Journal of Statistical Planning and Inference 16 (1987) 203-211
North-Holland

SOME INFERENTIAL ASPECTS OF FINITE POPULATION SAMPLING WITH ADDITIONAL RESOURCES

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Received 4 December 1985; revised manuscript received 12 June 1986
Recommended by A. Cohen

Abstract: The problem of extending a given sampling design, when additional resources are available, is considered. Some existing methods of improving an *initial* sampling strategy, so that the use of the additional resources is justified, are critically reviewed. Admissibility of the existing strategies is questioned. In the process, improved strategies are suggested in various cases.

AMS Subject Classification: Primary 62D05; Secondary 62C10.

Key words and phrases: Finite population sampling designs; Extension rules; Admissible strategies.

1. Introduction

Suppose a survey statistician is interested in estimating a parametric function $\theta(Y)$ of a character Y for a finite labelled population $U=(1,2,\dots,N)$ of size N. Given the initial resources, the statistician has decided to adopt the strategy (P_n, e_n) where P_n refers to a fixed sample design of size n and e_n refers to an estimator for $\theta(Y)$. Let s_n denote a typical sample of size n from U and S_n denote the sample space consisting of samples of the type s_n for which $P_n(s_n)$ is positive. Additional resources are subsequently made available to the statistician which may be used to obtain another sample of k units. We assume throughout $n+k<N$. In this paper we discuss the choices for the sampling design for the second sample. We also present *reasonable* sampling strategies (P_{n+k}, e_{n+k}) based on the combined sample.

We use the following notations and definitions in this paper.

(i) An estimator e_n is said to be $(P_n$-$)$ unbiased for $\theta(Y)$ if

$$\sum_{s_n \in S_n} e_n(s_n) P_n(s_n) = \theta(Y)$$

for all $Y = (Y_1, \ldots, Y_n)$.

(ii) A strategy (P, e) is said to be unbiased for $\theta(Y)$ if the estimator e is $(P\cdot)$ unbiased for $\theta(Y)$ in the above sense.

(iii) An unbiased sampling strategy (P, e) is said to be at least as good as another unbiased sampling strategy (P^*, e^*) if

$$\sum_{s \in S} e^2(s) P(s) \le \sum_{s \in S^*} e^{*2}(s) P^*(s) \tag{1.1}$$

for all Y, where S and S^* are the sample spaces corresponding to the sampling designs P and P^* respectively. We say (P, e) is better than (P^*, e^*) if strict inequality holds in (1.1) for at least one Y. The sampling strategy (P, e) is said to be admissible if there is no other sampling strategy that is better than (P, e). It is said to be inadmissible otherwise. In this paper we consider unbiased strategies only.

Let P_k denote a fixed size sampling design of size k on U. Suppose the statistician uses the sampling design P_n to obtain s_n and when additional resources are available, uses P_k to obtain an independent sample s_k. Then the combined sample, $s_n \cup s_k$, may be of size varying from n to $n+k$. We denote the sampling design obtained by taking an independent sample in the second stage by $P_n \cup P_k$. Given s_n is selected in the first stage using the sampling design P_n, an alternative procedure is to select a sample of size k from $U - s_n$. Let $\{Q_k(\cdot \mid s_n) \mid s_n \in S_n\}$ denote a family of such fixed size sampling designs of size k. If the second sample s_k is selected from $U - s_n$ using $Q_k(\cdot \mid s_n)$, following the selection of s_n as the first stage sample, then the combined sample s_m is of fixed size $m = n + k$. Let P_m denote the underlying fixed size sampling design of size $m = n + k$. Note that

$$P_m(s_m) = \sum_{s_n \subset s_m} P_n(s_n) Q(s_m - s_n \mid s_n) \tag{1.2}$$

for all s_m. Let S_m denote the sample space corresponding to the sampling design P_m. In Section 2 we will show that for a given sampling strategy $(P_n \cup P_k, e)$, we can find a fixed size sampling strategy (P_m^*, e^*) that is better.

If the sampling designs P_n and Q_k correspond to simple random sampling without replacement (SRSWOR) procedures, then P_m is also an SRSWOR design. It is well known that the sampling strategy $(P_m, \bar{y}_m)$ is better than $(P_n, \bar{y}_n)$ where $\bar{y}$ is the sample mean and P_n and P_m are SRSWOR sampling designs of sizes n and m respectively. Therefore, in this situation, the use of additional resources for selecting k more units is justified. This characteristic of improving the efficiency by using additional sampling units, however, is *not* shared by *all* sampling strategies. Cochran (1963), Prahbu-Ajgaonkar (1967), Chaudhuri (1977) and Chaudhuri and Mukhopadhayay (1978) considered the properties of the sample mean and/or the Horvitz-Thompson estimator (HTE) under different sampling designs P_n and P_m. It was observed that the sampling strategies (P_m, HTE) and $(P_m, \bar{y}_m)$ are not necessarily better than (P_n, HTE) and (P_n, $\bar{y}_n$), respectively. For the case where the sampling design Q_k is SRSWOR, Sinha (1980) presented simple conditions on the first and second order inclusion probabilities of the sampling design P_n so that (P_{n+k}, HTE) is better than (P_{n+k-1}, HTE) simultaneously for all $k=1, 2, \ldots$.

Lanke (1975) considered extending an arbitrary sampling strategy (P_n, e_n) to another strategy (P_m, e_m) via Q_k so that (P_m, e_m) is better than (P_n, e_n) irrespective of the choice of Q_k. He proposed the estimator

$$e_m(s_m) = [P_m(s_m)]^{-1} \sum_{s_n \subset s_m} e_n(s_n) P_n(s_n) Q_k(s_m - s_n \mid s_n) \tag{1.3}$$

We will refer to the estimator (1.3) as Lanke's estimator. Notice that Lanke's estimator is in some sense a Rao–Blackwellization of the estimator e_n. Lanke (1975) established that the estimator e_m in (1.3) is at least as good as e_n no matter what P_n, e_n and Q_k are.

Sengupta (1982) extensively studied the properties of Lanke's estimator for various choices of e_n, P_n and Q_k. In particular, he observed that (i) Lanke's estimator, even though it improves over the estimator e_n, may itself turn out to be inadmissible, and (ii) if the estimator e_n is the sample mean (or HTE) then there may not exist a sampling design Q_k such that Lanke's estimator based on e_n is again the sample mean (or HTE). He also showed that when e_n is the sample mean and the sampling design Q_k is SRSWOR, Lanke's estimator will again be the sample mean if and only if the sampling design P_n is itself SRSWOR.

In Section 2, we critically review Lanke's estimator and point out some of its demerits in the present form. We then consider different versions of this estimator to explore the scope for further improvement. Section 3 contains some concluding remarks.

2. Main results

In this section, we first show that it is better to use an appropriate fixed size sampling design of total size $m=n+k$ than to use two *independent* sampling designs P_n and P_k of sizes n and k respectively.

Theorem 2.1. *Let $P_n \cup P_k$ denote the sampling design obtained by taking two independent samples of sizes n and k using the sampling designs P_n and P_k respectively. Suppose e is an unbiased estimator for $\theta(Y)$ based on the composite sampling design $P_n \cup P_k$. Then, there exists a fixed size sampling design P^* of size $m=n+k$ and an estimator e^* such that the sampling strategy (P^*_m, e^*) is better than ($P_n \cup P_k, e$).*

Proof. We assume $n+k<N$ as otherwise the claim is trivially justified. We prove the theorem by constructing the sampling strategy (P^*_m, e^*). Define

$$P_{m}^{*}(J) = P_n \cup P_k(J) + \sum_{l=n}^{m-1} \sum_{I \subset J} [a(I)]^{-1} P_n \cup P_k(I) \tag{2.1}$$

where

$$I = (i_1, i_2, \ldots, i_l), \qquad J = (j_1, j_2, \ldots, j_m)$$

and

$$a(I) = \text{number of } J \text{ with } P_n \cup P_k(J) \text{ positive and } I \subset J.$$

It is easy to see that P_m^* is a fixed size sampling design of size $m = n + k$. Define, now, the estimator

$$e^*(J) = [P_m^*(J)]^{-1} \left[e(J) P_n \cup P_k(J) + \sum_{l=n}^{m-1} \sum_{I \subset J} \{a(I)\}^{-1} P_n \cup P_k(I) e(I) \right].$$

Then it is easy to verify that e^* is unbiased for $\theta(Y)$. Also, using the Cauchy-Schwartz inequality, one can show that (P_m^*, e^*) is better than $(P_n \cup P_k, e)$. □

It is thus advisable to obtain additional sampling units with additional resources. Note further that given the sampling designs P_n and P_k, the sampling design P_m^* has the same form as (1.2) for some conditional sampling design Q_k.

For the remainder of this paper, we will be dealing with sampling designs of the type P_m given by (1.2) with components given by P_n and $\{Q_k(\cdot \mid s_n) s_n \in S_n\}$. We next show that it is not possible to obtain an estimator for $\theta(Y) = \bar{Y}$ based on the sampling design P_m that is better than *every* possible estimator based on the sampling design P_n.

Theorem 2.2. *Let P_n be a connected sampling design (see Patel and Dharmadhikari (1977)). Let P_m be any fixed size sampling design of size $m = n + k$ $(< N)$ obtained by extending P_n as in (1.2) via an arbitrary Q_k. Then there does not exist any estimator e based on P_m that is (uniformly) better than every estimator based on P_n.*

Proof. Let e_q denote a homogeneous linear unbiased estimator of $\bar{Y}$ such that the variance of the estimator e_q is zero at the point q, where q is an $N \times 1$ vector. (See Patel and Dharmadhikari (1977) to ensure the existence of such estimators.) Suppose there exists an unbiased estimator e based on P_m that is better than every estimator based on P_n. Then the estimator e, in particular, will be better than the collection of estimators $\{e_q : q \text{ an } N \times 1 \text{ vector}\}$. Therefore, the variance of the estimator e must be identially zero. However, since $m < N$ this is not possible. □

Even though there does not exist an estimator based on P_m that is better than every estimator based on P_n, for given sampling designs P_n and P_m and an estimator e_n based on P_n, there always exists an estimator e_m based on P_m that is better than e_n. For example, Lanke's estimator serves this purpose. So if we wish

Table 1

s_4	$P_4(s_4)$	f_4	e_4
{1, 2, 3, 4}	0.1	a_1	$(a_1+a_2)/2$
{2, 3, 4, 5}	0.2	a_2	$(3a_2+a_3)/4$
{3, 4, 5, 6}	0.3	a_3	$(5a_3+a_4)/6$
{4, 5, 6, 1}	0.2	a_4	$(3a_4+a_5)/4$
{5, 6, 1, 2}	0.1	a_5	$(a_5+a_6)/2$
{6, 1, 2, 3}	0.1	a_6	$(a_6+a_1)/2$

to obtain a sampling strategy that is better than (P_n, e_n) we may use (P_m, e_m) where e_m is the Lanke's estimator.

Now suppose e_n and f_n are two estimators for $\theta(Y)$ based on P_n. Let e_m and f_m denote the corresponding Lanke versions of e_n and f_n respectively. The following example demonstrates that even in situations where the estimator e_n is uniformly better than the estimator f_n, it is *not* generally true that the estimator e_m is better than the estimator f_m.

Example 2.1. Consider the sampling designs and the estimators e_4 and f_4 of $\bar{Y}$ given in Table 1, where

$$a_1=(280Y_1+210Y_2+140Y_3)/504, \qquad a_2=(210Y_2+140Y_3+120Y_4)/504,$$

$$a_3=(140Y_3+120Y_4+140Y_5)/504, \qquad a_4=(120Y_4+140Y_5+210Y_6)/504,$$

$$a_5=(140Y_5+210Y_6+280Y_1)/504, \qquad a_6=(210Y_6+280Y_1+210Y_2)/504.$$

It is easy to show that the estimate e_4 is better than the estimator f_4.

Table 2

s_4	Extension with $k=1$	Q_1	s_5	$P_5(s_5)$	f_5	e_5
{1, 2, 3, 4}	5	0.5	(1, 2, 3, 4, 5)	0.15	$\frac{(a_1+2a_2)}{3}$	$\frac{(a_1+4a_2+a_3)}{6}$
	6	0.5				
{2, 3, 4, 5}	6	0.5	(2, 3, 4, 5, 6)	0.25	$\frac{(2a_2+3a_3)}{5}$	$\frac{(3a_2+6a_3+a_4)}{10}$
	1	0.5				
{3, 4, 5, 6}	1	0.5	(3, 4, 5, 6, 1)	0.25	$\frac{(3a_3+2a_4)}{5}$	$\frac{(5a_3+4a_4+a_5)}{10}$
	2	0.5				
{4, 5, 6, 1}	2	0.5	(4, 5, 6, 1, 2)	0.15	$\frac{(2a_4+a_5)}{3}$	$\frac{(3a_4+2a_5+a_6)}{6}$
	3	0.5				
{5, 6, 1, 2}	3	0.5	(5, 6, 1, 2, 3)	0.10	$\frac{(a_5+a_6)}{2}$	$\frac{(a_5+2a_6+a_1)}{4}$
	4	0.5				
{6, 1, 2, 3}	4	0.5	(6, 1, 2, 3, 4)	0.10	$\frac{(a_6+a_1)}{2}$	$\frac{(a_6+2a_1+a_2)}{4}$
	5	0.5				

Consider now the extension of the sampling design and the corresponding Lanke's estimators as given in Table 2.

It can be shown that the variance of f_5 is smaller than that of e_5 at the point $g=(0, 0, 0, 0, 1, 0)$ and hence e_5 is not better than f_5.

In the next example we demonstrate that Lanke's extension of an admissible estimator may turn out to be inadmissible. This was first demonstrated by Sengupta (1982). We, however, present a different example here.

Example 2.2. Consider the sampling designs for a population of size six given in Table 3.

Let e_3 denote the HTE for $\theta(Y)$ based on P_3. Consider the estimator

$$e(1, 2, 3, 4, 5)=ae_3(1, 2, 3)+be_3(1, 3, 5)+ce_3(2, 3, 4),$$

$$e(1, 2, 3, 4, 6)=de_3(2, 3, 4)+ee_3(1, 2, 3)+fe_3(2, 4, 6),$$

$$e(2, 3, 4, 5, 6)=ge_3(2, 4, 6)+he_3(2, 3, 4),$$

$$e(1, 3, 4, 5, 6)=e_3(1, 3, 5),$$

$$e(1, 2, 3, 5, 6)=ie_3(1, 2, 5)+je_3(1, 2, 3)+ke_3(1, 3, 5).$$

Note that Lanke's extension of e_3 is obtained by setting $a=\frac{10}{31}$, $b=\frac{21}{31}$, $d=g=i=1$ and $c=e=f=h=j=k=0$. However, it is easy to show that the estimator e with $a=\frac{10}{31}$, $b=\frac{21}{31}$, $c=0$, $d=\frac{3}{4}$, $e=0$, $f=\frac{1}{4}$, $g=\frac{1}{4}$, $h=\frac{3}{4}$, $i=1$ and $j=k=0$ *is better than* Lanke's estimator. In fact, one can construct several other estimators that are better than Lanke's estimator.

Note that in the above example (as in any other example) Lanke's estimator assigns non-zero weight to only those subsamples s_n of s_m for which $Q_k(s_m-s_n|s_n)$ is positive. However, positive weights could be assigned to *all* subsamples s_n of s_m for which $P_n(s_n)$ is positive. We investigate this possibility below and present some improvements over Lanke's estimator. Consider

Table 3

s_3	$P_3(s_3)$	Extension with $k=2$	Q_2	s_5	$P_5(s_5)$
(1, 2, 3)	0.1	(4, 5)	1	(1, 2, 3, 4, 5)	0.31
(1, 2, 5)	0.2	(3, 6)	1	(1, 2, 3, 5, 6)	0.20
(2, 4, 6)	0.1	(3, 5)	1	(2, 3, 4, 5, 6)	0.10
(2, 3, 4)	0.3	(1, 6)	1	(1, 2, 3, 4, 6)	0.30
(1, 3, 5)	0.3	(4, 6)	0.3	(1, 3, 4, 5, 6)	0.09
		(2, 4)	0.7		

$$e_m(s_m) = \sum_{s_n \subset s_m} e_n(s_n) W(s_n, s_m) \tag{2.2}$$

where $\{W(s_n, s_m)\}$ are nonnegative constants. Note that if

$$\sum_{s_m \supset s_n} W(s_n, s_m) P_m(s_m) = P_n(s_n) \tag{2.3}$$

for all s_n, then the estimator e_m is unbiased for $\theta(Y)$ whenever e_n is so. Also, using the Cauchy-Schwartz inequality, it can be shown that the estimator e_m is as good as e_n if

$$\sum_{s_m \supset s_n} W(s_n, s_m) P_m(s_m) \left\{ \sum_{s_n' \subset s_m} W(s_n', s_m) \right\} \le p_n(s_n) \tag{2.4}$$

for all s_n. It is now easy to see that (2.4) holds, under the condition (2.3), if and only if,

$$\sum_{s_n \subset s_m} W(s_n, s_m) = 1 \tag{2.5}$$

for all s_m. We can also express the estimator e_m in (2.2) as

$$e_m(s_m) = [P_m(s_m)]^{-1} \sum_{s_n \subset s_m} e_n(s_n) P_n(s_n) Q_k^*(s_m - s_n \mid s_n) \tag{2.6}$$

where Q_k^* is given by

$$Q_k^*(s_m - s_n \mid s_n) = P_m(s_m)[P_n(s_n)]^{-1} W(s_n, s_m).$$

Then, from (2.3) and (2.5) we get

$$\sum_{s_m \supset s_n} Q_k^*(s_m - s_n \mid s_n) = 1 \quad \text{for all } s_n \tag{2.7}$$

and

$$\sum_{s_n \subset s_m} P_n(s_n) Q_k^*(s_m - s_n \mid) = P_m(s_m) \tag{2.8}$$

for all s_m.

In summary, the general extension e_m in (2.2) that improves over the estimator e_n has the same form as Lanke's estimator. The only difference is that the estimator (2.2) uses possibly a different sampling design Q_k^* generating the same final design P_m. In some situations it is possible to find a different Q_k^* so that the estimator (2.2) is better than Lanke's estimator computed with Q_k. The choice of Q_k^* would be such that the estimator (2.2) assigns positive weights to all $e_n(s_n)$ for which $s_n \subset s_m$ and $P_n(s_n) > 0$. We now present one such choice in the next theorem.

Theorem 2.3. *Suppose the sampling designs P_n, Q_k and P_m are as defined in Section 1. Define*

$$A(s_m)=\{s_n : s_n \subset s_m,\ Q_k(s_m-s_n\,|\,s_n)>0\}.$$

Suppose there exist two samples $s_m^{(1)}$ and $s_m^{(2)}$ such that

(i) *$B_m^{(1)} \cup B_m^{(2)}$ is not empty, and*

(ii) *if $s_n \in B_m^{(i)}$ then s_n is a subset of $s_m^{(3-i)}$, $i=1,2$, where*

$$B_m^{(1)}=A(s_m^{(1)})-A(s_m^{(2)}) \quad \text{and} \quad B_m^{(2)}=A(s_m^{(2)})-A(s_m^{(1)}).$$

Then Lanke's estimator $e_m(s_m)$ in (1.3) is inadmissible.

Proof. Define

$$e_m^*(s_m)=\begin{cases} e_m(s_m) & \text{for } s_m \neq s_m^{(1)} \text{ or } s_m^{(2)}, \\ [P_1 e_m(s_m^{(1)})+P_2 e_m(s_m^{(2)})]/(P_1+P_2) & \text{if } s_m=s_m^{(1)} \text{ or } s_m^{(2)}, \end{cases} \tag{2.9}$$

where $e_m(s_m)$ is Lanke's estimator in (1.3) and $P_i=P_m(s_m^{(i)})$, $i=1,2$. It is easy to verify that e_m^* is better than e_m and hence e_m is inadmissible. □

Note that Lanke's estimator $e_m(s_m^{(i)})$ puts zero weight to $e_n(s_n)$ for s_n in $B_m^{(3-i)}$, $i=1,2$, whereas the estimator in (2.9) assigns positive weights. Note also that the estimator in (2.9) is a Lanke-type estimator with a different choice of Q_k^*. In fact,

$$Q_k^*(s_m-s_n\,|\,s_n)=\begin{cases} Q_k(s_m-s_n\,|\,s_n) & \text{for } s_n \in A(s_m^{(1)}) \cap A(s_m^{(2)}) \\ & \quad \text{and } s_m=s_m^{(1)} \text{ or } s_m^{(2)}, \\ Q_k(s_m-s_n\,|\,s_n) & \text{for } s_m \neq s_m^{(1)} \text{ or } s_m^{(2)}, \\ Q_k(s_m^{(1)}-s_n\,|\,s_n)\dfrac{P_1}{P_1+P_2} & \text{for } s_m=s_m^{(1)} \text{ and } s_n \in B_m^{(1)}, \\ Q_k(s_m^{(1)}-s_n\,|\,s_n)\dfrac{P_2}{P_1+P_2} & \text{for } s_m=s_m^{(2)} \text{ and } s_n \in B_m^{(1)}, \\ Q_k(s_m^{(2)}-s_n\,|\,s_n)\dfrac{P_1}{P_1+P_2} & \text{for } s_m=s_m^{(1)} \text{ and } s_n \in B_m^{(2)}, \\ Q_k(s_m^{(2)}-s_n\,|\,s_n)\dfrac{P_2}{P_1+P_2} & \text{for } s_m=s_m^{(2)} \text{ and } s_n \in B_m^{(2)}. \end{cases}$$

Note that the definition of the estimator in (2.9) assumes that every $s_n \in A(s_m^{(1)}) \cup A(s_m^{(2)})$ is a subset of both $s_m^{(1)}$ and $s_m^{(2)}$ which is guaranteed by the condition (ii) of Theorem 2.3. Lanke's estimator seems to distinguish between the two samples $s_m^{(1)}$ and $s_m^{(2)}$ while using $e_n(s_n)$. On the other hand, the revised estimator $e_m^*(s_m)$ does, in fact, the 'averaging' or 'unordering' and, consequently, performs better than that of Lanke. The generalization of this result to other complicated 'structures' is *not* difficult and hence is *not* included here. However, the improved estimator is seen to be again a Lanke type estimator with a revised extension rule Q_k^* but with the same over-all sampling design P_m.

3. Concluding remarks

The following observations have been made in this paper.

(a) Lanke's formula yields a strategy (P_m, e_m) which is better than any *given* strategy (P_n, e_n) irrespective of the choice of the extension rule Q_k where $m = n + k$. Also, for any given strategy $(P_n \cup P_k, e)$ governed by a combination of two *independent* sampling designs P_n and P_k, there exists a strategy (P_m^*, e^*) which performs better.

(b) It is difficult to set out the estimator at the initial stage as the ordering is *not* generally preserved by Lanke type improved estimators.

(c) Lanke's formula may sometimes lead to inadmissible estimators due to faulty selection of the extension rule Q_k. The structure of the samples underlying P_n and P_m may be studied and suitable recommendations made in some cases.

The following problems need further investigation:

(i) Order-preserving improved estimators using suitable/given extension rules.

(ii) Admissible improved estimators using suitable given extension rules.

As mentioned earlier, Sinha (1980) and Sengupta (1982) have some interesting preliminary results on characterizations of original sampling strategies ensuring (ii) with the extension rules given by SRSWOR designs. Is it possible to construct improved estimators in general terms which are essentially different from those given by Lanke's formula?

Acknowledgements

We thank the referee for some comments which led to improvements in the presentation of our ideas.

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