C25706.

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: (2005 – 2006)

B. Stat. (Hons.) II Year

Economics I

Date: 7.9.05 Maximum Marks: 100 Duration: 3 hours

Note: The paper carries 115 marks. You may attempt any part of any question. The maximum you can score is 100.

(1) Show that the law of diminishing marginal utility implies that a consumer will allocate his income to equate ratio of marginal utilities with price ratio.

(10)

(2) Can a consumer's budget set contain only inferior goods? Justify your answer.

(1+5)

(3) Show that under usual assumptions a consumer's preference relation can be represented by a utility function.

(12)

(4) Clearly distinguish between

(a) Hicks substitution effect and Slutsky substitution effect

(4)

(b) Income consumption curve and Engel curve

(4)

(c) Demand curve and price offer curve

(4)

(5) (a) State two sufficient conditions that will ensure that a consumer's indifference curve is strictly convex to the origin. Prove your claim.

(2+6)

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(b) Do you think that in such a case a consumer's optimal choice is unique? Why?

(2+2)

(6) Consider the utility function $U(x_1,x_2) = x_1^{1/2} + x_2$

(a) Find the demand functions and verify that they are homogeneous of degree zero.

(8)

(b) Find the indirect utility function and the expenditure function.

(6)

(c) Consider the transformation

$$V(x_1,x_2) = (U(x_1,x_2))^{1/4}$$

Will the demand functions change? (Do not calculate. Argue intuitively.)

(2)

(d) Let $(p_1 = 1, p_2 = 1, m = 20)$. Consider also the situation ($p_1 = 2, p_2 = 1,$ $m = 20$). Determine the Hicks and Slutsky substitution effects here.

(6)

(e) Why are the income effects equal?

(2)

(f) Determine two points on each of the Hicks and Slutsky demand curves for good 1.

(4)

(7) A consumer's demand function for a good is given by x = 10 – p. Find the loss of consumer's surplus as price increases from 2 to 3.

(5)

(8) (a) Define first order stochastic dominance and state a theorem showing its equivalence with the utilitarian rule.

(4)

(b) Write a short note on the Arrow-Pratt absolute risk aversion measure.

(3)

(9) Clearly state the expected utility theorem by giving necessary preliminaries.

(6)

(10) (a) Show that for a two-good world $s_1 e_{1m} + s_2 e_{2m} = 1$, where e_{im} is the income elasticity of demand for good i and s_i is the proportion of income spent on the consumption of good i, $i = 1,2$.

(5)

(b) Suppose that a consumer's utility function is of the form $U(x_1,x_2) = f_1(x_1) + f_2(x_2)$, where $f_i' > 0$, $f_i'' < 0$, $i = 1,2$. Show that none of the goods is inferior.

(7)

(c) Consider the individual demand functions $d_1(p) = 10 - p$ and $d_2(p) = 5 - p$. Find the market demand function.

(5)

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination :(2005-2006)

B. Stat II Year

Physics I

Maximum Marks: 100, Duration 3 hrs.

Group A : (Answer Group A & Group B in seperate answerscripts) Date: 7.9.05

Answer any three:

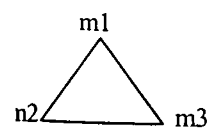
(1)(a)What is meant by constraints in classical mechanics? What are the main characteristics of the constraints ? 2+3

(b)The constraint relations for the gas molecules inside an expanding (with time) container of radius R are

$|\vec{r}| \leq R(t)$ where $\vec{r}$ is measured from the center of the container. What is the nature of constraints for this system? If R(t) = constant, does it remain the same? 2+2

(c)What is virtual displacement? Show that the virtual work done on a system of particles in static equilibrium due to any virtual displacement must vanish. 1+3

(d)Three point particles with masses m1,m2 and m3 interact with each other through the gravitational force as shown in the figure.



What is the equation of motion? 5

(2)(a)What is D'Alembert's principle? What do you mean by Lagrangian of a conservative system ? 2+3

(b)Show that

(i) $\delta L = \frac{d}{dt}(p_i \delta q_i)$ 5

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(ii) $\frac{\partial T}{\partial p_i} = \dot{q}_i$ 4

(iii) $\delta T + \delta V = 0$ 4

Where L is the Lagrangian and q_i, p_i are the generalised coordinates and momentum respectively.

(3)(a)What is Hamilton's principle? Lagrange's equations of motion can be derived both from D'Alembert's principle and Hamilton's principle. What are the advantages if one starts from Hamilton's principle ?
2+3

(b)Assume the Lagrangian for a certain on-dimensional motion is given by

$L = e^{\beta t}(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2)$

Where β, m, k are constants?

What is the Lagrange's equation of motion? 3

Suppose a point transformation is made to another generalized coordinate S, given by

S = exp(at/2) q

What is the Lagrangian in terms of S ? What is the Lagrange's equation? 2+3

How would you describe the relationship between the two solutions? 5

(4)(a)Show that the quantity

$$J = \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L = p_i\dot{q}_i - L = constant$$

5

where the system is supposed to be non-dissipative and Lagrangian is explicitly time independent.

(b)State the conditions under which this corresponds to the Hamiltonian of the system.
3

(c) Consider a particle of mass m in a bound orbit with potential

$V(r) = -\frac{K}{r}$

Using polar coordinates in the plane of the orbit, find

p_r, p_θ as functions of $r, \theta, \dot{r}, \dot{\theta}$.

Is either one constant?

10

(5)(a)What is Gauge function for Lagrangian? 3

(b)Find the equation of motion corresponding to the Lagrangian

$$L(x,\dot{x}) = e^{-x^2}(e^{-\dot{x}^2} + 2\dot{x}\int_0^{\dot{x}} e^{-\alpha^2} d\alpha)$$

Find the energy integral for the system. 9

Construct another Lagrangian which can give rise to the same equation of motion. 6

Group B : Questions 1-5 compulsory

1(a) Find the projection of the vector **A**=**i** -2 **j** + **k** on the vector **B**=4**i** - 4 **j** +7**k** 2

1(b) If **A**=2**i**-**j**+**k**, **B**= **i**+3**j**-2**k**, **C**= -2**i**+**j**-3**k** and **D**=3**i**+2**j**-3**k**, 3

find the scalars a, b, c such that **D** = a**A** + b**B** + c**C**.

1(c) If **A** has constant magnitude, show that **A** and d**A**/dt are perpendicular to each other provided d**A**/dt ≠ 0 2

2. If **A** = A_1**i** + A_2 **j** + A_3 **k**, **B**=B_1**i** + B_2**j** + B_3**k**, **C**=C_1**i** + C_2**j** + C_3**k**, prove that

(i) **A** X (**B** X **C**) + **B** X (**C** X **A**) + **C** X (**A** X **B**) = **0** 3

(ii) (**A** X **B**) . (**B** X **C**) X (**C** X **A**)= (**A** .**B** X **C**)^2 3

3.(a) If $\Phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\Phi$ at the point (1, -2, -1) 2

3.(b). Determine the constant 'a' so that the vector **V** = (x+3y) **i** + (y-2z) **j** + (x + a z) **k** is solenoidal. 3

3 (c) If **V** = **W** x **r**, prove that **W** = (1/2) curl **V** where **W** is a constant vector. 3

4(a). Calculate the line integral of the function **V** = y^2**i** + 2x(y+1) **j** from the point A(1, 1, 0) to the point B(2, 2, 0) along the path (1) and (2) (see fig.1). 4

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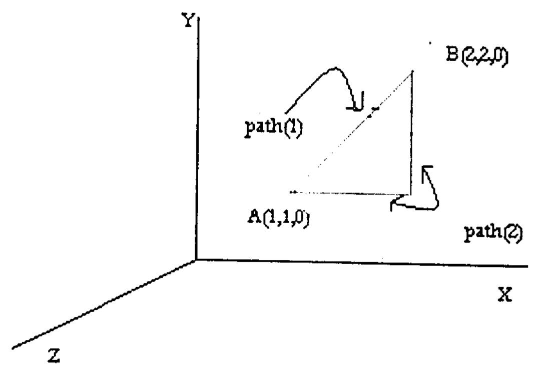

fig. 1

4 (b) Find the total work done in moving a particle in a force field given by **F**=3xy **i** -- 5z **j** + 10x **k** along the curve $x=t^2+1$, $y= 2t^2$, $z =t^3$ from t=1 to t=2

3

5. State and explain Stoke's theorem. 3

Answer any 3(three) from the questions 6,7,8,9 5 X 3

6. Verify Divergence theorem for the following function

F = 4xz **i** $-y^2$ **j** + yz **k** on the surface S where S is the cube bounded by x=0, x=1, y=0,y=1,z=0 z=1. 5

7. State Coulomb's law . Find the electric field of the source charges (q_i) when we have several point charges q_1, q_2,....q_n at distances r_1, r_2...r_n from Q(the test charge).
Mention the assumption, if any. 2+3

8.Find the electric field at a distance z above the midpoint between two equal charges q, a distance d apart. 5

9.(a) State Gauss's law in its integral and differential form 2
(b) A long cylinder carries a charge density that is proportional to the distance from the axis, ρ= kr, for some constant k, find the electric field inside the cylinder. 3

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination : 2005-2006
B. Stat. - Second Year
Analysis III

Date : 13. 09. 2005 Maximum Score : 100 Time :3 Hours

This paper carries questions worth a total of 115 marks. Answer as much as you can. The maximum you can score is 100 marks.

(1) (a) Let $X = \mathbb{R}^m$, $Y \subset \mathbb{R}^n$ compact and $C \subset X \times Y$ be closed. Show that the projection $\pi_X(C)$ is closed in X.

(b) Let $Y \subset \mathbb{R}^n$ be compact and $\{f_n\}$ a sequence of real-valued continuous maps on Y monotonically decreasing to the constant map 0. Show that the sequence $\{f_n\}$ converges to 0 uniformly on Y.

[12 + 18]

(2) (a) Let $f, g : \mathbb{R} \to \mathbb{R}^n$ be differentiable maps and $h : \mathbb{R} \to \mathbb{R}$ be defined by

$$h(t) = f(t) \cdot g(t).$$

Show that h is differentiable and compute it's derivative.

(b) Let U be a neighborhood of $z_0 = x_0 + iy_0$ in the complex plane $\mathbb{C}$ (also identified with $\mathbb{R}^2$ canonically). Let $f = u + iv : U \to \mathbb{C}$ be such that

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

exists.

(i) For any real α, show that

$$f'(z_0) = e^{-i\alpha}[u_a(z_0) + iv_a(z_0)],$$

where $a = \cos\alpha + i \sin\alpha$.

(ii) If $b = \cos(\alpha + \frac{\pi}{2}) + i\sin(\alpha + \frac{\pi}{2})$, show that

$$u_a(z_0) = v_b(z_0) \text{ and } u_b(z_0) = -v_a(z_0).$$

[12+(10+3)]

(3) (a) Let $x_0 \in \mathbb{R}^2$ and U an open neighborhood of x_0. Suppose $f : U \to \mathbb{R}$ is such that $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ exist on U and are continuous at x_0. Show that

$$\frac{\partial^2 f}{\partial x \partial y}(x_0) = \frac{\partial^2 f}{\partial y \partial x}(x_0).$$

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(b) Find the maximum value of $(x_1 \cdots x_n)^2$ under the restriction that $\sum_{i=1}^{n} x_i^2 = 1$.

[18 + 12]

(4) Let $U \subset \mathbb{R}^2$ be a connected open set and $f : U \to \mathbb{R}$ a continuously differential map.

(a) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ vanish on U, show that f is constant on U.

(b) Show that f is not one-to-one.

[12 + 18]

Periodical Examination
Class: B.Stat II
Subject: C & Data Structures
Full Marks: 100
Time: 3hrs
Date: 16. 9. 05

1. What will be the output of the following programs :-

(a)

```
main(){
float a = 7.999999;
float *b,*c;
b = &a;
c = b;
printf("\n%u%u%u",&a,b,c);
printf("\n%d%d%d%d",a,*(&a),*b,*c);
}
```

(b)

```
int check (int i, int j){
int *p,*q;
p = &i;
q = &j;
if(i >= 45)
  return (p);
else
  return(q);
}

main(){
int *c;
c = check (10,20);
printf("\nc = %u",c);
}
```

5+5 = 10

P. T. O

2. For each of the following pairs of functions f(n) and g(n), either f(n) = O[g(n)] or g(n) = O[f(n)], but not both. Determine which is the case :

(a) f(n) = (n*n - n)/2,g(n) = 6n
(b) f(n) = n + 2*sqrt(n),g(n) = n*n
(c) f(n) = n + n lg(n),g(n) = n*sqrt(n)
(d) f(n) = n*n+3*n+4,g(n) = n*n*n
(e) f(n) = n lg n,g(n) = n*sqrt(n)/2
(f) f(n) = n + lg(n),g(n) = sqrt(n)
(g) f(n) = 2*(lg(n)*lg(n)),g(n) = lg(n) + 1
(h) f(n) = 4*n*lg(n) + n,g(n) = (n*n - n)/2

8 x 2.5 = 20

3. Given two positive integers a and b, give an algorithm to calculate gcd(a,b). Assuming that a and b have k and l bits respectively in their binary representations (k >= l),can you give an estimate of the number of steps required? 10

4. How do you place eight queens on a chessboard so that no queen can take another? Discuss in detail mentioning the relevant data structures. Give an estimate of the amount of work done by your program / algorithm.

15

5. Define a "sparse polynomial" as a polynomial with high degree but very few non-zero coefficients.

(a) How do you represent a sparse polynomial using a linked list?
(b)How do you add two such polynomials?
(c)How do you multiply two such polynomials?

4+6+10 = 20

6.(a) Describe how you can generate permutations using a stack. Is it possible to generate all possible permutations using a stack? Justify.

(b)How do you implement the "waiting list" in a railway reservation system using a proper data structure?

15+10 = 25

INDIAN STATISTICAL INSTITUTE
Final Semester Examination (2005-2006)
B. Stat II Year
Physics I

Date: 25.11.05 Full Marks- 100 Duration 3 hrs

(Answer **Group A** and **Group B** in separate answer-scripts)

Group A
(Marks will be deducted for untidy answer-scripts)

Answer any 5 questions

1.i) If $\mathbf{A}=5t^2\,\mathbf{i}+t\,\mathbf{j}-t^3\,\mathbf{k}$ and $\mathbf{B}=\sin t\,\mathbf{i}-\cos t\,\mathbf{j}$
find (a) d /dt **(A.B)**
(b) d/dt **(A.A)** and
(c) d/dt **(A × B)** 2+2+4

ii) Prove the identity: $\nabla\cdot(\nabla\times\mathbf{A})=0$ 2

2. (a) If $\mathbf{F}=\nabla\phi$, prove **F** is irrotational 3

(b) If $\mathbf{A}=x^2y\,\mathbf{i}-2xz\,\mathbf{j}+2yz\,\mathbf{k}$, find : $\nabla\times(\nabla\times\mathbf{A})$ 3

(c) Find out a unit vector perpendicular to the two vectors 2
$-\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}-3\mathbf{j}+2\mathbf{k}$
(d) Check whether the field $\mathbf{E}=4y\,\mathbf{i}-2x\,\mathbf{j}+\mathbf{k}$ is electrostatic. 2

3. (a) State First and Second **Uniqueness Theorems.** 3
b) Verify that the vector potential (A) due to a uniform magnetic induction **B** is given by $\mathbf{A}=-\frac{1}{2}(\mathbf{r}\times\mathbf{B})$ 2
c) Derive $\nabla\cdot\mathbf{E}=(\rho/\varepsilon_0)$ where the symbols have their usual meanings. 3

(d) Hence derive Poisson's equation and Laplace's equation 2

P. T. O

4. (a) If $\phi = 2xyz^2$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from t=0 to t=1, evaluate $\int_C \phi \, d\mathbf{r}$ 2

(b) Prove $\int_C (3x + 4y)dx + (2x-3y)\, dy = -8\pi$

where C is a circle of radius 2 with centre at the origin of the xy-plane and is traversed in the positive sense 3

(c) Show that $\iint_S \mathbf{r.n}\, dS = 3V$

where S is a closed surface and V is the volume enclosed by S. 3

(d) Write down the expression for the electric field and charge density when the potential is given by

$$\phi(\mathbf{r}) = (1/4\pi\varepsilon_0)\,(q/r)$$ 2

5. a) Starting from the expression of

$$\mathbf{B(r)} = (\mu_0/4\pi) \int (\mathbf{J(r')} \times \wp\,) / (\wp^2)\, d\tau'$$

($\wp$ is the separation vector and other symbols have their usual meanings) show that

i) $\nabla.\mathbf{B} = 0$ 2

ii) Which law does the expression of **B(r)** represent ? 1

iii) What is the physical significance of the expression (i)? 1

(b) A charged particle is moving with a velocity **v** in an electric and magnetic field, what is the the total force acting on it ? Does it gain energy from the field? Give reasons for your answer 1+2

c) Derive the equation of continuity. What is the physical significance of it. 3

6 a) What is the work done to assemble a configuration of point charges ? 5

b) Find the energy of a uniformly charged spherical shell of total charge q and radius r. 4

c) Does the electrostatic energy obey Superposition principle? Give reasons for your answer 1

7 a) Find the capacitance of a parallel plate capacitor consisting of two metal surfaces of area A and distance d apart. 2

b) State Ampere's law in differential form. Prove it for a long straight wire carrying a current. 4

c) Prove that the dipole potential $\phi(r) = (1/4\pi\varepsilon_0)\,(\mathbf{p.r})/r^3$ where **p** is the electric dipole moment and **r** represents the vector from the dipole to the point of interest. 4

Group B

Answer any 5 questions.

(1)(a) Briefly describe the dilemma which necessitated the development of the special theory of relativity. 5

(b) Given that an atom at rest emits light of angular frequency ω_0 and that this atom is travelling at velocity v either directly towards or away from man observer, use the Lorentz transformation to derive a formula for the frequency observed by the observer for the two case (towards or away from the observer). 5

(2) A spaceship is moving away from the earth at a speed v =0.8c. When the ship is at a distance of $6.66 \times 10^8\, km$ from earth as measured in the earth's reference frame, a radio signal is sent out to the spaceship by an observer on earth. How long will it take for the signal to reach the ship.

(a) As measured in the ship's reference frame? 5

(b) As measured in earth's reference frame? 5

(3) A Lagrangian for a particular system can be written as

$$L = \frac{m}{2}(a\dot{x}^2 + 2b\dot{x}\,\dot{y} + c\dot{y}^2) \quad - \frac{K}{2}(ax^2 + 2bxy + cy^2)$$

where a,b and c are arbitrary constants but subject to the condition that $b^2 - ac \neq 0$. What are the equations of motion ? 6

Examine particularly the two cases a = 0 =c and b=0, c= - a. 2

What is the physical system described by the above Lagrangian ? 2

(4)(a) A particle of mass m moves in one dimension such that it has the Lagrangian

$$L = \frac{m^2}{12}\dot{x}^4 + m\dot{x}^2 V(x) - V_1(x)$$

where V and V_1 are differential functions of x. Find the equation of motion for x(t). 4

(b) By what arguments and using what measurable quantities can one determine the following quantities with good accuracy?

(a) The mass of the earth. 2

(b) The mass of the moon. 2

(c) The distance from the earth to the sun. 2

(5) A particle is constrained to be in a plane. It is attracted to a fixed point P in this plane; the force is always directed exactly at P and is inversely proportional to the square of the distance from P.

(a) Using the polar co-ordinates, write the Lagrangian of this particle. 3

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(b) Write Lagrangian equations for this particle for this particle and find constant of motion. 4+3

(6)(a) State Keplar's laws of planetary motion. 6

(b) Consider a particle of mass m moving in a plane under central force

$$F(r) = -\frac{k}{r^2} + \frac{k'}{r^3}$$

(Assume k >0).

What is the Lagrangian for the system in terms of the polar coordinates r, θ and show that their velocities. 2+2

(7)(a) What is Hamilton's principle? 1

(b) Find the equations of motion for the Lagrangian

$$L = e^{\gamma t}\left(\frac{m\dot{q}^2}{2} - \frac{kq^2}{2}\right)$$

Where γ, m, k are constants. 3

(c) Are there any constant of motion? 2

(d) Suppose a point transformation is made of the form

$$s = e^{\gamma t} q$$

What is the effective Lagrangian in terms of s? Find the the equation of motion for s. 1+3

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INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2005-06

B.Stat. II

Biology-I

Date: 25.11.05 Full marks: 50 Duration: 2hr 30min

(Answer any five questions. All questions carry equal marks)

1. Distinguish between DNA and RNA with respect to their (a) chemical nature (b) function and (c) location within the cell. (4+4+2)
2. A man with X-linked color blindness marries a woman with no history of color blindness in her family. The daughter of this couple marries a normal man and their daughter also marries a normal man. What is the chance that this last couple will have a child with color blindness ? If this couple has already a child with color blindness what is the chance that their next child will be color blind ? (5+5)
3. A man, who is color blind and possesses "O" blood, has children with a woman who has normal color vision and "AB" blood. The woman's father had color blindness. X-linked and autosomal genes determine color blindness and blood group, respectively.

   (a) What are the genotypes of the man and the woman? (b) What proportion of the children will have color blindness and type "B" blood? (c) What proportion of their children will be color blind and have type "AB" blood? (4+4+2)

4. (a) What is "Tm" and why does "Tm" depend directly on the "GC" content of the DNA? (2+2)

   (b) Indicate whether each of the following statements about the structure of double stranded DNA, is true or false:

   (i) A+T=G+C; (ii) A/T=C/G; (iii) A+G=C+T; (iv) A=T within each single strand; (v) when separated the two strands are identical, and (vi) once the base sequence of one strand is known, the base sequence of the second strand can be deduced. (6)

P.T.O

5. (a) Define, with examples, mitotic and meiotic cell division in humans. If two cells with genotypes (A/a) and (A/a, B/b) undergo mitotic and meiotic cell divisions, what will be the genotypes or gene compositions of the resultant cells with respect to these alleles ? (5+5)
6. (a) The pathways leading to "oxidations of fatty acid and glucose meet at a point for energy production" – show the meeting point. (3)

   (b) Compare the efficiency of ATP generation when one molecule of palmitic acid and one molecule of glucose are completely metabolized. (7)
7. (a) Why is the genetic code a triplet instead of being singlet or doublet? In what sense and to what extent the genetic code is degenerate, ordered and universal ? (6)

   (b) If the average molecular mass of an amino acid is assumed to be 100 daltons, then how many nucleotides will be present in an mRNA coding sequence specifying a single polypeptide with a molecular mass of 27,000 daltons. (4)

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INDIAN STATISTICAL INSTITUTE
First ~~Second~~ Semestral Examination : (2005-2006)
B. Stat. - Second Year
Analysis III

Date : 29. 11. 2005 | Maximum Score : 100 | Time :3 1/2 Hours

This paper carries questions worth a total of 115 marks. Answer as much as you can. The maximum you can score is 100 marks.

(1) Let (X, d) be a complete complete metric space and $\{U_n\}$ a sequence of dense open sets.

(a) Show that for each $n \geq 1$, there is a non-empty open ball B_n of radius at most $1/n$ such that
$$\overline{B_{n+1}} \subset U_n \cap B_n$$
for each n.

(b) Show that $\cap_{n=1}^{\infty} U_n \neq \emptyset$.

(c) Show that $\cap_{n=1}^{\infty} U_n$ is dense in X.

[8 + 8 + 5]

(2) Let U be an open set in $\mathbb{R}^n$, V open in $\mathbb{R}^m$ and $x \in U$. Suppose $f : U \to V$ is differentiable at x and $g : V \to \mathbb{R}^p$ differentiable at $y = f(x)$. Show that $h = g \circ f$ is differentiable at x and compute the total derivative $h'(x)$.

[14]

(3) Let $f : [0,1]^k \to \mathbb{R}$ be a bounded map such that the set
$$B = \{x \in [0,1]^k : f \text{ is not continuous at } x\}$$
is of measure zero. Show that f is Riemann-integrable.

[15]

(4) (a) Let T be a k-multilinear map and S a l-multilinear map on a finite dimensional vector space V such that $\text{Alt}(S) = 0$. Show that
$$\text{Alt}(T \otimes S) = 0.$$

(b) Let $c = (c_1, \cdots, c_n) : [0,1] \to \mathbb{R}^n$ be a smooth curve. Define the tangent vector v to c at t as
$$v = (c_1'(t), \cdots, c_n'(t))_{c(t)}.$$
If $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable, show that the tangent vector to $f \circ c$ at t equals $f_*(v)$.

[20 + 5]

(5) (a) Let I^k be the standard k-cube in $\mathbb{R}^k$, $f : [0,1]^k \to \mathbb{R}$ a differentiable map and $1 \leq i \leq k$. Show that
$$\int_{I^k} d(f dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots \wedge dx_k) = \int_{\partial I^k} f dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots \wedge dx_k.$$

(b) State and prove Stokes' theorem.

[10 + 8]

P.T.O

(6) Let $A = \mathbb{R}^2 \setminus \{(0,0)\}$ and $B = \{(x,y) \in \mathbb{R}^2 : y \neq 0 \text{ or } x < 0\}$. Consider the 1-form

$$\omega = \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$$

on A.

(a) Show that $d\omega = 0$.

(b) Show that there is a 0-form θ on B such that $d\theta = \omega$ on B.

(c) Show that there is no 0-form f on A such that $df = \omega$.

[2 + 8 + 12]

Semestral Examination
Class: B.Stat II
Subject: C & Data Structures
Full Marks: 100
Time: 3hrs

Date: 9.12.05

Instruction: Please write the answers of group A and group B questions in separate answer books.

Group A

1. Compute the average complexity of (a) Binary search algorithm.(b) Modified binary search algorithm. 8 + 7 = 15

2. (a) Write down the quicksort algorithm.
   (b)Modify it to find the k-th largest element in an array. 10+5 = 15

3. Prove that that Huffman's algorithm for coding an alphabet is optimum with respect to average length of the code. 15

4. Illustrate computations of 7^{1139} (mod 12) using the fast exponentiation algorithm. 5

Group B

1. What are the desirable properties of a hash function? What is clustering in a hash table? Describe two methods for minimising clustering. 3+2+10 = 15

2. Write complete C programs for calculating the GCD of two integers
   (a) recursively (b) non-recursively 7+8 = 15

3. How does the worst-case performance of an AVL tree compare with the worst-case / average-case performance of a random binary search tree? How does the average-case performance of an AVL tree compare with that of a random binary search tree? 15

4. Define a B-tree. 5

INDIAN STATISTICAL INSTITUTE
First Semester Back Paper Examination: 2005-06
B. Stat. II Year
Probability Theory III

Date: 23.1.06 Maximum Marks: 100 Duration: 3 Hours

Answer all questions.

1.(a) Let $X_1, \ldots\ldots, X_{k+1}$ be independent random variables having Gamma distributions $G(1,\alpha_1), \ldots G(1,\alpha_{k+1})$ with the same scale parameter 1. Find the joint density of $(Y_1, \ldots, Y_k)$ where

$$Y_i = \frac{X_i}{X_1 + \cdots + X_{k+1}}, \quad i = 1,2,\ldots k$$

(b) If $X_1, \ldots\ldots X_n$ has the multivariate Normal distribution with parameters $(\underset{\sim}{\mu}, \Sigma)$, Find the conditional distribution of $X_1, \ldots\ldots X_k$ given $X_{k+1} = x_{k+1}, \cdots, X_n = x_n$, where $k < n$.

[8+8=16]

2.(a) Let $X_1, X_2, \ldots\ldots$ be a sequence of random variables on a probability space $(\Omega, \mathcal{A}, P)$, show that the set $\{w : \lim_{n\to\infty} X_n(w) \text{ exists}\}$ is an event.

(b) Show that if the random variables $X_n \to X$ with probability one, then $X_n \to X$ in probability.

[8+8=16]

3.(a) Show that if $F_1, \ldots\ldots F_n, \ldots$ be a sequence of distribution functions, then there exists a subsequence $\{F_{n_k}\}$ converging weakly to a (possibly improper) distribution function F.

(b) Let $X_1, X_2, \ldots X_n, \ldots$ be independent and identically distributed with the Cauchy distribution having density $\frac{1}{\pi(1+x^2)}$. Let $Y_n = \max(X_1, \ldots X_n)$, $n = 1,2,\ldots$. Show that the random variables $\frac{\pi}{n} Y_n$ converge in distribution.

[8+8=16]

[P.T.O.]

4.(a) Let X have the normal distribution $N(\mu, \sigma^2)$. Find the characteristic function of X.

(b) Show that if $E(|X|)$ is finite, then the function $\frac{d}{dt}\varphi_X(t)$ is continuous, where φ_X is the characteristic function of X.

[8+8=16]

5.(a) Let $A_1, A_2, \ldots\ldots$ be a sequence of independent events such that $\sum_{n=1}^{\infty} P(A_n) = \infty$. Find $P\left(\bigcap_{n=1}^{\infty} \bigcup_{k \geq n}^{\infty} A_k\right)$.

(b) Prove the Weierstrass approximation theorem.

[8+8=16]

6.(a) Show that the characteristic function φ of a random variable X having a density vanishes at infinity.

(b) If φ is the characteristic function of the random variable X, show that

$$P(X = 0) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \varphi(t)\, dt$$

(c) Show that the function

$$\begin{aligned} \varphi(t) &= 0 \quad if \quad t \leq -1 \\ &= 1+t \quad if \quad -1 \leq t \leq 0 \\ &= 1-t \quad if \quad 0 < t \leq 1 \\ &= 0 \quad if \quad t \geq 1 \end{aligned}$$

is a characteristic function.

[7+7+6=20]

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INDIAN STATISTICAL INSTITUTE

First Semestral Backpaper Examination, 2005-06

B. Stat. (Hons.) II

Statistical Methods III

Date: 27.1.06 Maximum Marks : 100 Time: 3 hours

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1. Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent, $Y_i \sim N(\mu_i, \sigma^2)$, with μ_i following the linear regression model

$$\mu_i = \beta_0 + \beta_1 z_i, \quad 1 \leq i \leq n,$$

where it is assumed that all z_i's are not identical. Obtain sufficient statistics for this model.

[15]

2. Let X have a Poisson(θ) distribution with θ having a Gamma($K+1, \frac{1}{2}$) prior where K is a nonnegative integer. Find the Bayes estimate of θ. Next assume that K has a discrete uniform distribution over $\{0, 1, 2, \ldots, N\}$ where N is a large positive integer. Find an approximation to the posterior distribution of K given X in terms of a known pmf.

[5+15 =20]

3. Let $X_1, X_2, \ldots, X_n$ be iid $N(0, \sigma^2)$. Find a c such that $E(\frac{d}{\sigma^2} - 1)^2$ is minimum where $d(X_1, \ldots, X_n) = c\sum_1^n X_i^2$. Is d unbiased?

[15]

4 (a) Define various components of a time series and discuss how to eliminate their effects by using simple linear combinations of the observed values.

(b) Suppose that $Y_t = a + bt + \epsilon_t$, $t = 0, 1, \ldots, N$. Further assume that $\epsilon_0 = 0$, and $\epsilon_t = 0.5\,\epsilon_{t-1} + w_t$ for $t > 0$, where $w_1, w_2, \ldots, w_N$ are iid $N(0, \sigma^2)$. Find the maximum likelihood estimate of a and b.

[5+20=25]

5. The following are times until breakdown in days of air monitors operated under two different maintenance policies (A and B) at a nuclear plant. Experience has shown that both types of samples can be assumed to be exponentially distributed with possibly different means. Give a 90% confidence interval for the ratio, Δ of the mean failure times under two policies.

Policy A	3, 150, 40, 34, 2, 31, 6, 5, 4, 150, 4, 6, 10
Policy B	8, 26, 10, 8, 29, 20, 10

Is $H_0 : \Delta = 1$ rejected at level $\alpha = 0.10$?

[25]

Statistical Methods IV
B-Stat 2^{nd} Year 2005-2006
Mid-Semester Examination

Date: February 21, 2006 **Time: 2 hours**

This paper carries 35 marks. Answer all questions.

1. Suppose $\mathbf{X} = (X_1, X_2, \ldots, X_p)$ is a random vector with density:

$$f(\mathbf{x}) = \frac{\Gamma(\frac{p}{2}+1)}{\pi^{p/2} r^p}, \quad \mathbf{x}'\mathbf{x} \le r^2$$

   Obtain the density of $Y = \mathbf{X}'\mathbf{X}$ and show that $E(Y) = pr^2/(p+2)$. [5]

2. Consider the usual regression set-up:

$$y_i = \beta_0 + \beta_1 x_i + e_i; \quad i = 1, 2, \ldots, n$$

   where, x_is are fixed and e_is are i.i.d. N$(0,\sigma^2)$.

   Suppose that $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are the least squares estimators of β_0 and β_1, respectively and $Q = \sum_{i=1}^{n} \left(y_i - \widehat{\beta}_0 - \widehat{\beta}_1 x_i\right)^2$.

   (a) Show that $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are the m.l.e.s of β_0 and β_1, respectively.

   (b) Using an orthogonal transformation on the vector of y_is, show that $\bar{y}$, $\widehat{\beta}_1$ and Q are independent and Q/σ^2 is distributed as chi-square with (n-2) degrees of freedom.

   (c) Obtain the variance-covariance matrix of $(\widehat{\beta}_0, \widehat{\beta}_1)$.

   (d) Obtain a level α test for $H_0 : \beta_1 = b$ vs $H_1 : \beta_1 < b$. Give a rough sketch of the power function of the above test. [15]

3. Suppose $\overline{\mathbf{X}}$ and S are respectively, the sample mean vector and sample variance-covariance matrix based on a random sample of size n from a $N_p(0, I)$ population. Obtain the distribution of:

   (a) the sum of the eigenvalues of S

   (b) $\frac{\mathbf{a}'\,\overline{\mathbf{X}}\,\overline{\mathbf{X}}'\,\mathbf{a}}{\mathbf{a}'S\mathbf{a}}$, $\mathbf{a}$ is a fixed non-null vector. [4+4]

4. In genetic linkage analysis, it is known that under certain conditions, an offspring can have one of four gametic combinations where the probabilities of the combinations are $(\theta^2+2)/4, \theta(1-\theta), (1-\theta)^2/2$, and $\theta^2/4$, respectively. Suppose that in a random sample of offspring satisfying the conditions, there are n_1, n_2, n_3 and n_4 individuals having the four gametic combinations. Explain how you can use an EM algorithm to obtain the m.l.e. of θ. Show all your computational steps clearly. [7]

INDIAN STATISTICAL INSTITUTE

B. STAT SECOND YEAR

Elements of Algebraic Structures

Date : 24.2.06 Periodical Examination Time : 3 hrs.

This paper carries 120 marks. Maximum you can score is 100. You may use any theorem proved in the class.

1. Let M, N be normal subgroups of a group G.

   (a) Show that MN is a subgroup of G, M is normal in MN and $M \cap N$ is normal in N. [10]

   (b) Show that $\frac{MN}{M}$ is isomorphic to $\frac{N}{M \cap N}$. [10]

2. Let $J = \{a + bi : a, b \text{ integers}\}$.

   (a) Show that $(J, +)$ is a group. [5]

   (b) Show that J is not cyclic. [5]

   (c) Show that there exist cyclic groups G, H such that J is isomorphic to $G \times H$. [10]

3. What are the homomorphisms from $\mathbf{Z_{12}}$ into $\mathbf{Z_{16}}$? Justify your answer. [20]

4. Let $n \geq 3$.

   (a) Show that the alternating group A_n is generated by the set of 3-cycles. [10]

   (b) Show that if a normal subgroup of A_n contains a 3-cycle then it equals A_n . [10]

5. If p is prime show that there are exactly 2 groups of order p^2 (upto isomorphism). [20]

6. Let S be a p-Sylow subgroup of a group G. If B is a subgroup of G of order p show that $xBx^{-1} \subset S$ for some x in G. [20]

determine the energy level to which it corresponds.

(b) Find the constants a and b such that $\phi_1(x) = \frac{1}{\sqrt{2\sqrt{\pi}}}(ax^2 - 1)e^{-x^2/2}$ and $\phi_2(x) = \frac{1}{\sqrt{6\sqrt{\pi}}}x(bx^2 + 1)e^{-x^2/2}$ are othogonal to $\psi(x)$. Determine the energy levels corresponding to $\phi_1(x)$ and $\phi_2(x)$.

INDIAN STATISTICAL INSTITUTE
Mid-semester Examination : (2005-2006)
Course Name: B.Stat. II Year
Subject Name : Physics II **Group B**

X

Date: 1.3.06 Maximum Marks: 25

Note: All questions carry 5 marks. Please answer any four of them. All symbols carry their usual meanings.

(1)
One *mole* of a mono-atomic perfect gas initially at temperature T_0 expands from volume V_0 to volume $2V_0$ at constant pressure. Calculate the work done and the heat absorbed by the gas.

(2)
Calculate the change in entropy in heating a gram-atomic weight of Silver at constant volume from 0^0 C to 30^0 C. The value of C_v is constant over this temperature and is given by *p* cal /deg.mole.

(3)
Large heat sources (*H* and *C*) are available at temperatures 900^0 K (*H*) and 300^0 K (*C*).

(a) 100 cal. of heat is removed from *H* and added to *C*. What is the entropy change of the universe?
(b) A reversible heat engine operates between *H* and *C*. Find the work done for each 100 cal. of heat removed from *H*.

(4)
(a) Give an example of a model for a real gas.
(b) How are the real nature of gas particles taken in to account in that model?
(c) Draw qualitative graphs of the isotherms and discuss its essential features briefly.

(5)
(a) Prove the Maxwell's relation $(\partial p/\partial T)_V = (\partial S/\partial V)_T$ where the symbols have their usual meanings.
(b) Using the above relation show that for an ideal gas, U depends only on T.

INDIAN STATISTICAL INSTITUTE

Mid semester Examination: (2005-2006)
B.Stat 2nd Year
Physics II
Total time: 2 hrs.
Date: 1.3.06
Group A

Total Marks 20

Answer any four questions. Each question carries equal marks.

1. (a) Calculate the wavelength of de Broglie waves associated with electrons accelarated through a potential difference of 200 Volts. [Given, charge of an electron = 1.6×10^{-19} Coulomb, mass of electron = 9.1×10^{-31}kg, h= 6.62×10^{-34} Joule sec].

(b) Energy levels A, B and C of a certain atom correspond to increasing energyvalues E_A, E_B and E_C respectively. If λ_A, λ_B and λ_C are wavelengths corresponding to the transitions $C \to B$, $B \to A$ and $C \to A$ respectively, then show that $\lambda_C = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$.

2. A particle is moving inside an infinite square well potential of length L. It's wave function at time $t = 0$ is $\psi(x,0) = c\, sin(\frac{\pi x}{L}) + \sqrt{\frac{2}{3L}} sin(\frac{3\pi x}{L}) + \sqrt{\frac{1}{3L}} sin(\frac{3\pi x}{L})$, where c is a constant. If a measurement of energy is carried out, what are the probable values and the corresponding probabilities?

3. Consider a Hamiltonian $H = -\frac{1}{2m}\frac{d^2}{dx^2} + \frac{k}{2}x^2$. Evaluate $\frac{d<x>}{dt}$, $\frac{d<p>}{dt}$ and hence determine $<x>(t)$ (Given $<x>(0) = x_0$).

4. A particle is moving inside a thin tube of length L. Calculate $\Delta x \Delta p$ in the nth state and hence estimate the ground state energy [For any operator A, $\Delta A = \sqrt{<A^2> - <A>^2}$].

5. For the Hamiltonian $H = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2$,
(a) Show that $\psi(x) = \frac{e^{-x^2/2}}{\sqrt{\sqrt{\pi}}}$ is an eigenstate of the above Hamiltonian and

P.T.O

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : (2006)

B.Stat. II Year

Economics II

1.3.06..............MaximumMarks-40...........Duration-120 minutes

Answer all questions

1a.What is value added? Show that aggregate demand for final goods and services equals the sum of the value added of all enterprises in an open economy.

b.. For producing current period output the firm B incurs the following costs:- It purchases **Rs.50, 000** worth of goods from firm A, holds half of it in inventory and uses the other half as raw material for current production ,**Rs.40,000** is paid as wage bill of which half is paid to the labour contractor, **Rs,35,000** is paid as interest to banks, **Rs.5,000** is paid as interest on bonds sold to households, **Rs.15,000** is paid as rent to households,**Rs.500,000** is distributed as dividend, **Rs.20,000** paid as profit tax, the firm pays **Rs.75,000** as net indirect tax and donates **Rs.1 0,000** to Ramkrishna Mission's relief fund.(assume social security contribution is zero), undistributed profit of the firm B amounts to **Rs.200,000.**

i)What is the intermediate input cost incurred by the firm B? What is the value of output produced by the firm? What is the gross value added of the firm?

ii) what is the firm B's contribution to National income?

iii)what is the firm B's contribution to personal income? (10+15)

2a)How do you define saving (S) (domestic saving)? What are the three components of saving (domestic saving)? Can you show if saving exceeds investment in the domestic economy then the rest of the world investment exceeds rest of the world saving by the same magnitude.

b) Suppose in an economy GNP is **1600** units, private disposable income is **1,000** units, government budget deficit is **20** units, consumption is **850** units, trade deficit is**10** units, net factor income earned from abroad is **30** units, net transfer earning from abroad is nil, depreciation is **50** units.

a) How large is saving S?
b) What is the size of investment I ?
c) How large is government spending? (7+8)

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination
B.Stat (Hons.) – II
Biology – II

Date...1.3.06..............Maximum no. 30 Duration: 2.00 hrs.

1. Name the different meteorological variables that are related to crop production. Write down the names of apparatus used to measure those parameters.

   10

2. Classify rice and rice culture depending on eco-geographical situation. Critically highlight the variation in yield of winter (*Aman*) rice and summer (*Boro*) rice.

   5+3

   OR

   Define drought. Write about different types of drought. Write in details about any two of the drought indices

3. Write short notes on any four 4 X 3 =12
   a) Onset of monsoon
   b) Withdrawal of monsoon
   c) Field capacity
   d) Evapotranspiration
   e) Soil texture
   f) Phytoclimate

INDIAN STATISTICAL INSTITUTE

Mid-Semestral II Examinations 2005 – 2006

B. Stat (Hons.) II : 2005 – 06

Marks : 100

Date : 03 March 2006 Time : 10:30 A.M. to 1:30 P.M.

GROUP A

DEMOGRAPHY

50 Marks

Attempt Q.4 and any TWO from the rest

1. a) Define Census
   b) Describe various method of enumeration in Census operation and their merits and demerits. What is the method of enumeration in Indian Census 2001 ?
   c) What do you mean by system of census ? Dicuss various system of Census. State the system of Census adopted in Indian Census 2001.

3 + 7 +2 +6 +2 = 20

2. a) How is the age of an individual recorded in Census ?
   b) How can you detect age heaping ?
   c) Describe Myers' Index for detection of errors in age returns and discuss its merits and demerits.
   d) Give an account of Chandrasekaran – Deming's formula for completeness of data from the vital registration system.

2 + 2 + 8 + 8 = 20

3. a) Show that the geometric growth rate is approximately equal to the exponential growth rate up to a certain degree.

   b) Derive the logistic curve :

$$P_t = d + \frac{A}{1 + e^{-ct}}$$

where symbols have their usual meanings.

c) By employing Hoetelling's method, fit the logistic curve as given in b).

5 + 7 + 8 = 20

4. Show that

$$C_x = \frac{6 + AR_x}{7AR_x}$$

where age heaping spread over six year age interval. AR_X is the age ratio score at age 'x' and C_x is the correction factor at age 'x'.

10

SQC and OR

Group 'B'

X

Maximum Marks: 50

(i) Begin this group on a new answer script.

(ii) This group carries 55 marks. You may answer as much as you can, but the maximum you can score is 50.

1. Two alloys A and B are made from four different metals I, II, III and IV, according to the following specifications :-

Alloy	Specification	Selling Price ($)/ ton
A	at most 80% of I at least 30% of II at least 50% of IV	200
B	between 40% & 60% of II at least 30% of III at most 70% of IV	300

The four metals, in turn, are extracted from three different ores with the following data:

Ore	Max Quantity (tons)	Constitutents (%)					Purchase Price ($)/ton
		I	II	III	IV	others	
1	1000	20	10	30	30	10	30
2	2000	10	20	30	30	10	40
3	3000	5	5	70	20	0	50

The company has to decide how much of each alloy should be produced to maximize the profit. Formulate this company's problem as a LP model.

[15]

2. Consider the following problem P :

P : Minimize $x_0 = 2x_1 + 3x_2 + 5x_3 + 2x_4 + 3x_5$

Subject to

$$x_1 + x_2 + 2x_3 + x_4 + 3x_5 \geq 4$$
$$2x_1 - 2x_2 + 3x_3 + x_4 + x_5 \geq 3$$
$$x_i \geq 0 \quad i = 1, \ldots, 5$$

P.T.O.

(a) Write down the dual D of the above problem.
(b) Solve the dual problem D.
(c) Hence obtain the solution of the original problem P.

[5+5+5=15]

3. (a) Express the following linear programme into a standard form :

Maximize $z = x_1 + 2x_2 - x_3$

subject to

$$x_1 + x_2 - x_3 \le 5$$
$$-x_1 + 2x_2 + 3x_3 \ge -4$$
$$2x_1 + 3x_2 - 4x_3 \ge 3$$
$$x_1 + x_2 + x_3 = 2$$
$$x_1 \ge 0,\ x_2 \ge p \qquad x_3 \text{ unrestricted.}$$

(b) Mention the range of p in the standard LP.

[5+3=8]

4. Answer the following questions.

(a) Why is it sometimes more advantageous to obtain the optimal solution of the primal by solving the dual ?

(b) What is the ISO definition of Quality ?

(c) What are Quality Costs ? Mention the broad categories of quality costs.

[3+2+(3+2)=10]

5. Choose the best answer (Do NOT copy the statements).

(i) The j^{th} constraint in the dual of a LPP is satisfied as strict inequality by the optimal solution. The the j^{th} variable of the primal will assume a value.

(a) $\neq 0$ b) ≤ 0 c) ≥ 0 d) 0

(ii) If the j^{th} constraint in the primal is an equality, then the corresponding dual variable is

(a) unrestricted in sign (b) restricted to ≥ 0 (c) restricted to ≤ 0.

(iii) The optimum of a LPP occurs at X = (1, 0, 0, 2) and Y = (0,1,0,3). Then the optimum also occurs at (a) (2, 0, 3, 0) (b) $\left(\frac{1}{2}, \frac{1}{2}, 0, \frac{5}{2}\right)$ (c) (0,1,5,0) (d) none of the above.

(iv) If in any simplex iteration the minimum ratio rule fails, then the LPP has

(a) non degenerate BFS (b) degenerate BFS (c) unbounded solution (d) infeasible solution

(v) A LPP in standard form has m constraints and n variables. The number of basic feasible solutions will be

(a) ${}^{n}C_{m}$ (b) $\le {}^{n}C_{m}$ (c) $\ge {}^{n}C_{m}$ (d) none of these

(vi) Who is regarded as the father of SQC methodology ?

(a) Box (b) Romig (c) Taguchi (d) Shewhart

(vii) Who is associated with the PDCA wheel ?

(a) Ishikawa (b) Deming (c) Junan (d) Feigenbaum

Statistical Methods IV
B-Stat 2^{nd} Year 2005-2006
End-Semester Examination

Date: May 3, 2006 **Time: $3\frac{1}{2}$ hours**

This paper carries 55 marks. Answer all questions.
The maximum you can score is 50

1. (a) A DNA sequence comprises nucleotides A, T, G, C; each occurring independently with probabilities p_1, p_2, p_3 and p_4, respectively, where $\sum_{i=1}^{4} p_i = 1$. Given a DNA sequence comprising n nucleotides, obtain the Fisher's information matrix for the vector (p_1, p_2, p_3).

(b) Suppose $\Sigma_{a,c} = aI + c1' + 1c'$ where a is a scalar and c is a vector. Under what necessary and sufficient conditions on a and c is $\Sigma_{a,c}$ a valid dispersion matrix? [5 + 5]

2. (a) Suppose $\mathbf{x}$ is distributed as $N_p(0, \sigma^2 I)$. If A is a symmetric, idempotent matrix, obtain the distribution of $\mathbf{x}'A\mathbf{x}/\mathbf{x}'\mathbf{x}$.

(b) Suppose $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}$ is a random sample from $N_p(\mu, \Sigma)$, obtain an unbiased estimator of $\mu'\Sigma^{-1}\mu$ as a linear function of its m.l.e.
[5 + 5]

3. (a) Using random observations from $U(0,1)$, explain how you would generate a pair of observations (X, Y) such that the marginal distributions of X and Y are chi-squares with 2 d.f. and 3 d.f., respectively, and the correlation coefficient between between X and Y is 0.6.

(b) It has been postulated that the variance of the end-semester scores in a subject is twice that of the mid-semester scores. A student, who suspects that this ratio is in reality less than what is postulated, collects data on the mid-semester and end-semester scores of 4 randomly chosen students as follows: (13,29), (18,31), (18,36) and (19,33). Is the student's suspicion justified? State all your assumptions clearly.

(c) There is a lot of debate on whether Indian cricketers perform as well outside the sub-continent as in sub-continent pitches. The performances of 10 cricketers were studied in terms of the following variables:

X_1: mean first innings score in sub-continent pitches
X_2: mean second innings score in sub-continent pitches

P.T.O

X_3: mean first innings score outside the sub-continent
X_4: mean second innings score outside the subcontinent

Based on the sample of 10 cricketers, the mean vector and dispersion matrix of (X_1, X_2, X_3, X_4) were estimated as:

$$\begin{pmatrix} 36.12 \\ 33.49 \\ 33.93 \\ 20.44 \end{pmatrix}, \begin{pmatrix} 9.8 & 8.2 & 13.3 & 7.2 \\ & 14.4 & 18.4 & 9.6 \\ & & 28.2 & 14.8 \\ & & & 9.9 \end{pmatrix}$$

Do the above data support the hypothesis that the mean first innings score in sub-continent pitches and outside the sub-continent are equal, but the mean second-innings score in sub-continent pitches is twice that outside the sub-continent? State all your assumptions clearly. [5 + 7 + 8]

4. (a) Consider two populations: $N_p(\mu_1, \Sigma)$ and $N_p(\mu_2, \Sigma)$. If an observation $\mathbf{x}$ is classified to that population which yields the greater likelihood, show that the above rule is equivalent to classifying $\mathbf{x}$ to Population 1 iff $(\mathbf{x} - \mu_2)'\Sigma^{-1}(\mu_1 - \mu_2) \geq \Delta^2/2$, where Δ^2 is the Mahalanobis distance between the two populations.

(b) The lifetime of Nokia cellphone batteries is distributed as exponential. 30 randomly chosen batteries were studied and the five maximum lifetimes were 5.6, 5.37, 5.21, 5.17 and 4.97 years, while the mean lifetime was 3.62 years. Based on these data, obtain a 95% confidence interval for the 0.9^{th} quantile of the lifetime distribution of Nokia batteries.

(c) Suppose X and Y are both binary random variables, assuming values 0 or 1. If the dependence of Y on X is modelled via a linear logistic function with slope parameter β, show that the Odds Ratio based on (X, Y) can be expressed as a function of β. Hence, based on data on (X, Y), explain how you would test $H_0 : \beta = 1$ vs $H_1 : \beta > 1$ at level 0.01.

[5 + 5 + 5]

INDIAN STATISTICAL INSTITUTE

B. Stat Second Year

Elements of Algebraic Structures

Date : 9. 5. 06. Semestral-I Examination Time : 3 hrs.

This paper carries 120 marks. Maximum you can score is 100. You may use any theorem proved in the class.

1. (a) How many non isomorphic groups are there of order 9 ? [10]

(b) How many non isomorphic groups are there of order 45 ? [20]

Justify your answers.

2. Let R be a commutative ring and A an ideal of R. Let $N(A) = \{a \in R : a^n \in A$ for some positive integer $n\}$

(a) Show that $N(A)$ is an ideal of R. [10]

(b) Show that $N(N(A)) = N(A)$. [5]

3. Let $a_1, a_2, \ldots a_n \in$ an extension of a field F .

(a) Show that $F(a_{i_1})(a_{i_2})\ldots(a_{i_n}) = F(a_1)(a_2)\ldots(a_n)$ where $(i_1, i_2, \ldots i_n)$ is any rearrangement of $(1, 2, \ldots n)$ [15]

(b) If $[F(a_i) : F] = p_i$ where $(p_i, p_j) = 1, 1 \leq i < j \leq n$, show that $[F(a_1)(a_2)\ldots(a_n) : F] = p_1 p_2 \ldots p_n$. [15]

4. Let α be algebraic of odd degree over a field F. Show that

(a) α^2 is algebraic of odd degree over F. [15]

(b) $F(\alpha^2) = F(\alpha)$ [15]

5. Let Q be the field of rational numbers. Show that $Q(\sqrt{2})$ and $Q(\sqrt{3})$ are not isomorphic . [15]

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INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2005 – 2006)
B. Stat II Year
Biology II

Date : 12.5.06 Maximum Marks 50 Duration 2:30 hours .

(Attempt any five questions)

1. Write in brief about different types of rice. Briefly describe the cultural practices associated with rainfed lowland rice cultivation.
3+7

2. What are the differences between Manures and Fertilizers? Calculate the quantity of VC, Urea, SSP and KCL required for 1 ha. rice crop to supply the nutrient requirement of 100 kg N, 50 kg P_2O_5 and 50 kg K_2O per hectare. 25% of required N should be given through VC.
3+7

3. Describe the suitable agrotechniques for rice nursery bed preparation. Estimate the expected yield of rice grain in t/ha from the following data.
i) Spacing = 20x10cm ii) Average no. of tillers/hill = 9 iii) Average no. of effective tillers/hill = 7 iv) Average no. of grain/panicle = 145 v) Average no. of unfilled grain/panicle = 30 vi) Test weight = 20 g.
4+6

4. Write short notes on any five of the following: 2 x 5
   a) Monsoon onset
   b) Moisture availability index
   c) Cup counter anemometer
   d) Reproductive stages in rice
   e) Phytoclimate
   f) Field capacity
   g) Capillary water

5. Write in brief about any two of the following: 2 x 5
   a) Weather forecasting.
   b) Available soil moisture.
   c) Histological Defense and Chemical Defense
   d) Inoculum Potential and Functional Resistance

6. Give salient features of enzymes. How enzymes are classified? Name different types of enzymes. What are the factors affecting the enzyme activity?
3+2+2+3

7. (a) Define stability of a dynamical system $\frac{dX}{dt} = f(X)$.

   (b) Transform the following model into non-dimensional form

$$\frac{dN}{dt} = rN - cNP$$

$$\frac{dP}{dt} = bNP - mP$$

Here N and P are the population densities of prey and predator respectively. Find the dynamical behavior of the system around the positive interior equilibrium.

2+3+5

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INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2005-06
B. Stat. I Year
Probability Theory II

Date: 12.5.06 **Duration: 3 Hours**

This paper is set for 70 Marks. Maximum you can score is 60.

1.(a) X is a random variable with density

$$f(x) = \frac{C}{e^{10x} + e^{-10x}} \qquad -\infty < x < \infty.$$

Find the value of C. Find the median of X.

(b) $X \sim Uniform(0,10)$. Find the density of the random variable $Y = Min(X, 10 - X)$.

[4+4]

2. X, Y are independent $Uniform(0,1)$ random variables. Find the density of $Z = 1 - X - Y + XY$.

[10]

3. Let X be a random variable having all moments finite. Let $\mu_i = EX^i$. Show that the following matrix is nonnegative definite

$$\begin{pmatrix} \mu_{10} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{14} & \mu_{15} \\ \mu_{13} & \mu_{15} & \mu_{16} \end{pmatrix}$$

[10]

4. $X \sim Uniform(0,1)$. Given $X = x$, the conditional density of Y is $Uniform\ (x-1, x+1)$. Find the joint density of (X, Y). Find the conditional density of X given Y.

[7]

5. X, Y are independent standard Cauchy variables. Show that XY has density $\frac{2}{\pi^2} \frac{\log|x|}{x^2 - 1}$.

[10]

P.T.O.

6. X is a random variable having density $f(x)$, $-\infty < x < \infty$. Find the joint distribution function of (X, Y), where $Y = |X|$. Find the conditional distribution of X given Y. You should show that your conditional distribution satisfies the defining equation.

[10]

7. Calculate the moment generating function of a standard normal variable X. Use this to give an explicit formula for the moments $\mu_{2n} = E(X^{2n})$ for $n \geq 1$.

[5]

8. X is a symmetric random variable such that $X^2 \sim Unif(0,5)$. Calculate the density of X.

[10]

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Syllabus for the end semester examination : **Simple Keynesian model, IS-LM model, money supply.**

Course: B-Stat II Year 2006
End Semester
Subject: Economics II

Date: 12.5.06
Time 3 Hours — Full Marks 60
All questions carry equal marks
Answer any three questions

1. Suppose that a closed economy is given by the following equations: C = 100 + .8(Y-T), I = 20, G = T=10 . Find out the equilibrium level of Y. What is the value of the involuntary change in inventory at Y = 500? By how much will Y rise following an increase in G by 10 units, accompanied by an equal increase in T? Does your answer to the last question change if marginal propensity to consume out of disposable income is .9 instead of .8? Explain. **(20)**

2. Suppose that an economy is given by the following equations: C = 100 + .8(Y-T), I = 20, G = 10,T=10. The full employment level of output of the economy is Y_f=800. (i)By how much will the government have to change its expenditure G to achieve full employment equilibrium? (ii)If the government wants to achieve the same target by changing the level of lump sum tax, by how much will it have to change T? How can you explain the difference in answer to part (i) and (ii)(both sign and magnitude) ? **(20)**

3a.Is the value of the investment multiplier in the IS-LM model different from that in the simple Keynesian model? Explain your answer.

3b.Following an unit increase in autonomous expenditure the IS curve is found to shift horizontally by $\frac{5}{2}$ and vertically by 50 units respectively. Derive the slope of the IS curve. An increase in money supply is found to increase y by 250 units in this IS-LM model. What is the accompanying change in interest rate? (Here 1 unit change in r implies 1percent change in r).

(10+10)

4.Suppose that in an economy currency-deposit ratio of the public is (1/4), cash-reserve ratio of the banks is (1/10). In this economy the government borrows Rs.100 from the central bank to finance its expenditure. Will it have any impact on the supply of money? If your answer is yes, explain the whole process of change in money supply. **(20)**

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Second semestral Examination: (2005-2006)
B.Stat 2nd Year
Date: 12.5.06 Physics II Time: 2 hrs.
Use seperate answerscripts for Gr. A & B.
Group A

Total Marks: 20

Answer any four questions.

1. Let H be a Hamiltonian such that $H\psi_n = n^2\psi_n$ where $<\psi_m|\psi_n>= \delta_{mn}$. Consider a state $\phi(x) = \frac{1}{2}\psi_1(x) + A\psi_2(x) + \frac{1}{4}\psi_3(x)$. Then
   (a) Determine the constant A,
   (b) Find $\psi(x,t)$,
   (c) Obtain the probability of finding the system at a time t in the state $\psi_3(x,t)$. (1+2+2)

2. Starting from the relations $a|n>=\sqrt{n}|n-1>$, $a^\dagger|n>=\sqrt{n+1}|n+1>$, obtain matrix representation of the operators a^2 and $a^{\dagger 2}$. (2+3)

3. A particle is moving in a potential
$$V(x) = V_0, \quad x > 0$$
$$= 0, \quad x < 0$$
Find the reflection (R) and transmission (T) coefficient when $E > V_0$ and verify that $R + T = 1$. (2+2+1)

4. Using creation and annihilation operators (or otherwise) show that the ground state of the harmonic oscillator saturates the uncertainty relation $\Delta x \Delta p \geq \frac{\hbar}{2}$ where $\Delta x = \sqrt{<x^2> - <x>^2}$. (5)

5. The Schrödinger equation of a particle moving in a potential $V(x)$ is given by
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$
where

$$V(x) = 0, \; 0 < x < L$$
$$= \infty, \; \text{elsewhere}$$

(a) Obtain the eigenvalues and the corresponding eigenfunctions,

(b) Using the result $\Delta x \Delta p \sim \hbar$, explain why the ground state energy is non zero.

(4+1)

GROUP - B

Maximum Marks: 20

Answer any four questions.

1)

Consider a classical ideal gas at temperature **T** in equilibrium in a volume **V** and **m** denotes mass of the gas molecules and **N** is the number of gas molecules.

(a) Find partition functions for a single particle and also for the entire system.

(b) Find the mean pressure and hence deduce the ideal gas law.

3+2=5

2)

Consider a harmonic oscillator in one dimension with **m** and **k** denoting the particle mass and spring constant respectively. Treat the problem in a quantum mechanical way.

(a) What are the energy levels for the above quantum harmonic oscillator?

(b) Let the above oscillator be in thermal equilibrium with a heat bath at temperature **T**. Find an expression for the average energy.

(c) Find the classical (high temperature) limit of the energy and show that it agrees with classical equipartition of energy theorem.

1+2+2=5

3)

Consider a substance having **N** spin **half** atoms per unit volume at temperature **T**. The atoms have an intrinsic magnetic moment μ. The system is placed in an external magnetic field **H**. Treat the system in a quantum mechanical way.

(a) Calculate the mean value of magnetic moment μ for a single atom and the magnetization (the mean magnetic moment per unit volume).

(b) In the above problem take the atoms to be of spin **one**. Find the partition function for a single atom.

3+2=5

4)

A molecule has three energy levels, $\mathbf{E_1 = 0}$, $\mathbf{E_2 = \varepsilon}$ and $\mathbf{E_3 = 10\varepsilon}$ and **N** such molecules are placed in a heat bath at temperature **T**. The particles follow classical Boltzmann statistics.

(a) Find the ratio of the number of particles in energy levels $\mathbf{E_2}$ and $\mathbf{E_3}$.

(b) Find the average energy of a single molecule.

2+3=5

5)

Consider a gas of photons at temperature **T**.

(a) Derive the photon distribution function, that is the average number of photons in a quantum state s, having energy ε_s.

(b) Write down the expression for Bose-Einstein distribution function. On what condition does it reduce to the photon distribution function?

(c) How is the Pauli-exclusion principle for fermions reflected in the Fermi-Dirac distribution function?

3+1+1=5

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INDIAN STATISTICAL INSTITUTE
Second Semester Examination: 2005-06
M. S. (Q.E.) I Year & II Year
Modern Growth Theory

Date: 15.05.2006 **Maximum Marks: 60** **Duration: 3 Hours**

Answer any three questions. All questions carry equal marks.

1) Derive the rate of growth of income along the balanced growth path in the model of Barro (1990) when the tax rate is exogenously given.

2. In the presence of negative externalities of environmental pollution, competitive equilibrium rate of growth differs from the socially efficient rate of growth-Examine the validity of this statement in terms of an appropriate endogenous growth model.

3. Show that, in the absence of externalities, the steady-state growth equilibrium in the Lueas (1988) model is a saddle point.

4. How does the model of Benhabib and Farmer (1994) differ from the Ramsey-Solow model ? Derive the condition of multiple equilibria in athe model of Benhabib and Farmer (1994).

5. Consider a two country dynamic model with perfect international capital mobility and analyse the stability property of its steady-state growth equilibrium.

□□□□□

INDIAN STATISTICAL INSTITUTE

B. Stat II: 2005 – 2006

Second Semester Examination

Economic Statistics and Official Statistics

Duration: 3 hours | Maximum marks: 100 | Date: 15.05.2006

[Answer Group A and Group B in separate answer scripts. Answer question no. 1 and any ***three*** *of the rest of the questions of Group A and all questions of Group B. Allotted marks are given in brackets[] at the end of each question.]*

Group A: Economic Statistics

1. Derive (i) Elteto-Fryges Measures and (ii) Relative Mean Deviation About Mean for Pareto distribution. Hence find values of these measures for the Pareto distribution having the inequality parameter $\nu = 2.0$. [12+12+2=26]
2. Derive Sen's measure of Poverty. Show why this measure is superior to Head Count Ratio and Income Gap Ratio. Examine whether Sen's measure of Poverty satisfies the strong transfer axiom. [18]
3. Write down the five axioms of the economic-theoretic approach to Price Index Numbers. Prove that none of these is superfluous. Give examples of at least six index number formulae satisfying all these five axioms. [5+10+3=18]
4. State the principle of Diminishing Transfer of Income. Examine CV, LR and RMD in the light of this principle. [18]
5. Explain how Prais and Houthakker accommodate the phenomenon of economy of scale in Engel curve analysis to examine the effect of household size? How can one estimate these parameters? [12+6=18]
6. Write short notes on any three of the following:
   (i) Measures of Concentration in Business and Industry
   (ii) Fixed base indices vs. Chain base indices.
   (iii) Estimation of two-parameter lognormal distribution.
   (iv) Linear Expenditure System.
   (v) Law of Proportionate Effect. [6×3=18]

Group B: Official Statistics

1. Briefly describe the basic functions of National Sample Survey Organisation. [5]
2. Briefly compare the role of Index of Industrial Production and Annual Survey of Industries in understanding industrial growth of the country. [5]

   Or

   Indicate the production boundary used for compilation of GDP of India. [5]
3. Calculate national income by expenditure and income methods from the following data. [10]

Sl. No.	Item	Value in Rs. Crore
01.	Compensation to employees	5200
02.	Government final consumption expenditure	1500
03.	Net indirect taxes	1400
04.	Operating surplus	2000
05.	Net exports	(-) 400
06.	Gross fixed capital formation	2500
07.	Private final consumption expenditure	12000
08.	Net change to stocks (addition)	400
09.	Net factor income from abroad	400
10.	Consumption of fixed capital	1000
11.	Mixed income of the self-employed	6400

INDIAN STATISTICAL INSTITUTE
SEMESTER II EXAMINATION : 2005-06
B. STAT. (H) II YEAR : 2005-06

Subject : Demography and SQC & OR

Date: 19.5.06 | Full Marks: 100 | Time: 3 hours

(Answer each group in a separate answer script)

GROUP –A : DEMOGRAPHY

Marks : 50

Attempt Question no. 1 and any two from the rest.

1. Indicate the correct answer :

a) Death rates are standardised:

(i) to eliminate the differential influence of one or more variables.
(ii) to obtain an estimate of the ideal rate.
(iii) to determine the future rates that may be expected.
(iv) to obtain a correct statement of the actual or experienced rates.
(v) to correct for under registration of the phenomenon in question.

b) The difference between a generation life table and a period life table is that.

(i) the radix is different.
(ii) one refers to a true birth cohort and the other does not.
(iii) one uses a different method for calculating q_0 than the other.
(iv) none of the above.

c) If country – A has a higher life expectancy and also a higher crude death rate than country– B, it is likely that:

(i) A's population is younger than that of B.
(ii) A's population is older than that of B.
(iii) A's population has a higher infant mortality rate.
(iv) none of the above.

d) The stationary population is a model that :

(i) excludes migration.
(ii) holds fertility constant.
(iii) has fixed mortality rates.
(iv) is not very good as descriptive model and is mainly useful for analytic purpose.
(v) only (i), (ii) and (iii) are true
(vi) only (i), (ii), (iii) and (iv) are true.

e) Survival ratios may be use for :

(i) making projections of the future population.
(ii) comparing the mortality of several countries or the same country at different points of time.
(iii) estimating the effects of different levels of q_x on future population sizes.
(iv) Only (i) and (ii) are true.
(v) Answers (i), (ii) and (iii) are true.

P.T.O

f) A crude rate of 3 percent growth of the population leads to the doubling of population in approximately:

(i) 15 years.
(ii) 23 years.
(iii) 35 years.
(iv) 50 years.
(v) 70 years.

g) The net reproduction rate is a measure of the:

(i) annual excess of births over deaths.
(ii) annual rate at which women are replacing themselves on the basis of prevailing fertility and mortality assuming no migration.
(iii) decennial growth rate of the population.
(iv) per generation growth rate assuming current age-specific fertility and mortality rates and no net migration.
(v) none of the above.

h) Period birth rates and cohort birth rates may exhibit large differences under which of the following conditions:

(i) when most couples plan their fertility.
(ii) when the mean age at marriage is increasing.
(iii) when the mean age at marriage is decreasing.
(iv) (i) and (ii) of the above are true.
(v) (i), (ii) and (iii) of the above are true.

i) The crude birth rate of India per thousand population at present is approximately:

(i) 10
(ii) 15
(iii) 20
(iv) 32
(v) 40
(vi) 60

j) The net reproduction rate per woman in India is now approximately :

(i) less than 1.0
(ii) 1.0 - 1.5
(iii) 1.5 – 2.0
(iv) 2.0 – 3.0
(v) more than 3.0

[1 x 10 = 10]

2. Write briefly why there is a need for evaluation and adjustment of basic demographic data. Discuss a method of evaluating age data given by quenquennial age-groups.

[8+12=20]

3. Define crude and standardised death rates. In what way are the standardised rates superior ? Explain briefly the differences between the direct and indirect methods of standardising death rates.

[4+4+12=20]

4. a) Deduce :

$$ {}_nq_x = \frac{{}_nm_x}{\frac{1}{2} + {}_nm_x\left[\frac{1}{2} + \frac{n}{12}\left({}_nm_x - {\cdot}09\right)\right]}, $$

where ${}_nq_x$ = Probability of dying between age x and x + n

& ${}_nm_x$ = Death rate between age x and x + n.

Explain all the symbols you use and the assumptions you make.

b) If between ages x to $x+t$, ${}_tp_x$ is the probability of surviving and μ_{x+t} is the force of mortality, then show that.

$$ \int_0^\infty {}_tp_x\, \mu_{x+t}\, dt = 1 $$

c) Deduce :

$$ \int_0^x \gamma(t)dt = -\log s(x), $$

where : $\gamma(x)$ = Force of nuptiality at age x

& s(x) = Proportion single at age x.

Explain all symbols you use and the assumptions you make.

[8+4+8=20]

5. Distinguish between fecundity and fertility. Describe a fertility model which incorporates various socio-cultural factors affecting human fertility. From this model, estimate the effect of (i) non-marriage, (ii) use of contraceptions and abortions within marriage, and (iii) postpartum abstinence after child birth within reproductive years of women in reducing live-births.

[4+10+6=20]

P.T.O

Group B
(SQC & OR)

This group carries 55 marks. You may answer as much as you can, but the maximum you can score is 50.

1. Subgroups of $n = 6$ items are taken from a manufacturing process at regular intervals. A normally distributed quality characteristic is measured and $\bar{x}$ and S values are calculated for each sample. After 50 subgroups have been analysed, we have

$$\sum_{j=1}^{50} \bar{x}_j = 1000 \quad \text{and} \quad \sum_{j=1}^{50} S_j = 75$$

(a) Compute the control limits for the $\bar{x}$ and S control charts.

(b) Assume that all points on both the charts lie within the control limits. What are the natural tolerance limits for the process?

(c) Suppose the specification limits are 19 ± 4. Assuming that if an item exceeds the upper specification limit it can be reworked, while if it is below the lower specification limit, it must be scrapped, what percent scrap and rework is the process now producing?

(d) If the process were centered at $\mu = 19.0$, what would be the effect on percent scrap and rework?

[4+2+5+4=15]

2.(a) Suppose that a single sampling acceptance rectification plan $n = 150, c = 1$ is being used for inspection where the vendor ships the product in lots of size $N = 3{,}000$. Find the $AOQL$ for this plan.

(b) Show that for an acceptance rectification plan, $AOQ = p(1 - AFI)$

[10+5=15]

3. Write down the dual (D) of the problem P.

P : Maximize $z = 4x_1 + 5x_2$

Subject to

$$3x_1 + 2x_2 \le 20$$
$$4x_1 - 3x_2 \ge 10$$
$$x_1 + x_2 = 5$$

$x_1 \ge 0$, x_2 unrestricted in sign.

[7]

4. Choose the correct answer that is most complete:

(a) Which of the following is NOT a benefit of SPC charting?

(i) Charting helps in evaluation of system quality.
(ii) It helps identify unusual problems that might be fixable.
(iii) It encourages people to make continual adjustments to processes.
(iv) It encourages a principled approach to process meddling (only after evidence).
(v) Without complete inspection, charting still gives a feel for what is happening.

(b) When considering sampling policies, the risks associated with accepting an undesirable lot grows with

(i) Larger rational subgroup size.
(ii) Decreased tolerance of non-conformities in the rational subgroup (e.g., lower c).
(iii) Increased tolerance of non-conformities in the overall lot (e.g. higher c).
(iv) Decreased overall lot size.
(v) None of the above.

(c) A constraint in an LP model restricts

(i) Value of objective function
(ii) Value of a decision variable
(iii) Use of the available resource
(iv) All of the above

(d) A constraint in an LP model becomes redundant because

(i) two iso-profit lines may be parallel to each other
(ii) the solution is unbounded
(iii) this constraint is not satisfied by the solution values.
(iv) none of the above

(e) The role of artificial variables in the simplex method is

(i) to aid in finding an initial solution.
(ii) to find optimal dual prices in the final simplex table
(iii) to start phases of the simplex method
(iv) all of the above.

(f) If for a given solution, a slack variable is equal to zero, then

(i) the solution is optimal
(ii) the solution is infeasible
(iii) the entire amount of resource with the constraint in which the slack variable appears has been consumed.
(iv) all of the above.

[$6\times 1\frac{1}{2}=9$]

5.(a) Give the full form of the following

(i) LTPD
(ii) AQL
(iii) ARL

(b) Distinguish between Type-A and Type-B OC –Curves.

(c) Assume that $X \sim N(\mu,\sigma^2)$. Define $S^2 = \frac{1}{n-1}\sum(X_i - \bar{X})^2$

Assume $E(S) = \alpha_n\sigma$. Show that $\alpha_n \le 1 \ \forall n$.

[3+3+3=9]

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INDIAN STATISTICAL INSTITUTE
SEMESTER II EXAMINATION : 2005-06
B. STAT. (H) II YEAR : 2005-06

BACK PAPER

Subject : Demography and SQC & OR

Date: 21.7.06 Full Marks: 100 Time: 3 hours

(Answer each group in a separate answer scripts)

GROUP –A : DEMOGRAPHY

Marks : 50

Attempt Question no. 1 and any two from the rest.

1. Determine whether each of the following statements is true or false:

a) On the average, age- specific death rates are high only in persons over age 65.
b) A period life table is a hypothetical model because mortality rates actually change from one time period to the next.
c) In a life table, the life-table death rate is twice the life table birth rate.
d) It would be possible to construct an age standardised rate of natural increase.
e) The net reproduction rate can never be higher than the gross reproduction rate.
f) For all practical purposes, the gross reproduction rate is equal to the product of the total fertility rate times the proportion of live births that are female.
g) Death rates are standardised to correct for the under registration of the phenomenon in question.
h) Geometric growth rate is approximately equal to the arithmetic growth rate.
i) General marital fertility rate is always higher than general fertility rate.
j) The mean age at marriage of a population is always equal to the singulate mean age at marriage of that population.

[1 x 10 = 10]

2. Write briefly why there is a need for evaluation and adjustment of basic demographic data. Describe Myers' index in evaluating age data with its relative merits and demerits. Derive Zelnik's method in correcting age data stating the assumptions. .

[4+8+8=20]

3. Derive, under suitable assumption(s) to be stated clearly, the equation of the logistic curve. Enumerate the different properties of the logistic curve. Will the logistic curve give a satisfactory fit to Indian data ? Discuss.

[10+5+5=20]

P.T.O.

4. a) For a certain life table :

$$\ell_x = 20900 - 80x - x^2$$

i) What is the ultimate age in the table ?
ii) Find μ_x, q_x and ${}_{10}p_{20}$.

b) Describe different methods of computing the infant mortality rate. Comment on their me
and demerits.

[8+12=

5. a) Discuss a procedure for calculating the singulate mean age at marriage, if it is known t
5 percent women never marry at all.

b) Describe various columns of 'gross' nuptiality table along with formula.

c) What are the additional columns you need to compute in 'gross' nuptiality table to const
'net' nuptiality table ?

[8+10+2=

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Group B: SQC & OR

Marks: 50

Answer all questions.

1.a) List down the seven tools of QC.
b) Define Quality costs.
c) How does the ISO define Quality?

[4+3+3=10]

2. Consider a single sampling acceptance rectification plan. Suppose that the consignments come in lots of size 10,000. A random sample of 89 units is inspected; and the consignment is considered to be acceptable if the number of defectives in the sample is at most 2. For such a plan, find the AOQ and the ATI if the vendor's process operates at 1%.

[15]

3. $\overline{X} - R$ charts, based on a subgroup size of 5 rings, are used to monitor rings produced by a forging process. The parameters of the $\overline{X} - R$ charts are the following :-

Parameter	$\overline{X}$-chart	R-chart
UCL	74.014	0.049
Central Line	74.001	0.023
LCL	73.988	0.000

These charts are used at the shop floor. Since the process exhibited good control, manufacturing engineering personnel want to reduce the subgroup size to three rings. Find the new parameters of the $\overline{X} - R$ charts.

[10]

4. A company manufactures three grades of points: Venus, Diana and Aurora. The plant operates on a three-shift basis and the following data are available from the production records:

Requirement of Resources	**Grade**			**Availability (capacity/month)**
	Venus	**Diana**	**Aurora**	
Special additive (kg/litre)	0.30	0.15	0.75	600 tonnes
Milling (kl/machine Shift)	2.00	3.00	5.00	100 machine Shifts
Packing (kl/Shift)	12.00	12.00	12.00	80 Shifts

P.T.O.

There are no limitations on other resources. The particulars of sales foreca
and estimated contribution of overheads and profits are given below:

	Venus	Diana	Aurora
Maximum possible Sales per month (Kilolitres)	100	400	600
Contribution (Rs./Kilolitre)	4,000	3,500	2,000

Due to commitments already made, a minimum of 200 Kilolitres per month
Aurora has to be necessarily supplied the next year.

Just as the company was able to finalize the monthly production program for the next 12 months, an offer was received from a nearby contractor hiring 40 machine shifts per month of milling capacity for grinding Di paint, that could be spared for at least a year. However, due to additio handling at the contractor's facility, the contribution from Diana will reduced by Re 1 per litre.

Formulate this problem as an LP model for determining the mon production programme to maximize contribution.

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$mx - 2.0 = 0$.

INDIAN STATISTICAL INSTITUTE
B. STAT SECOND YEAR
Elements of Algebraic Structures
Date: 24.7.06 Backpaper Examination Time : **3** hrs.

Answer all questions. This paper carries 100 marks. You may use any theorem proved in the class.

1. G_1, G_2 are isomorphic groups with isomorphism ϕ on G_1 onto G_2. H_1 is a normal subgroup of G_1 .Show that
   (a) $\phi(H_1)$ is a normal subgroup of G_2 . [10]
   (b) G_1/H_1 is isomorphic to $G_2/\phi(H_1)$. [10]
2. D is an integral domain. Let m be the smallest positive integer such that there is $a \neq 0$ in D with $ma = 0$. Show that
   (a) $mx = 0$ for all x in D . [10]
   (b) m is prime. [10]
3. Let F be the field of rationals. What are the automorphisms of $F(\sqrt{2})$ onto itself ? [20]
4. Let p be any prime, $a \in Z_p$. Show that $x^p + a$ is not irreducible over Z_p . [20]
5. Let E be an extension of a field F with $[E : F] = m$. Let $f(x) \in F[x]$ be irreducible of degree n where $(m, n) = 1$. Show that $f(x)$ is irreducible over E. [20]

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Mid-semester Examination
B. Stat (II year): SemesterI
September 5, 2005.
Probability Theory III

Maximum Marks 30

Time 2:30 hrs

per carries a total of 32 marks: answer as many questions as you can. your score will be limited to 30.

(a) Let $X_1, X_2, \ldots, X_m$ have the multivariate normal distribution with the parameters $(0, \Sigma)$. Find the Conditional distribution of $X_{n+1}, X_{n+2}, \ldots, X_m$ given $X_1 = a_1, X_2 = a_2, \ldots, X_n = a_n$ where $n \leq m$. 5

(b) Let $Y_1, Y_2, Y_3, \ldots, Y_n, \ldots$ be a sequence of independent random variables with expectation 0. Define

$$X_1 = Y_1, \;\; X_2 = X_1 + Y_2Y_1, \;\; \ldots \;\; , X_{n+1} = X_n + Y_nY_{n-1}\ldots Y_1$$

Find $E(X_{n+1} \mid X_n, X_{n-1}, \ldots X_1)$. 3

(a) Given that (X_1, X_2) have uniform distribution on the unit square, find the conditional density of X_1 given $X_1 + X_2 = s$. 4

(b) The random vector $(X_1, X_2, \ldots, X_n)$is said to have an exponential type distribution if it has a joint density of the form $h(\mathbf{x})exp[\mathbf{x}'\mathbf{t} - q(\mathbf{t})]$ for a suitable constant vector $\mathbf{t}$ called parameters of the distribution and some function q of the parameters. Show that any marginal distributions of an exponential type distribution is again of exponential type. 4

(a)Show that if $X_1, X_2, \ldots, X_n$ are independent observations from a normal population with parameters (μ, σ^2), then the statistics

$\bar{X} = \frac{X_1 + X_2 + \ldots X_n}{n}$

$S^2 = \frac{\Sigma_1^n (X_i - \bar{X})^2}{n-1}$ are independent. 4

(b) Give an example of a bivariate density which is not normal but has normal marginals. 4

(a)Show that the function

$$f(x_1, x_2, \ldots, x_k) = \frac{\Gamma(\nu_1 + \nu_1 \ldots\ldots + \nu_{k+1})}{\Gamma(\nu_1)\Gamma(\nu_2)\ldots\Gamma(\nu_{k+1})} x_1^{\nu_1 - 1} x_2^{\nu_2 - 1} \ldots x_k^{\nu_k - 1}(1 - x_1 - x_2 \cdots - x_k)^{\nu_{k+1} - 1}$$

if $(x_1, x_2, \ldots, x_k) \in S_k = \{(x_1, x_2, \ldots, x_k) : x_i \geq 0, i = 1, \ldots, k, \Sigma x_i \leq 1\}$ and zero otherwise, where the ν_i are positive reals,
is a density function. 4

(b) Show that the function

$F(x, y) = \min\{x, y\}$ if $0 \leq x, y \leq 1$

and defined obviously elsewhere is a bivariate distribution function (only the last property needs to be checked). Show also that the distribution is concentrated on the diagonal. 4

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Mid- Semestral Exams (B. Stat, Biology I, 2005)

Date: 7th Sept., 2005

Answer any five; All questions carry equal marks; Full marks = 40; Time = 2.5 hours

1. a) Distinguish between photosynthetic and heterotrophic cells.
   b) How does ATP function in the transformation of energy by living cells?

2. a) What are enzymes?
   b) Discuss the special features of enzyme-catalyzed reactions.

3. What features distinguish a prokaryotic cell from an eukaryotic cell?

4. a) If pancreatic cells synthesizing digestive enzymes for export are supplied with radioactive amino-acids, the pathway of proteins from synthesis to exocytosis can be followed. In what order does the radioactivity appear in the organelles involved in exocytosis?
   b) The function of circulating white blood cells is to engulf invading bacteria in a process known as phagocytosis. Suggest what happens to the bacteria after they are engulfed.

5. Describe how the phenotypes (a) galactosemia, (b) alkaptonuria (c) albinism and (d) phenylketonuria are related to the metabolism to carbohydrate and protein.

6. "Pyruvate is not always metabolized to produce ATP" - give reasons to validate this statement showing the generation of ATP (if any). What is the efficiency of energy trapping when glucose is metabolized in anaerobic condition.?