BIOLOGY AND GEOMETRY OF THE CHAMBERED NAUTILUS

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The pearly nautilus attracts artists and biologists alike

ONE of the most acclaimed natural beauties is the shell of the chambered nautilus (*Nautillia* sp.). Painted, drawn, and photographed innumerable times, saluted in poetry, its graceful shape appeals to all viewers, even the artistically ungifted. There is an international science journal named *The Nautilus* founded in 1889 by Pilsbry, and the Delaware Museum of Natural History at Greenville (U.S.A.) is publishing a quarterly, *Chambered Nautilus Newsletter* which is now in its third year of publication. The remarkable nautilus lent its name to the deep-diving submarine in Jules Verne's adventure, *Twenty Thousand Leagues Under the Sea*. The air-tight chambers of the shell act as ballast tanks. Nautilus is also the name of one of the rafts used by the Indo-United States Ganga Expedition to navigate an unchartered gorge in the Ganges in 1976.

The great family of nautilus

Cephalopoda, to which the nautilus belongs, is a group of highly organised invertebrate animals of exclusively marine distribution constituting a class of the phylum Mollusca. They appeared late in the Cambrian, and with shells four metres long, they were the largest shelled invertebrates that ever lived. Some 3,500 different nautiloid species once flourished in the shallow seas that covered the prehistoric earth. The fossil of a long extinct species whose shell measured 2.7 metres was discovered in an Arkansas river-bed. About 130 genera and 650 species of living cephalopods are known of which octopus, squid, cuttle-fish and nautilus are the most familiar representatives. Cephalopods are characterised by a head with eight or more tentacles and highly developed eyes. The sexes are separate.

Now there exist only half a dozen species of nautilus; the main region of their survival is the Pacific Ocean from the Philippines to Fiji. They live in the sea as deep as 600 metres but ascend closer to the surface during nights. Although evolution has whittled these descendants to not more than 25 cm—only a small fraction of the size of their giant ancestors—their form has remained remarkably the same for ages.

Distribution

Nautilus is a popular name applied to two distinct genera of cephalopod molluscs—the pearly nautilus, also called the chambered nautilus, because of the many chambers in its shell, and the paper nautilus, *Argonauta*, a cosmopolitan genus of open ocean octopod.

Although the pearly nautilus lives mostly between the Philippines and

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Fig. 1. Shells of baby nautilus. Note the prominent hole at the centre of the shell from where the coiling begins

Fiji Islands, dead shells are found on many beaches of the Indo-Pacific region. Traders in shell and shell products at Port Blair (Andaman Is.) flourish on the sale of nautilus shells which their agents collect locally. During our two expeditions to the uninhabited South Sentinel Island in 1973 and 1974, we collected a large number of nautilus shells, their sizes ranging from 2.5 cm to 22.5 cm (Figs. 1 and 2). The animal produces a series of ever-larger chambers, at an estimated rate of a new chamber every 2 or 3 weeks. The first four chambers of the shell are formed while the young nautilus is still within the egg. The mollusc fashions as many as 38 chambers, increasing in size with a mathematical consistency (*see cover*). The baby nautilus begins to coil its shell in such a manner that a hole, wide enough to insert a needle, is left at the centre (Fig. 1). But this hole gets covered later when more chambers are built with the secretion of pearly material (which the animal daubs on the shell in such a way that the central point of the shell is no longer visible). All the chambers are connected by a tube called siphuncle (Fig. 3) through which gases are released into or absorbed from the chambers as necessary. The shell thus acts as a float or hydrostatic organ that aids in ascending from sea bed to upper layers of water or descending to lower layers. When the nautilus swims, the largest (also the newest) chamber is the lowermost. The main food hunting region of this mollusc is the muddy ocean floor which abounds in small crabs, shrimps and shellfish, its main diet.

Unlike the octopus and the squid, the chambered nautilus has about 90 tentacles, most of them are small. They may be separated into three groups. Each eye has two tentacles, one in front and the other at the back. There is an outer ring of nineteen pairs of tentacles and an inner circle of many short tentacles around the mouth and beak. The first and third of these tentacles are said to be capable of being withdrawn into their sheaths. The tentacles of the nautilus are ridged to give them good grip on anything they hold, and they look different from the tentacles of octopuses and squids as they do not have any sucker. Some of the tentacles are extended while searching for food. As soon as they grasp a fish or a crab, they pass it to the ten tantacles near the mouth. It is now known that these appendages do not help the animal to crawl as was believed earlier. Two trailing tentacles support the nautilus while it tramples over the uneven surface of the ocean floor (Fig. 4). Locomotion is effected by jetting out of water through the funnel located below the layer of the tentacles (Fig. 3). When the animal has to move in a reverse direction, it aims its funnel forward and spurts off water. There is a prominent leathery hood with which the mouth of the shell is closed after drawing in the tentacles. The animal rests usually covering itself under the hood.

Use of nautilus shell

As mentioned earlier, the many-chambered spiral shell acts as ballast tanks and helps the nautilus ascend

Fig. 2. Empty shells of two adult chambered nautilus. Flame markings on the shell are conspicuous on the older portion

or descend in the ocean. It offers, together with the hood, a good shelter for the resting animal. But perhaps the most important function the shell has is to protect the boneless, succulent animal from its predators. Judging from the injuries seen on many shells (one had upto 13 injury marks), it is clear that predators try to reach the prey by breaking the rim of the shell. But as the shell offers tough resistance to the smaller predators, the nautilus escapes from such attacks with minor injuries. The animal has even the capacity to repair and reshape the broken shell (Figs. 5 and 6). The animal daubs the shell in narrow strips with a special secretion of its body. These strips remain as permanent lines. When a bit of the broken shell is patched up with new material, the construction lines of the new surface do not match with the markings on the original shell as revealed in Figs. 5 and 6.

The shell, with its reddish brown flame markings on a white background, is a thing of beauty and consequently an object of commercial importance. When polished, it reveals its rare mother-of-pearl lustre. The shells are fashioned into lamps and other decorative items. They are also worked into cups, saucers, spoons, ear-rings, studs and bangles. In Polynesia and the Philippines, the shell is cut into fish lures.

Pioneer investigators

Although the shell of the nautilus has been known from the 16th century, the animal was not studied until 1831. The first animal ever studied was obtained from the New Hebrides in the South Pacific by George Bennett who gave it to his fellow student, Richard Owen, at the Royal College of Surgeons in London. Owen made a detailed anatomical study of the specimen and wrote a book on the results of his dissection. This brought him fame as the greatest anatomist of his time.

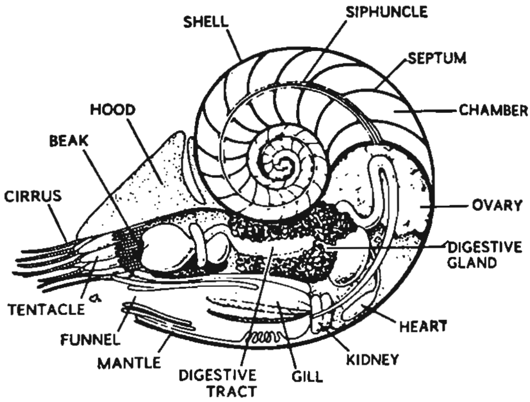

Fig. 3. A section through a live nautilus showing the important organs (*Courtesy*: *National Geographic Magazine*, Washington)

He died in 1892 as Sir Richard Owen. Among other early workers on nautilus, Anna Bidder of Cambridge University who went to New Hebrides to study the animal, E.J. Denton of the Plymouth Laboratory, J. B. Gilpui-Brown of the University of Auckland, are the more important ones. H. K. Dugdale publishes regularly the recent work on nautilus in his *Chambered Nautilus Newsletter*.

Fig. 4. Lone nautilus 'walking' with the help of two tentacles (*Courtesy* : *National Geographic Magazine*, Washington, Black and white reproduction of a colour plate with permission)

The paper nautilus

The paper nautilus (*Argonauta* sp.) is found in all tropical and subtropical seas, living near the surface. However, specimens of this nautilus have also been captured below 900 metres. It belongs to the eight-armed order octopoda, but differs from the octopods in having a thin unchambered, coiled shell, unlike either the internal shell of the squid or cuttlefish, or that of the chambered nautilus. The *Argonauta* shell is formed by large flaps or membranes found on the dorsal arms of the females (the males do not have shells). It is in this shell, cradled by the flaps, that the eggs are laid and the young hatch out. Large shells, which attain a diameter of 30 to 40 cm, are very fragile and highly prized by collectors. They are often found on the Florida coast in U.S.A. In contrast to the other octopods, the male of *Argonauta* is only about 1/20th the size of the female and possesses no shell. The male was once thought to be parasitic in the shell of the female. For many years the argonauts were pictured as sailing about the surface

Figs. 5, 6. Portions of shells of nautilus showing patched up injuries

of their seas with the arm flaps extended as sails, until the true function of het flaps was discovered. In all other essentials, the female resembles the octopus.

Incidentally, the word Argonaut is derived from the Greek *Argonautes* which means the sailor in the 'Argo'. According to a famous story in Greek mythology, it refers to legendary heroes who sailed with Jason in the Argo in search of the golden fleece.

The geometry of the nautilus shell

It is known that a rectangle of certain proportions has an appeal to a wider population than a rectangle of any other shape. Such a more appealing rectangle is known as the golden rectangle, e.g., *AFGD*, in Fig. 7. It is constructed in the following way. The side *AB* of a square *ABCD* is bisected at *E*. With center *E* and radius *EC*, an arc of a circle is drawn cutting *AB* produced in *F*. *FG* is drawn perpendicular to *AF* meeting *DC* produced in *G*. The resultant figure *AFGD* is said to be a golden rectangle with its charming and aesthetically pleasing appeal. The proof is also very simple. Let $AB=2$ units of length. Then $EC=EF=\sqrt{5}$ units. *AF* is divided by *B* in the golden section. *B* is also known as the 'golden cut'. It is associated with the idea of the 'mean proportional'. *AB* is the mean proportional of *AF* and *BF*. $AB/BF=AF/AB$, i.e. $AB^2 = AF.BF$. From the golden rectangle, almost an unlimited number of squares, progressively decreasing in area, can be obtained as per procedure shown in Fig. 8. For the rectangle *ABCD*, $AB : BC = \phi = 1$. Through *E*, the golden cut of *AB*, *EF* is drawn perpendicular to *AB* cutting off from the rectangle the square *AEFD*. Then the remaining rectangle *EBCF* is a golden rectangle. If from this, the square *EBGH* is lopped off, the remaining figure *HGCF* is also a golden rectangle. This process can be repeated indefinitely until the limiting rectangle *O*, indistinguishable from a point, is reached. Such a figure offers the following interesting features :

1. The limiting point *O* may be called the *pole* of a unique spiral known as the equiangular spiral (logarithmic spiral) which passes through the golden cuts *D*, *E*, *G*, *J*,...The general equation of this spiral is $r=ae \cos a$. The sides of the rectangle are nearly, but not wholly, tangential to the curve. This shows the connection between the logarithmic spiral and the golden section.

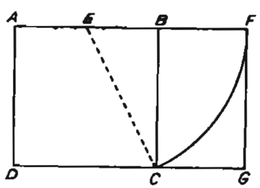

Fig. 7. Construction of a golden rectangle

2. Alternate golden cuts on the *rectangular* spiral *ABCFH*...lie on the diagonals *AC* and *BF*. This suggests a convenient method of constructing the figure.

3. The diagonals *AC* and *BF* are perpendicular to each other.

4. The points *E*, *O*, *J* are colinear, as also are the points *G*, *O*, *D*.

5. The four right-angles at *O* are bisected by *EJ* and *DG* so that these lines are mutually perpendicular.

6. $AO/OB=OB/OC=OC/OF= \ldots$ There is an infinite number of similar triangles, each being one half of a golden rectangle.

The relationship of the spiral to the Fibonacci series is evident from Fig. 8, for, the spiral is seen to pass through diagonally opposite corners of successive squares such as *DE*, *EG*, *GJ*,....... The lengths of the sides of these squares form a Fibonacci series. If the smallest square shown in Fig. 8 has a side of length d, the adjacent square also has sides of length d, the next square has sides of length 2d, the next 3d and so on, giving the series 1d, 1d, 2d, 3d, 5d, 8d,.... Therefore, it is very easy to draw the golden rectangle whose sides will have the measurements of two consecutive Fibonacci numbers. Say, in a rectangle if the length is 89 units and width 55 units, the rectangle is said to be a golden one. We can obtain from this figure, a continuous number of squares commencing with one having the side 55 units length, and others with units of lengths 34, 21, 13, 8,... etc.

Another interesting property of this charming spiral becomes obvious. However different two segments of the curve may be in size, they are not different in shape. Suppose a photograph were taken with the aid of a microscope of the convolutions near the pole *O*, too small to be visible by the unaided eye. If such a copy were suitably enlarged, it could be made to fit exactly on a spiral of the size of Fig. 8. The spiral is without

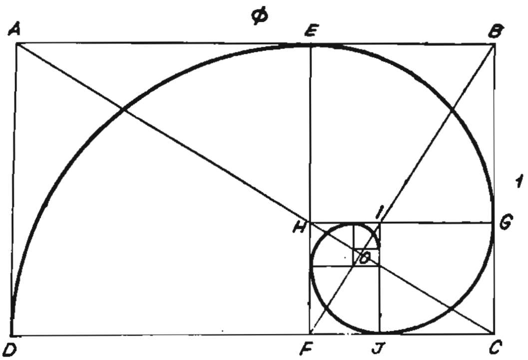

Fig. 8. Producing an equiangular spiral

a terminal point. That is, it may grow outwards (or inwards) indefinitely, but its shape always remains unchanged.

This is one of the examples of the Fibonacci series in nature. The successive chambers of the nautilus, as we have already seen, are built on a framework of a logarithmic or equiangular spiral. As the shell grows, the size of the chambers increases but their shape remains unchanged.

Equiangular spiral

A spiral may be defined as a curve which, starting from a point of origin, continually diminishes in curvature as it recedes from that point. In other words, its *radius of curvature* continuously increases. After a stage, the diminishing curvature tends to turn up in a straight line. Such a curve, of course, excludes the simple screw or the cylindrical helix, which neither starts from a definite origin nor changes its curvature as it proceeds. But true spirals are noticed in the horns of mammals, shells of molluscs, in the florets of a sunflower head, in the outline of a cordiform leaf, in the coil of an elephant's tusk and the like. Two important spirals of relevance for the present are the equable spiral (spiral of Archimedes), and the equiangular spiral (logarithmic spiral). Spiral of Archimedes appears when a rope of uniform thickness is coiled tightly on a horizontal surface. In this spiral, each whorl is of the same breadth as that preceding it and that which follows it.

In contrast to the spiral of Archimedes, in the equiangular spiral of the nautilus the whorls continuously increase in breadth, and do so in a steady and unchanging ratio. D'Arcy Thompson, the author of the classic, *On Growth and Form*, defines such a spiral as follows: 'If instead of travelling with a uniform velocity, our point moves along the radius vector with a velocity increasing as its distance from the pole, then the path described is called an equiangular spiral'. Each whorl which the radius vector intersects will be broader than its predecessor in a definite ratio. The radius vector will increase in length in geometric progression as it sweeps through successive equal angles, and the equation to the spiral will be r=ae.

The equiangular spiral was first recognised and designated so by the French philosopher Descartes in 1638 because the angles at which a radius vector cuts the curve at any point is constant. Mathematician Halley, noting that the lengths of the segments cut off from a fixed radius by successive turns of the curve were in continued proportion, named it the proportional spiral. Jakob Bernoulli (1654-1705) called it the logarithmic spiral.

The fundamental mathematical property of the equiangular spiral corresponds precisely to the biological principles that govern the growth of the mollusc's shell. This is the simplest principle possible. That is, the size increases but the shape remains unaltered. The mollusc's shell grows longer and wider to accommodate the growing animal, but the shell remains always similar to itself. It grows at one end only, each inc-ement of length being balanced by a proportional increase of radius so that its form is unchanged. The shell grows by accretion of material which it accumulates rather than due to biological growth. The only mathematical curve to follow this pattern of growth is the logarithmic spiral.

Logarithmic spiral and golden triangle

A golden triangle may be easily reached after a brief look at what is called a *gnomon*. A gnomon is a portion of a figure which has been added to another figure so that the whole is

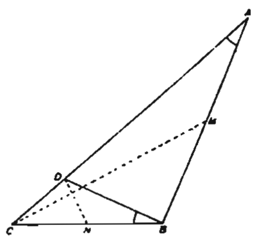

Fig. 9. Gnomons

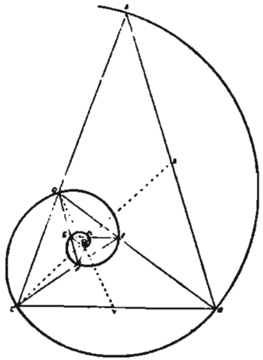

Fig. 10. Golden triangles and logarithmic spiral

of the same shape as the small figure (Fig. 9). Mathematicians have shown that in any triangle ABC, triangle ABD is a gnomon to triangle BCD, if angle CBD = angle A. If we add to or subtract from this triangle a series of gnomons, it turns out that all the apices lie upon an equiangular spiral. Radial growth (dr) and intrinsic growth in the direction of the curve (ds) bear a constant ratio to each other : $dr/ds = \cos a$ = constant. The equiangular spiral is the only curve to possess this property.

An application of the gnomon principle which interested the contemporaries of Pythagoras concerns the isosceles triangle ABC (Fig. 11) which has base angles 72° and apex angle 36°. Here $AB : BC = \phi/1$. Hence the triangles of this figure may be termed 'golden triangles'. The bisector of angle B meets AC in D, so that D is the golden cut of AC. By this the triangle ABC has been divided into two isosceles triangles both of which could be called 'golden', their apex angles being 36° and 108°, and the ratio of their areas $\phi = 1$. Bisecting angle C, we obtain E, the golden cut of BD, and two more golden triangles. This process, producing a series of gnomons, converges to a limiting point O, which is the pole of a logarithmic spiral passing successively and in the same order through the three vertices of each of the series of triangles, $A, B, C, D, E, \ldots\ldots$

Aesthetic appeal

H.E. Huntley, author of the fascinating book, *The Divine Proportion*, quotes the opinion of marine biologists, artists, poets, psychologists and above all, mathematicians, on how they regard the beauty of the smooth curvature of the shell of the chambered nautilus. Poet Oliver Wendell Holmes wrote the following poem, entitled *The Chambered Nautilus*:

This is the ship of pearl, which, poets feign,
Sails the unshadowed main—
The venturous bark that flings
On the sweet summer wind its purpled wings
In gulfs enchanted, where the siren sings,
And coral reefs lie bare,
Where the cold sea-maids rise to sun their streaming hair.

The poet goes on to draw a moral in his last stanza on the life-history of the mollusc.

Build thee more stately mansions, O my soul,
As the swift seasons roll!
Leave thy low-vaulted past!
Let each new temple, nobler than the last,
Shut thee from heaven with a dome more vast
Till thou at last are free,
Leaving thine outgrown shell by life's unresting sea.

The pearly nautilus attracts the artist both by the tints of its lustrous exterior and by the perfection of its spiral curve. Aesthetic appreciation of any sort has a dual aspect. Beauty evokes an immediate sensuous pleasure which is a common human experience. The sensuous satisfaction which is produced by simple lines has been studied by psychologists. Results of some of Lundholm's experiments summarised by H. E. Huntley are reproduced below :

Expressiveness of lines. When asked to draw a beautiful line, Lundholm's (1921) subjects tried to make one that was smooth, curved, symmetrical, continuous with rhythm or repetition and expressive of a single idea. For an ugly line they drew an unorganised mass without continuity, with mixed angles and curves and unrelated spaces ..., and when they wished to express merriment, playfulness, agitation or fury, they drew sharp waves of zigzags.

The subjects said : 'Small waves make the movement of a line go more quickly. The calm line has long slow curves.'

The long slow curve of the equiangular spiral, according to the above, must be evocative of calm feelings which may be regarded as a part fo the mathematicians' aesthetic experience.

Mathematician Jakob Bernoulli was so fascinated by the beauty of the logarithmic curve that he asked that it might be engraved on his tombstone!