INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR:1998-99
SEMESTRAL-I EXAMINATION
PHYSICS-I

Date:2.11.98 Maximum Marks:60 Time: 3 Hours

Note: Answer any five questions. All questions carry equal marks.

1.(a) Define and explain:-
i) a thermodynamic system,
ii) a thermodynamic process,
iii) thermodynamic equilibrium.

(b) State and explain in all essential details the ''zeroeth'' and the ''first'' laws of thermodynamics. Critically comment on their scopes and limitations. How are the limitations overcom

2.(a) Describe with neat diagrams, the working of a model heat engine operating in a reversible (carnot's) cycle and obtain an expression for its efficiency.

(b) Give a brief account of the ''Maxwellian distribution'' of molecular velocities in a gas kept at a steady temperature of $T^{o}K$. Describe an experiment for its verification.

3.(a) What are Newton's rings? How are they produced? Give a brief theoretical account of them.

(b) Give a brief account of the Michelson's interferometer and its working.

4. Explain the phenomenon of ''diffraction'' of light. How does it differ from interference? Describe with necessary theory, the intensity distribution of light in the diffraction (Fraunhofer) spectra exhibited by a narrow slit of width d, when it is illuminated by a monochromatic light beam having warelength λ. What happen if

i) $d \cong \lambda$? ii) $d >> \lambda$?

5. Define ''Polarization'' of light. Show that any polarized light is equivalent to superposition of two mutually perpendicular, linearly polarized light waves.
What do you mean by ''unpolarized'' light? Briefly explain the working of a Polaroid.

6.(a) What do you mean by constraints of a mechanical system? Define holonomic and non-holonomic constraints. Give an example for ea

(b) Starting from D'Alembert's Principle, derive the Lagrange's equations of motion for an N-particle mechanical system having K no. of holonomic constraints, assuming that forces are conservative.

p.t.o.

6. (c) Define generalized force. Derive the expression for radial and transverse components of acceleration for a particle m in a plane by the use of Lagrange's equations of motion.

7. (a) Under what conditions, the total angular momentum of a sys of N-particles is conserved?

(b) Write down the equation of motion of a particle subjected t an inverse square law force using proper arguments. Comment on the nature of the trajectory described by a particle und an inverse square law force.

(c) Write down the Hamiltonian for a particle of mass m, movin in a central force field (whose potential is V(r)) in polar coordinates and hence obtain the equations of motion. What the characteristic conserved quantity in this motion?

Indian Statistical Institute

B.Stat. (Hons.) II year

First Semestral Examination, 1998-99

Biology-I

Maximum marks: 60 *Date: 03.11.98* *Duration: 2 hr*

Answers should be brief, All question carry equal marks.

1. (a) "Quality and quantity of proteins are important for growth of human" –Justify

   (b) "Quality of rice protein could be improved by supplementation" – explain

2. Describe oxidation/catabolism and energetics of fat and fatty acids with an example.

3. State Mendel's Laws of Inheritance. Explain dominant and recessive traits with examples.

4. Distinguish between :

   (a) Cilia and flagella

   (b) Chromatin and chromosome

   (c) Plasma membrane and cell-wall

   (d) Mitochondria and chloroplast.

5. What are the differences between :

   (a) plant cell and animanl cell

   (b) eukaryotes and prokaryotes.

6. What are the differences between DNA and RNA. Explain replication of DNA in the context of cell division.

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INDIAN STATISTICAL INSTITUTE

B-Stat (Second Year)

Statistical Methods III

Semestral Examination (November 6, 1998)

TOTAL MARKS : 100 TIME ALLOWED : 3 hours

Answer All Questions.

(1). Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations with a common $N(\mu, \sigma^2)$ distribution, where $-\infty < \mu < \infty$ and $0 < \sigma < \infty$ are both unknown parameters. Derive expressions for maximum likelihood estimates for μ and σ^2 based on these observations (justify your answer). Are the maximum likelihood estimates for μ and σ^2 unbiased for the respective parameters ? (Justify your answer). What will be the forms of the maximum likelihood estimates for μ and σ^2 if the observations are independent but X_i has distribution $N(\mu, \sigma^2/i)$ for $i = 1, 2, \ldots, n$? (Justify your answer).

[$8 + 4 + 8 = 20$]

(2). Consider i.i.d. observations $X_1, X_2, \ldots, X_n$ with a common probability density function $f(x - \theta)$. Here $f(t)$ is a symmetric density around $t = 0$, and it is continuous and positive at $t = 0$. Prove that the sign test for $H_0 : \theta = 0$ against $H_A : \theta \neq 0$ in this set-up is a consistent test for any specified level $0 < \alpha < 1$ as the sample size $n \longrightarrow \infty$.

[15]

(3). The initial weights (X) and gains in weight (Y) (in grams) of 13 female rats on a high protein diet that was administered from the 24th day to the 84th day of their age are given below :

$X_1 = 50, Y_1 = 128$; $X_2 = 64, Y_2 = 159$; $X_3 = 76, Y_3 = 158$;
$X_4 = 64, Y_4 = 119$; $X_5 = 74, Y_5 = 133$; $X_6 = 60, Y_6 = 112$;
$X_7 = 69, Y_7 = 96$; $X_8 = 68, Y_8 = 126$; $X_9 = 56, Y_9 = 132$;
$X_{10} = 48, Y_{10} = 118$; $X_{11} = 57, Y_{11} = 107$; $X_{12} = 59, Y_{12} = 106$;
$X_{13} = 46, Y_{13} = 82$.

The interest is in studying the extent of dependence of the weight gain on the initial weight. Assuming a simple linear regression model $Y_i = \alpha + \beta X_i + e_i$, where the e_i's are i.i.d. $N(0, \sigma^2)$ random variables ($0 < \sigma < \infty$ is unknown), compute 95% confidence intervals for the unknown parameters α and β based on this data.

[25]

(4).

(a). Suppose that the observed value of the χ^2 statistic based on a 2×2 table for testing independence against association is 3.97. What is the corresponding P-value ?

(b). Consider Wilcoxon's rank statistic for the two sample problem with no ties. Assume that the two sample sizes are $m = n = 3$, and the observed value of Wilcoxon's statistic (= rank sum of the data values in the first sample) is 6. If the objective is to test the null hypothesis that the two samples originated from the same distribution against the alternative hypothesis that the distribution of the first sample is stochastically smaller than that of the second sample, what is the P-value corresponding to this observed value of Wilcoxon's statistic ?

[$4 + 6 = 10$]

(5). Suppose that our observation X has double exponential distribution with density $(1/2)\exp(-|x-\theta|)$, where $-\infty < \theta < \infty$ is the unknown parameter. If θ has prior density $(1/2)\exp(-|\theta|)$, compute the expression for Bayes estimate for θ based on the observation X and this prior. Your estimate should be in a form as simplified as possible.

[15]

(6). Assignments

[15]

INDIAN STATISTICAL INSTITUTE

B.Stat (Hons). II year (1998-99)

Probability 3 Semestral Exam

Date 9-11-98 Maximum Marks- 60 Duration: 3 hours

1. k is an integer > 1. Let $X_1, X_2, \ldots, X_{k+1}$ be i.i.d Standard Normal variables. For $1 \le i \le k$ define $Y_i = X_i^2 / \sum_{i=1}^{k+1} X_i^2$. Show that $Y = (Y_1, Y_2, \ldots, Y_k)$ has Dirichlet distribution. What are its parameters ? [8]

2. Let $Y_1 < Y_2 < Y_3$ be an order statistic of size 3 from the distribution having density : $f(x) = 2x \ for \ 0 < x < 1$. Show that $\frac{Y_1}{Y_2}, \frac{Y_2}{Y_3}, Y_3$ are independent. [8]

3. X is uniform (0, 1). For $x \in (0, 1)$, given $X = x$ the conditional distribution of Y is uniform $(x - \frac{1}{2}, x + \frac{1}{2})$.
   a) Find the joint pdf of (X, Y).
   b) Find the marginal density of Y.
   c) For what values of y is the conditional density of X given $Y = y$ defined ? For such a y what is the conditional distribution of X given $Y = y$. [3 + 3 + 4 = 10]

4. F_n, and F are one dimensional cdfs. Assume that $F_n(r) \to F(r)$ for every rational r. Show that $F_n \Rightarrow F$. [8]

5. $X_n \Rightarrow X$ and $P(X = 5) = 1$.
   a) Show that $X_n \to X$ in Probability.
   b) Show that X_n need not converge to X almost everywhere. [3 + 5 = 8]

6. X is a random variable with c.f φ. I have a sequence of numbers t_n, each $t_n \neq 0$, $t_n \to 0$ and $|\varphi(t_n)| = 1$ for each n. Show that there is a real number c such that $P(X = c) = 1$. [8]

7. I have a coin whose chance of heads in a single toss is p, $0 < p < 1$. Let T_n be the proportion of heads in n tosses of the coin. Show that $\sqrt{n}(Sin^{-1} T_n - Sin^{-1} p)$ is asymptotically normal. What is the asymptotic variance ? [8]
   Here Sin^{-1} is the inverse function of $Sin x$ $[0, \frac{\pi}{2}] \to [0, 1]$.

8. $X \sim N_k(\underline{0}, \Sigma)$. Show that the Quadratic form $X'AX$ is χ^2 if and only if $A\Sigma A = A$. In such a case what is the degrees of freedom of the χ^2 variable ? [8]

INDIAN STATISTICAL INSTITUTE

Final Examination : Semester I (1998-99)

B. Stat. II

Calculus III

Date : 11.11.98 Maximum Marks : 100 Duration : $3\frac{1}{2}$ hrs.

This paper carries 110 marks. Maximum you can score is 100. Precisely justify all your steps. Carefully state all the results you are using.

1. Decide whether the following statements are true or false. In each case, give reasons for your answer.

   (i) If R is the region enclosed by a smooth Jordan curve C parametrized by $(X(t), Y(t))$, $t \in [a, b]$. Then the area of R is given by

   $$\frac{1}{2}\int_a^b [X(t)Y'(t) - X'(t)Y(t)]dt$$ [8]

   (ii) The centroid of the region enclosed by the curve $y = \sin x$ on $[0, \pi]$ and the x-axis is given by $(\pi/2, \pi/8)$. [6]

   (iii) The function

   $$f(x,y) = \begin{cases} \dfrac{xy^3}{x^3 + y^6} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{otherwise} \end{cases}$$

   has directional derivative $f'(0; a)$ for every vector $a \in \mathbb{R}^2$ and f is continuous at the origin. [6]

   (iv) The vector field $f(x,y) = (3x^2y, x^3y)$ is a *gradient* on any open subset of $\mathbb{R}^2$. [5]

2. Let S be the region in $\mathbb{R}^2$ enclosed by the curve whose polar equation is given by

   $$r = 1 - \cos\theta, \quad \theta \in [0, 2\pi]$$

   Sketch the region in the xy-plane and find the integral of the function $f(x,y) = x^2$ over S. [5 + 10 = 15]

3. Find the volume common to the two cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. [15]

4. Let $\mathbf{r}(x, y, z) = (x, y, z)$ and $r = \|\mathbf{r}\|$.

   (a) Let $g : \mathbb{R}^+ \to \mathbb{R}$ be continuously differentiable. Define a scalar field ϕ on $S = \mathbb{R}^2 \setminus \{0\}$ by $\phi(\mathbf{r}) = g(r)$. Show that

$$\nabla\phi(\mathbf{r}) = \frac{g'(r)}{r}\mathbf{r}$$

   (b) Let $G(\mathbf{r}) = r^p\mathbf{r}$, $p \in \mathbb{R}$. Show that G is a gradient on $\mathbb{R}^3$ and find the corresponding potential.

   (c) Show that $\phi(\mathbf{r}) = \dfrac{1}{r}$ is *harmonic* on S, i.e., $\nabla^2\phi = 0$. [5 + 4 + 6 = 15]

5. Let D denote the open unit disc in $\mathbb{R}^2$. Let f be harmonic on D (i.e., $\nabla^2 f = 0$). Assuming continuity of the mixed partial derivatives, show that for any $r \in (0, 1)$,

$$f(0,0) = \frac{1}{2\pi}\int_0^{2\pi} f(r\cos\theta, r\sin\theta)d\theta$$

[Hint : Define the RHS as a function of r, find its derivative by differentiating under the integral sign (justify this!) and evaluating it using the Green's Theorem on the disc of radius r.] [20]

6. On $\mathbb{R}^2$ or $\mathbb{R}^3$, let r denote the distance from the origin. Let $n \geq 0$. Let

   (a) S be the region in $\mathbb{R}^2$ between two concentric circles of radius a and b respectively, with $0 < a < b$; or,

   (b) S be the volume in $\mathbb{R}^3$ between two concentric spheres of radius a and b respectively, with $0 < a < b$.

   Let $I_n(a, b)$ denote the integral of $f(\mathbf{r}) = \dfrac{1}{r^n}$ over S. In both cases, find $I_n(a, b)$ and the values of n for which $I_n(a, b)$ converges to a limit as $a \to 0$. [(7 + 3) × 2 = 20]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year: 1998-99

Economics - I

Date: 13.11.1998 Maximum Marks: 60 Time: 3 hours

Note: Answer ALL questions from each group. The question paper is set for full marks of 70, but the maximum that you can score is 60.

Answer Group A and Group B in separate answerscripts.

GROUP - A

1.(a) The long-run cost function for each firm that supplies Q is $C = q^3 - 4q^2 + 8q$. Firms will enter the industry if profits are positive and leave the industry if profits are negative. Describe the industry's long-run supply function. Assume that the corresponding demand function is $D = 2{,}000 - 100P$ where P is the price of the product. Determine equilibrium price, aggregate quantity and number of firms.

(2+3) = [5]

(b) A firm has a production function $y = x_1 x_2$. If the minimum cost of production at factor prices $w_1 = 1 = w_2$ is equal to 4, what is y equal to ?

[3]

2. A good Q is produced using only one input x. Let P be the price of the good. The market for Q is supplied by 100 identical competitive firms, each of which has the production function $q = \sqrt{x}$. Each firm behaves as if the price of x (denoted by r) were constant. Derive the industry's supply curve in each of the following cases. Also comment on the slope of the industry's supply curve.

(a) The industry as a whole faces a horizontal supply curve for input x such that r = 10 at any level of supply.

(b) The industry as a whole faces a vertical supply curve for input x such that its supply is 100 units irrespective of the price of x.

(7+8) = [15]

3. Consider the market for a homogeneous product. There are only two firms, 1 and 2, to supply the good. The market demand curve is given by $P = 20 - q_1 - q_2$ where P is the

contd..... 2/-

price of the product and q_i is quantity supplied by the i-th firm. The i-th firm's cost function is :

$$C_i(q_i) = \begin{cases} F + a_i q_i & \text{if} \quad q_i > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose firms have Cournot conjectures and they choose quantities simultaneously and non-cooperatively. Find t equiliberium quantity and profit for each firm under the following two situations;

(i) $F = 5, \quad a_1 = 11 = a_2$

(ii) $F = 5, \quad a_1 = 5, \quad a_2 = 11.$

If, instead of quantities, firms would choose prices simu neously and non-cooperatively, what would be the correspo ding equilibrium values in each of the above two situati

(2 x (2+4)]= [12

GROUP - B

4. A utility maximising consumer has strictly convex indiffe ence curve between quantities of two goods x_1 and x_2. Each good has a price of 1. He cannot consume negative amount of any good and has an income of m every year. Hi current level of consumption is (x_1, x_2) where $x_1 > 0$ a $x_2 > 0$. Suppose that next year he will be given a grant $g_1 (= x_1)$ which must be spent on good 1 only. (If he wish he can refuse to accept the grant).

(a) Show graphically the budget line of the consumer for the next year.

(b) Explain (graphically or otherwise) whether the following statement is true or false:

If good 1 is an inferior good for the consumer at all inc $m > x_1 + x_2$, then the effect of the grant on his consump tion bundle must be the same as the effect of an uncons- trained lumpsum grant of an equal amount.

(2+10) = [1

5. A monopolist in a certain country has its domestic market protected by law from import competition. The domestic demand curve for its product is given by

contd..... 3/-

$p_\alpha = 120 - \frac{q_\alpha}{10}$ where p_α and q_α are price and quantity demanded in the domestic market. The firm can sell <u>any</u> amount q_w in the world market at a constant price p_w (that is, the firm is a price taker so far as the world market is concerned).

His MC is given by : $MC = 50 + \frac{Q}{10}$, where $Q = q_\alpha + q_w$.

(a) Find the optimum overall output Q and its division between the two markets. Show it graphically.

(b) Compare demand elasticities and price in the domestic versus world market.

(c) Suppose the monopolist could sell only in the domestic market. Will it then <u>necessarily</u> produce more or less output than in (a) ? Explain.

(8+2+3) = [13]

Person A has a utility function: $U_A(x_1, x_2) = \max(x_1, x_2)$ and person B has a utility function: $U_B(x_1, x_2) = x_1 + x_2$ (Here x_1, x_2 are quantities of two goods consumed by the person in question).

Further, A and B have identical endowments of $(\frac{1}{2}, \frac{1}{2})$.

(a) What is the shape of the indifference curve of person A ? Show it graphically.

(b) Draw the Edgeworth box diagram and find a Walrasian equilibrium, i.e., Walrasian relative price vector and allocation of two goods.

(2+8) = [10]

INDIAN STATISTICAL INSTITUTE

Final Examination : Semester II (1998-99)

B. Stat 2nd Year

Statistical Methods IV

Date: 19.4.99 Maximum marks: 100 Duration: 3 hours.

Note: This paper carries 110 marks. The maximum you can score is 100.

. If the rows of $\mathbf{X}$ are i.i.d. $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and C_1 and C_2 are symmetric matrices, show that $\mathbf{X}'C_1\mathbf{X}$ and $\mathbf{X}'C_2\mathbf{X}$ are independent if $C_1C_2 = 0$. [10]

. (For this problem you may use results based on partitioned Wishart matrices proved in class, but state them very clearly.)

(a) Derive the relationship between the Hotelling T^2 and F distributions.

(b) Derive the one-sample Hotelling T^2 statistic and show that it is invariant under any nonsingular linear transformation.

(c) Show that the distribution of the Mahalanobis distance between two p-dimensional multivariate normal distributions with common unknown dispersion matrix $\boldsymbol{\Sigma}$ is a constant multiple of an F distribution. [7 + 7 + 6] = [20]

. Suppose that $\mathbf{X}$ follows a $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ distribution, and we want to test the hypothesis $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_1$ against the alternative $H_1 : \boldsymbol{\mu} = \boldsymbol{\mu}_2$ based on a single observation when $\boldsymbol{\Sigma}$ is known. Let $\mathbf{U} = (\mathbf{X} - \frac{1}{2}(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2))'\Sigma^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)$.

(a) Express the likelihood ratio test statistic in terms of $\mathbf{U}$.

(b) Find the distribution of $\mathbf{U}$ under H_0 and H_1.

(c) Find the probabilities of type I and type II errors and express them in terms of the c. d. f. of the univariate normal distribution and the Mahalanobis distance between

4. Let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be i.i.d. observations from $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. We want to test the hypothesis $H_0 : \mathbf{R}\boldsymbol{\mu} = \mathbf{r}$.

   (a) Describe the likelihood ratio test procedure for testing H_0.

   (b) Show that the union intersection test procedure yields the same test statistic in this case. [10 + 10] = [20]

5. Consider the multivariate regression model $\mathbf{Y} = \mathbf{XB} + \mathbf{U}$ where rows of $\mathbf{U}$ are i.i.d. from a $N_n(0, \boldsymbol{\Sigma})$ distribution.

   (a) Find the maximum likelihood estimates of $\mathbf{B}$ and $\boldsymbol{\Sigma}$.

   (b) Show that the above estimates are independent.

   (c) Derive the distributions of these estimates. [8 + 5 + 7] = [20]

6. Suppose $X_1, \ldots, X_n$ is a random sample from Uniform$(0, \theta)$ distribution. Find an estimate of θ which has asymptotic normal distribution with variance equal to $\frac{1}{n}$. [10]

7. In a demographic study of 1436 women who were married at least once it was found that 205 of them got married more than once. Of these 205 women 61 had a college degree while 550 of the rest of the women had a college degree. Test whether marital stability is independent of educational level based on this data. [10 points]

INDIAN STATISTICAL INSTITUTE
B.STAT-I (1998-99)
Theory of Probability and its Applications - II
Semestral-II examinations
Maximum marks: 100. Time: 3 hours.

Date : 21 April, 1999.

Note: Answer as many questions as you wish. The whole question paper carries 115 marks. The maximum you can score is 100.

1. (a) Prove that the set of all discontinuity points of a distribution function (D.F.) $F(x)$ of a random variable is a countable set.
   (b) Show that if X is a random variable with a continuous D.F. $F(x)$ then the random variable $Y = F(X)$ has uniform distribution over the interval $(0,1)$.

[5 + 10]

2. (a). Let X be a random variable having uniform distribution $\mathcal{U}(0,1)$. Let
$$U = \text{minimum}\,(X, 1-X)$$
and
$$V = \text{maximum}\,(X, 1-X).$$
Find the distribution and the density function of the random variable $\frac{V}{U}$.
   (b). Let Θ be a random variable which is uniformly distributed in $(-\frac{\pi}{2}, \frac{\pi}{2})$. Let
$$Y = \begin{cases} \cos\Theta & \text{if } \Theta < 0 \\ \sin\Theta & \text{if } \Theta \geq 0. \end{cases}$$
Find the D.F and the density function of the random variable Y.

[10+10]

3. Let X and Y be independent random variables having $\Gamma(1,\lambda)$ and $\Gamma(2,\lambda)$ densities respectively ($\Gamma(\alpha,\lambda)$-density $= \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1}$ for $x > 0$ and $= 0$ otherwise).
   (a). Find $P(X + Y \leq a)$ for any $a > 0$.
   (b). Find $P(\frac{X}{Y} \leq b)$ for any $b > 0$.
   (c). Find $P(X + Y \leq a, \frac{X}{Y} \leq b)$.

[7+8+10]

4. (a). Write down the bivariate Normal density with parameters $(\mu, \nu, \sigma^2, \tau^2, \rho)$. Find the conditional distribution of X given $Y = y$ when (X, Y) have this bivariate Normal density.
   (b). Let
$$f(x,y) = \begin{cases} [(1+ax)(1+ay) - a]\, e^{-x-y-axy} & \text{if } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$
(where $0 < a \leq 1$ is a constant).
   i) Prove that $f(x,y)$ is a density.
   ii) Show that both the marginal densities are exponential $\mathcal{E}(1)$. Find the conditional expectation $E(X \mid Y = y)$, for $y > 0$.

[(5 + 10) + (3 + 5 + 7)]

(P.T.O)

5. (a). Let X be a random variable with $M_X(t) = E(e^{tX})$ finite , $t \geq 0$. Show that

$$P(X \geq a) \leq e^{-ta} M_X(t).$$

(b). Let $X \sim \Gamma(\alpha, \lambda)$ - distribution . Find the set of all t in $\mathbb{R}$ such that $M_X(t)$ is finite. Find $M_X(t)$ for all such values of t. Prove that

$$P[X \geq \frac{2\alpha}{\lambda}] \leq 2^{\alpha} e^{-\alpha}$$

(Hint : Find $\underset{t \geq 0}{\text{Minimum}} (e^{-ta} M_X(t))$, for $a = \frac{2\alpha}{\lambda}$.)

[5+(10+10)]

INDIAN STATISTICAL INSTITUTE

Final Examination: Semester II (1998-99)

B. Stat.(Hons) II Year

Elements of Algebraic Structures

21 April 1999 Maximum Marks: 100 Duration: $3\frac{1}{2}$ hrs.

Note: Answer as much as you can. The whole paper carries 117 marks but the maximum you can score is 100. Marks allotted to each question are indicated near the right margin. While answering problems, state clearly the theorems you are using.

1. Let H be a subgroup of a group G. Let J be the set of all right cosets of H in G. Define $A \cdot B = \{ab : a \in A \text{ and } b \in B\}$. Show that J forms a group under this operation iff $aH = Ha$ for all $a \in G$. (Do not assume any results on normal subgroups.) [10]

2. For every even integer $n \geq 6$, show that the dihedral group with n elements has the property that it has a subgroup of order d for every positive divisor d of n. [8]

3. Prove that every finite abelian group is a direct product of cyclic groups. [15]

4. (a) Define inner automorphisms of a group G and show that they form a group (under composition) which is isomorphic to $G/Z(G)$. [10]

   (b) Find the centre of S_3 and show that the group of all automorphisms of S_3 is isomorphic to S_3 itself. [8]

5. Let R be a Euclidean domain.

   (a) Show that every ideal in R is of the form aR for some $a \in R$.[8]

   (b) Show that if a and b belong to R and at least one of them is nonzero, then there exists an element d of R such that (i) d divides a and b and (ii) if c divides a and b then c divides d. [8]

6. (a) Prove that if L is a finite extension of K and if K is a finite extension of F, where F, K and L are fields, then L is a finite extension of F. [10]

(b) Prove that every finite extension of a field is an algebraic extension. Is the converse true? Justify your answer. [7]

7. (a) If $f(x)$ is a polynomial over a field F and if $f(x)$ and its derivative $f'(x)$ are relatively prime in $F[x]$, prove that $f(x)$ cannot have a multiple root in any extension of F. [9]

(b) Assuming the existence of splitting fields for all polynomials over all fields, prove that for any prime power, p^n, there exists a field of order p^n. [12]

(c) Prove that for any positive integer n and any prime number p, there exists an irreducible polynomial of degree n over $\mathbb{Z}_p$. [12]

INDIAN STATISTICAL INSTITUTE

B.Stat. (Hons.) II Year : 1998 - 99

SEMESTRAL - II EXAMINATION

Economic Statistics & Official Statistics

ate: 23.4.1999 Maximum Marks: 60 Time: 3 hours

Note: This question paper carries a total of 66 marks. Answer as much as you can. The maximum marks that you can score is 60.

,(a) Define True Cost of Living Index Number (TCLIN) stating the assumptions involved and comment on their measurability. How do Laspeyres and Passche Consumer Price Index Numbers compare with the corresponding TCLIN'S ?

(4+4) = (8)

(b) Derive an Index Number for comparing the levels of industrial production in two situations. Using Laspeyres formula show how the sectorspecific indices can be combined to get the index of industrial production for the economy as a whole.

(4+4) = (8)

(c) The following data relate to a manufacturing firm:

year	average monthly value added at 1980 prices (Rs.)	no. of workers		nominal wage rate (Rs./month)	
		skilled	unskilled	skilled	unskilled
1980	1,50,000	15	10	3,000	1,000
1990	4,25,000	12	15	6,000	3,000

Calculate an Index of Labour Productivity for the firm for the year 1990 taking 1980 as the base year.

(6)

a) Define Lognormal Distribution and state its properties. Derive the Lorenz Curve and the Lorenz ratio for the Lognormal Distribution and demonstrate the properties of the Lorenz Curve.

(4+6) = (10)

b) State and explain the Law of Proportionate Effect and its modification.

(6)

c) For a Lognormal Distribution the Lorenz ratio is 0.3. Given that $\Phi(-0.385) = 0.15$, where $\Phi(.)$ denotes the area under $N(0,1)$, find the value of the AKS inequality measure assuming the value of the inequality aversion parameter to be 0.5.

(6)

a) Explain briefly the Additive and Multiplicative models of descriptive time series analysis. Describe how a set of seasonal indices can be

contd.... 2/

estimated for a given time series of monthly observations. How would you examine whether the assumption of constant seasonal indices is valid or not ? Mention the possible uses of the estimated seasonal indices.

(3+3+1+1) = (8)

(b) Define covariance stationarity of a stochastic time series. Why do you need this property for analysis of a time series ? Define the ARMA (p,q) process and derive the condition for its stationarity.

(2+2+2+2) = (8)

(c) For a covariance stationary process $\{X_t\}$ the autocorrelation function is defined to be $\rho_s = h(s)$, where $\rho_s = \frac{\gamma_s}{\gamma_0}$, $\gamma_s = Cov(X_t, X_{t-s})$, $s = 0, 1, 2, \ldots$, being the sth order auto covariance of X.

Sketch the autocorrelation function of the following processes:

(1) $X_t = a_0 + a_1 X_{t-1} + \epsilon_t$, $|a_1| < 1$.

(2) $X_t = a_0 + \epsilon_t + b\epsilon_{t-1}$, where ϵ_t's are iid with $E(\epsilon_t) = 0$, $V(\epsilon_t) = \sigma^2$ for all t.

(3+3) = (6)

:bcc:

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1998-99
SEMESTRAL-II EXAMINATION
DEMOGRAPHY AND SQC AND OR

Date: 26.4.99 | Maximum Marks: 50 | Time: 2 Hours

Note: Answer all Questions

STATISTICAL QUALITY CONTROL

1. An instrument is to be used as part of a proposed SPC implementation programme. 20 units of the product are obtained, and the process operator who will actually take the measurements for the control chart uses the instrument to measure each unit of product twice. The data are as follows:-

Part Number	Measurements I	Measurements II
1.	21	20
2.	24	23
3.	20	21
4.	27	27
5.	19	18
6.	23	21
7.	22	21
8.	19	17
9.	24	23
10.	25	23
11.	21	20
12.	18	19
13.	23	25
14.	24	24
15.	29	30
16.	26	26
17.	20	20
18.	19	21
19.	25	26
20.	19	19

The part used has USL=60 and LSL=5

(a) Draw an $\overline{X}$-R chart for the above data
(b) How much is the gauge capability in terms of precision - to - tolerance ratio ? Is it adequate?
(c) How much is the measurement error expressed as a percentage of the product characteristic variability?
(d) What is your interpretation of $\overline{X}$-chart in this situation?

[9+2+1+3+2]

2. An assembly consists of 3 components. The specifications on this assembly are 6.00 ± 0.06 inches. Let each component x_1, x_2 and x_3 be normally and independently distributed with means u_1=1.00 inch, u_2=3.00 inch, and u_3=2.00 inch, respectively. Suppose that the natural tolerance limits for each component as well as the final assembly are defined such that the fraction of components or final assemblies falling outside these limits is 0.0027. Assuming equal variability for the components, find out the specification limits for each component. [8]

OPERATIONS RESEARCH

3. A newsboy pays 8 cents per paper and sells them for 15 cents each. If he has any left over, he can return them for 1 cent credit each. Lost sales involve no direct cost. Assuming that demand is normally distributed with a mean of 150 papers and a s.d. 25, determine the optimal order quantity. If the newsboy finds an alternative buyer who is willing to pay 5 cents per paper late in the day then what will be the change in optimal order quantity? [5+5]

4. A large departmental store chain sells blank recording tape under its own brand name. The tape itself is identical to that of a well-known recording tape manufacturer, and is infact supplied to the chain by that manufacturer. Because of the special labeling and packaging, however, there is a lead time of 5.2 weeks, or 1/10 year. Assume that the demand during a period of this length is normally distributed with a mean of 1000 tapes and a s.d. of 250. The cost of paper work and handling associated with placing an order is $ 100, and the holding cost is 15 cents per tape per year. Suppose back orders are taken with a penalty of $ 1.00 per tape back ordered. Determine as to what should be the optimal order quantity and reorder point. [15]

INDIAN STATISTICAL INSTITUTE

Final Examination : Semester II (1998-99)

B. Stat. (H) II Year

Demography

Date : 26.04.1999 *Maximum Marks : 50* *Duration : 1 ½ hours*

Note : Attempt question 6 and any three from the rest

1. a) Define a population census.
   b) Define and discuss various systems of census operations.
   c) Describe U. N. Joint Score for evaluating age data and also indicate their possible ranges to measure the accuracy of the data.

2+5+6+2=15

2. a) Definite and derive the logistic low of population growth .
   b) Interpret the parameter in a logistic law
   c) Why do we prefer the logistic curve to other curves for describing population growth ?

6+6+3=15

3. a) Following demographic is information available in a year :

Item	Country A	Country B	Country C
Population by age	$P^A{}_x$	$P^B{}_x$	$P^C{}_x$
Deaths by age	$D^A{}_x$	$D^B{}_x$	--
Total population	$P^A = \Sigma P^A{}_x$	$P^B = \Sigma P^B{}_x$	$P^C = \Sigma P^C{}_x$
Total Deaths	$D^A = \Sigma D^A{}_x$	$D^B = \Sigma D^B{}_x$	D^C

(i) Estimate crude death rates for these three countries. What can you say about the relative mortalities for these three countries through crude death rates ?

ii) Suggest and derive other suitable methods for comparison of mortality between these three countries.

b) Find the relationship between crude birth rate and total fertility rate

1+2+6+6=15

[Turn over]

4. a)

(i) What conditions must a survivorship function l_x satisfy ?

(ii) Show that $1-(x/106)$ satisfies these conditions.

(iii) What is the ultimate age in the table with survivorship functions (ii) ?

(iv) What is the formula for μ_x with survivorship function (ii) ?

(v) With the survivor function in (ii), what is the probability that a life aged 13 will die before age 43 ?

b) In a certain life table,

$\mu_x = 0.15 - 0.10\,x \qquad$ for $0 < x < ½$,

and

$\mu_x = (0.01)^x \qquad$ for $½ < x < 1$.

Find l_1 when $l_0 = 100,000$.

3+2+1+[illegible]

5. a) If the minimum age at marriage for female is 18 years and the maxi[illegible] marriage for female is 50 years and about 10 per cent women never [illegible] life time, then derive an expression to estimate the singulate [illegible] marriage.

b) Show that

$$\int_0^x \upsilon(t)\,dt = -\log s(x)$$

Where

s (x) = the proportion of person single at age x, and
υ (x) = the force of nuptiality.

c) Describe various columns of a net nuptiality table .

5+5+5=15

6. ***Write Short notes (Any one)***

(i) Census Measures of Fertility

(ii) Replacement Index

(iii) Chiang's Method in estimating ${}_nq_x$

(iv) Chandresekaran-Deming formula

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1998-99
SEMESTRAL-II EXAMINATION
ECONOMICS-II

Date: 28.4.99 | Maximum Marks: 100 | Time: 3 Hours

Note: Answer any five question. Each question carries equal marks.

1.(a) Explain why one should get the same estimate of GDP whether measured by value added method or expenditure method.

(b) Assume GDP is Rs.1200, disposable income is Rs.1000, government budget deficit is Rs.70, consumption is Rs.850, the trade surplus is Rs.20.
Find out the magnitudes of (i) saving, (ii) investment and (iii) government spending. [8+12]

2.(a) Consider a Simple Keynesian economy where the government spends all its tax revenue and tax is proportional to income.
(i) Is the investment multiplier larger or smaller compared to the case when government spending is exogenous ? Explain.
(ii) When t increases, does Y increase, decrease, or stay the same? Explain your answer.

(b) In the above economy suppose people want to save more at every level of income. What effect will this have on the new equilibrium level of saving? Explain your answer. [10+10]

3. Consider two alternative equations of the LM curves :

$$\frac{\bar{M}}{\bar{P}} = .9Y - 900\, r \text{ and}$$

$$\frac{\bar{M}}{\bar{P}} = .9Y - 950\, r \text{ respectively.}$$

(a) What is the slope of each LM curve? Explain why the slopes are different?

(b) In which of the two cases monetary policy will be more powerful? Explain. [10+10]

4. What is speculative demand for money? How does Tobin explain the functional relationship between speculative demand for money and the rate of interest? [20]

5. Consider a small open economy with flexible exchange rate. Suppose there is no capital mobility. Discuss and compare the qualitative effect on Y and exchange rate of (a) an exogenous increase in G and (b) an exogenous increase in export. [10+10]

. Consider a small open economy facing perfect capital mobility under fixed exchange rate regime. Explain the precise mechanism involved in the process by which a balance of payment surplus surfaces as money supply increase in the economy. Describe how the Central Bank's balance sheet is affected in the process. [20]

. Consider two small open economics A and B, each facing perfect capital mobility under flexible exchange rate regime. The import and export elasticities are smaller for economy A than that for economy B such that a currency depreciation deteriorates the trade balance in economy A but not in economy B. What does this suggest about the effectiveness of monetary policy on output and employmen in the two economics? [20

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS.) II YEAR: 1998-99
SEMESTRAL-II EXAMINATION
DIFFERENTIAL EQUATION

28.4.99 Maximum Marks:100 Time: $3\frac{1}{2}$ Hours

Note: You may answer as many questions as you please but the maximum that you can score is 100 marks. Marks assigned to questions are given in parenthes

1.(a) Let the function $f(x,y)$ be continuous in the strip $S=J\times\mathbb{R}$, $J=[0,a]$ and satisfy the condition

$$|f(x,y)-f(x,z)|\leq \frac{q}{x}|y-z| \text{ for } 0<x\leq a \text{ and } y, z\in\mathbb{R}, \text{ with}$$

$0<q<1$. Show that the initial value problem

$$y'=f(x,y) \text{ in } J,\ y(0)=\eta$$

has exactly one solution. [15]

(Hint. In the Banach space B of all functions $u\in C(J)$ with fini norm $||u||:=\sup\left\{\frac{|u(x)|}{x} : 0<x\leq a\right\}$, define a suitable linear operator that satisfies a Lipschitz condition.)

(b) Apply the method of successive approximation to $y'=y^2, y(0)=1$. The functions $y_n(x)$ thus defined are polynomials $(n\geq 1)$. Find their degrees and show that the coefficients of each $y_n(x)$ are non-negative.. Show that $y_n(x)$ converges to a limit for $-2\leq x\leq 1$ uniformly. What is the limit?

(Hint. show by induction that $|y_n(x)|\leq 1$ in $[-2,0]$). [15

2.(a) Define the Laplace transform $\bar{f}(p)=\mathcal{L}\{f(x):p\}$ of a function $f(x)$ and prove that

(i) $\mathcal{L}\{f'(x):p\}=p\,\bar{f}(p)-f(0)$ [3]

(ii) $\mathcal{L}\left\{\int_0^x f(x-y)g(y)\,dy:p\right\}=\bar{f}(p)\,\bar{g}(p)$ [5]

(iii) $\mathcal{L}\left\{\frac{d}{dx}\int_0^x f(x-y)g(y)\,dy:p\right\}=p\,\bar{f}(p)\,\bar{g}(p)$ [2]

(b) Show that the differentiable solution of

$$f(y)+\int_0^y f(x)\,f'(y-x)\,dx=\cos y,\ y>0,$$

$f(0)=1$, in $y\geq 0$, is

$$f(y)=\frac{2}{\pi}\int_0^{\pi/2}\cos(y\sin\theta)\,d\theta.$$

(Hint. Show first that the solution is actually $J_0(y)$ and from the relation $e^{ix\sin\theta}=\sum_{n=-\infty}^{\infty}J_n(x)\,e^{in\theta}$, deduce that

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\theta-x\sin\theta)\,d\theta.)$$ [1

2.(c) Use Laplace transforms to find y(t) from the simultaneous differential equations

$$2\left(\frac{dx}{dt} - 3\ \frac{dy}{dt}\right) = e^t$$

$$\frac{d^2y}{dt^2} + x = 2y,$$

given that $x(0)=y(0)=0$ and $\frac{dy}{dt}(0) = 1$.

3.(a) Define the symbol F(a,b,c,x) associated with the hypergeom series. Discuss the convergence behaviour of this series a prove the following:

(i) $e^x = \lim_{b\to\infty} F(a,b,a,\frac{x}{b})$ (ii) $F'(a,b,c,x)=\frac{ab}{c} F(a+1,b+1,c+1,x)$

(iii) the general solution of $(2x^2+2x)y''+(1+5x)y'+y=0$

near x=0 is given by

$$y=C_1 F(\tfrac{1}{2},1,\tfrac{1}{2},-x)+C_2(-x)^{\frac{1}{2}} F(1,\tfrac{3}{2},\tfrac{3}{2},-x)$$

and simplify this expression. [2+4+6

(b) A function y(x) satisfies the differential equation

$$x\frac{d^2y}{dx^2} + y = 0.$$

If the variables x,y are changed to t,z by the transformat

$$x = at^p,\ y = t^q z$$

where a,p,q are constants. Show that z(t) satisfies

$$t^2\frac{d^2z}{dt^2} + (1+2q-p)t\frac{dz}{dt} + \left\{q(q-p)+ap^2t^p\right\}z=0$$

Show that by a suitable choice of a,p,q this reduces to Be equation and hence obtain a solution of the original equat terms of a Bessel function.

4.(a) Solve the euqation $y'=\exp(\frac{y}{x})+\frac{y}{x}$.

(b) Solve the equation

$$y'+\frac{y}{1+x}+(1+x)y^4=0,\ y(0)=-1.$$

(c) Find an integrating factor of the equation

$$dy+[f(x)y-g(x)]dx=0.$$

contd. ...

4. (d) Show that the solution $y(x)$ of the special

Riccati equation $y' = x^2 + y^2$, $y(0)=1$, satisfies

$$y(x) > \frac{1}{1-x} \text{ for } x>0.$$

(Hint. Try $y(x) = \sum_{n=0}^{\infty} a_n x^n$ as a solution.) [9]

(e) If $y = u(x)$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} + q(x)y = 0,$$

prove that a second solution is given by

$$y = u(x) \int \frac{dt}{u^2(t)}$$

If $y(0) = a$ and $y'(0)=b$, show that the solution is

$$y(x) = \frac{a}{u(0)} u(x) + [bu(0) - au'(0)]u(x) \int_0^x \frac{dx}{u^2(x)} .$$ [10]

(f) One solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{2y}{1+x^2} = 0$$

is a simple polynomial of degree two: find the solution satisfying the conditions $y(0) = y'(0)=1$. [5]

INDIAN STATISTICAL INSTITUTE
B.STAT.(HONS) II AND III YEAR: 1998-99

Semester II (1998-99)

Sociology

Maximum Marks- 100 Date: 28.4.99 Duration 3 hours

Answer five questions with at least one from each group.
Questions carry equal value

Group : A

1. a) Which of the following three concepts Karl Marx has emphasised
   (i) Division of labour in society
   (ii) Class structure and class struggle
   (iii) Individual motivation.

   b) Discuss the relevance of your answer to (a) above in the context of studying rural development in India.

2. Choose any one of the three Indian social scientists, M.N Srinivas, Ramkrishna Mukherjee and Nirmal Kumar Bose and explain whether you consider his approach to the study of society in India is on the whole, Weberian, Durkheimian or Marxian.

3. If you are planning to undertake a project to study trend of socio-economic changes in rural India after independence, what will be your theoretical framework for the study? Discuss your answer in the light of any sociological theory.

Group : B

1. Define Social Science Research. What decisions are to be taken before designing a work-plan for social science research ?

2. What are the techniques of study? Discuss two different techniques which are used in social science research.

3. Formulate a specific research proposal of your interest to be examined by a board of experts.

Group : C

1. What are the experiences of planned economic development in India especially in the rural areas ? Would you argue for decentralised planning for rural development ? If so, why ?

2.(a) Discuss different forms of marriage generally found in our society.

(b) State how does divorce affect the stability of a family.

3. What is social network ? what are the main charecteristics of social network analysis ?

4. Write short notes on any two of the following :

   (a) Ethnic group (b) Functions of a family (c) Women's status in our society
   (d) Land Reforms in India.