

The Modified and Generalised Henry George Theorem and Inter-Regional Equity : A Theoretical Note

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1. Introduction :

In the realm of taxation, the name and idea of Henry George is associated with a single tax on land rents to support local government. This idea is grounded on the reasoning that land rent is 'unearned' by efforts on the part of the land owner, and thereby it appears as an especially fit object of taxation.

In the recent past, the idea of Henry George has been gaining considerable attention in models that deal with determining optimum population in the presence of local public goods.¹ For instance, Stiglitz (1977) showed in his single region model that at the optimum population the equality between aggregate land rent and total cost of local public good holds. This equality is the modern vindication of the above stated idea of Henry George, and is dubbed as Henry George Theorem (HGT) by Stiglitz (1977).

Stiglitz's single region HGT has been extended to city models [Arnott (1979), Arnott and Stiglitz (1979)]. In these extensions, the basic problem is to determine optimal size for a circular city. The HGT obtained in these extensions read that when the city population is optimal, differential land rents equal expenditure on the Samuelsonian public good.

Further, Stiglitz's single region HGT has been extended to multiregions economy also. Assuming a perfectly symmetric situation where each region would be like any other region in the economy, and each would be self-sufficient, Kanemoto (1983) has shown that the optimisation problem can be reduced to one of finding the efficient number of regions. In this symmetric situation, the basic result of Stiglitz is shown to hold.

* The author is associated with the Institute of Socio Economic Planning, University of Tsukuba, Japan. The author wishes to thank Professor Noboru Sakashita, Institute of Socio Economic Planning, University of Tsukuba, Japan, for all his help and guidance in the preparation of this note. The financial help in carrying out the above research from the Ministry of Education, Government of Japan, is gratefully acknowledged.

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In our opinion, two remarks on the above extensions deserve special mention. First of all, the above studies invariably assume that the local public good in question is a pure one, and optimum population or number of regions could be characterised in terms of aggregate land rents and expenditure on local public goods without reference to congestion/crowding cost. Moreover, the treatment of local public good as an endogenous variable, and therefore the derivation of Samuelsonian condition, in the above models seems to have no necessary relation in obtaining HGT. In other words, it can be shown that, in the above models, the HGT holds whether or not the supply of local public good is optimal. Secondly, the assumption of one type city or identical communities in the above models has shadowed the implications of the Theorem for inter-regional equity. In a federal model of differing regions, for instance, Hartwick (1980) showed that the HGT has implications for inter-regional equity through the mechanism of rent sharing.

The purpose of this theoretical note is to develop a two-region federal model, in an asymmetric situation, for the purpose of demonstrating a few modifications and generalisations to Stiglitz's single region HGT, and to derive its implications for inter-regional equity. We mainly emphasise the necessary modification that is needed to derive and interpret when impure (congested) local public good enter as an argument in the utility function. Further, we demonstrate the generality of the HGT by deriving the Theorem independent of (i) the nature of public good in question, and (ii) where the benefits of public good in question are concentrated between regions.

The main result of this theoretical note is that with congested local public good as an argument of the utility function, there is no way possible to derive the HGT unless the level of impure local public good supply is optimal. This result is new in the literature on Henry George Theorem, and thereby contributes to a new understanding on the role of Samuelsonian optimality condition in deriving the HGT in the context of impure local public goods. Secondly, the results of our generalisation of the HGT clearly demonstrate that the derivation of HGT does not depend on the nature of public goods (local or national) if only if they are pure public goods. Finally, our generalisations also show that HGT holds regardless of where the benefits of pure public goods are located between regions.

The rest of the materials of this note is organised in the following way. The section 2 the basic model is outlined. Section 3 derives a Modified Henry George Theorem. In Section 4, few generalisations of HGT are attempted. Finally, in Section 5, the implications of the HGT for inter-regional equity are stated.

2. The basic model :

We consider a federal economy which consists of two regions : Region 1 and

Region 2. The total population (or total labour supply) of region 1 and region 2 is denoted by L_1 and L_2 respectively. The total population in the federation is assumed perfectly inelastic and equal to L .

We assume that each labourer has one unit of labour which is intrinsically tied to him/her (and is therefore mobile). All labourers are assumed to be treated equally as citizens. This equality of treatment requires the same utility level for all labourers within a region and between regions. Further, mainly for the sake of simplicity, it is assumed that the utility function of all labourers are identical. We define the utility function in region i ($i=1, 2$), which is assumed to be continuous and strictly quasi-concave, as follows.

$$U[C_i, Q_i], U_{C_i} > 0, U_{Q_i} > 0, \quad (1)$$

where

$$U_{C_i} = \partial U / \partial C_i \text{ and } U_{Q_i} = \partial U / \partial Q_i;$$

C_i = per capita consumption of private good in region i , and

$Q_i = G_i L_i^{-\alpha}$ = services of local public good in region i .

We assume that parameter α is an index of publicness or congestion of local public services. If $\alpha = 0$, we have the case of pure public good and utility is independent of population size and variation. On the other hand, if $\alpha = 1$, we have the case of pure private good. Thus, by assuming that $0 < \alpha < 1$, we treat the services of local public good as "impure" or "congested" in each region (implicitly, here and throughout this paper, we assume that there are no spill-over effects between regions).² This treatment implies that the services of local public good are now allowed to depend on the number of persons with whom the good is to be consumed.³ This means that the utility is not independent of population variation or size. However, we follow Buchanan and Goatz (1972) in assuming that these goods remain "public, in the strict sense of non-exclusion.

Further, from the requirement that all labourers derive the same or equal utility between regions, we have

$$U[C_1, Q_1] = U[C_2, Q_2]. \quad (2)$$

which, in fact, can also be considered a market equilibrium condition under free migration.⁴

Next, we postulate that total output (X) in each region is an increasing function

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of labour (L_i). That is

$$X_i = F(L_i, \bar{N}_i) = f_i(L_i), \quad (3)$$

where

$F(L_i, \bar{N}_i)$ is the production function which is assumed to be concave and homogeneous of degree one. Also, F is assumed to be identical between regions; \bar{N}_i is the fixed land resource in region i . The land resource is assumed to be different between regions, i.e., $\bar{N}_1 \neq \bar{N}_2$.

As additional assumptions on f_i , we have $f_i'(L_i) > 0$, $f_i''(L_i) < 0$.

The output thus produced in (3) in two regions is assumed to be freely and jointly allocated as private goods or public goods in both regions. In other words, we assume an implicit cross subsidisation between regions. Thus, the equality of the aggregate produced and expended income in the federation can be written as

$$f_1(L_1) + f_2(L_2) = C_1 L_1 + C_2 L_2 + G_1 + G_2, \quad (4)$$

One key feature of the above stated two-region federal model is that all production and consumption take place within the region. Hence, the two regions in question do not trade with each other. Further, land being a fixed resource, labour is the only other factor that can be considered mobile between regions.

3. Derivation of a Modified Henry George Theorem (MHGT) :

We formulate below an optimisation problem wherein a federal planner determines a level of population for the federation that shall maximise per capita utility. This formulation is essential because HGT is basically a result derived from optimum population size (or level) condition.

Now, given the information regarding the utility function, production conditions, labour supply restrictions in the federation as outlined in the basic model of section 2, an omnipotent federal planner's task of determining optimum population for the federation can be formulated as follows.

$$\begin{aligned} & \text{Max} && U(C_1, Q_1), && (5) \\ & \{ C_1, C_2, G_1, G_2, L_1, L_2 \} \\ & \text{s.t.} && U(C_1, Q_1) = U(C_2, Q_2), && (6) \end{aligned}$$

$$f(L_1) + f(L_2) = C_1 L_1 + C_2 L_2 + G_1 + G_2 \quad (7)$$

The Lagrangean form of the above maximisation problem can be written as follows.

$$\begin{aligned} \text{Max } Z = & U(C_1, Q_1) + \lambda_1 \{U(C_2, Q_2) - U(C_1, Q_1)\} + \lambda_2 \{f(L_1) + f(L_2) \\ & - [C_1 L_1 + C_2 L_2 + G_1 + G_2]\}, \end{aligned} \quad (8)$$

with respect to C_1, C_2, G_1, G_2, L_1 and L_2 .

By partial differentiation, we have the following first order conditions. 5

$$\partial Z / \partial C_1 = (1 - \lambda_1) U_{C_1} - \lambda_2 L_{C_1} = 0, \quad (9)$$

$$\partial Z / \partial C_2 = \lambda_1 U_{C_2} - \lambda_2 L_{C_2} = 0, \quad (10)$$

$$\partial Z / \partial G_1 = (1 - \lambda_1) U_{G_1} - \lambda_2 G_{G_1} = 0, \quad (11)$$

$$\partial Z / \partial G_2 = \lambda_1 U_{G_2} - \lambda_2 G_{G_2} = 0, \quad (12)$$

$$\partial Z / \partial L_1 = -(1 - \lambda_1) U_{L_1} - \lambda_2 L_{L_1}^{-(\kappa+1)} + \lambda_2 \{f'(L_1) - C_1\} = 0, \quad (13)$$

$$\partial Z / \partial L_2 = -\lambda_1 U_{L_2} - \lambda_2 L_{L_2}^{-(\kappa+1)} + \lambda_2 \{f'(L_2) - C_2\} = 0, \quad (14)$$

$$\partial Z / \partial \lambda_1 = U(C_1, Q_1) - U(C_2, Q_2) = 0, \quad (15)$$

$$\partial Z / \partial \lambda_2 = \{f(L_1) + f(L_2) - C_1 L_1 - C_2 L_2 + G_1 + G_2\} = 0. \quad (16)$$

Next, solving (9) and (10), we get

$$\left[\frac{L_1^{1-\kappa}}{L_2^{1-\kappa}} \cdot \left\{ \frac{U_{C_1}}{U_{C_2}} \right\} \right] = 1. \quad (17)$$

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In the same way, solving (11) and (12), we obtain

$$\left[\frac{1-\alpha}{\alpha} \cdot \left\{ \frac{U_{11}}{U_{12}} \right\} \right] = 1. \quad (18)$$

Equation (17) and (18) are the familiar but modified Samuelsonian condition for the optimal level of public goods in region 1 and in region 2 respectively. It tells that each region has an optimal division of output into private and public goods when the sum of marginal rate of substitution equal the marginal rate of transformation [which is unity in (17) and (18)]. In other words, at the optimum, the marginal cost of supplying the last unit of Q in terms of C foregone just equals the sum of marginal benefits that all users of the increment of Q simultaneously obtain in terms of C in each region with a discounting factor introduced by congestion.

Next, substituting (11) into (13), and (12) into (15), and using the resulting equations in (16), we finally obtain the following.

$$\left[\frac{L_1}{L_2} \cdot f'(L_1) - C_{L_1} - \alpha G_1 \right] + \left[\frac{L_2}{L_1} \cdot f'(L_2) - C_{L_2} - \alpha G_2 \right] = 0. \quad (19)$$

Essentially, equation (19) is a condition derived from the optimum population level for the federation when the per capita utility levels are maximised for the federation as a whole. The reason for considering (19) as a condition derived from the optimum population comes from the following analysis. Suppose we were to consider the determination of optimum population level for a single region only. Then the optimum condition we would have got is as follows.

$$f'(L) - [\alpha G/L] = C. \quad (20)$$

What equation (20) tells is that at the optimum population the marginal product of labour equals per capita consumption of good plus marginal congestion cost. We note that $[\alpha G/L]$ is the marginal congestion cost because it is the marginal change in the utility benefit from public services as L increases. However, to facilitate a better interpretation, we consider $[\alpha G/L]$ as an implicit per capita congestion tax. In this case, optimality condition shows the equality between marginal product of labour less implicit per capita congestion tax and per capita private congestion.

The equality in (20) establishes optimality for a single region because of the following reason. Suppose that the level of public expenditure and private consumption are fixed, and consider a case where the marginal product of labour less implicit per capita congestion tax is greater than per capita consumption of private good. Now allow one more labourer to come in. Then, the addition to the production is greater

then what is required to maintain all labourers at the previous standard of living, and thereby someone can be made better off without making anyone worse off. In this situation, it is optimal to increase population. On the other contrary, if the marginal product of labour less implicit per capita congestion tax is less than per capita consumption of private good, then it is optimal to decrease population. Hence, equation (20) is a condition for optimal population for a single region.

In short, it is precisely in line with the above reasoning, we consider equation (19) as a condition derived from the optimum population level for the federation.

Next, using (16) in (19), we get

$$L_{i1} \cdot f'(L_{i1}) + L_{i2} \cdot f'(L_{i2}) - f'(L_{i1}) - f'(L_{i2}) - \alpha[G_1 + G_2] + G_1 + G_2 = 0. \quad (21)$$

Letting $R_i = f(L_{i1}) - L_{i1} \cdot f'(L_{i1})$, as the aggregate land (resource) rent in region i , equation (21) can be rewritten as follows.

$$R_1 + R_2 + \alpha[G_1 + G_2] = [G_1 + G_2]. \quad (22)$$

Equation (22) tells that when the level of population is optimised to maximise per capita utility of the federation, we have a condition that the aggregate land rent plus implicit congestion tax equals total cost of local public goods in the federation. Since equation (22) modifies the original HGT by inclusion of congestion factor, we call it the Modified Henry George Theorem (MHGT) for the two-region federal economy.

We notice an important implication of the MHGT, viz., the MHGT is dependent on the Samuelsonian optimality condition of impure local public good. This comes from the fact that in deriving equation (19), we have explicitly used equations (11) and (12). In other words, equations (11) and (12) are indispensable in obtaining (19) and hence equation (22) of MHGT. In short, with congested local public good as an argument of the utility function, there is no way possible to derive HGT unless the level of impure local public good is optimal.

4. Some generalisations of the Henry George Theorem :

In this section we consider two important generalisations to the Modified HGT derived in section 3 under case 1 and case 2.

Case 1 :

Suppose that local public goods are provided by local governments. Then, in a

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federal set up (say, of two-tier structure), the existence of only such local public goods makes the presence of federal government functionally inoperative. In order to eliminate it and to make the analysis more realistic, we extend the optimisation model in section 3 by introducing one more public good, viz., a national public good denoted by P.

By a national public good we mean the following. It is a pure public good provided by a federal government. Its cost is equally shared and its benefit is equally enjoyed by all in a federation. National defence may be a good example for such a public good.

Following the above definition, we note that such a national public good need not affect the migration behavior of an individual because neither cost nor benefit varies between regions. However, from the point of view of optimum population an interesting problem exists with regard to treating the national public good as exogenously given or endogenously determined. In what follows, we demonstrate that in either of the cases, the Modified HGT is invariant since the national public good is assumed to be a pure public good.

An omnipotent federal planner's task of determining optimum population for the federation in the presence of impure local public good and pure national public good can be formulated in the following Lagrangean form.

$$\begin{aligned} \text{Max } Z = & U[C_1, Q_1, P] + \lambda_1 \{U[C_2, Q_2, P] - U[C_1, Q_1, P]\} + \lambda_2 [f(L_1) + f(L_2) - \\ & [C_1 L_1 + C_2 L_2 + G_1 + G_2 + P]], \end{aligned} \quad (23)$$

with respect to $C_1, C_2, G_1, G_2, L_1, L_2$ and P.

By partial differentiation, we have

$$\partial Z / \partial C_1 = (1 - \lambda_1) \cdot U_{c_1} - \lambda_2 \cdot L_1 = 0, \quad (24)$$

$$\partial Z / \partial C_2 = \lambda_1 \cdot U_{c_2} - \lambda_2 \cdot L_2 = 0, \quad (25)$$

$$\partial Z / \partial G_1 = (1 - \lambda_1) \cdot U_{g_1} \cdot L_1^{-\alpha} - \lambda_2 = 0, \quad (26)$$

$$\partial Z / \partial G_2 = \lambda_1 \cdot U_{g_2} \cdot L_2^{-\alpha} - \lambda_2 = 0, \quad (27)$$

$$\partial Z/\partial L_1 = -(1-\lambda_1) U_{e_1} \cdot \alpha G_1 L_1^{-(\alpha+1)} + \lambda_1 \cdot [f'(L_1) - C_1] = 0, \quad (28)$$

$$\partial Z/\partial L_2 = -\lambda_2 \cdot U_{e_2} \cdot \alpha G_2 L_2^{-(\alpha+1)} + \lambda_2 \cdot [f'(L_2) - C_2] = 0, \quad (29)$$

$$\partial Z/\partial P = (1-\lambda_1) U_p + \lambda_1 U_p - \lambda_2 = 0, \quad (30)$$

$$\partial Z/\partial \lambda_1 = [U[C_1, Q_1] - U[C_2, Q_2]] = 0, \quad (31)$$

$$\partial Z/\partial \lambda_2 = [f(L_1) + f(L_2) - C_1 L_1 - C_2 L_2 - G_1 - G_2 - P] = 0. \quad (32)$$

From the above first order conditions, we first obtain the following three results.

$$\left[L_1^{(1-\alpha)} \cdot \{U_{e_1}/U_{e_1}\} \right] = 1, \quad (33)$$

$$\left[L_2^{(1-\alpha)} \cdot \{U_{e_2}/U_{e_2}\} \right] = 1, \quad (34)$$

$$L_1 [U_p/U_{e_1}] + L_2 [U_p/U_{e_2}] = L_1 [U_p/U_{e_1}] + L_2 [U_p/U_{e_2}] = 1. \quad (35)$$

Equations (33) and (34) are the familiar but modified Samuelsonian conditions for impure local public good in each region. On the other hand, equation (35) is the Samuelsonian condition for national public good.

Next, from equation (30), we get the following relation.

$$\lambda_2 = U_p. \quad (36)$$

Using this relation in (26) and (27), and after some manipulations, we finally obtain the following condition derived from the optimum population level for the federation.

$$L_1 f'(L_1) + L_2 f'(L_2) - f(L_1) - f(L_2) - \alpha [G_1 + G_2] + G_1 + G_2 + P = 0. \quad (37)$$

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Now, it is interesting to consider another case. Suppose, we consider that P is exogenously given. In this case, equation (30) disappears, and the condition (37) derived from the optimum population level for the federation still holds.

Next, Letting $R = f(L_i) - L_i f'(L_i)$, as the aggregate land (resource) rent in region i ,

we get

$$R_1 + R_2 + \alpha[G_1 + G_2] = [G_1 + G_2] + P. \quad (38)$$

The above equation tells that with the level of population that maximises per capita utility in the federation, we have a condition that the aggregate land rent plus congestion tax equals the total cost of local public goods and national public good. Since the result in (38) generalises the Modified HGT beyond one type of public good, we call it the Generalised HGT.

The main implication of the above model is that the HGT holds whether or not the pure national public good supply is optimal. This implication is not restricted only to pure national public good. In other words, the above implication holds for all types of public good (local and national) so as they are characterised by no congestion phenomenon.

Case 2 :

Let a public good, Y , provided by the federal government is called a national public good. It is a national public good in the sense that it is financed by taxing all over the federation. We further assume that Y is a pure public good and is located in one of the two regions of the model. Finally, we suppose that the benefits of Y are restricted only to residents of the region where it is located. Given all these assumptions, in the strict sense of the term, Y must be considered as a nationally financed local public good. In the following, we consider a model which incorporates this type of public good and derive another version of generalised HGT from the optimum population condition for the federation.

The model, in this case also, is basically an extension of the model in section 3. The task of the federal planner is assumed to be the determination of optimum population for the two-region federal economy. Stated clearly, the problem is to

$$\text{*Max} \quad U^1 [C_1, Q_1, Y], \quad (39)$$

$$\{C_1, C_y, G_1, G_y, L_1, L_y, Y\}$$

s. t.

$$U^1 [C_1, Q_1, Y] = U^2 [C_y, Q_y] \quad (40)$$

$$f_1(L_1) + f_y(L_y) = C_1 L_1 + C_y L_y + G_1 + G_y + Y \quad (41)$$

The equality in (40) comes from the assumption that all individuals are treated equally as citizens in the federation. However, it can be justified in either of the following ways. First, if two regions are far between each other, the existence of nationally financed local public good in the utility function of region 1 may simply be ignored. Second, if there exists some form of compensation payments from region 1 to region 2 (for having contributed towards the cost of Y but without any benefits), say in the form of inter-regional transfers. Since the constraint in (41) allows for implicit cross-subsidisation between regions, the equality in (41) is indisputable.

Next, the Lagrangean form of the above maximisation problem can be written as follows.

$$\begin{aligned} \text{Max } Z = & U^1 [C_1, Q_1, Y] + \lambda \{ U^2 [C_y, Q_y, Y] - U^1 [C_1, Q_1, Y] \} + \lambda \{ f_1(L_1) + f_y(L_y) - \\ & [C_1 L_1 + C_y L_y + G_1 + G_y + Y] \}, \end{aligned} \quad (42)$$

with respect to $C_1, C_y, G_1, G_y, L_1, L_y$ and Y.

By partial differentiation, we have the following first order conditions.

$$\partial Z / \partial C_1 = (1 - \lambda) U_{C_1}^1 - \lambda U_{C_1}^2 = 0, \quad (43)$$

$$\partial Z / \partial C_y = \lambda U_{C_y}^2 - \lambda U_{C_y}^1 = 0, \quad (44)$$

$$\partial Z / \partial G_1 = (1 - \lambda) U_{G_1}^1 - \lambda U_{G_1}^2 = 0, \quad (45)$$

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$$\partial Z / \partial G_y = \lambda \frac{2}{U_{yy}} \frac{-\alpha}{L_y} - \lambda_y = 0, \quad (46)$$

$$\partial Z / \partial L_1 = -(-\lambda_1) U_{x1} \frac{1}{\alpha G_1 L_1} \frac{-(\alpha+1)}{+ \lambda_1} [f'_1(L_1) - C_1] = 0, \quad (47)$$

$$\partial Z / \partial L_y = -\lambda_y \frac{2}{U_{yy}} \frac{-\alpha}{L_y} \frac{-(\alpha+1)}{+ \lambda_y} [f'_y(L_y) - C_y] = 0, \quad (48)$$

$$\partial Z / \partial Y = (1 - \lambda_1) \frac{1}{U_{y1}} - \lambda_y = 0,$$

$$\partial Z / \partial \lambda = \{U_{11} C_1 - U_{1y} C_y\} - \{U_{yy} C_y - U_{yy} C_y\} = 0, \quad (49)$$

$$\partial Z / \partial \lambda = \{f'_1(L_1) + f'_y(L_y) - C_1 L_1 + C_y L_y + G_1 + G_y + Y\} = 0. \quad (50)$$

Further, solving (43) and (45), we get

$$[L_1 \frac{(1-\alpha)}{\{U_{y1} / U_{y1}\}}] = 1. \quad (51)$$

In the same way, solving (44) and (46), we obtain

$$[L_y \frac{(1-\alpha)}{\{U_{yy} / U_{yy}\}}] = 1. \quad (52)$$

Equations (51) and (52) are the modified Samuelsonian conditions which have the same interpretation as equations (17) and (18). In addition, we have the following basic optimality condition for nationally financed local public good.

$$\frac{1}{U_y} \frac{1}{U_{y1}} = \frac{1}{U_y} \frac{1}{U_{y1}} = 1 \quad (53)$$

Next, substituting (45) into (47), (46) into (48) and after some manipulations, we finally obtain

$$(R_1 + R_2) + (G_1 + G_2) = G_1 + G_2 + Y, \quad (54)$$

where R_1 and R_2 are the same as defined earlier.

Equation (54) tells that the level of population that maximises per capita utility for the federation as a whole is such that aggregate land rent plus the congestion tax equal the total cost of not only the local public goods but also nationally financed local public good. Thus, equation (54) is another version of generalised HGT.

Moreover, equation (54) tells that the HGT is invariant as to the location of pure public good because it is perfectly equivalent to equation (38). This means that regardless of where the benefits of pure public good are concentrated, the final equality between aggregate land rent and total cost of public good from optimum population condition holds.

5. Implications of modified and generalised HGT for inter-regional equity :

We briefly discuss a few implications of the HGT for inter-regional equity in a federal set up. These implications hold common both for the modified and the generalised HGT we derived above.

First of all, the equality between aggregate land rent and total cost of public good implies that a single tax on land covers total expenditure on public good/s in a federation.⁶ This implication is in conformity with a tradition in federalism, viz., resource rich regions share their rent with resource poor regions with the ultimate purpose of equalising welfare levels between regions. More particularly, rent sharing through a single tax on land, the federal government may collect more revenue than what is needed to financing public good in region 1 (2) and may transfer the excess to region 2 (1). In short, it is precisely here the implication of HGT for inter-regional equity lies.

However, with the introduction of congestion taxes, the above implication does not differ in any significant way. In other words, the basic procedure of rent sharing holds in the presence of congestion-tax-adjusted-cost of public good in the federation.

FOOTNOTES

1 Throughout this note we deal with only the HGT in spatial models with local public goods. For details of other types of HGT, see the footnote 7 in Kanemoto (1984).

2 This representation of impure public good is adopted from Borcharding and Deacon (1972).

3 We are treating congestion effects from the benefit side of the public goods. In fact, congestion effect can be incorporated into the cost side also. See, for instance, Pestieau (1983). However, both the approaches yield qualitatively similar results.

4 With additional assumptions on the institutional set up of the federal economy in question, equation (2) can be derived as a case of decentralised free-market equilibrium [essentially of Tiebout (1956) type]. This type of market equilibrium is descriptively formulated by Buchanan and Goetz (1972). For a mathematical translation of these descriptive models, see Narayana (1985).

5 Here and in the rest of the essay, we assume that the second order conditions are fulfilled.

6 This theoretical result is not without critics [Johnson (1973), Wildasin (1986)]. For instance, Johnson (1973) argues most eloquently in the following way. The fiscal notion that taxes should be levied on rents rather than the income from effort is a mixture of theoretical wisdom and practical ignorance. Theoretically, an optimal tax system should fall on rents, and not distort marginal choices in the allocation of resources. The practical problem is to devise taxes that accomplish this, since the income on which taxes are usually levied almost invariably involves an element of marginal choice; also, even if one could specify a source of pure rent for taxation purposes, there would be no guarantee that source would suffice to finance government's needs for revenue. [page 74]

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