OPTIMAL ROUTING BEFORE MAINTENANCE

By J. P. Saksena

The problem is as follows: "A vehicle traverses a distance d (i, j) whenever it goes from city i to city j. Before travelling for more than D units, it must receive maintenance of a kind available at only some cities. How should the vehicle be routed, from a given origin, so that the total profit before maintenance is maximised?" The method extends to the case of several types of maintenance available respectively at several different subsets of cities.

The Problem

Consider the network of points shown in Fig. I.

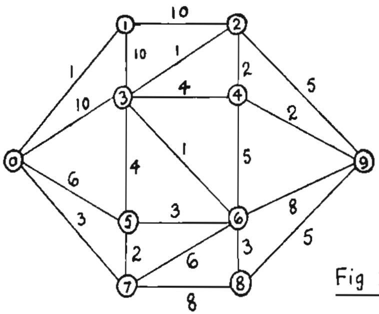

Fig 1

Let the points (cities) at which the maintenance is available be denoted by 2, 4, 6, 8. Our object is to trace the path from 0 to any of these cities so as to maximise the total profit given by the vehicle. The values of d (i, j) are given in Fig. I.

Case I (when $p(i, j) \propto d(i, j)$)

The first step of our method consists of finding the shortest distance from 0 to all these nodes (2, 4, 6, 8). This can be done, as given in ref. (1). The next step consists of finding the minimum positive difference of "D" and the values just obtained (D is already defined in Sec. I). The node which gives the least positive difference is the city (node), at which the maintenance will be optimal.

Example:

Let 'D', the distance after which maintenance is necessary, be denoted by, say, 10. Then we find d (o, j), j=2, 4, 6, 8. This is given in Table I.

j / i	2	4	6	8
0	10 (7,5,3)	12 (7, 5, 3, 2)	8 (7, 5)	11 (7 or 7, 5, 6)

TABLE I

(Numbers inside the brackets denote the nodes which lie enroute). So, if D=10, the maintenance should be carried out at node 2, because D-d (o, j)=0.

Min (10—10, 10—12, 10—8, 10—11)

0 , —2 , +2 , —1

Case 2 (when p (i, j) $\not<$ d (i, j))

In this case when p (i, j) $\not<$ d (i, j), the values p (i, j) are known in advance and are given in Fig. 2.

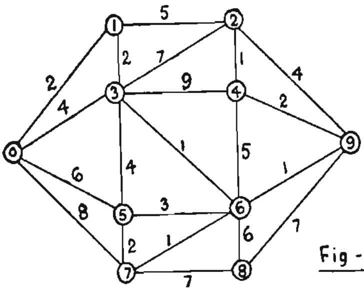

Fig -2

We shall calculate p (0, j) values simultaneously from Fig. 2, following the route given in Table I.

0, j	d (0, j)	D — d (0, j)	P (0, j)
0, 2	10 (7, 5, 3)	0	21
0, 4	12 (7, 5, 3, 2)	— 2	22
0, 6	8 (7, 5)	+ 2	13
0, 8	11 (7 or 7, 5, 3)	— 1	15

TABLE 2

Now we arrange p (0, j) values in descending order.

0, j	p (0, j)	D — d (0, j)
0, 4	22	— 2
0, 2	21	0
0, 8	15	— 1
0, 6	13	+ 2

TABLE 2

Rejected ; because we do not want the negative values of (D — d (0, j)) max (21, 13)=21. This value corresponds to node 2. Hence at the node number 2, the maintenance should be carried out.

When p (i, j) $\not<$ d (i, j) and D is very large.

In this case the following steps should be carried out serially ;

i. First we take any one of the cities (nodes) at which maintenance can be had and find out k shortest paths (2) from origin to this node. These paths may involve loops. The kth shortest path should be greater less than or equal to D. Then find the k value of p (i, j) (profit function) along these paths and choose the maximum.

ii. Repeat the process with each of the other nodes where a maintenance can be had and obtain the value of maximum profit function for that node in the manner shown above.

iii. Now these profit functions will form a column vector, say, P, and then choose the particular component of this vector which is greatest. The corresponding node is the required node at which maintenance should be carried out.

Case of Several Types of Maintenance

In this case of several types of maintenance, we find out which of these maintenances should come first (Call it M_1). Then arrange serially the maintenances as M_1, M_2,......M_n available respectively at cities A_1, A_2,... A_n. First we will find the shortest path from 0 to A_1 using ref. (f). If this is less than D, proceed as in Sec. 4, para 1. The path which gives the maximum value of the profit function will be the required path in the present case. If the shortest distance between 0 and A_1 is greater than D, the present technique fails to give any solution.

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