. We draw n balls without replacement from the urn. Let S denote the sum of all the numbers from among $\{1, 2, \cdots, r\}$ that did not appear in the sample. Find $E(S)$.

[5]

6. Five men and four women are ranked according to their scores in a test. Assume that no two scores are identical and that all the 9! possible rankings are equally likely. Find the distribution of the highest ranking achieved by a woman. Find its expected value.

[5]

7. (a) State and prove Cauchy-Schwarz inequality for two discrete random variables X, Y with finite second moments.
(b) Give an example of two random variables X, Y such that $Cov(X, Y) = 0$, but X, Y are not independent.

[5+5]

INDIAN STATISTICAL INSTITUTE

B.STAT I YEAR

FIRST SEMESTRAL EXAMINATION (2001-02)

Computing Techniques and Programming I

me : 3 and a half hours. Date : 20. 12. 2001

rks : Answer as many parts as you like. Maximum you can score is 100.

hat are good programming practices ?

xplain giving examples what are black-box testing and white-box esting of programs.

3+7

rite a program to read a five digit integer N and print the sum f the decimal digits in it.

rite a program to tabulate the function

$$f(X,Y) = (X^2 + Y^2 + 2XY)/ (X^2 + Y^2 + 8XY)$$

r the following set of values of (X,Y) :

0,-10), (2,-8), (4,-6), (6,-4), (8,-2), (10,0) and (12,2).

5+5

Write a program which calculates the electric bill per month after reading an input, according to the following rules :

i) Upto < 50 units : charge only meter rent of Rs.40.00, no other charge;
i) = 50 to < 100 : Rs.1.25/10 units for all units ;
i) =100 to < 150 : Rs.1.20/10 units for all units ;
v) =150 to < 200 : Rs.1.10/10 units for all units ;
v) > 200 units : Rs.0.90/10 units for all units ;

for cases ii) to v) the meter rent should be added.

Write a print routine for the output of the above exercise (3(a)) using a nice format.

6+4

Write a program to delete all vowels from a sentence assuming that the sentence is 80 characters long.

Write a program to read a five character string of decimal digits and to print the sum of the digits which form the character string.

5+5

Write a subroutine to multiply a (NxN) square matrix by a (Nx1) vector. The CALL statement will pass the size (N) of the matrix and the names of the matrix and vector arrays.

10

P.T.O

6. Write a program defining a Function subprogram with two arguments integer and an array of integers. The logical function returns TRU the integer is present in the array of integers, else it returns F

7. A point is defined by its two co-ordinates. A circle is defined by co-ordinates of its centre and its radius.
   Define a derived type point. Use this to define a derived type c

8. Write a program which will read a file Stud_file which records info about students in the following format :

Fields -	Roll-number	Name	Marks_1	Marks_2	Mar
Digits -	4 digit integer	25 char maximum	3 digit integer	3 digit integer	3 d int

The program should print a list of students who have failed in one subjects assuming 40 percent as pass marks in each subject.

9. Write a program to read a list of integers from a file of unknown si and store the integers in a linked list. Determine the size of an allocatable array from the number of integers in file and allocate array. Then store the integer values from the linked list to the a

10. Write a recursive subprogram to compute the Ackermann function A(m,n defined as follows :

```
A(0,n) = n+1
A(m,0) = A(m-1,1)              for m>0
A(m,n) = A(m-1, A(m,n-1))      for m,n > 0
```

11. Write short notes on (any two) :

a) Arithmetic operators, intrinsic functions and user-defined functi

b) The use of MODULES and INTERFACE to facilitate communication betw calling and called programs.

c) Parameter passing mechanisms of call-by-value and call-by-value-r and the IN and IN-OUT arguments of FORTRAN subroutines.

d) Generic functions and operator overloading.

5 x

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B.STAT. - II YEAR

MID-SEMESTER EXAMINATION (2001 - 2002)

ELEMENTS OF ALGEBRA

TE : 26.02.02 MAXIMUM MARKS : 100 DURATION : 3 hrs.

a) Show that the cosets of a normal subgroup H of a group G forms a group under coset multiplication. (10)

b) In a group, if $a^m = e$ for an element a of the group, show that order of a divides m. (10)

a) In H_n is the group of integers under addition modulo n, find $H_n \cap H_m$ $(n \neq m)$ (10)

b) If f is a group homomorphism from a group G and a is an element of G of finite order, show that order of f(a) is a divisor of the order of a. (10)

a) Let A, B be subgroups of a group G. For x, y in g define $x \sim y$ if $y = axb$ for some a in A, b in B. Show that $\sim$ is an equivalence relation on G. (10)

b) For real numbers a, b, define a function $\Phi_{a,b}$ on real numbers,

$$\Phi_{a,b}(x) = ax + b \text{ and let}$$

G be the set of all $\Phi_{a,b}$ where $a \neq 0$. Show that G is a group under function composition. (10)

c) If N is the set of all functions above of the form $\Phi_{1,b}$ show that N is a normal subgroup of G. (10)

a) If product of any two left cosets of a subgroup H in a group G is again a left coset of H in G. Show that H is normal in G. (10)

b) If N is a normal subgroup of a group G and H is any subgroup of G, show that NH is a subgroup of G. (10)

c) If $(ab)^3 = a^3b^3$ for any two elements a, b in a finite group G, where 3 is not a factor of order of G, show that each element of G can be written as x^3 for some x in G. (10)

INDIAN STATISTICAL INSTITUTE
B. Stat II: 2001 – 2002
Mid-semester Examination
Economic Statistics and Official Statistics

Duration: 3 hours   Maximum marks: 100   Date: 20 February 2002

[Answer any three questions]

1. (a) What do you mean by concentration in business and industry? Write down the criteria for a good measure of concentration in business and industry. Show how these criteria are satisfied by (i) Herfindahl – Hirschman index and (ii) Hall and Tideman index.
(b) How is it different from a measure of inequality? [26+4]

2. (a) State Pareto law. Give your comments on the universality of Pareto law stating the evidences for and against this law. How can you graphically test whether a given set of data is coming from a Pareto distribution? State and prove some properties of Pareto distribution.
(b) Suppose the mean incomes of bottom and top 50% people in a community are 1.2 and 4.8 units respectively. Assuming that income follows a Pareto distribution, (i) estimate the parameters of the distribution. Hence find (ii) the income below which there are 10% people and (iii) the share of income of the bottom 10% people in the community. [15+15=30]

3. Describe Positive and Normative Measures of Inequalities. Write down the desirable properties of a measure of inequality. Examine Coefficient of Variation in the light of these properties. [8+5+17=30]

4. Define Lorenz Curve (LC). State its properties. Derive LCs and Lorenz Ratios for Pareto and Lognormal distributions. Also state and prove the properties of LCs in each case. [1+4+20+5=30}

Home Assignments and Practical Assignments: [10]

INDIAN STATISTICAL INSTITUTE

B II Semester II

Statistical Methods IV

Midterm Examination

Total points 30

Clearly explain your assumptions and notations used so that all answers are self-contained and to the point

Date: February 22, 2002 Time: 3 Hours

1. Let X and Y have bivariate normal distribution with parameters $\mu_x = 5$, $\mu_y = 10$, $\sigma_x^2 = 1$ and $\sigma_y^2 = 25$. If $\rho > 0$, find ρ when $\Pr[4 < Y < 16 \mid X = 5] = 0.954$. [5]

2. Suppose that two random variables X and Y have a bivariate normal distribution and that $Var(X) = Var(Y)$. Are $X + Y$ and $X - Y$ independent? Justify your answer. [5]

3. Establish the algebraic identity $(\mathbf{Y} - \boldsymbol{\mu})' \Sigma^{-1} (\mathbf{Y} - \boldsymbol{\mu}) =$
$(\mathbf{Y}_2 - \boldsymbol{\mu}_2)' \Sigma_{22}^{-1} (\mathbf{Y}_2 - \boldsymbol{\mu}_2) +$
$((\mathbf{Y}_1 - \boldsymbol{\mu}_1) - \Sigma_{12} \Sigma_{22}^{-1} (\mathbf{Y}_2 - \boldsymbol{\mu}_2))' \Sigma_{11.2}^{-1} ((\mathbf{Y}_1 - \boldsymbol{\mu}_1) - \Sigma_{12}\Sigma_{22}^{-1} (\mathbf{Y}_2 - \boldsymbol{\mu}_2))$
where $\mathbf{Y} = (\boldsymbol{Y}_1, \boldsymbol{Y}_2)'$ follows p-variate normal with mean vector $\boldsymbol{\mu} = (\boldsymbol{\mu}_1, \boldsymbol{\mu}_2)'$ and dispersion matrix

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

and $\Sigma_{11.2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$. [8]

4. Suppose $\boldsymbol{X} \sim N_n (\mu \mathbf{1}, \sigma^2 [(1 - \rho) I + \rho J])$, where J is a matrix all of whose elements are 1. Show that $\sum (X_i - \bar{X})^2 \sim (1 - \rho) \chi^2_{n-1}$. [8]

5. If $\boldsymbol{u_i} \sim N_p (\mathbf{0}, \Sigma)$ independently for all $i = 1, \dots m$, what is the distribution of $A = \sum_{i=1}^m \boldsymbol{u_i} \boldsymbol{u_i'}$? Further, if Σ is nonsingular, what is the distribution of $\Sigma^{-1/2} A \Sigma^{-1/2}$? [4 = 1 + 3]

INDIAN STATISTICAL INSTITUTE
B.STAT-I (2001-2002)
Probability - II (Mid-Semester Examinations)
Max. marks:35 Time:$2\frac{1}{2}$ hours.

Date: 27 February, 2002

Note: ***Answer all questions. The maximum you can score is 35.***

1. Let F be the Distribution Function (D.F.) of a random variable X.
(a) Prove that F is right continuuous.
(b) Prove that for any real number x, $F(x) - F(x-) = P(X = x)$. Prove that the set of discontinuity ponts of F is a countable set.
[As usual $F(a-)$ denotes the left limit of F at a].
(c) Let $a > 0$ and $b \in \mathbb{R}$ be constants. Show that the function G defined by
$$G(x) = \begin{cases} 0 & \text{if } x < -\frac{b}{a} \\ F(\sqrt{ax+b}) - F((-\sqrt{ax+b})-) & \text{if } x \geq -\frac{b}{a} \end{cases}$$
also a D.F. (Hint: Try to find a functiion h such that the random variable $Y = h(X)$ has D.F. given by G.).

[3+5+4]

2. The D.F. of a random variable is given by
$$F(x) = \begin{cases} \frac{1}{8}e^{x} & \text{if } x < 0 \\ \frac{1}{8} & \text{if } 0 \leq x < 1 \\ \frac{1}{8}x + \frac{1}{16} & \text{if } 1 \leq x < 2 \\ \frac{1}{64}x^2 + \frac{1}{4} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3 \end{cases}$$
(a) Find the discontinuity points of this D.F.
(b) Find $P(-0.5 \leq X \leq 2.5)$ and $P(X^2 \geq 3X)$.
(c) Find the D.F. of the random variable e^{-X}.

[3+2+5]

3. Show that if U is a uniform $(0,1)$ random variable then $X = (-\frac{1}{\lambda}logU)^{\frac{1}{\alpha}}$ ($\lambda > 0$ and $\alpha > 0$), has the density given by
$$f(x) = \begin{cases} \lambda\alpha x^{\alpha-1}e^{-\lambda x^{\alpha}} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$
(Incidentally this density is called the Weibull(λ, α) density.).

[5]

(Please Turn overleaf)

4. Suppose X follows $N(0,\sigma^2)$ .Write down the density of X . Let

$$Y = \begin{cases} -X & \text{if } X < 0 \\ X^2 & \text{if } X \geq 0 \end{cases}$$

Find the density of Y.

[5]

3. Let X and Y be independent random variables each having the exponential distribution with parameter λ.

(a) Find the $P(X - Y \leq a)$, for $a \in \mathbb{R}$ (consider the two cases $a \geq 0$ and $a < 0$ separately). Hence find the density of the random variable $Z = X - Y$..

(b) Find $P(X + Y \leq a,\ \frac{X}{Y} \leq b)$, for $a > 0,\ b > 0$..

[(5 + 2) + 6]

INDIAN STATISTICAL INSTITUTE

Second Semestar Examination - 2001-2002

B. Stat (Hons.) - I Year

Subject : Analysis II

Date : 24.4.02 **Time : 3 Hours**

Maximum marks : 70
Answer five questions.

1. (a) Show that $\int_1^\infty \frac{\sin x}{\sqrt{x}} dx$ exists.

(b) Compute $\int_0^\infty \frac{\sin x}{x} dx.$ [7+7=14]

2. A sequence of functions f_n on an interval I is said to be equicontinuous if for every $x \varepsilon I$, given $\varepsilon > 0$, there exists $\delta > 0$ such that $|f_n(x) - f_n(y)| < \varepsilon$ for all $y \varepsilon I, |y - x| < \delta$ and $n = 1,2,...$

Let the sequence of functions f_n on the closed and bounded interval [a,b] be equicontinuous and let $\lim_{n \to \infty} f_n(x) = f(x)$, for all $x \varepsilon [a,b]$. Show that f is continuous on [a,b] and $f_n \to f$ uniformly on [a,b]. [14]

3. (a) Show that the function $\phi(t) = \log \Gamma(t)$ is convex on $(0, \infty)$.

(b) Show that (with ϕ as in (a))

$$\text{for } 0 < h < 1, \ \lim_{x \to \infty} \big(\phi(x+h) - 2\phi(x) + \phi(x-h)\big) = 0$$ [6+8=14]

4. If c and x are real, test for convergence the power series $\sum a_n x^n$ where

$$a_n = \frac{c(c+1)......(c+n-1)}{n!}$$ [14]

5.(a) Show that

$$\sum_{k=3}^{n} \frac{\log k}{k} = \frac{1}{2} \log^2 n + A + 0\left(\frac{\log n}{n}\right)$$

where A is a constant.

(Hint : compare with $\int_3^n \frac{\log t}{t} dt$)

P.T.O

(b) If p and q are fixed integers, $1 \leq q \leq p$ and $x_n = \sum_{k=qn+1}^{pn} \frac{1}{k}, n = 1,2,.....$

show that $\lim_{n\to\infty} x_n = \log\left(\frac{p}{q}\right)$ [7+7=14]

(Hint : use Euler's constant).

6. (a) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

(b) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$. Given that $\lim_{x\to 1-} f(x) = l$, and that $a_n \geq 0$ for all $n = 0,1,2,....$, show that $\sum_{n=0}^{\infty} a_n = l.$ [7+7=14]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination - 2001-2002

B.Stat (Hons.) - II Year

Subject : Elements of Algebraic Structures

Date : 26.4.02 **Time : 3 Hours**

Maximum marks : 100
You may answer all questions. Max. score would be 100.

1. (a) Show that a finite group G with atlest 3 elements, having an element a with exactly two conjugates in G, must have a nontrivial normal subgroup. [8]

 (b) Show that a group G of order 28 having a normal subgroup of aorder 4 is Abelian. [10]

2. (a) Prove that a group of order 30 has a normal subgroup of order 15. [10]

 (b) Let a,b be elements of an integral domain and let m,n be positive integers which are relatively prime. If $a^m = b^m$ and $a^n = b^n$, show that $a = b$. [10]

3. Prove or disprove

 (a) A subgroup of the multiplicative group of a field is cyclic [8]

 (b) A group of order p^2, p a prime , is Abelian [8]

 (c) The field of rational numbers has atleast one nontrivial automorphism [8]

4. (a) In a commutative ring R, an ideal P is called a prime ideal if

$$ab \varepsilon P, a,b \varepsilon R \text{ implies } a \varepsilon P \text{ or } b \varepsilon P$$

 Show that P is a prime ideal of R if and only if R/P is an integral domain [10]

 (b) If R is a finite commutative ring with unit element, show that every prime ideal of R is a maximal ideal in R. [10]

 (c) Let k be the smallest field containing the rationals Q and $\sqrt{2}+\sqrt{5}$. Find the dimension of k over Q. - Substantiate [10]

5. (a) Explain how to construct a field of size p^n for a prime p and positive integer n. Construct GF(16) and find a primitive element. [5+10]

 (b) Find all nonisomorphic Abelian groups of order, $2^4 3^3$. [10]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2001- 2002)

B.Stat (Hons) – II Year

Physics - II

Date : 29 . 4 . 02 Maximum Marks : 60 Duration : 3.00 Hrs

Note : The questions are divided into two groups - A and B. Students are required to answer five questions in all selecting three from group A and two from group B. All questions carry equal marks.

QUESTIONS

Group A (Introduction to quantum theory and quantum mechanics)

Answer any three questions

1. What is a black body? Show that when a black body is in equilibrium with radiation, the energy density of radiation is proportional to fourth power of temperature. Give an outline of derivation of the Rayleigh – Jean's energy distribution for black body radiation. Discuss the defect of this law.

(1 + 5 + 5 + 1)

2. What are the basic assumptions of Planck's theory of black body radiation? At what point does it differ from classical ideas? Derive Planck's energy distribution law. Derive Wein's displacement law from Planck's distribution law.

(1 + 1 + 7 + 3)

3. What are the main features of photo-electric effect? Describe how these features can be explained by quantum theory of light. Describe how the particle nature of light is revealed in Compton effect. Find the expression for change in wavelength in Compton scattering as function of the scattering angle θ (say).

(2 + 3 + 2 + 5)

4. Solve the Schrodinger equation for a particle in a one dimensional box with the specification

$$V(x) = \infty \quad ; \; x < 0 \text{ and } x > L$$
$$V(x) = 0 \quad ; \; 0 \leq x \leq L$$

and find out the energy eigenvalues and eigenfunctions. Find the expectation values for the position and momentum of the particle in this box.

(8 + 4)

P. T. O

Group B (Classical thermodynamics)

Answer any two questions

1. Following Maxwell, deduce the expression for the molecular velocity distribution function (in the case of a perfect gas kept at a steady temperature of T degree Kelvin) in the framework of kinetic theory of gases. How is this result experimentally verified?

(8 + 4)

2. State and explain the first law of thermodynamics in the context of thermodynamic system. Apply this law to obtain the efficiency of a heat engine working in a reversible Carnot's cycle, using a perfect gas as the working substance.

(2 + 2 + 8)

3. Define and explain - irreversibility , reversibility, and entropy, in the context of thermodynamics. Obtain an expression for the entropy of a perfect gas.

(6 + 6)

INDIAN STATISTICAL INSTITUTE
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B. STAT. (Hons.) II Year (2001-02)

SECOND SEMESTRAL EXAMINATION

ECONOMICS – II

Date : 29.04.02 Maximum Marks : 100 Time : 3.5 hrs.

Answer any five questions taking at least two from each group. Use separate answer books for Group A and Group B.

GROUP – A

1. a) Consider the following data for an economy :

	Period 1	Period 2
New buildings produced	5	5
New equipment produced	10	10
Consumer goods produced	110	90
Estimated depreciation on existing buildings	10	10
Estimated depreciation on existing equipment	10	10
Inventories of consumer goods at the beginning of the year	30	50
Inventories of consumer goods at the end of the year	50	30
Consumer goods consumed	90	110

Compute for each period GDP, break up of GDP into C and gross I: NDP and break up of NDP into C and net I.

b) This is a problem on national income accounting. Consider an economy where personal disposable income is 1,,000 units. Undistributed profits are 65 units. Gross investment is 180 and net investment is 140. The government makes transfer payments of 100 units and interest payment of 30 units. The corporate profit tax amounts to 50. Indirect business taxes are 190. The government collects 150 units in personal income taxes and social security contributions. Public sector enterprises make a loss of 4 units. Business transfer payments are 0. Given this information, what is GDP?

(10+10=20)

P.T.O

2. a) Consider a simple Keynesian Model for a closed economy with government where the tax function is given by $T = \overline{T} + t.y$, and investment is autonomous. $(0 < t < 1)$

If the government wants to increase total tax collection by a given amount (say Δ) by altering lumpsum tax $\overline{T}$, keeping t unchanged then it will have to raise $\overline{T}$ by an amount larger than Δ. Explain.

b) In the same model an increase in G by 10 units is found to increase budget deficit by 3.75 units. If the marginal propensity to spend out of disposable income is 0.8, estimate the tax function provided $\overline{T} = 40$.

(10+10=20)

3. a) Explain the concept of Paradox of Thrift in a simple Keynessian Model.

b) Consider a simple Keynessian Model for a closed economy where investment is an increasing function of GDP (y), the marginal propensity to invest being 0.1. A shift in the saving function by 10 units reduces aggregate planned saving by 5 units. Compute the autonomous expenditure multiplier in the model.

(10+10 = 20)

GROUP – B

1. a) Let K_e be the expenditure multiplier, b, the interest sensitivity of private sector spending, h the interest sensitivity of the demand for money and K the transaction demand for money as a proportion of income. From the following sets of values for K_e, b, h and K, find one in which a change in the supply of money will have the largest multiplier effect on output.

(i)	$K_e = 5$,	$b = 5$,	$h = 5$	and	$K = .20$
(ii)	$K_e = 4$,	$b = 1$,	$h = 5$	and	$K = 0.20$
(iii)	$K_e = 5$,	$b = 10$	$h = 1$	and	$K = 0.20$
(iv)	$K_e = 4$,	$b = 5$,	$h = 10$	and	$K = 0.10$

b) From the following set of values for K_e, b, h and K, find the set in which a change in government spending has the largest multiplier effect on output.

(i)	$K_e = 5$,	$b = 5$,	$h = 5$	and	$K = 0.20$
(ii)	$K_e = 10$,	$b = 5$,	$h = 10$	and	$K = 0.20$
(iii)	$K_e = 5$	$b = 10$	$h = 1$	and	$K = 0.20$
(iv)	$K_e = 5$,	$b = 5$,	$h = 1$	and	$K = 0.20$

(20)

2. a) Establish the direction and magnitude of the shift of LM curve when (1) K = 0.20, and there is a \$ 20 increase in the money supply (2) K = 0.50 and there is a \$20 decrease in the money supply (3) K = 0.25 and there is a \$ 20 decrease in the money supply.

b) Discuss the concept 'Crowding Out' and 'lequidity trap'

c) Take into account the roles of the money market and goods market together in obtaining the equilibrium of income and interest rate. Consider a change in autonomous expenditure (ΔG) by the government.

When do you expect the multiplier effect of ΔG to be materialised to the full extent. Assume a situation of less than full employment to start with.

(20)

3. a) Consider an economy (closed) with a marginal propensity to consume ½ and an accelerator of 2. Let us assume further that a rise in income in one period affects consumptions only in the following period. Let net investment rise by K 100 per period. Show the process by which the growth and fluctuations of income take place under the above circumstances.

b) "A stationary economy though may have some positive gross investment, net investment will be zero". Explain and comment.

(20)

4. a) Show how an economy having a constant rate of investment will have accumulation of unutilised capacity over time.

b) Consider the following Macro economic system.

Supply of labour function : $L = 20 + 5\dfrac{w}{p}$

Demand function for labour : $N = 80 - 10\dfrac{w}{p}$

Being consistent with the following assumed aggregate production employment relationship which exhibits classical diminishing returns,

$Y = 8N - 0.05N^2$

And the quantity theory equation

$M = 100 = 0.5PY$

Obtain values of equilibrium W, P, Y,N.

Now compute the effect of rise in supply of Money from M = 100 to M = 150.

Note : The notations have the usual menaings.

(20)

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination (2001 – 2002)

B. Stat II (Hons.)

Group - A

Subject : Demography

Date 2.5.02 **Maximum Marks : 50** **Duration : 2 hours.**

Note : Answer Question No. 6 and any Two from the rest.

1. What are the main sources of demographic data ?
   Discuss the pitfalls of birth and death registration in India.

   Describe the sample registration system of India and discuss the procedure adopted to estimate birth rate for India and various states for each year.

   2+6+4+8=20

2. Describe the procedure of calculation of preference indices for terminal digits by Myers Blended Method.

   Calculate Myers' Index when

   $P_x = a+bx$, x=10,11,12 ...., 109.

   10+10=20

3. a) Explain the term 'Person Years' lived in a population during a calendar year and its importance in computation of vital rates. Why is it estimated by the mid-year population ?

   b) Distinguish between the direct and indirect methods of standardisation of death rates.

   c) What do you mean by infant mortality rate ? Describe the different methods of computing the infant mortality rate. Comment on their merits and demerits.

   6+6+8=20

P.T.O

4 a) Derive $p(t) = p(o)\, e^{rt}$, assuming growth rate of population is constant in the interval (o,t)

where : $p(o)$ = Population at base year,
$p(t)$ = Population at time 't', and
t = Interval of time in years.
r = Growth rate of the population over o to t.

b) Distinguish between 'period' and 'cohort' lifetables

c) If $\mu_x = A+BC^x$, then show that

$$l_x = KS^x g^{c^x}$$

where μ_x = force of mortality at age x, and
l_x = number of persons at exact age x .

d) Derive mathematically the relationship between total fertility rate and crude birth rate when there is a correlation between age structure and fertility of the female population.

5+5+5+5=20

5. a) What is the difference between 'mean age at marriage' and ' singulate mean age at marriage' ? Discuss a method for calculating the singulate mean age at marriage if it is known that 2 per cent women will never marry at all.

b) Define and explain **any two** of the following :
i) Net Reproduction Rate
ii) Parity Progression Rate
iii) Sex age adjusted birth rate

12+8=20

6. Write short notes on **any two** of the following :
a) Lexi's diagram;
b) Zelnik's method of smoothing data;
c) Logistic model of population growth;
d) Stationary population and stable population;

5+5=10

Group B: SQC & OR

Maximum Marks: 50 Time :1 hr. 30 mts.

NOTE: Answer any **two** questions in a separate script.

1. A control chart is to be set up on a process producing refrigerators.The inspection unit is one fully assembled refrigerator , and a chart for defects per unit of refrigerator is to be used. As preliminary data, 15 defects were counted in inspecting 30 refrigerators.
   (a) What control chart is to be used here?
   (b) What are the corresponding 3-sigma control limits here?
   (c) A sample point violates the Upper Control Limit.What is the probability of falsely concluding that change in the level of defects has occurred even though there is no real change in the level of defects?
   (d) All the points plotted so far in the control chart are within the control limits. What is the probability of falsely concluding that no change has occurred when actually the average number of defects has gone up to 2.0?
   (e) Find the Average Run Length if the average number of defects has increased to 2.0.

(3+5+5+5+7)

2. (a) What is Acceptance Sampling ?
   (b) What is an Acceptance Sampling Plan used for deciding acceptability/rejectability of lots of products submitted for inspection by a customer?
   (c) The quality characteristic (X) of a product is Normally distributed with known σ. The Upper Specification Limit for X is U.The variable sampling plan for lot acceptance/rejection is operated as follows :

   Accept the lot if $\bar{x} + k\sigma < U$, otherwise reject the lot,
   $\bar{x}$ being the sample average based on n measurements.

   Derive the expressions for **n** and **k** under usual assumptions.

(5+5+15)

3. (a) State the characteristic features of a Linear Programming problem.
   (b) A company makes two products P1 and P2.The profit per tonne of the two products are Rs.50 and Rs.60 respectively. Both the products require processing in three machines A, B, and C.The relevant information are as follows.

Machine	Machine-hour requirement /tonne for P1	P2	Total available machine hrs. per week
A	2	1	300
B	3	4	509
C	4	7	812

Formulate the problem as a Linear Programming Problem and solve it by Simplex Algorithm, to determine the amount of P1 and P2 to be manufactured per week to maximise the profit.

(5+20)

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INDIAN STATISTICAL INSTITUTE

B II Semester II

Statistical Methods IV

Semestral Examination: Spring 2002

Total points 70

Date: 6.5.02 Time: 3:00 hours

Clearly explain your assumptions and notations used so that all answers are self-contained and to the point

1. Let X and Y be distributed in the bivariate normal form with means equal to 0, variances σ_x^2 and σ_y^2 and correlation coefficient ρ. Show that $\rho_{X^2,Y^2} = \rho^2$. [6]

2. Given the random variables $X_1, \ldots, X_p$ with associated dispersion matrix Σ,

   (a) Define multiple and partial correlation coefficients. [2 + 2 = 4]

   (b) If $\rho_{1j} = \rho$, $j = 2, 3, \ldots, p$ and $\rho_{ij} = \rho'$, $i, j = 2, 3, \ldots, p, i \neq j$, what are the values of the multiple correlation coefficient $R_{1.23\ldots p}$ and partial correlation coefficient $\rho_{12.34\ldots p}$? [4 + 4 = 8]

3. Consider a sample of size n, (X_i, Y_i), $i = 1, \ldots, n$ from an arbitrary bivariate distribution whose moments of all orders (r, s) exist such that $r + s \leq 4$, and $\mu_{20} > 0, \mu_{02} > 0$.

   (a) Find asymptotic distribution of sample correlation coefficient. [8]

   (b) Suppose now the joint distribution of (X, Y) is bivariate normal $N_2(0, 0, 1, 1, \rho)$. Indicate and justify a method for construction of confidence interval for the population correlation coefficient. [6]

4. Let $\boldsymbol{X} \sim N_n(\boldsymbol{\mu}, I_n)$. Define $Q_i = \boldsymbol{X}' A_i \boldsymbol{X}$ where A_i is a symmetric matrix of order n and rank r_i for $i = 1, \ldots, k$. Show that if $A_i A_j = 0, i \neq j, i, j = 1, \ldots, k$ then Q_i, Q_j are independently distributed for all pairs of i, j. [10]

P. T.

5. Suppose that $X_1, \ldots, X_n$ represent a sample from a uniform distribution on (0, 1). Find the correlation coefficient between $X_{(r)}$ and $X_{(s)}$, $r < s$, where $X_{(i)}$ is the i-th smallest order statistic. [10]

6. 30 subjects were examined in a clinical comparison of the effects of 3 drugs (labeled 1, 2 and 3) on controlling blood sugar levels. Each subject used each drug for a two month period. At the end of each two month period measurements were made on each subject. The sample mean and sample covariances are $\bar{X} = \begin{bmatrix} 15 \\ 10 \\ 12 \end{bmatrix}$ and $S = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 2 & 6 \end{bmatrix}$ We will like to test whether the mean response levels are the same for all the three drugs. Write down the appropriate hypothesis and perform a test for it at level $\alpha = 0.05$. [10]

7. Suppose that $Y_1, \ldots, Y_n$ are independent random variables such that $Y_i = \alpha + \beta x_i + \epsilon_i$, where α and β are unknown parameters, x_i are known fixed constants, and ϵ_i, $i = 1, \ldots, n$ are independent $N(0, \sigma^2)$ random variables. Let $\widehat{\alpha}$ and $\widehat{\beta}$ be the least squares estimates of α and β. Define $\sum_{i=1}^{n} (Y_i - \widehat{\alpha} - \widehat{\beta} x_i)^2$ to be the residual sum of the squares, and let

$$\widehat{\sigma}^2 = \frac{\sum_{i=1}^{n} (Y_i - \widehat{\alpha} - \widehat{\beta} x_i)^2}{(n-2)}.$$

Show that

$$\frac{(n-2)\,\widehat{\sigma}^2}{\sigma^2} \sim \chi^2_{(n-2)}$$

and is independent of $\widehat{\alpha}$ and $\widehat{\beta}$. [8]

INDIAN STATISTICAL INSTITUTE

B. Stat II: 2001 – 2002

Semester Examination

Economic Statistics and Official Statistics

Duration: 3 hours　　Maximum marks: 100　　Date: 9 May [redacted]

*[Answer any **four** questions from Group A and any **one** from Group B. Al[redacted] questions carry equal marks.]*

<u>Group A: Economic Statistics</u>

1. Write down the desirable properties of a measure of inequality. Give examples of some positive measures of inequality. Derive Atkinson's measure of inequality based on Social Welfare Function Approach. Do you think it is a superior measure to Positive Measures of inequality? Discuss. [5+5+7+3=20]

2. Is it true that the demand for a commodity will always increase if the mean income increases and the inequality of income decreases for a given group of people? Explain your answer assuming a suitable form of the Engel curve and a specific income distribution. [20]

3. When will you test for autocorrelation while estimating Engel Curve? Write down important steps towards derivation of bounds for the distribution of DW statistic and hence state DW test. Why is the test so popular in Engel Curve Analysis? [2+15+3=20]

4. Write down the five axioms of the economic-theoretic approach to Price Index Numbers. Prove that none of these is superfluous. Give examples of at least five index number formulae satisfying all these five axioms. [7.5+10+2.5=20]

5. Write short notes on any two of the following:

   (a) Law of Proportionate Effect.
   (b) Measures of Poverty.
   (c) Fixed base indices vs. Chain base indices. [10+10=20]

<u>Group B: Official Statistics</u>

1. (a) Give an account of household budget data collected by NSSO, mentioning the important concepts, definition and procedures adopted for such data collection.
   (b) How is absolute poverty in India measured utilizing this body of data? [12+6=20]

2. (a) What type of information is collected in the decennial censuses of India's population? How reliable are the data on various items collected in these censuses?
   (b) Write a short note on the Sample Registration System. [14+6=20]