ON THE SEAT OF ACTIVITY IN THE UPPER AIR

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In recent years W. H. Dines has pointed out several remarkable relationships between upper air elements and from these several important and very general conclusions have been drawn. In 1912 Sir Napier Shaw adopted "the working hypothesis that the chief outlines of pressure distributions of our (*i.e.*, British) latitudes are governed by pressure changes initiated just under the stratosphere "[1] and concluded "it appears that we must regard the sub-stratosphere and the regions above as the dynamical laboratory of the atmosphere where the main causes of pressure change originate, and the troposphere beneath the sub-stratosphere as the physical laboratory of the atmosphere where cloud, rainfall, and other physical phenomena are produced by local causes induced, in some cases, by the effect of dynamical changes in the upper regions "[2].

2. The arguments which have led to the formulation of the Shaw-Dines theory of the sub-stratosphere and the regions above as the seat of origin of meteorological causes, may be broadly divided into two groups :—physical and statistical.

F. M. Exner[3] has discussed Dines's theory from the physical standpoint. He believes the horizontal shift of air masses to be a large causative factor of temperature-fluctuations in the free atmosphere. Hesselberg has considered the problem in greater detail and comes to the conclusion that "the statistically determined relations between pressure-changes and temperature-changes and between pressure-changes above and below do not justify the formulation of Shaw's hypothesis. These may be derived, with the help of the first law of thermodynamics, from the assumed movements in cyclones and anti-cyclones, without any assumption about the 'seat' of the pressure changes."[4]

Among others who have contributed to the subject on the physical side are Köppen and Wedemayer[5] and C. K. M. Douglas[6] whose conclusions are not accepted by Dines[7].

3. It will be seen that the whole subject, important as it is, is in a highly controversial state. Considerable emphasis has been laid by both Shaw and Dines

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[1] M. O. No. 210b. Geophysical Memoirs, No. 2. 1912, p. 17.

[2] The same paper. p. 22.

[3] F. M. Exner: "Über Luftdruckschwankungen in der Höhe und am Erdboden." Met. Zeit. Sept. 1913, pp. 429-436.

[4] Th. Hesselberg : Met. Zeit. July 1915, p. 318. Translation is my own.

[5] Met. Zeit. 1914, pp. 1-15 and 75-87.

[6] Quar. Jour. Roy. Met. Soc., Vol. XLVII, Jan. 1921, p. 43.

[7] Quar. Jour. Roy. Met. Soc., Vol. XLVII, Jan. 1921, p. 44.

on the statistical evidence; and I propose in the present paper to examine the question from the statistical, not the physical, standpoint.

4. The argument behind the 9 km. hypothesis may, I think, be fairly summed in the following way:—

(*i*) The average numerical value of the correlation co-efficients of "pressure at 9 km. level" is *higher* than the average values of the co-efficients of the other variables.

(*ii*) Therefore "pressure at 9 km. level" is statistically more important than the other variables (*e.g.*, P_o, T_m, H_c, etc.).

(*iii*) Hence, it is inferred that "pressure at 9 km. level" is the controlling variable.

Dines says—"The average value of the 12 numerical co-efficients of δP_9 is, ·94, of δH_c it is ·78, of δT it is ·60, of δP_s it is ·32 and of δT_c it is ·25. From this it appears that P_9 is the most important of all the variables, since its standard variation is capable of producing very nearly the full standard variation of all the other quantities." [1]

Dines was however not dogmatic as to the exact height, for on p. 226 of the paper just quoted he speaks of the pressure at a height somewhere between 7 km. and 11 km. as dominant in the changes of the atmosphere.

5. Let us now turn to the problem of determining which is the height that gives the numerically greatest set of correlation co-efficients. Adopting the notation of Dines in No. 2 of the Geophysical Memoirs[2], we have

$P_s = P_o =$ pressure at sea level (denoted by variable x_1)

$T_m =$ mean temperature of air-column from 1 km. to 9 km. (x_2)

$P_9 =$ pressure at 9 km. (x_3)

$H_c =$ height of the stratosphere (x_4)

$T_c =$ temperature of the stratosphere (x_5)

Let $P_z =$ pressure at height z km.

P_9 or P_z is not a directly observed quantity but is calculated from observed values of P_o and T_m with the help of Laplace's formula—

$$Z = \frac{K\,(1+\alpha\,\theta_z)\,(1+Z/R)}{(1-0{\cdot}378\,\phi/\eta)\,(1-k\cos 2\gamma)}\,\log_{10} P_o/P_z \quad \ldots \quad \ldots \quad \ldots \; (1)$$

where $Z =$ altitude, $\gamma =$ latitude, $R =$ terrestrial radius, $\theta_z =$ mean temperature up to z km., ϕ = mean pressure of aqueous vapour, α=co-efficient of expansion of air, $k =$ constant of variation of gravity, $K =$ barometric constant and $\eta = \frac{1}{2}\,(P_z + P_o)$.

Neglecting as Dines does ([3]), the effect of variation of gravity, correction for moisture and correction for altitude in comparison with R, *i.e.*, putting $k=\phi=Z/R=0$ and changing to absolute temperatures, *i.e.*, substituting $67{\cdot}4\,T_m \equiv (1+\alpha\theta_z)\,K$ we get

$$Z = 67{\cdot}4\,T_m \log_{10} P_o/P_z \quad \ldots \quad \ldots \quad \ldots \quad \ldots \; (1{\cdot}1)$$

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[1] Beiträge zur Physik der freien Atmosphäre, 1913, p. 223.

[2] Computer's Handbook, M. O. No. 223, Section I 2, page 19.

[3] Geophysical Memoirs No. 2, M. O. 210b, 1912, page 33.

Taking logarithmic differentials

$$dP_z = \frac{P_z}{P_o} \cdot dP_o + \frac{Z\,P_z}{29{\cdot}3\,T_z^2}\,dT_z \qquad \ldots \qquad \ldots \qquad \ldots \; (2)$$

where T_z = mean temperature up to height z km.

Writing $dT_z \equiv dT_m$ in accordance with the present notation

$$dP_z = \frac{P_z}{P_o} \cdot dP_o + \frac{ZP_z}{29{\cdot}3\,T_z^2} \cdot dT_m$$

$$= a\,dP_o + b.\,dT_m \qquad . \quad . \quad . \quad . \quad . \quad . \; (2{\cdot}1)$$

And $$B \equiv b/a = \frac{P_o}{29{\cdot}3}\;\frac{Z}{T_z^2} \qquad . \quad . \quad . \quad . \quad . \quad . \; (3)$$

Z is fixed uniquely as soon as B is given. Turning to equation (2·1)[1], dividing by a and changing notation to x_1, x_2, etc., we get

$$dP_z/a = dP_o + B.\,dT_m = dx_1 + B\,dx_2 \qquad . \quad . \; (2{\cdot}2)$$

Squaring (2·2)

$$dP_z^2/a^2 = dx_1^2 + B^2.\,dx_2^2 + 2\,B\,dx_1\,dx_2 \qquad . \quad . \; (4{\cdot}1)$$

Multiplying (2·2) by dx_n (deviation of any other variable x_n) we have

$$dP_z.\,dx_n/a = dx_1\,dx_n + B\,dx_2\,dx_n \qquad . \quad . \quad . \; (4{\cdot}2)$$

Summing (4·1) and (4·2), dividing by the total number of records and writing s_1, s_2, s_z as the standard deviations of x_1, x_2 and P_z respectively and r_{12}, r_{1n}, r_{2n} and r_{zn} as the co-efficients of correlation between x_1 and x_2 (P_o and T_m), x_1 and x_n (P_o and x_n), x_2 and x_n (T_m and x_n), P_z and x_n respectively, we get

$$s_z^2/a^2 = s_1^2 + B^2 s_2^2 + 2\,B\,s_1\,s_2\,r_{12}$$

and $$s_z\,s_n\,r_{zn}/a = s_1\,s_n\,r_{1n} + B\,s_2\,s_n\,r_{2n}$$

Writing $s'_z \equiv s_z/a$, we get finally

$$s'^2_z = s_1^2 + B^2 s_2^2 + 2\,B\,s_1\,s_2\,r_{12} \qquad . \quad . \quad . \quad . \; (5{\cdot}1)$$

and $$r_{zn} = \frac{s_1\,r_{1n} + s_2\,r_{2n}}{s'_z} \qquad . \quad . \quad . \quad . \quad . \quad . \; (5{\cdot}2)$$

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[1] (2·1) Dines for example takes (Geophysical Memoirs No. 2, p. 31)

$$dP_9 = {\cdot}33\,dP_o + 1{\cdot}08\,dT_m$$

He says on p. 43 of the same paper "dP_9 instead of being entered from the figures given for each ascent was written down at once from the (above) equation." On p. 222 of Beitrage z. P. d. fr. Atm., 1913, he says "P_9 is not an observed quantity like T_m or H_o but is calculated by a definite formula from the values of P_s (*i.e.*, P_o) and the mean temperature of the air column" and gives the formula as $dP_9 = {\cdot}30\,dP_o + 1{\cdot}08\,dT_m$. If we take Dines's values of $P = 231$, $T = 18°C = 255$ absolute, (quoted on p. 33 of Geo. Mem. No. 2) we get: $dP_9 = {\cdot}304\,dP_o + 1{\cdot}09\,dT_m$ which is much nearer the second of the two formulæ given by Dines.

6. In order to get the *numerically* greatest set of correlation co-efficients it is obvious that we must neglect the sign of r_{zn} and make $S(r_{zn})$, *i.e.*, the sum of all r's a maximum, *i.e.*, choose B in such a way that $\frac{d}{dB} S(r_{zn}) = 0$. This value of B will lead by equation (3) to a definite value of Z, the height which will give the numerically greatest set of co-efficients.

Writing $S(r_{1n}) = R_1$, $S(r_{2n}) = R_2$ and $S(r_{zn}) = R_z$

we have $R_z = (s_1 R_1 + B s_2 R_2) / s'_z$

$$\text{Then} \quad \frac{dR_z}{dB} = \frac{s'_z \, s_2 \, R_2 - (s_1 R_1 + B s_2 R_2) \frac{d s'_z}{dB}}{s'^2_z} = 0$$

$$\text{Differentiating (5·1)} \quad \frac{d s'_z}{dB} = \frac{B s_2^2 + s_1 s_2 r_{12}}{s'_z}$$

$$\text{Hence} \quad s'^2_z \, s_2 \, R_2 - (s_1 R_1 + B \, s_1 \, s_2) \, (B \, s_2^2 + s_1 \, s_2 \, r_{12}) = 0$$

$$\text{leading to} \quad B = \frac{s_1 \, (r_{12} R_1 - R_2)}{s_2 \, (r_{12} R_2 - R_1)} \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

7. We can now use equation (6) to find appropriate values of B for the different series of data given by Dines.

Consider the 66 English ascents (series 66E) on p. 33 of Geo. Mem. No. 2. We have

$r_{12} = \cdot 47$	$r_{21} = \cdot 47$	Thus
$r_{13} = \cdot 68$	$r_{23} = \cdot 90$	$R_1 - r_{12} \, R_2 = 1{\cdot}9967$
$r_{14} = \cdot 78$	$r_{24} = \cdot 74$	$R_2 - r_{12} \, R_1 = 1{\cdot}7027$
$-\, r_{15} = \cdot 71$	$-\, r_{25} = \cdot 28$	Also $s_1 = 10{\cdot}5$, $s_2 = 7{\cdot}6$
$r_{11} = 1{\cdot}00$	$r_{22} = 1{\cdot}00$	Hence $B = 1{\cdot}1785$
$R_1 = 3{\cdot}59$	$R_2 = 3{\cdot}39$	

Using equations (5·1) and (5·2) I calculate the co-efficients of correlation with pressure at z km. Dines's co-efficients at 9 km. are also given for comparison.

Height z km.	Height 9 km. (Dines).
$r_{z1} = \cdot 9587$	$r_{31} = \cdot 68$
$r_{z2} = \cdot 9053$	$r_{32} = \cdot 90$
$r_{z3} = \cdot 9907$	$r_{33} = 1{\cdot}00$
$r_{z4} = \cdot 9316$	$r_{34} = \cdot 86$
$-\, r_{z5} = \cdot 6515$	$-\, r_{35} = \cdot 48$
$R_z = 4{\cdot}4377$	$R_3 = 3{\cdot}92$

Thus our "constructed" set gives 4·4377 against Dines's 3·92, a result which is 13% better.

8. In the following Table (Table I) the "constructed" co-efficients are given under P_z and Dines's co-efficients for 9 km. under P_9 for 4 series discussed by Dines. The name of each set is as mentioned by Dines with the exception of *B. f. A.* which is the set given by Dines in Beitr. z. P. d. fr. Atm., 1913. The percentage difference between the "constructed" and the 9 km. set is shown under each series.

In one case in the B. f. A. series marked (*a*) in the Table my constructed co-efficient exceeds 1·00. This is due to the fact that the B. f. A. series does not represent any actual set of observations but merely gives a set of values considered "most likely" by Dines. Want of statistical consistency is hence not impossible.

I should also mention that I have taken r_{33} to be 1·00 in every case. Actual figures given by Dines do not give this result in all cases. For example in series 106 C the directly calculated value of r_{33} comes out to be only ·92 instead of 1·00. This is the reason why I have not included 106 C in Table I.

The constructed sets are on the average of the 4 series included in Table I, 7·6% better than the 9 km. sets.

TABLE I.

(Values of correlation co-efficients including P_9.)

	1. 66 E.		2. E. A.		3. 29 E.		4. B. f. A.	
	P_z	P_9	P_z	P_9	P_z	P_9	P_z	P_9
1. P_o	·9587	·68	·8849	·80	·9314	·88	·8270	·65
2. T_m	·9053	·90	·8756	·94	·9336	·90	·9522	·90
3. P_9	·9907	1·00	·9867	1·00	·9970	1·00	1·0(*a*)	1·00
4. H_c	·9315	·86	·8476	·77	·9590	·92	·8977	·80
5.—T_e	·6515	·48	·7104	·71	·6967	·76	·5345	·45
Average	·8875	·784	·8614	·844	·9035	·892	·8563	·76
Difference	13·2%	...	2·1%	...	1·3%	...	12·5%	...
B − b/a	1·1785	...	1·7594	...	2·0320	...	1·5612	..

9. In the above Table I have included P_9 itself as one of the variables. This is scarcely justified. What we want to find out is whether any height Z exists at which the pressure P_z gives a numerically greater set of correlation co-efficients than that given by pressure at 9 km. For this purpose it seems desirable to exclude P_9. Doing this we get Table II. Here *G. 2* is the mean series given in Geophysical Memoirs No. 2 and *B. f. A.* the set of most likely values already included in Table I, otherwise the names are as given by Dines.

It will be noticed that omission of P_9 lowers the "best" height still more while the co-efficients are now 14·7% better than Dines's co-efficients.

TABLE II.

(Values of correlation co-efficients omitting P_9.)

	1. 66 E.		2. E. A.		3. 100 C.		4. 106 C.		5. G. 2 (Mean).		6. B. f. A.	
	P_8	P_9	P_8	P_9	P_8	P_9	P_8	P_9	P_8	P_9	P_8	P_9
1. P_8 . . .	·9163	·68	·9060	·80	·9402	·67	·7452	·29	·9484	·66	·9352	·65
2. T_m . . .	·7840	·90	·8518	·94	·9735	·96	·7105	·88	·9103	·92	·9172	·90
3. H_8 . . .	·8443	·86	·8527	·77	·9120	·79	·7759	·82	·9325	·83	·6926	·80
4.—T_8 . . .	·6361	·48	·7118	·71	·4319	·27	·4086	·22	·6309	·49	·5464	·46
Average . .	·7927	·73	·8308	·83	·8144	·6725	·6600	·5525	·8555	·725	·8226	·70
Difference .	8·6%	...	...	...	20·7%	...	19·3%	...	17·8%	...	17·4%	...
B	0·8487	...	1·4743	...	1·0330	...	0·7286	...	1·2019	...	1·2668	...

10. We can now proceed to find Z from the above values of B. In equation (2) the mean temperature T_m must be determined from the known values of temperatures at different heights. Dines for example[1] takes $3\ T_m = t_{rs} + t_{so} + t_{rs}$ or $3\ dT_m = dt_{rs} + dt_{so} + dt_{rs}$. The actual temperatures t_{rs}, t_{so}, t_{rs} will of course be widely different but the fluctuations $dt_{rs}, dt_{so}, dt_{rs}$, etc., may be taken to be sensibly equal on the average of a large number of cases[2]. Thus we may take for statistical averages $dt_1 = dt_2 = dt_3 = dt_4 \ldots = dt_s = dT_m$.

In equation (2) putting $Z_1 = 1$ and $Z_2 = 2$ and writing t_1, t_2, as mean temperatures between 0 and 1 km., 1 and 2 km., respectively

$$dP_1 = \frac{P_1}{P_0} \cdot dP_0 + \frac{Z_1 P_1}{29{\cdot}3\ t_1^2} \cdot dt_1$$

$$dP_2 = \frac{P_2}{P_1} \cdot dP_1 + \frac{(Z_2 - Z_1) P_2}{29{\cdot}3\ t_2^2} \cdot dt_2$$

Putting $dt_1 = dt_2$ generally and remembering $Z_1 = 1,\ Z_2 = 2$

$$dP_2 = \frac{P_2}{P_0} \cdot dP_0 + \frac{P_2}{29{\cdot}3} \cdot dt_2 \left(\frac{1}{t_1^2} + \frac{1}{t_2^2}\right)$$

If we put $\frac{1}{t_1^2} + \frac{1}{t_2^2} = \frac{2}{T_2^2}$

We get $$dP_2 = \frac{P_2}{P_0} \cdot dP_0 + \frac{2\ P_2\ dt_2}{29{\cdot}3\ T_2^2}$$

[1] M. O. No. 210b, Geophysical Memoir No. 2, p. 32.
[2] Beitr. z P. d. freien Atmos. 1913, p. 222.

Proceeding in the same way, we get

$$dP_z = \frac{P_z}{P_0} \cdot dP_0 + \frac{Z\,P_z\,dT_m}{29{\cdot}3\;T_z^2}$$

where

$$\frac{1}{T_z^2} = \frac{1}{Z}\left(\frac{1}{t_1^2} + \frac{1}{t_2^2} + \frac{1}{t_3^2} + \;.\;\;.\; + \frac{1}{t_z^2}\right)$$

11. We can easily find $1/T_z^2$ for different values of Z from published data. Using Table X, Geophysical Memoir No. 13 and taking $P = 1014$ *mb* $= 760{\cdot}6$ *mm.*, I get the following table of values of B at different heights above sea level.

TABLE III.

(Tabulated values of $1/T_z^2$ for South East England.)

Km	t_z	$\frac{1}{T_z^2} = \frac{1}{z} S \left(\frac{1}{t_z^2}\right)$	Z	$B = \frac{zP_0}{29{\cdot}3} \cdot \frac{1}{T_z^2}$
0·5	260·0	0000 12 75 51	1·0	0·33 11
1·5	275·5	12 96 52	2·0	0·67 32
2·5	270·5	13 19 90	3·0	1·02 79
3·5	265·0	13 45 93	4·0	1·39 76
4·5	258·5	13 76 04	5·0	1·78 60
5·5	251·5	14 10 20	6·0	2·19 64
6·5	244·5	14 47 71	7·0	2·63 07
7·5	237·5	14 88 35	8·0	3·09 09
8·5	231·0	15 31 21	9·0	3·57 74
9·5	226·1	15 73 87	10·0	4·08 56
10·5	221·2	16 16 93	11·0	4·61 71

12. From column 5 of the above Table the height Z is calculated for each value of B given in Tables I and II and is shown in Table IV.

TABLE IV.

(Heights in Km. for "best" correlations.)

Series.		66 E	E. A.	29 E.	B. f. A	100 C.	106 C.	G. 2	Average "best" height.
I	B	1·1785	1·7594	2·0320	1·5612	...	...	...	...
	Z	3·4 Km.	4·9 Km.	5·5 Km.	4·4 Km.	..	..	...	4·6 Km.
II	B	0·8487	1·4743	...	1·2668	1·0330	·7285	1·2019	...
	Z	2·5 Km.	4·2 Km.	...	3·6 Km.	3·0 Km.	2·2 Km.	3·5 Km.	3·2 Km.

13. We notice that the height for "best" correlation varies from 3·4 to 5·6 km., if we include P_9 and from 2·2 to 3·6 km. if we omit P_9. The average "best" height is 4·6 and 3·2 km. respectively.

On the data discussed by Dines it thus seems likely that the region from 2 to 4 km. is statistically more important than the region from 7 to 11 km. If statistical importance implies any causal or physical significance then the region from 2 to 4 km. seems to have better claims to be considered the seat of activity rather than the sub-stratosphere and the regions above.

14. Dines has discussed the partial correlation and the partial regression co-efficients of various orders. A few remarks on this point may not be quite out of place.

If we construct a function $z = a\,x_1 + b\,x_2$, where a and b are constants and x_1 and x_2 are the independent variables, then the partial co-efficients involving z can be easily calculated. If p and q are any two variables, s_1 s_2 s_p s_q and s_z the standard deviations of x_1, x_2, p, q, and z, respectively, and if r_{12}, r_{1p}, r_{pq}, r_{zp}, etc., are the co-efficients of correlation between x_1 and x_2, x_1 and p, p and q, z and p, etc., respectively, then

$$r_{zp} = (a\,s_1\,r_{1p} + b\,s_2\,r_{2p})/s_z$$

$$r_{zq} = (a\,s_1\,r_{1q} + b\,s_2\,r_{2q})/s_z$$

Hence

$$r_{zp.q} = \frac{a\,s_1\,(r_{1p} - r_{1q}.r_{qp}) + b\,s_2\,(r_{2p} - r_{2q}.r_{qp})}{(1-r_{pq}^2)^{\frac{1}{2}}\,\{a^2\,s_1^2\,(1-r_{1q}^2) + b^2\,s_2^2\,(1-r_{2q}^2) + 2\,ab s_1\,s_2\,(r_{12} - r_{1q}.r_{2q})\}^{\frac{1}{2}}}$$

Writing $d_q = a^2\,s_1^2(1-r_{1q}^2) + b^2\,s_2^2\,(1-r_{2q}^2) + 2\,ab s_1\,s_2\,(r_{12} - r_{1q}.r_{2q})$

and d_p = a similar expression in p, we have

$$r_{pq.z} = \frac{a^2\,s_1^2\,(r_{pq} - r_{p1}r_{1q}) + b^2\,s_2^2\,(r_{pq} - r_{p2}.r_{2q}) + ab s_1\,s_2\,(2r_{12}\,r_{pq} - r_{1p}.r_{2q} - r_{1q}r_{2p})}{\{d_p\,d_q\}^{\frac{1}{2}}}$$

$$r_{z1.q} = \frac{a s_1\,(1-r_{1q}^2) + b\,s_2\,(r_{12} - r_{1q}.r_{q2})}{\{(1-r_{1q}^2)\,d_q\}^{\frac{1}{2}}},\quad r_{z1.p} = \frac{a s_1\,(1-r_{1p}^2) + b s_2\,(r_{12} - r_{1p}.r_{p2})}{\{(1-r_{1p}^2)\,d_p\}^{\frac{1}{2}}}.$$

Also
$$r_{1q.z} = \frac{a s_2(r_{12}r_{1q} - r_{2q}) + b\,s_2(r_{1q} - r_{12}\cdot r_{2q})}{\{(1-r_{12}^2)d_q\}^{\frac{1}{2}}}$$

$$r_{2q.z} = \frac{a s_1(r_{2q} - r_{12}r_{1q}) + b\,s_2(r_{12}r_{2q} - r_{1q})}{\{(1-r_{12}^2)d_q\}^{\frac{1}{2}}}$$

Writing $r\ (1q.z)$ for $r_{1q\cdot z}$, $r(2q\cdot z)$ for $r_{2q\cdot z}$ etc.

$$r\ (1q\cdot z) + r\ (2q\cdot z) = 0,\ r\ (zp\cdot 1) = r\ (2p\cdot 1) \text{ and } r\ (zp\cdot 2) = r\ (1p\cdot 2)$$

Also
$$r\ (z1\cdot 2) = r\ (11\cdot 2) = r\ (z2\cdot 1) = r\ (22\cdot 1) = +\ 1{\cdot}00.$$

And
$$r\ (12\cdot z) = -\ r\ (22\cdot z) = -\ 1{\cdot}00$$

Dines has assumed $dP_9 = {\cdot}33\ dP_8 + 1{\cdot}08\ dT_m$, that is $dx_3 = a\ dx_1 + b\ dx_2$. Hence we should have $r(31\cdot 2) = r(32\cdot 1) = +\ 1{\cdot}00$, $r(12\cdot 3) = -\ 1{\cdot}00$ and $r(1q\cdot 3) = -r(2q\cdot 3)$ generally.

But a glance at Table IV, Geophysical Memoir No. 2, p. 37, will show that none of the above relations is actually satisfied by any one of the set of values quoted by Dines. This discrepancy is puzzling and throws doubt on the accuracy of Dines's figures. Absence of raw data makes checking impossible. I may note that all my constructed sets of "best" co-efficients do satisfy the above relations.

15. Passing on to the partial correlations of the second order I get the following results by straightforward algebraic simplification.

$$r\,(z1{\cdot}24) = r\,(z2{\cdot}14) = r\,(z1{\cdot}25) = r\,(z2{\cdot}15) = +\,1{\cdot}00$$

$$r\,(z4{\cdot}12) = r\,(z5{\cdot}12) = r\,(14{\cdot}2z) = r\,(15{\cdot}2z) = r\,(24{\cdot}z1) = r\,(25{\cdot}z1) = 0$$

$$r\,(12{\cdot}z4) = r\,(12{\cdot}z5) = -\,1{\cdot}00$$

$$r\,(z4{\cdot}15) = r\,(24{\cdot}15) \quad , \quad r\,(z5{\cdot}14) = r\,(25{\cdot}14)$$

$$r\,(z4{\cdot}25) = r\,(14{\cdot}25) \quad , \quad r\,(z5{\cdot}24) = r\,(15{\cdot}24)$$

$$r\,(25{\cdot}4z) = -\,r\,(15{\cdot}4z) \quad , \quad r\,(24{\cdot}5z) = -\,r\,(14{\cdot}5z)$$

$$r\,(45{\cdot}1z) = r\,(45{\cdot}2z) = r\,(45{\cdot}12)$$

Also $r\,(24{\cdot}51)$, $r\,(25{\cdot}41)$, $r\,(15{\cdot}42)$, $r\,(12{\cdot}45)$, $r\,(14{\cdot}52)$ and $r\,(45{\cdot}12)$ are quite independent of a and b, *i.e.*, of z.

Hence $r(z1{\cdot}45)$, $r(z2{\cdot}45)$, $r(25{\cdot}4z)$, and $r(24{\cdot}5z)$ are the only four second order co-efficients (for 5 variables) which depend on the numerical values of a and b, *i.e.*, depend upon the particular height considered.

I have to note again that Dines's figures given in Table V, page 40 of the memoir quoted above are quite discrepant with the above results.

16. I find for the third order partial co-efficients.—

$$r\,(z1{\cdot}245) = r\,(z2{\cdot}145) = +\,1{\cdot}00, \qquad r\,(12{\cdot}z45) = -\,1{\cdot}00$$

$$r\,(45{\cdot}z12) = r\,(45{\cdot}z1) = r\,(45{\cdot}z2) = r\,(45{\cdot}12).$$

And
$$r\,(z4{\cdot}125) = r\,(z5{\cdot}124) = r\,(14{\cdot}z25) = r\,(15{\cdot}z24)$$
$$= r\,(24{\cdot}z15) = r\,(25{\cdot}z14) = 0.$$

Thus all the third order co-efficients are quite independent of a and b, *i.e.* quite independent of the height.

17. In conclusion I wish to acknowledge with gratitude my indebtedness to Dr. Gilbert T. Walker, F.R.S., Director-General of Observatories, for the help and encouragement he has given me at all times. He suggested the problem, gave me every facility for work in his department and made valuable criticisms. I also wish to thank Dr. Normand, Imperial Meteorologist, and other members of the staff at Simla who were kind enough to help me in various ways.

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