INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2007-08

I

B. Stat. II Year
Statistical Methods III

BOOKS & CLASS NOTES ARE ALLOWED

Date: 03.09.07 **Maximum Marks:100** **Duration: 3 Hours**

1. Let a be a positive integer, a < 100.

   $X_1, X_2,...X_n$ are i.i.d. uniform over $\{a, a+1,...,100\}$.

   Provide a sufficient statistic for a and prove that it is sufficient.

   Suggest an unbiased estimate for a and improve it on the basis of the sufficient statistic.

[3+5+2+5=15]

2. a) $X_1, X_2,...X_n \overset{i.i.d}{\sim} N(\mu,\sigma^2)$.

   Consider the following estimators for σ^2.

   $$T_1 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

   $$T_2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

   which estimator is better with respect to MSE criterion ? Justify.

   b) Can you find another estimator whose MSE is less than or equal to that of both T_1 and T_2 ? Justify.

[8+7=15]

3. Let $X_1, X_2,...X_n \overset{i.i.d.}{\sim} U[\theta-1,\theta+1],\theta$ unknown. Provide an MLE for θ.

[10]

P.T.O

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2007-08
B. Stat. II Year
Statistical Methods III

Date: 23.11.07 Maximum Marks: 100 Duration: 3 Hours

Answer all questions
Class notes and text books are allowed.

1. Let $X_1, X_2, \ldots, X_n$ i.i.d. $U(a,2a)$ for given constant a.

a) Find MLE for a.
b) Here are 3 natural estimators for a

$$T_1 = X_{(n)}/2$$
$$T_2 = X_{(1)}$$
$$T_3 = X_{(n)} - X_{(1)}$$

Compare these 3 estimators.
c) Find an unbiased est. for a.

(5+10+5=20)

2. Consider a population with 3 kinds of individuals labeled 1,2,3 and occurring in the proportions

$$p(1,\theta) = \theta^2, p(2,\theta) = 2\theta(1-\theta), p(3,\theta) = (1-\theta)^2$$

We observe a sample of 100 individuals and find 18 of them belong to label 1, 42 of them belong to label 2 and 40 of them belong to label 3.

a) Find an unbiased estimate for θ.
b) Work out the MLE for θ.

(3+12=15)

3. Let $X_1, X_2, X_3, X_4 \overset{i.i.d.}{\sim} N(\mu,1)$.

What is the distribution of $\dfrac{(X_1 - X_2 + X_3 - X_4)}{|X_1 + X_2 - X_3 - X_4|}$? Justify.

(10)

P.T.O.

4. Sourav Ganguly scored 21, 105, 37, 53 (n.0.), 2 (n.0), 41, 79, 113, 67, 59, 23, 30 (n.0) in last 12 one day matches. Please suggest a "statistical" method to obtain his average score per match and compute it using the method.

[10+15=25]

5. Let $X_1, X_2, \ldots X_{100} \overset{i.i.d}{\sim} N(0,\sigma^2)$

Describe a procedure to test the following hypothesis

$$H_0 : \sigma^2 = 1 \quad \text{vs.} \quad H_1 : \sigma^2 = 5$$

with level of significance 0.10.

What is the type II error for your test ?

[10+5=15]

6. $X_1, X_2, \ldots X_n \overset{i.i.d}{\sim}$ Bernoulli (p)

we would like to test.

$$p = \frac{1}{2} \quad \text{vs.} \quad p = \frac{3}{4}$$

so that both type I and type II errors are less than or equal to 0.05.

Can you perform such test? Under what condition? Justify.

[20]

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4. Find the following probabilities approximately using statistical tables.

a) $X \sim N(50,15)$, $P(38 \le X \le 55)$.
b) $X \sim X^2_{(12)}$, $P(X \ge 20)$.
c) $X \sim t_{(8)}$, Find x such that $P(X > 1)$.
d) $X \sim F_{10,3}$, Find $P(2 \le X \le 4)$.

(10)

5. Let $X \sim Bin(8, p)$.

We like to test $H_0 : p = \frac{1}{4}$ vs $H_1 : \frac{1}{2}$.

Construct a test with $\alpha \le .10$.
What is the power of your test ?

(10+5=15)

6. $X_1, X_2, ..., X_n \overset{iid}{\sim} \exp(\lambda)$. Find Fisher's information for λ. Can you suggest an MVUE for λ or $\frac{1}{\lambda}$? Justify.

(10+5=15)

7. We have the following observations from a $N(\mu,1)$ distribution where μ is believed to have a prior distribution which is normal with mean 5 and standard deviation 2.

-1.4, -0.9, -0.4, 0.5, 2.2, 2.8, 3.1, 3.2, 3.3, 3.7, 4.0, 4.2, 4.3, 4.5, 5.2, 6.8, 7.9, 8.0, 8.1 .

What is the posterior probability that $\mu > 5$?

What would be your '**Bayes**' estimate for μ?

(15)

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and zero elsewhere. Let $Z = X + Y$, show that $\phi_Z(t) = \phi_X(t)\phi_Y(t)$ holds, although X and Y are not independent. What is the difference of this from bivariate c.f's?

4. Suppose $\Omega = (0, 1)$ and X_n denote the nth digit of the dyadic expansion of a number ω chosen uniformly from Ω. Does X_n converge with probability 1? Does X_n converge in distribution? Does X_n converge in probability?

5. (a) For any sequence of random variables $\{X_n\}$ show that

$$\max_{1\le k\le n} |X_k| \xrightarrow{P} 0 \text{ implies } S_n/n \xrightarrow{P} 0.$$

(b) Let X_n be a sequence of random variables with common finite variance σ^2. Suppose that the correlation coefficient between X_i and X_j is $< 0, \forall i \neq j$. Show that the WLLN holds for the sequence $\{X_n\}$.

7 + 8 = 15 pts.

6. (a) Let $X_1, \cdots, X_n$ be iid with mean zero, variance σ^2 and finite fourth moment. Argue if the following limit exists, and if it does then find it

$$\lim_{n\to\infty} E(\frac{S_n}{\sigma\sqrt{n}})^4.$$

(b) Suppose X_n are iid random variables with zero means and finite fourth moment. If $S_n = \sum_{k=1}^n X_k$ then show that S_n/n converges to zero with probability 1. A complete proof from first principles is required.

7 + 8 = 15 pts.

7. (a) Suppose X_n has the following density

$$f_n(x) = \frac{\lambda^n}{\Gamma(n)} e^{-\lambda x} x^{n-1}, x > 0,$$

where $\lambda > 0$ is a fixed number. Find sequence of numbers a_n, b_n such that

$$P(\frac{X_n - a_n}{b_n} \le x) \to \Phi(x), x \in R,$$

where Φ denotes the standard normal distribution function?

Indian Statistical Institute
Semester 1 (2007-2008)
B. Stat 2nd Year
Backpaper
Probability Theory 3

Date: 31.1.08 Time: 3 hrs. Total Points $15 \times 7 = 105$

The maximum you can score is 100. Answers must be justified with clear and precise arguments. More than one uncrossed answers to the same question or part of a question will not be entertained and only the first uncrossed answer will be graded. If you use any theorem or lemma proved in class state it explicitly.

1. (a) Let (X, Y) be bivariate normal with zero means, unit variances and correlation coefficient ρ. Suppose (U, V) are random variables independent of (X, Y) such that $P((U, V) = (0, 0)) = 0$. Find the distribution of

$$\frac{UX + VY}{\sqrt{U^2 + 2\rho UV + V^2}}.$$

(b) Let $\mathbf{X}_1, \mathbf{X}_2$ be two random p-vectors and let $\mathbf{X} = (\mathbf{X}_1', \mathbf{X}_2')'$. If $\mathbf{X}$ has a nondegenerate multivariate normal distribution with null mean vector and covariance matrix

$$\begin{pmatrix} A & B \\ B' & C \end{pmatrix},$$

where A, B, C are all $p \times p$ matrices, show that a necessary and sufficient condition for $\mathbf{X}_1 + \mathbf{X}_2$ to be independent of $\mathbf{X}_1 - \mathbf{X}_2$ is $A = C, B = B'$.

$7 + 8 = 15$ pts.

2. A random sample of size n is drawn from $Exp(\beta)$. (a) Show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independent where $s > r$. (b) Find the pdf of $X_{(r+1)} - X_{(r)}$. (c) Let $Z_1 = nX_{(1)}, Z_2 = (n-1)(X_{(2)} - X_{(1)}), Z_3 = (n-2)(X_{(3)} - X_{(2)}), \cdots, Z_n = (X_{(n)} - X_{(n-1)})$. Show that $Z_1, \cdots, Z_n$ and $X_1, \cdots, X_n$ are identically distributed.

3. Let the joint distribution of (X, Y) be given by the pdf

$$f(x, y) = \frac{1}{4}[1 + xy(x^2 - y^2)], |x| \leq 1, |y| \leq 1,$$

P.T.O

(b) Let $X_1, X_2, \cdots$ be iid random variables with mean μ and variance σ^2. Discuss whether the following sequence converges mentioning the appropriate mode of convergence and find the limit if possible

$$\binom{n}{2}^{-1} \sum_{1 \le i < j \le 1} X_i X_j.$$

$7 + 8 = 15$ pts.

Indian Statistical Institute
Semester 1 (2007-2008)
B. Stat 2nd Year
Final Exam
Probability Theory 3

Monday 3.12.2007, 10:30-1:30 Total Points $6 \times (5+5) = 60$

Answers must be justified with clear and precise arguments. More than one uncrossed answers to the same question or part of a question will not be entertained and only the first uncrossed answer will be graded. If you use any theorem or lemma proved in class state it explicitly.

1. For a random sample $X_1, X_2, \cdots, X_n$ from the distribution

$$f(x) = \exp(-(x-\theta)), x > \theta,$$

show that

(a) $X_{(1)}, X_{(2)} - X_{(1)}, X_{(3)} - X_{(2)}, \cdots, X_{(n)} - X_{(n-1)}$ are independent, where $X_{(1)}, \cdots, X_{(n)}$ denote the order statistics of $X_1, X_2, \cdots, X_n$.

(b) $2n[X_{(1)} - \theta]$ has the $\chi^2(2)$ distribution, and $2\sum_{i=2}^{n}[X_{(i)} - X_{(1)}]$ has the $\chi^2(2n-2)$ distribution. Are the two statistics independent?

2. Suppose, for $n \ge 1$, X_n are independent random variables with the probability distribution

$$\begin{aligned} X_n &= 0 \text{ with probability } 1 - \frac{1}{n} \\ &= n \text{ with probability } \frac{1}{n^2} \\ &= \frac{1}{n} \text{ with probability } \frac{1}{n}(1 - \frac{1}{n}) \end{aligned}$$

(a) Discuss whether the sequence $\{X_n\}$ converges with probability 1 or not.

(b) Consider the sum $\sum_{k=1}^{\infty} X_k$. Can this sum be finite with a positive probability? (Hint: all the terms are nonnegative, mgf may be useful.)

P.T.O

3. (a) If ϕ defined on the real line is not constant and near zero $\phi(t) = 1 + o(t) + o(t^2)$ with $o(t)$ denoting an odd function such that $o(t)/t \to 0$, as $t \to 0$, then can ϕ be a characteristic function?

(b) Construct a sequence of random variables X_n on $(0,1)$ taking values in $\{0,1\}$, such that X_n converges in distribution to zero, but X_n does not converge with probability 1.

4. Suppose X_n are iid random variables taking values ± 1 with probability 1/2 each and consider the partial sums

$$S_n = \sum_{k=1}^{n} \frac{X_k}{2^k}.$$

(a) Show that the partial sums converge with probability 1.

(b) Identify the distribution of the limit $\sum_{k=1}^{\infty} \frac{X_k}{2^k}$.

5. Let $X_2, X_3, \cdots$ be a sequence of independent random variables such that

$$P(X_n = \pm n) = \frac{1}{2n \log n}, P(X_n = 0) = 1 - \frac{1}{n \log n}.$$

(a) Recall how to express X_n/n in terms of S_n and S_{n-1}, and discuss whether S_n/n converges to zero with probability 1 or not.

(b) Does S_n/n converge to zero in probability?

6. (a) If Y_n is geometric with $p = \lambda/n$ then does Y_n/n converge in distribution? If possible identify the limiting distribution.

(b) Find the limit, if possible, of

$$e^{-n}(1 + n + \frac{n^2}{2!} + \cdots + \frac{n^n}{n!})$$

as $n \to \infty$. (Hint: Think of Poisson(n) as a sum of Poisson(1)'s.)

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2007-2008)

B. Stat. II Year

Physics II

Use separate answerscripts for Group A and Group B.

Group A (Statistical Mechanics)

ate: 16.5.2008 Maximum Marks 25 Duration: $1\frac{1}{2}$ hours

Attempt any five questions.

(a) Two identical bodies of constant heat capacity C_P have initial temperatures T_1 and T_2 respectively and final temperature T_f. If the bodies are at constant pressure without any change of phase, show that the work obtainable is maximum when $T_f = \sqrt{T_1 T_2}$.

(b) One mole of an ideal gas (in thermally insulating container) is allowed to expand freely so that its volume doubles. Calculate the entropy change for this irreversible process. (3 + 2)

2. The pressure on 1 Kg of copper is increased reversibly and isothermally from very near 0 atm. to 1000 atm. at 0^oC. Taking the density $\rho = 8 \times 10^3 Kg/m^3$, volume expansivity $\beta = 5 \times 10^{-5} K^{-1}$, and specific heat $C_P = 285 JK^{-1}/Kg$ to be constant, calculate

(a) heat transfer during the process.

(b) temperature change if the compression had been reversible and adiabatic. (2.5+2.5)

3. The expression for entropy S of an ideal gas having N particles, each of mass m, contained in volume V at temperature T is $S = Nkln(V) + \frac{3}{2}Nk[1 + ln(\frac{2\pi mkT}{h^2})]$, where k is the Boltzmann constant.

(a) If two samples of the same ideal gas at same termperature T and same particle density (N/V) are mixed together, find the change in the entropy of the system before and after the mixing.

P. T. O

(b) Modify the given expression for S in such a way that the process in part (a) above becomes explicitly reversible. (3 + 2)

4. A single particle with energy $\epsilon \le E$ where $E = P^2/2m$, is enclosed in a volume V.

(a) Determine (asymptotically) the number of accessible microstates in the energy range ϵ to $\epsilon + d\epsilon$.

(b) Using the above result, obtain the partition function of the system if the energy varies between zero and infinity. (3 + 2)

5. Consider N one dimensional harmonic oscillators in a canonical ensemble.

(a) Obtain the Helmholtz free energy of the system.

(b) Evaluate the mean energy U. (3.5 + 1.5)

6. A system exchanges energy with a heat reservoir such that thermal equilibrium is reached at a common temperature T. Assume that the energy of the reservoir is much greater than that of the system. Find the probability P_r that the system, at any instant of time, would be in a state specified by energy E_r. How would P_r change if the physical nature of the reservoir is altered? (4 + 1).

7. Find the relative root-mean-square fluctuation in the particle density in a grand canonical ensemble. When do these fluctuations become large? (4 + 1).

Questions for Final Semester BII (2008)
(Quantum Mechanics)

Group B

Answer any three **Total Marks : 24**

1. Consider a 1-D oscillator with the Hamiltonian

$$H = \frac{p^2}{2m} + m\omega^2 x^2/2$$

and the position and momentum operators are

$$x_0 = x\cos\omega t - (p/m\omega)\sin\omega t$$
$$p_0 = p\cos\omega t + m\omega x\sin\omega t$$

(a) Do these operators commute with the Hamiltonian ?
Illustrate your answer. 3
(b) Compute the commutator $[p_0, x_0]$ 3
What is its significance in terms of measurement ? 2

2. (a) Let the solution to the 1-D free particle time dependent Schrodinger equation of definite wavelength λ be $\psi(x,t)$ as described by some observer O in a frame with coordinates (x, t). Now consider the same particle as described by wave function $\psi'(x',t')$ according to observer o' with coordinates (x',t') related to (x,t) by the Galilean transformations.
Do $\psi(x,t), \psi'(x',t')$ describe waves of the same wavelength?
Illustrate your answer. 3
(b)Estimate the binding energy of ${}_2He^4$ nucleus. 2
(c) Quantum phenomena are often negligible in the macroscopic world. Show this numerically for the following case :
The diffraction of a tennis ball of mass m = 0.1 kg moving at a speed v = 0.5 m/sec by a window of size $1\text{x}1.5m^2$ 3

3. (a) Explain Radioactivity phenomena with examples. 2
(b)The radioactive isotope ${}_{83}Bi^{212}$ decays to ${}_{81}Ti^{208}$ by emitting an alpha particle with energy E = 6.0 Mev.
Calculate the transition probability $|T|^2$ in the limit T<< 1. 6

P.T.O

4.(a) What are the conditions for endothermic and exothermic reactions? Illustrate your answer with examples. 2

(b)Consider a free particle that moves in 1-D. Its initial (t=0) wave function is

$$\psi(x,t) = (\frac{\alpha}{\pi})^{1/4} e^{ik_0 x - \alpha x^2/2}$$

where α and k_0 are real parameters.

(i)Calculate the momentum wave function at all times $t > 0$ and the corresponding momentum probability density. 3

(i) Write down the position probability density $\rho(x,0) = |\psi(x,0)|^2$ and show that $\lim_{\alpha \to \infty}[\rho(x,0)] = \delta(x)$ 3

(5) Consider 1-D normalized wave functions $\psi_0(x), \psi_1(x)$ with the properties

$$\psi_0(-x) = \psi_0(x) = \psi_0^*(x), \psi_1(x) = N\frac{d\psi_0}{dx}$$

Consider also the linear combination

$$\psi(x) = C_1\psi_0(x) + C_2\psi_1(x)$$

with $|C_1|^2 + |C_2|^2 = 1$. The constants N, C_1, C_2 are assumed to be known.

(a) Show that ψ_0 and ψ_1 are orthogonal and that $\psi(x)$ is normalized. 4

(b) Compute the expectation value of the kinetic energy T in the state ψ_0 and demonstrate that

$$\langle\psi_0 | T^2 | \psi_0\rangle = \langle\psi_0 | T | \psi_0\rangle | \langle\psi_1 | T | \psi_1\rangle$$

4

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: (2007 – 2008)
B. Stat II Year
Biology II

Date 16.05.08 Maximum Marks 50 Duration Three hours .

(Attempt any five questions)

(Number of copies of the question paper required Twenty)

1. Name different meteorological variables that are related to crop production. Write down the names of apparatus used to measure those variables. 5+5

OR

1. Define onset and cessation of southwest monsoon. Draw a suitable rice calendar with the following data. 2+8

Week No.	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*
Rainfall (mm) at 0.5 Prob.	*5*	*0*	*15*	*10*	*9*	*37*	*18*	*35*	*18*	*20*	*98*	*142*	*95*	*80*	*15*	*32*	*2*	*0*	*0*
PET (mm)	*45*	*42*	*33*	*37*	*33*	*25*	*22*	*20*	*22*	*20*	*19*	*17*	*19*	*20*	*25*	*28*	*31*	*33*	*34*

2. What are the differences between Manures and Fertilizers? Calculate the quantity of VC, Urea, Single super phosphate and Muriate of potash required for 1 ha. rice crop to meet the nutrient requirement of 120kg N, 60 kg P_2O_5 and 60 kg K_2O per hectare (25% of required N should be given through VC). 3+7

3. Describe suitable agro-techniques for direct seeded rice. Estimate the expected yield of rice grain in t/ha from the following data.
i) Spacing - 20x10cm ii) Average no. of tillers/hill –10 iii) Average no. of effective tillers/hill – 8 iv) Average no. of grain/panicle –160 v) Average no. of unfilled grain/panicle –20 vi) Test weight -24 g. 6+4

4. Write short notes on any five of the following: 2 x 5
   a) Moisture availability index
   b) Soil pH
   c) Reproductive stages in rice
   d) Bulk density of soil
   e) Transgenic rice
   f) Capillary water
   g) Intercropping and Mixed cropping

5. Write in brief about any two of the following: 2X5
   a) Variation in yield of winter (*Aman*) rice and summer (*Boro*) rice.
   b) Soil texture.
   c) Micro and macro nutrients

P.T.O

6. a) Determine the values of *a* and *b* so that (0, 0) is a sink of

$$\frac{dx}{dt} = ax - by$$
$$\frac{dy}{dt} = bx + 2y$$

b) Write down the Lotka – Volterra model for predator-prey interaction. Find the dynamic behavior of the model around biologically feasible equilibria.

3+2+5

INDIAN STATISTICAL INSTITUTE

Second Semester Examination (2007-2008)
Course: B-Stat II Year
Subject: Economics II (Macroeconomics)

Date-16.05.08 Maximum Marks-60 Duration-2.5hrs.

Answer all questions.

1. Consider a simple Keynesian model for a closed economy without government. At GDP (Y) = 1000, producers have to sell 20 units from their stock to meet the customers' demand fully. It is given that the equilibrium level of GDP is 1040. **(i)** Find out the impact of unit amount of increase in autonomous expenditure on the equilibrium level of Y. **(ii)**Suppose due to technological progress input requirement per unit of output becomes half of what it was before. What impact is it likely to have on the equilibrium level of output? Explain. (16+6)

2. In a simple Keynesian model for a closed economy without government, there are two groups of income earners. Group 1 earns 800, while the income of group 2 is Y – 800, where Y denotes NDP. Average consumption propensities of Group 1 and Group 2 are 0.6 and 0.5 respectively. Investment function is given by $I = 400 + 0.1Y$.**(i)** Derive the aggregate saving function and the equilibrium amount of saving. **(ii)**Now suppose there takes place a transfer of income of 100 units from Group 1 to Group 2. How will it affect the aggregate saving function? Do you observe paradox of thrift here? Explain. (8+14)

3. (a) In an IS-LM model, planned investment, I, is a function of interest rate; r, alone and $[dI/dr] = -50$. An increase in demand for real balance is found to shift the LM curve horizontally and vertically by $-(1/0.25)$units and $(1/62.5)$ respectively. Now, following a shift in the import function, Y in equilibrium is found to have increased by 500 units.
(i) Derive the slope of the LM curve and explain its meaning. **(ii)**Compute the change in the planned level of investment from the initial to the new equilibrium.
(b) Suppose that the government takes an additional loan of Rs.100 from the central bank. Explain how this will lead to an increase in the stock of high-powered money by the same amount. Assuming CRR to be unity, show how people will come to hold an additional amount of money of the same amount at the end of the operation of the money multiplier process when high-powered money goes up by Rs.100. (6+6+10)

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: 2007-2008

B. Stat. II Year

Elements of Algebraic Structures

ate: May 2, 2008 **Maximum Marks: 60** **Duration: 3 hours**

There are TEN questions on this test. The marks for each question are indicated on the right . Answer ALL questions. JUSTIFY your answers. You can assume that ALL integral domains question paper have multiplicative identity.

1. Find all group homomorphisms from the dihedral group of order six to the cyclic group of order six. [7]
2. Let H and K be normal subgroups of a group G such that $H \cap K = \{e\}$. Show that HK is a normal subgroup of G. [6]
3. Let G be a group of order 30. Show that G has either a normal Sylow 3-subgroup or a normal Sylow 5-subgroup. [7]
4. Prove that every ideal in an Euclidean domain R is principal. [7]
5. Show that every nonzero prime ideal in a principal ideal domain R is maximal. [7]
6. Show that the set $J = \{p(x) \in \mathbf{Q}[x] \mid p(1) = 0\}$ is a maximal ideal of $\mathbf{Q}[x]$. [6]
7. Show that the polynomial $x^3 + x + 1$ is irreducible in $(\mathbf{Z}/2\mathbf{Z})[x]$. Is it irreducible in $\mathbf{Q}[x]$? [7]
8. Consider the subring $S = \{a + b\sqrt{-5} \mid a, b \in \mathbf{Z}\}$ of the field $\mathbf{Q}(\sqrt{-5})$. Show that the elements 3, $2 + \sqrt{-5}$ and $2 - \sqrt{-5}$ are irreducible in S. Then show that S is not a unique factorization domain. [9]
9. Find the degree of the extension field $\mathbf{Q}(\sqrt{2}, \sqrt{3})$ over the field $\mathbf{Q}$. [7]
10. Construct an extension field of $\mathbf{Q}$ containing a root of the polynomial $x^3 + 4x^2 + 6 \in \mathbf{Q}[x]$. [8]

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination : (2007-2008)

B. Stat. II Year

Physics II *Total Time: 2 hrs.*

Group A

(Thermodynamics)

Date: *25.2.08* Maximum Marks: 12 Duration: 1 hour

Attempt any three questions. All carry equal marks.

1. Your hostel room contains about 2500 moles of air. Find the change in the internal energy of this much air when it is cooled from $25C$ to $15C$ at a constant pressure of 1 atm. Treat the air as an ideal gas with $\gamma = 1.4$, $R = 8.314\ J/mole\ K$.

2. After drinking hot chocolate containing $10Kcal$, you want to work off the energy you have taken in, by running up the stairs. How high do you have to climb? Assume that your mass is $60Kg$.

3. $1Kg$ of water at $100C$ is placed in thermal contact with $1Kg$ of water at $0C$, such that heat transfer takes place and the system attains equilibrium.

(a) Is this a reversible or an irreversible process? State reason for your answer.

(b) Calculate the total change in entropy given that $C_P = 4190\ J/Kg\ K$.

4.(a) A Carnot engine with the sink at $10C$ has an efficiency of 30%. By how much must the temperature of the source be changed to increase its efficiency to 50%.

(b) An ideal gas is compressed adiabatically to 1/10th of its original volume at $27C$. What is the final temperature after the compression?

P.T.O

Group B

Total Marks: 13.5

Duration: 1 hour.

Answer any three:

(1)(a)The explanation of black body radiation spectrum led Planck to introduce the idea of quanta of energy. Explain why it needs so. 2.5

(b)Calculate the wavelength associated with a 1 eV (i) photon, (ii) electron 1+1

(2)Consider an experiment in which a beam of electrons is directed at a plate containing two slits, labeled A and B. Beyond the plate is a screen equipped with an array of detectors which enables one to determine where the electrons hit the screen. For each of the following cases draw a rough graph of the relative number of incident electrons as a function of position along the screen and give a brief explanation.

(a) Slit A open and B closed 1.5

(b)Slit B open and A closed 1.5

(c) Both slits open 1.5

(3)(a)Define Poisson Bracket for two variables. 1

(b)Show that the Poisson Bracket for three variables satisfy the following relation :

$$[u+v, w] = [u,w] + [v,w]$$ 1

(c)Show that the following commutation relations hold in Quantum Mechanics:

$$[p, x^2] = -2i\hbar x$$

where $p = \frac{\hbar}{i}\frac{d}{dx}$ 1

and

$$[p, x^n] = -in\hbar x^{n-1}$$

for $n > 1$. 1.5

(4) $\psi(x,t)$ is a solution of the Schrödinger equation for a free particle of mass m in one dimension and

$$\psi(x,0) = A\exp\left(-\frac{x^2}{a^2}\right)$$

(a)At time t = 0, find the probability amplitude in momentum space. 2

(b)Find $\psi(x,t)$ 2.5

Biology II

Date: 25.2.08 B.Stat.-II yr.

Questions for Mid Term examination

Please answer for 30 marks. Time limit: 01 hour 30 minutes

1. What are the major objectives of classical plant breeding? Compare self and cross pollinated crops with respect to their major characteristics? (3+3 marks)
2. What is *Agrobacterium tumifaciens*? Briefly outline the procedure for making a transgenic plant. (3+3 marks)
3. Briefly describe why *Arabidopsis thaliana* is considered as model organism for plant development studies? What is the `modular' construction of plants? (3+3 marks)
4. What are the defining properties of a stem cell? What is the major difference between a `unipotent' and a `pluripotent' stem cell? Describe briefly the salient features of mutation breeding? (2+2+2 marks)
5. Three broad classes of signaling events take place during mammalian development. Describe them briefly. (2+4 marks)
6. What are the three classes of `segementation genes' in early *Drosophila* development? The developmental control genes of *Drosophila* have homologues in vertebrates. Name one and what they code for in vertebrates? (3+3)
7. What are the major determinants of plant breeding? What are the different types of plant breeding based on mode of reproduction? What are the different sexual groups in plant breeding? (2+2+2)
8. What is the definition of biodiversity? What are the major causes for the loss of biodiversity? Define alpha-, beta- and gamma- diversity with examples. (1+3+2)

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: 2007-2008

B. Stat. II Year

Elements of Algebraic Structures

te: 22nd Feb. 2008 **Maximum Marks: 40** **Duration: 3 hours**

te: Attempt ALL questions. **Justify your answers.**

1. Find all group homomorphisms from
   (a) $\mathbb{Q}$ to $\mathbb{Z}$ [4]
   (b) $\mathbb{Z}$ to $\mathbb{Z}_5$. [4]

2. Suppose H is a subgroup of G and index$(H) = 2$. Show that H is a normal subgroup of G. Use this to show that the dihedral group D_{10} is not simple. [8]

3. Suppose σ is an element of the permutation group S_n having cycle decomposition $\sigma = \tau_1\tau_2$ where τ_i has length k_i. What is the order of σ? Show that S_{10} has an element of order 30. [8]

4. Show that the center $Z(G)$ of a group G is a normal subgroup of G. In addition if $o(G) = p^n$, where p is a prime number and $n \geq 1$, show that $Z(G)$ is a nontrivial subgroup. [7]

5. Show that there exists a surjective group homomorphism from G to $\mathbb{Z}_7$ if $o(G) = 77$. [8]

6. List all abelian groups of order 20 up to isomorphism. [6]

Indian Statistical Institute
Mid Semester Examination (2008)
B-Stat. II Year
Economocs II
25.2.08 Maximum Marks-40 Duration-150 minutes

Answer all questions

1a. What is value added? Show that aggregate demand for final goods and services equals the sum of the value added of all enterprises in an open economy.

b. This is a question on national income accounting. The following information are given about a closed economy.
Personal disposable income is 1,000 units (in domestic currency). Undistributed profits are 65 units. Gross investment is 170units, aggregate saving is 130 units. The government makes a transfer payment of 100 units and interest payment of 30units.The corporate profit tax amounts to 50 units. Indirect business taxes are 190units.The government collects 150 in personal income taxes and social security contributions. Public sector enterprises incur a loss of 4 units. Business transfer payments are 9 units. Given this information ,compute GDP of the economy.
(8+12)=20

2a. Define investment (I) in national income accounting.
How do you define saving (S) (domestic saving)? What are the three different components of saving? Derive the relationship between S and I in an open economy and hence show that the sum of the savings (in the domestic and the rest of the world economy) is identically equal to total investment (in the domestic economy and the rest of the world economy).

b. Following information are given for the domestic economy in a certain year. Machines and equipment (for business use) produced in the economy is worth Rs.7,000 cr. consumption goods produced is worth Rs.20,000 cr. of which ¼ is sold to domestic households and ¼ is exported. Domestic households also consume Rs.3,000 cr. worth of imported goods. Machines ad equipments purchased from abroad is worth Rs.4,000 cr. Raw materials imported of which ½ is held in stock is worth Rs.10,000 cr. Raw materials produced in the domestic economy and fully used for current production is worth Rs.3,000 cr. Construction output is worth Rs.1,000 cr. Depreciation is worth Rs.1,000 cr.

It is found that for the rest of the world economy saving exceeds investment by Rs.10,000 cr. Government budget deficit for the domestic economy is Rs.4,000 cr.

i) Compute gross domestic investment
ii) Compute consumption
iii) Compute private disposable income. (10+16)=26

Mid-sem. Exam.(2007-08)

Questions of B-Stat-II
Official Statistics
Marks: 40 Time: 2$\frac{1}{2}$ hrs. Date: 27.2.08

QUESTION PAPER

1. What is official statistics? Describe its importance in economic planning. 8

2. Indicate the responsibilities of CSO. Name some important official statistics published by CSO. 8+2

3. How many divisions are there in NSSO? Describe the function of any one of them 4+8

4. Describe the use of IIP and its shortcomings as an indicator of industrial growth. 10

or

Describe the item basket, weight, selection of base year and methodology adopted in compilation of Index of Industrial Production.

INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: (2007 - 2008)

B. STAT, II YEAR
Demography

Date: 29 February, 2008 **Maximum Marks: 60** **Duration: 3 hrs**

Note: (i) Desk calculators are allowed in the exam, (ii) Symbols and notations have their usual meaning.

Answer the following questions

1. Given the Lotka's integral equation, $\int_0^\beta exp(-ra)\lambda(a)k(a)da = 1$, where $\lambda(a)$ is age specific fertility rates, $k(a)$ is probability of surviving at age a and β is oldest age of non-zero fertility.
a) Prove that the integral equation has one real solution, $r = r_0$.
b) Prove that if any complex roots $\{r_j\}$ occur is complex conjugate pairs, then $r_0 > Real(r_j)$. [5, 5]

2. a) What are the three basic axioms of the *pure birth process?* Prove that the number of births in a time interval $(0, t)$ is a random variable which follows *Poisson process.*
b) If μ_x follows Gompertz law of mortality, then find an expression for l_x. [8, 2]

3. Suppose $l_x = s(x).l_0$ and $s(x) = \frac{1}{10}(100 - x)^{1/2}$, where $s(x)$ is survival function and $l_0 = 100,000$. Then construct the life table columns d_x, q_x, L_x, T_x and e_x^0 columns of the life table for ages $x = 0, 1, 2, ..., 10$. [10]

4. a) Obtain an expression for μ_x if $l_x = ks^x b^{x^2} g^{c^x}$, where k, s, b, g, c are constants.
b) What are the basic axioms of *birth - death process?*
c) In a *birth - death process* prove that $P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$, where P_n is the probability that there are n individuals in the system at any point in time,λ is birth rate, μ is death rate. [3, 2, 5]

5. a) Show that ${}_nq_x = \int_0^n {}_tp_x\mu_{x+t}dt$. b) If ${}_{n/m}q_x$ denotes the probability that individual aged x will die between ages $x + n$ and $x + n + m$, then deduce a similar integral equation as in a) for ${}_{n/m}q_x$. [8, 2]

6. a) Given $s(x) = 1 - 0.005x - 0.00005x^2$ and $s(x)l_0 = l_x$. Construct the l_x, d_x, and $q_x$columns of the corresponding life table for ages $x = 0, 1, 2$ using a radix of $100,000$.
b) The following values of q_x have been derived from a mortality experience: $q_0 = 0.011$, $q_1 = 0.005$ and $q_2 = 0.003$. Construct the corresponding the l_x, d_x, and $q_x$columns using a radix of $10,000$. [5, 5]

7. a) Assuming the relation ${}_nK_{x+t}^{(t)} =_n K_x^{(0)} \frac{{}_nL_{x+t}}{{}_nL_x}$, explain the method of population projection.
b) Write down the difference equations and Matrix containing the survival rates involved in the method in (a). [4, 6]

INDIAN STATISTICAL INSTITUTE, KOLKATA
B-Stat (Hons.), 2nd Year
STATISTICAL METHODS IV
Semestral Examination : May 6, 2008

Maximum Marks : 100 Time Allowed : 2 Hours 30 Minutes

Answer All Questions. No Probability Table/Chart Will Be Supplied.

(1). Consider i.i.d. observations $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ with a common bivariate normal distribution with all parameters unknown. Let $\rho = Corrln.(X_i, Y_i)$ and consider $H_0 : \rho = 0$ against $H_A : \rho \neq 0$. Derive the form of the likelihood ratio test statistic and the distribution of that statistic under H_0.

[8 + 12 = 20]

(2). 400 i.i.d. observations are drawn from the double exponential distribution with p.d.f. $(1/2)\exp(-|x - \theta|)$, where θ is an unknown parameter. Suppose that the median of those 400 observations is 15. Obtain a 95% confidence interval for θ.

[10]

(3). 125 individuals are treated with a medicine, and their blood pressures are measured before and after the application of the medicine. Consider the following variables.

s_i = the systolic pressure after taking the medicine − the systolic pressure before taking the medicine for the i-th individual.

d_i = the diastolic pressure after taking the medicine − the diastolic pressure before taking the medicine for the i-th individual.

Suppose that in the data, we have $\bar{s} = -12$, $\bar{d} = -6$, $Var(s) = 16$, $Var(d) = 9$ and $Corrln.(s, d) = 0.6$. Based on this data, test whether there is significant evidence to conclude that the medicine has an effect of changing the blood pressure. State your assumptions, the null and the alternative hypotheses clearly and report the P-value.

[20]

P. T. O.

(4). The value of the χ^2-statistic for testing independence based on data in a 3×3 contingency table is 5.8. Compute the P-value.

[10]

(5). Suppose that $(Y, X_1, X_2, \ldots, X_p)$ follows multivariate normal distribution with all parameters unknown, and we have n i.i.d. observations from that distribution. With adequate justification, derive the distribution of the sample multiple correlation coefficient between Y and $(X_1, X_2, \ldots, X_p)$ computed from the data, when Y and $(X_1, X_2, \ldots, X_p)$ are independent.

[20]

(6). Problem sessions and assignments.

[20]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination: (2007-2008)
B. Stat. (Hons.) II Year

ECONOMIC STATISTICS & OFFICIAL STATISTICS

Date: 9.5.08 **Maximum Marks: 100** **Duration: 3 hours**

Note: This question paper has 5 questions, each of 20 marks. These apart, a maximum of 15 marks is allotted to Assignments. Answer as many questions as you like. The maximum marks that you can score is 100.

1. (a) State and briefly explain the desirable properties of a price index number formula.

   (b) Examine if the following price index number formulas possess all the desirable properties:

$$\text{(i) } P_{0T} = \sqrt{\left(\frac{\sum p_{iT} q_{i0}}{\sum p_{i0} q_{i0}}\right)} \qquad \text{(ii) } P_{0T} = \frac{\sum p_{iT} q_{iT}}{\sum p_{i0} q_{i0}}$$

   (c) Based on a given price-quantity data set, the following index numbers are obtained:

$$P^L_{T0} = 85, P^F_{0T} = 125, V_{0T} = 100$$

   where the symbols have usual meanings. Calculate P^L_{0T} and Q^L_{0T}.

[10+6+4=20]

2. (a) Using a suitable linear regression model, derive the sampling variance of P^L_{0t} and briefly explain the result obtained.

   (b) Stating clearly the assumptions involved, derive True Cost of Living Index Numbers and show that P^L_{0T} may not correctly measure True Cost of Living Index Number.

[10+ (8+2)=20]

3. (a) Define Lognormal Distribution, derive its central tendency measures and variance and comment on its suitability as a description of a *size distribution*.

P.T.O

Explain the graphical test of Lognormality of the population distribution underlying a given observed grouped data set on a size variable.

(b) For a given Lognormal distribution $E(X)$ is 1.15 times $Mode(X)$. Find the value of Lorenz ratio of this distribution [given that $\Phi(-0.142241) = 0.443445$, $\Phi(z)$ being the area under the standard normal distribution function up to z].

[(10+6)+4=20]

4. (a) Distinguish between the concepts of variability and inequality of size distribution of income or allied variable.

Briefly explain the properties that an inequality measure should possess.

Write down the commonly used statistical measures of inequality.

(b) Examine whether the Relative Mean Deviation measure of inequality satisfies the Pigou-Dalton principle of transfer.

[(3+6+6)+4=20]

5. (a) Derive the Lorenz curve of the Lognormal distribution and enumerate its properties. Also, obtain the expression of the Lorenz ratio of the Lognormal distribution.

(b) For a certain population of households, X : per capita total consumer expenditure (pce) follows Lognormal distribution $\Lambda(\mu,\sigma^2)$ and Y : per capita food expenditure is given by $Y = \alpha X^{\beta}$, $\alpha, \beta > 0$. What can you say about the form of distribution of Y? What will be the expression of Lorenz ratio of Y?

[(10+4)+6=20]

6. Assignments [15]

INDIAN STATISTICAL INSTITUTE

Second Semestral Examination : (2007-2008)

B. Stat. (Hons.) II Year

Subject : **SQC & OR**

Date : **13.05.2008** Maximum Marks : **100** Duration : **3 hrs.**

Note :

1. Answer all questions.
2. Wherever necessary, all assumptions made, must be explicitly stated.
3. Write neatly. Marks will be deducted for illegible handwriting.

---

1. Consider the following constraints of a LPP written in standard form :

$$x_1 + x_2 + 4x_3 + 2x_4 + 3x_5 = 8$$
$$4x_1 + 2x_2 + 2x_3 + x_4 + 6x_5 = 4$$

$$x_i \geq 0 \qquad i = 1, \ldots, 5$$

Identify

a) a basic feasible solution
b) basic infeasible solution
c) infinity of solutions

[2 × 3 = 6]

2. The following table represents a specific simplex interaction for a maximization problem. All variables are non-negative.

	y_0	$-x_2$	$-x_4$	$-x_5$	$-x_6$	$-x_7$
x_0	620	-5	4	-1	-10	0
x_8	12	3	-2	-3	-1	5
x_3	6	1	3	1	0	3
x_1	0	-1	0	6	-4	0

P.T.O

a) Categorize the variables as basic and non-basic and write the current values of all the variables.

b) Identify the non-basic variables that have the potential to improve the value of x_0. If each such variable enters the basis, determine the associated leaving variable and the associated change in x_0.

c) Which non-basic variable(s) will not cause a change in the value of x_0 when selected to enter the basis?

[3 + 4 + 2 = 9]

3. Consider the following LP :

P : Maximize $z = 5x_1 - 12x_2 + 4x_3$

Subject to

$$x_1 + 2x_2 + x_3 \le 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$

a) Solve P.

b) Write down the dual D of P.

c) Obtain the dual optimal solution ***without*** any further iterations.

[15 + 5 + 5 = 25]

4. Piston rings for an automotive engine are produced by a forgoing process. The quality characteristic of interest is the inside diameter of the rings manufactured by this process. The production manager proposes to monitor this process using an appropriate control chart. He collect 25 preliminary samples of size 5 and arrives at the following results :

$\sum_{i=1}^{25} \bar{x}_i = 1850.028$ $\sum_{i=1}^{25} R_i = 0.581$

The specification limits on the diameter are 74.000 $\pm$0.05 mm.

a) Find the trial control limits.
b) Assuming that the process is in control, estimate the process mean and standard deviation.

c) Estimate the fraction of non-conforming piston rings produced. Mention clearly any assumption that you make.

d) Find the probability of detecting a shift of two standard deviation units on the first sample following the shift.

[6 + 4 + 6 + 4 = 20]

5. Consider the following acceptance sampling plans :-

Plan	N	n	c
I	1000	240	2
II	1000	170	1
III	1000	100	0

Suppose that the AQL has been fixed at 2.2%

a) As a consumer which plan would you prefer to use?
b) Find the probability of accepting lots of 1% defective for each of these plans.
c) If you are the producer, which plan would you prefer to use?

[5 + 5 + 5 = 15]

6. a) Write down the ISO definition of Quality.
b) Describe the procedure for operation of a double sampling plan.
c) Define Quality Costs. Mention their broad categories for a manufacturing organization.
d) In the context of acceptance sampling, distinguish between Type A and Type B OC-curves.
e) Who is/are credited with a the development of

(i) acceptance sampling plans
(ii) control charts
(iii) the seven tools of QC
(iv) loss related definition of Quality
(v) the simple algorithm for solving the LP.

[5 × 5 = 25]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : 2007-2008

B. Stat. - Second Year

Analysis III

Date : 11. 09. 2007 Maximum Score : 40 Time :3 Hours

This paper carries questions worth a total of 46 marks. Answer as much as you can. The maximum you can score is 40 marks.

(1) Let $A \subset \mathbb{R}^n$. Show that A is closed and bounded if and only if every continuous $f : S \to \mathbb{R}$ is bounded.

[5]

(2) Let $n > 1$. Show that there is no injective (one-to-one) continuous, real-valued function defined on any non-empty open subset of $\mathbb{R}^n$.

[6]

(3) Show that the set of all connected components of an open set in $\mathbb{R}^n$ is countable.

[6]

(4) (a) When is a $\mathbb{R}^m$-valued function defined on a neighbourhood a point $a \in \mathbb{R}^n$ called differentiable?

(b) State Taylor's formula for a real-valued C^k-function defined on an open ball in $\mathbb{R}^n$ with center at a.

(c) State the method of Lagrange' multipliers giving a necessary condition for points of extrema of a C^1 function with constraints.

[3+3+3]

(5) Let $a \in \Omega \subset \mathbb{R}^n$. Ω open, and $f \in C^2(\Omega)$. For $0 \neq u \in \mathbb{R}^n$ and $t \in \mathbb{R}$, whenever $a + t \cdot u \in \Omega$, define

$$g(t) = f(a + t \cdot u).$$

Show that g is defined in a neighbourhood of 0 and that $g''(0)$ exists. Further compute $g''(0)$.

[5]

(6) Let $\Omega \subset \mathbb{R}^3$ be a connected open set not containing the origin and $g : (0, \infty) \to$ a twice differentiable function. Define $f : \omega \to \mathbb{R}$ by

$$f(x, y, z) = g(\sqrt{x^2 + y^2 + z^2}).$$

P.T.O

Assume that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \equiv 0$$

on Ω. Show that there exist reals a and b such that for every $(x, y, z) \in \Omega$,

$$f(x, y, z) = \frac{a}{\sqrt{x^2 + y^2 + z^2}} + b.$$

[7]

(7) Let $a \in \Omega \subset \mathbb{R}^n$, Ω open, and $f : \Omega \to \mathbb{R}^n$ be a C^1 map whose Jacobian at a is non-zero. Show that there is an open set $V \subset \Omega$ containing a on which f is injective.

[8]

INDIAN STATISTICAL INSTITUTE
1st Semester Examination
B. Stat. II year : 2007–2008
C & Data Structures

Date : 14. 09. 2007 | Marks : 50 | Time : 3 Hours

The paper is of 60 marks (six questions of ten marks each). Answer any part of any question. The maximum marks you can get is 50.

1. Implement a function in C programming language that calculates the GCD of two positive integers m, n $(m > n)$. Estimate the maximum number of divisions required by your method during the execution of the function.

2. Write a function in C that finds a given substring in a circular string. As an example, the substring "pqr" is absent in the string "rxyzsdfgpq", but exists when the string is considered in a circular manner.

3. Implement a variable argument function f(int n, ...) in C programming language as follows. The first argument n denotes the number of data (say $x_1, \ldots, x_n$) that the function f will accept. The function needs to print the maximum and the minimum values among $x_1, \ldots, x_n$ efficiently.

4. Explain how can you implement a Chess board in C. Consider a Chess board when a game is continuing. There can be m many pieces in the board $(2 \leq m \leq 32)$. Write a C program that will report the largest empty square given the current situation on the board.

5. Describe how a priority queue can be implemented using an array in C programming language. Explain the data structure and the insertion and deletion strategies with proper examples.

6. Write a C program that takes the preorder traversal of a binary search tree as input and outputs the tree itself.

INDIAN STATISTICAL INSTITUTE
1st Semester Examination
B. Stat. II year : 2007–2008
C & Data Structures

ate: 7. 12. 2007 Marks: 100 Time: 3 Hours

nswer any four questions. Please try to write all the part answers of a question at the same place.

1. (a) Write a non-recursive function in C that can evaluate $f(n) = f(n-1) + f(n-2)$ with initial conditions $f(0) = 0, f(1) = 1$.

   (b) Explain what happens when the following codes are executed.

      i. `char *p, *q; while (*p++ = *q++);`

      ii.
```
int i, k = 1, n = 5;
for (i = 0; k < n+1; i = k-i)
        { printf("%d\n", k); k = k+i; }
```

   (c) Write a function in C to check whether a string contains at least three vowels.

   (d) Write a C program without using recursion for inorder traversal in a binary search tree.

$5+5+5+10 = 25$

2. (a) Write down the algorithms for bubble sort and heap sort and explain which one should be preferred.

   (b) Construct a heap with the data set 151, 27, 5, 111, 71, 98, 67, 75.

   (c) Given a binary search tree, how do you get the keys in sorted order?

$14+6+5 = 25$

3. (a) Explain the meaning of hashing.

   (b) Write a specific hashing strategy including the collision resolution mechanism.

   (c) Implement a C function that can manage search and insertion in this strategy when the hash table is stored in the primary memory (RAM) of a computer.

   (d) How will you modify the C implementation if the hash table is stored in a file in the hard disk?

$5+5+9+6 = 25$

P. T. O.

4. (a) Clearly explain the insertion algorithm in a balanced binary search tree.
   (b) Write down the C routines for single and double rotations.
   (c) Provide specific examples to demonstrate single and double rotations while inserting a node in a balanced binary search tree. The tree should have at least eight nodes.
   (d) Derive the worst case time complexity to search a key in a balanced tree containing N nodes?

$10+5+5+5 = 25$

5. Given positive integers a, b, n, the integer b is called the inverse of a modulo n if $ab - 1$ is divisible by n.
   (a) Given a, n, write a function in C that finds the inverse of a modulo n.
   (b) Derive the time complexity of the algorithm you use to find the inverse?
   (c) Execute your function with (i) $a = 7, n = 51$ and (ii) $a = 9, n = 39$.
   (d) Describe the RSA public key cryptosystem and highlight where exactly the inverse finding algorithm is required.

$6+5+4+10 = 25$

6. Suppose you have to implement a software in C programming language that takes care of the tabulation of marks for current B. Stat. II year students.
   (a) How will you design the relevant input screens?
   (b) Explain the data structures useful for your implementation.
   (c) What will be the structures of the files where the data will be stored?
   (d) How will you format the final tabulation document and present the output?

$5+8+7+5 = 25$

Indian Statistical Institute
First Semestral Examination : 2007 -2008
B. Stat. - II Year
Biology I

Date : 30/11/07 ***Full marks = 50*** ***Time = 2.5 hours***

Answer **any five**; All questions carry equal marks.

1. What are the basic differences in DNA replication and transcription? (10)

2. Explain with pedigrees, how X-linked recessive and Y-linked diseases are transmitted and expressed in human? (10)

3. (A) Define (1) allele (2) genotype, with examples. (5)
   (B) Bacterial genome contains about $\sim 10^6$ base pairs whereas human genome contains about $\sim 10^9$ base pairs: what could be the possible reasons for the difference in sizes of the genomes. (5)

4. (a) Mention how the codons, amino acids and tRNAs are involved in protein synthesis. (b) How many different DNA sequences are possible with 6 codons: ATG, TAA, AGG, GGG, CGT, TAC such that the sequences start with the codon ATG, end with the codon TAA and rearrangements are possible within each codon. (4+6)

5. (a) How is palmitic acid oxidized to give energy and what is the efficiency of energy conservation? (5)
   (b) Define tyrosinase positive and negetive albinism. (5)

6. Albinism in human is caused by a recessive autosomal allele "a". In a family, the father is normal but carrier of the recessive allele, and the mother is an albino. What proportion of the children would be expected to suffer from albinism? If the family plans to have three children, what is the chance that one will be normal and two will suffer from albinism? (5)

P.T.O

(b) In a family of six children, what is the chance that at least three are girls? (5)

7. (a) Define, with examples, mitotic and meiotic cell division in humans. If two cells with genotypes (A/a) and (A/a, B/b) undergo mitotic and meiotic cell divisions, what will be the genotypes or gene compositions of the resultant cells? (5)

(b) Indicate whether each of the following statements, about the structure of double stranded DNA, is true or false: (i) A/T=C/G; (ii) A+G=C+T; (iii) A=T within each single strand; (iv) when separated the two strands are identical, and (v) once the bas sequence of one strand is known, the base sequence of the second strand can t deduced. (:

***

4. Write short notes on (any two)
   (a) Roy's identity
   (b) Properties of demand function
   (c) Axioms of consumer's choice [6×2=12]

Group B

The maximum you can score in this part is **30**.

1. (a) Suppose the market demand for a product is described by the equation $P = 300 - Q$ and competitive conditions prevail. The short-run supply curve is $P = -180 + 5Q$. Find the initial short-run equilibrium price and quantity. Let the long-run supply curve be $P = 60 + 2Q$. Verify whether the market is also in long-run equilibrium at the initial short-run equilibrium that you have worked out. Now suppose that the market demand at every price is doubled. What is the new market demand curve? What happens to the equilibrium in the *very* short-run? What is the new long-run equilibrium? If a price ceiling is imposed at the old equilibrium, estimate the perceived shortage. Show all your results in a diagram.

[2+2+2+2+2+2+3=15]

Or,

1. (b) Consider a competitive industry consisting of 100 identical firms. The cost function of each such firm is given by: $C(q) = 0.1q^2 + q + 10$. The market demand for the product is: $D(p) = -400p + 4000$. Find the equilibrium price of the product. Now assume that a specific tax of (Rs.) t is imposed. Derive the expression for equilibrium price as a function of t.

[7+8=15]

2. Suppose a monopolist faces two types of consumers. In type *I* there is only one person whose demand for the product is given by: $Q_I = 100 - P$, where *P* represents price of the good. In type *II* there are n persons, each of whom has a demand for one unit of the good and each of them wants to pay a maximum of Rs. 5 for one unit. Monopolist cannot price discriminate between the two types. Assume that the cost of production for the good is zero. Does the equilibrium price depend on n? Find the equilibrium price.

[10]

3. Write short notes on (any two):
   (a) Stability of equilibrium
   (b) Nash equilibrium
   (c) Labor supply curve

[5x2=10]

Indian Statistical Institute

First Semestral Examination: (2007–2008)

B.Stat.(Hons.) – II year

Economics I

Date: ***30/11/2007*** *Maximum Marks* ***–60*** *Duration:* ***3 hours***

Answer each Group on a separate answer script.

Group A

Answer any **three** questions. The maximum you can score in this part is **30**.

1. (a) A consumer's expenditure function is given by $e(p_1, p_2, u) = p_1^{\alpha} p_2^{1/2} u$. Using the appropriate properties of $e(.)$ calculate the value of α.

   (b) Can two indifference curves cross? Justify your answer.

   (c) Show that for a consumer with strictly convex preferences the utility maximizing bundle is unique.

   (d) What is a "corner solution"? How does one arrive at such a solution? Explain graphically using a two-commodity set up.

   [3×4=12]

2. (a) Define 'Giffen goods' and 'Inferior goods'. Show that all Giffen goods are inferior, but the converse is not true.

   (b) "Compensated demand curves are always negatively sloped". Justify.

   (c) In a two-commodity world, where both goods are normal, derive (graphically) the Demand Curve for good 1 from the Price Offer Curve and the Engel Curve from the Income Expansion Path.

   [4×3=12]

3. (a) State the Weak Axiom of Revealed Preference (WARP).

   (b) Check whether the WARP is satisfied in each of the following cases. Give reasons for your answer.

   (i) Income m=20, $(p_1, p_2) = (1, 1)$; the choice is (5, 15)
   Income m=20, $(p_1, p_2) = (2, 0.5)$; the choice is (8, 8)

   (ii) Income m=5, $(p_1, p_2) = (1, 2)$; the choice is (1, 2)
   Income m=5, $(p_1, p_2) = (2, 1)$; the choice is (2, 1)

   (iii) Income m=4, $(p_1, p_2) = (2, 1)$; the choice is (1, 2)
   Income m=4, $(p_1, p_2) = (1, 2)$; the choice is (2, 1)

   [3+3×3=1

P.T.

Indian Statistical Institute

Semester 1 (2007-2008)

B. Stat 2nd Year

Mid-semestral Exam

Probability Theory 3

Friday 7.9.2007, 10:30-1:00

Total Points $5 \times 8 = 40$

Answers must be justified with clear and precise arguments. More than one uncrossed answers to the same question or part of a question will not be entertained and only the first uncrossed answer will be graded. If you use any theorem or lemma proved in class state it explicitly.

1. Let $Y_1, \cdots, Y_n$ be iid $N(0, 1)$ random variables and denote $\mathbf{Y} = (Y_1, Y_2, \cdots, Y_n)'$. Let $\mathbf{X}_1$ and $\mathbf{X}_2$ be the following two matrices

$$\mathbf{X}_1' = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{pmatrix}, \mathbf{X}_2' = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ x_1^2 & x_2^2 & \cdots & x_n^2 \end{pmatrix},$$

where $x_1, \cdots, x_n$ are real numbers such that $\mathbf{X}_1, \mathbf{X}_2$ are of full rank. Find the density of the ratio

$$\frac{||\mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-1}\mathbf{X}_2\mathbf{Y} - \mathbf{X}_1(\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1\mathbf{Y}||^2}{||\mathbf{Y} - \mathbf{X}_2(\mathbf{X}_2'\mathbf{X}_2)^{-1}\mathbf{X}_2\mathbf{Y}||^2},$$

where $||\mathbf{v}||^2$ denotes the euclidean norm of a vector.

2. (a) Is $\phi(t) = \cos t^2, t \in R$, a characteristic function?

   (b) Let X, Y be independent $N(0, 1)$ and U, V be independent of X, Y. What is the distribution of $(UX + VY)/\sqrt{U^2 + V^2}$? $4 + 4 = 8$ pts.

3. Suppose $(\mathbf{X}_1, \mathbf{X}_2)$ is nondegenerate multivariate normal with zero means and variance covariance matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$$

P.T.O

and let us denote $\mathbf{Y} = \mathbf{X}_2 - \Sigma_{21}\Sigma_{11}^{-1}\mathbf{X}_1$. Let σ_i' denote the i-th row of Σ_{21} and $\beta_i' = \sigma_i'\Sigma_{11}^{-1}$. Identify the corresponding component of $\mathbf{Y}$ as $Y_i = X_i - \sigma_i'\Sigma_{11}^{-1}\mathbf{X}_1 = X_i - \beta_i'\mathbf{X}_1$. $4 + 4 = 8$ pts.

(a) Show that for any column vector α of real numbers, $Var(X_i - \alpha'\mathbf{X}_1) \geq Var(Y_i)$.

(b) Prove or disprove: for every such α as in part (a), $Corr(X_i, \beta_i'\mathbf{X}_1) \geq Corr(X_i, \alpha'\mathbf{X}_1)$.

4. Suppose $X_1, X_2, \cdots, X_n$ are iid exponential random variables with density $\lambda e^{-\lambda x}, x > 0$. Find the density of

$$\frac{nX_{(1)}}{\sum_{i=1}^{n}(X_i - X_{(1)})},$$

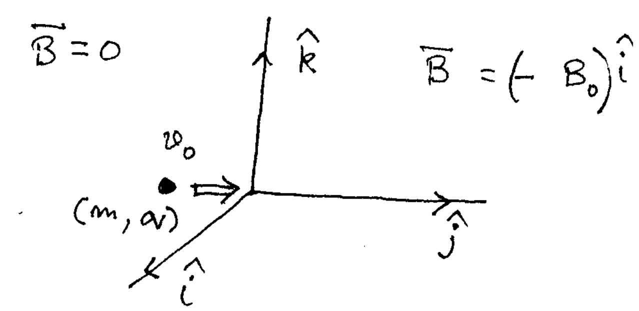
where $X_{(k)}$ denotes the k-th order statistic.

5. The random variables X_n have Poisson distribution with parameter n for $n = 1, 2, \cdots$. Decide whether the following limit exists and if it does then find it:

$$\lim_{n\to\infty} P\left(\frac{X_n - n}{\sqrt{n}} \leq x\right).$$

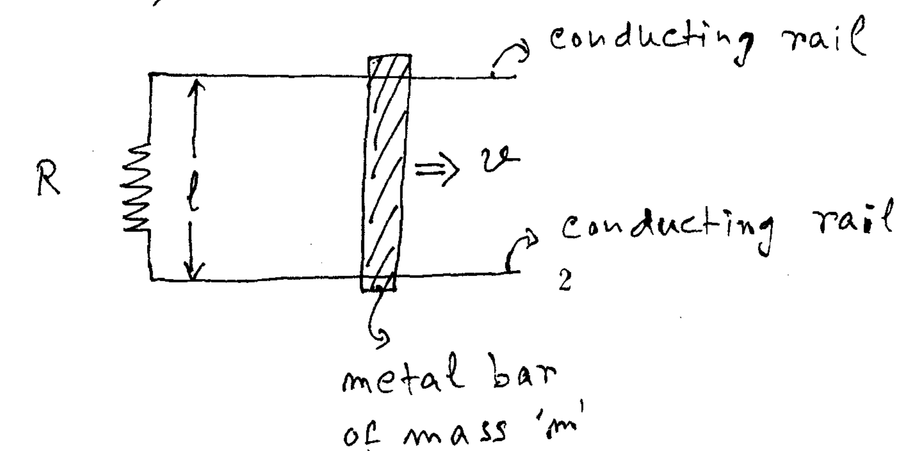
Group B: Answer any four questions

1) A particle of mass m and charge q with velocity $v_0\hat{j}$ enters a region of uniform magnetic field $(-B_0)\hat{i}$ at time $t = 0$.
(i) Set up the equations of motion.
(ii) Find the momentum in the $\hat{j}$-direction at a later time $t = t_0$. Draw the approximate trajectory when the charge q is positive.
2+3=5



2) (i) Consider two parallel wires of infinite length carrying uniform currents I_1 and I_2. Find the magnetic force per unit length on one of the wires.
(ii) Find the magnetic field at the centre of a circular loop of wire of radius R with a uniform current I in the wire loop.
2+3=5

3) A metal bar of mass m slides without friction on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field B, pointing into the page, fills the entire region.
(i) If the bar moves to the right with velocity v, find the direction and magnitude of the current in the resistor.
(ii) Find the direction and magnitude of the magnetic force on the bar.
(iii) If the bar starts out with speed v_0 to the right at time $t = 0$ and is left to slide, calculate its speed at a later time t.
1+1+3=5



P.T.O

4) (i) Obtain the current conservation law $\nabla.\mathbf{J} = -\frac{\partial\rho}{\partial t}$ from the Maxwell's equations of electrodynamics.
(ii) Explain the role of displacement current in the context of validity of Ampere's law in charging a capacitor.
(iii) Give an example where Newton's third law is violated when electromagnetic forces are involved. How is the conservation of momentum law restored?
2+1+2=5

5) (i) From Maxwell's equations derive the wave equation satisfied by the magnetic field in vacuum. Find out the expression for the speed of propagation of electromagnetic wave.
(ii) Consider plane wave solutions of the form

$$\mathbf{E} = \mathbf{E_0} e^{i(kz-wt)} \; ; \; \mathbf{B} = \mathbf{B_0} e^{i(kz-wt)}.$$

From the Maxwell equations show that electric and magnetic fields are mutually orhogonal.
(iii) Find the direction and magnitude of the energy flow from the Poynting vector for the system.
2+2+1=5

INDIAN STATISTICAL INSTITUTE

Semestral Examination : 2007-2008

B. Stat. - Second Year

Analysis III

Date : 27.11.07 Maximum Score : 100 Time : 3 Hours

This paper carries questions worth a total of 72 marks. Answer as much as you can. The maximum you can score is 60 marks.
You are free to use any theorem proved or presented without proof in the class. However, you must state such a result at least once in the answer script because the evaluation will take that into account.

(1) Which of the following statements are true and which are false? Give a brief justification to your answer.
  (a) There is a one-to-one continuous map from $\{x \in \mathbb{R}^n : |x| \leq 1\}$ onto $\{x \in \mathbb{R}^n : |x-1| \leq 1\} \cup \{x \in \mathbb{R}^n : |x+1| \leq 1\}$.
  (b) If A is a countable subset of $\mathbb{R}^n$, then $\mathbb{R}^n \setminus A$ is connected.

[5 + 5]

(2) (a) Let u and x be two vectors in $\mathbb{R}^n$ such that $|x| = 1$ and $u \cdot x = 1$. Show that there is a smooth curve $\alpha : (-1,1) \to \mathbb{R}^n$ such that $|\alpha(t)| = 1$ for all t, $\alpha(0) = x$ and $\alpha'(0) = u$
  (b) Show that every C^1 function defined on an open subset of $\mathbb{R}^n$ is differentiable.

[6 + 10]

(3) Let $f : [0,1]^n \to \mathbb{R}$ be a bounded function such that the set of all discontinuity points of f is of measure zero. Show that f is Riemann integrable.

[10]

(4) Let V be a finite-dimensional vector apace, T a p-tensor and S a q-tensor on V. Assume that $Alt(S) = 0$. Show that $Alt(T \otimes S) = 0$.

[14]

P. T. O.

(5) (a) For any smooth vector field F on an open set U in $\mathbb{R}^3$, show that both $curl(\nabla F)$ and $div(curl F)$ vanish.

(b) Let α be a singular k-cube and $p : [0,1]^k \to [0,1]^k$ be a bijective C^∞ function such that $|J_p(x)| > 0$ for every $x \in [0,1]^k$. For any k-form ω show that

$$\int_\alpha d\omega = \int_{\alpha \circ p} \omega.$$

(c) For any $(k-1)$-form ω on $[0,1]^k$ of the form $f dx_1 \wedge \cdots \wedge \widehat{dx_i} \wedge \cdots \wedge dx_n$, show that

$$\int_{I^k} d\omega = \int_{\partial I^k} \omega.$$

[(3+3) + 6 + 10]

Indian Statistical Institute

First Semestral Examination: (2007-2008)

B.Stat 2nd year

Physics 1

Total Marks: 40

Use seperate sheets for Group A and Group B

Time : 3 Hours

Date:- 30.11.07

Group A : Answer any four questions

1.(a) A particle moves in a circular orbit of radius a under the influence of an attractive central force $f(r) = \mu[b/r^2 + c/r^4]$. b, c being positive constants. Show that for $a^2b - c > 0$ the motion is a stable. (3)

(b) Show that the product of the maximum and minimum linear speeds of a particle moving in an elliptic orbit is $(2\pi a/T)^2$ where T is the period and a is the semi major axis. (2)

2. Show that (i) if a particle describes a circle under the influence of a central force directed towards a point on the circle. then the force varies as the inverse fifth power of the distance (ii) the total energy of the particle is zero (iii) Find the period of the motion. (2+1+2)

3. A particle of mass m is projected from infinity with velocity V_0 in a manner such that it would pass a distance b from a fixed centre of inverse square repulsive force k/r^2 if it were not deflected. Find (i) the distance of closest approach (ii) the differential scattering cross section for a homogeneous beam of particles scattered by this potential. (2+3)

OR

Show that for a particle of mass m under the influence of an attractive central force $f(r) = -k/r^2$. the vector $(\mathbf{p} \times \mathbf{L} - mk\frac{\mathbf{r}}{r})$ is conserved. Hence obtain an expression of the path of the particle. (3+2)

4. A meter stick is moving with speed $0.8c$ relative to a frame S. What is the stick's length as measured by an observer in the S frame (i) if the stick is parallel to it's velocity? (ii) if the stick is moving at an angle 60° to it's velocity as seen from the rest frame? (2+3)

5.(a) Two signal lamps are lighted simaltaneously at points 1km apart on a straight railroad track. How fast must a train move along the track in order that there is a time interval of 0.5s between the events of lighting in driver's frame? Which lamp is lighted first in this frame? (3)

P.T.O

(b) A spaceship is travelling to the star Alpha Centauri which is 4 light years away. If the spaceship is travelling with speed $.75c$ relative to earth, find the time taken to reach the star (i) as measured on earth (ii) as measured by an observer on the spaceship.

Indian Statistical Institute

Mid-Semester Examination: (2007-2008)

B.Stat 2nd year

Physics 1

Total Marks: 40

Use seperate sheets for Group A and Group B

Time : 2 Hours

Date: 5.9.07

Group A: Answer any four questions

1. (a) Prove that the angular momentum of a system of particles about a fixed point is equal to the sum of the angular momentum of the total mass concentrated at the centre of mass about that point and the angular momentum of the particles about the centre of mass. (5)

2. If the Lagrangian does not depend on time explicitly, then show that $L - \sum \dot{q}_i \frac{\partial L}{\partial \dot{q}_i}$ is a constant of motion. Further if the force is conservative, show that the total energy is constant. (5)

3. (a) Show that $\frac{\partial \dot{\mathbf{r}}_\nu}{\partial \dot{q}_j} = \frac{\partial \mathbf{r}_\nu}{\partial q_j}$. (2)

(b) A particle is falling vertically under the influence of gravity when frictional forces obtainable from a dissipation function $\frac{1}{2}k\dot{r}^2$ are present. Use Lagrange formalism to find velocity as a function of time. (3)

4. (a) For a particle moving under the influence of a central force, prove that $E = \frac{1}{2}m\dot{r}^2 + V_{eff}(r)$ where $V_{eff} = V(r) + \frac{L^2}{2mr^2}$, the symbols having their usual meaning. (3)

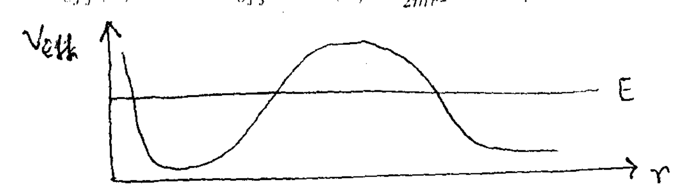


Fig 1.

(b) If V_{eff} is of the shape as in fig 1, then use the above result to identify the turning points, allowed, forbidden and scattering regions. (2)

5. (a) The path of a particle moving in a central force field is given by $r\theta = constant$. Determine the potential in terms of r. (2)

(b) The path of a particle is given by $x = a\ cos(\alpha t)$, $y = b\ sin(\beta t)$. Under what condition the force acting on the particle is a central one? In this case show that the total energy is constant. (3)

P.T.O

ALL QUESTIONS CARRY 5 MARKS. ANSWER ANY 4 QUESTIONS.

(1): Consider the surface

$$x^2 + y^2 + z^2 - 3 = 0.$$

(i) Find the unit normal to this surface at the points $(1,1,1)$ and $(0,0,1)$.
(ii) Consider two point charges $+a$ and $-b$ placed at $(\frac{1}{2}, 0, \frac{1}{2})$ and $(2,1,0)$ respectivly. Find the flux of electric field linked with the above surface.
(iii) Calculate

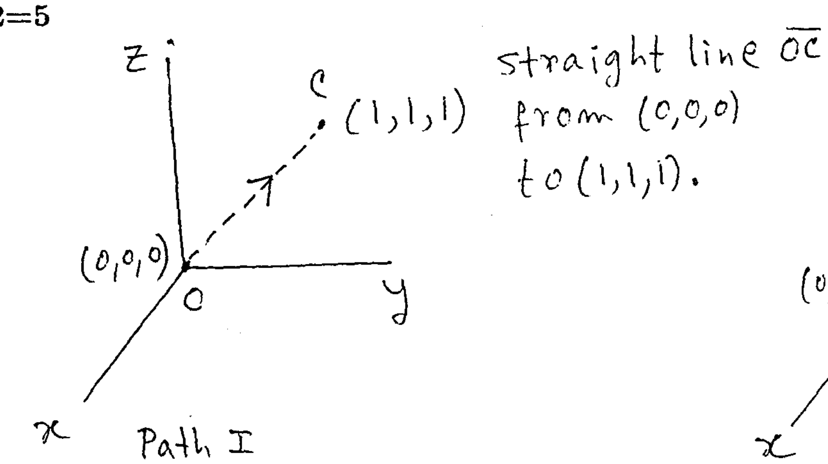
$$\vec{\nabla}.(f\vec{A})$$

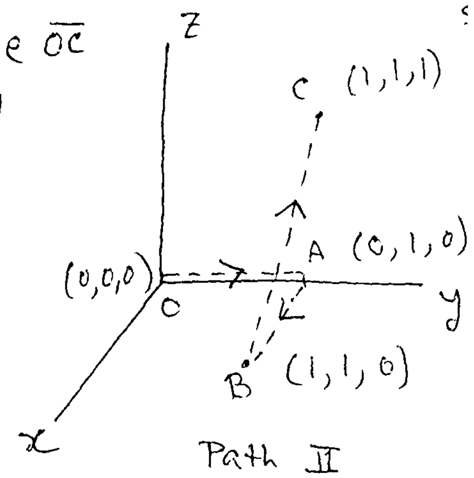
where $f = xy$ and $\vec{A} = z\hat{j}$.
3+1+1=5

(2): Consider the vector field

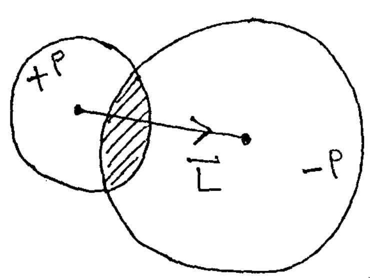
$$\vec{A}(x,y,z) = yz\ \hat{i} + xz\ \hat{j} + xy\ \hat{k}.$$

(i) Find the line integral $\int \vec{A}.\vec{dr}$ for the two paths shown in figure, between the endpoints $(0,0,0)$ and $(1,1,1)$.
(ii) Prove that for this $\vec{A}$ the line integrals are independent of the path chosen and recalculate $\int \vec{A}.\vec{dr}$ between $(0,0,0)$ and $(1,1,1)$ *without* performing any line integration.
3+2=5

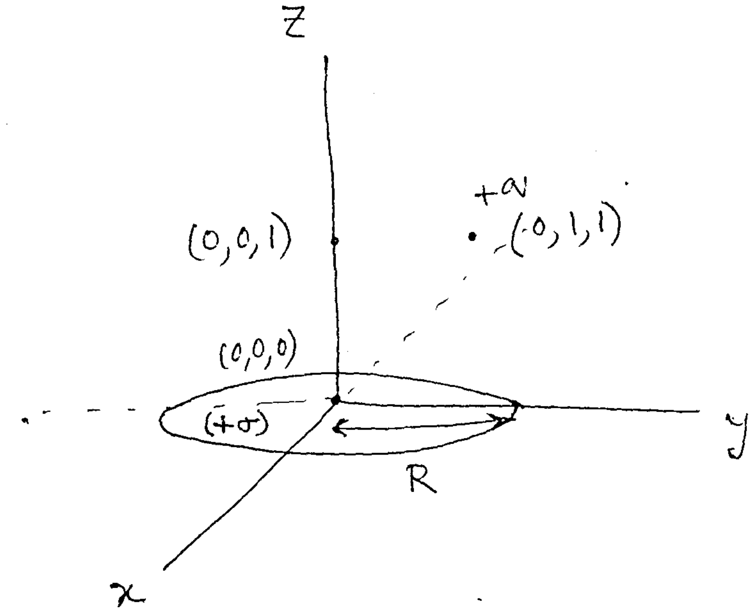




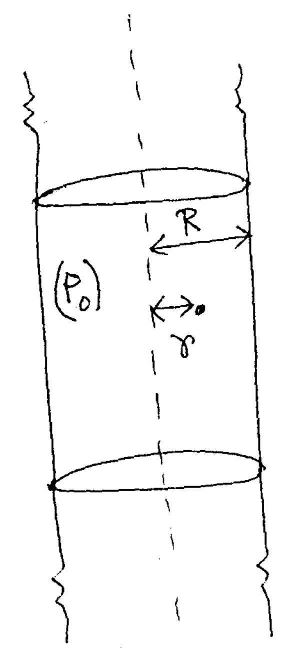
(3): In the figure the smaller sphere has *constant* volume charge density $+\rho$ and the larger sphere has *constant* volume charge density $-\rho$. The vector joining the centres of the spheres is $\vec{L}$. Find the electrostatic field in the *shaded region* which is the *intersection* of the two spheres.
5



(4): (i) Find the electrostatic field at $(0,0,1)$ for the following charge configuration - a disc radius R on the $x-y$ plane with its centre at $(0,0,0)$ having constant surface charge density *and* a point charge $+q$ at $(0,1,1)$ (see figure).
Find the total electrostatic energy due to a *spherical shell* of radius R with uniform surface rge density σ.
2=5



5): Consider an *infinite* cylinder of radius R, having *uniform* charge density ρ_0. ind the electrostatic field at a distance r from the axis of the cylinder with $R > r$ (see e).
Show that the infinitesimal form of Gauss law for electrostatics, $\vec{\nabla}.\vec{E} = \frac{\rho}{\epsilon_0}$, is satisfied.
he general form of divergence in cylindrical coordinates is $\vec{\nabla}.\vec{A} = \frac{1}{r}\partial_r(rA_r) + \frac{1}{r}\partial_\theta(A_\theta) + \partial_z(A_z)$.
+1=5



INDIAN STATISTICAL INSTITUTE
MID-SEMESTER EXAMINATION: 2007-08
B. STAT. II: 2007-08
ECONOMICS-I

Date: 5.9.07 | Maximum Marks: 40 | Duration: 2 Hours

Answer Each Group in a Separate Script.

Group A

This part of the question carries 25 marks. Answer as much as you can. The maximum you can score is 20.

1. (a) Define price elasticity of demand and supply. What are perfectly inelastic, inelastic, unit elastic and perfectly elastic items? Illustrate graphically with respect to both demand and supply.
(b) Show that an increase in the price of a good leads to an increase or a decrease in the total amount spent on purchases of the good according as the demand for the good is inelastic or elastic.

[9+3=12]

2. (a) What is an indifference curve? Justify the shape of an indifference curve using the relevant axioms of consumers' choice.
(b) Plot the indifference maps for each of the following cases, indicating the direction of preferences in each case.
   (i) Goods 1 and 2 are substitutes for a consumer. If the quantity of good 1 increases by 100% and that of good 2 decreases by 50%, the consumer is just as well off.
   (ii) Commodity 1 is 'good' and commodity 2 is 'garbage'.
(c) A consumer spends Rs. 450 on two goods, 1 and 2. His utility function is $U(x_1, x_2) = 0.5x_1x_2^2$, $p_1 = 5$, $p_2 = 3$. What quantities of the two goods does he buy?

[5+3+5=13]

P.T.O

Group B

Answer any **three** questions. Each question carries 5 marks.

1. Define average productivity (AP) and marginal productivity (MP) of an input. Establish the relationship between AP and MP. In particular, derive conditions for which MP is falling but AP is rising. Also show your results numerically.

[1+2+1+1=5]

2. Define marginal rate of technical substitution (MRTS). Does diminishing marginal productivities of factors necessarily imply diminishing MRTS? Prove your result. Show that along a ray from the origin, MRTS between two inputs is constant when the production function is homogeneous of any degree.

[1+3+2=5]

3. What is elasticity of substitution (σ)? Derive the expression of σ for the CES production function. Show that the production function associated with each of linear technology and Leontief technology follows as a special case of the CES technology.

[1+2+1+1=5]

4. Prove that if the production function is concave, then the iso-quants are convex to the origin. Explain the meaning of a convex iso-quant in economic terms.

[3+2=5]

5. Evaluate profit maximization hypothesis. Does profit maximization necessarily imply cost minimization? Explain.

[3+2=5]