A STATISTICAL STUDY OF CERTAIN ANTHROPOMETRIC MEASUREMENTS FROM SWEDEN.

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1. *Introduction.* The present paper consists of a statistical study of certain anthropometric data from Sweden, which have been taken from *The Racial Characters of the Swedish Nation*, edited by H. Lundborg and F. J. Linders, and published by the Swedish State Institute for Race Biology, Uppsala, in 1926*. My aim is to make a first application of the Coefficient of Racial Likeness to the discrimination of racial differences to be ascertained from measurements on the living. Hitherto the method of the C. R. L. has been applied chiefly to craniometric data†. As I have indicated in an earlier memoir (*Biometrika*, Vol. xx^{A}. pp. 1—31) the want of standardisation renders analysis of measurements on the living by this, or indeed by any other, method largely futile.

The material consists of measurements of 46,983 conscripts and regular soldiers belonging to the Swedish Army and Navy. The subjects were all born in Sweden and were over 20 and under 22 years of age. The measurements were taken in 1922 and 1923, each person being measured by two observers. Special precautions were taken to ensure the same standards being maintained by all observers. One of the examiners measured the entire naval force, and another examined nearly half of the persons included in the investigation. The total number of observers was small, and all of them were connected with the Swedish State Institute for Race Biology. It may be assumed therefore that the present series of measurements are standardised and comparable *inter se*. Measurements of 404 persons born in foreign countries are available, but they were excluded from my analysis.

The birthplace of the person examined was chosen as the basis for the regional grouping of the material into five territories:

(A) *North Sweden*, comprising the provinces of Lappland, Västerbotten and Ångermanland.

(B) *West Sweden*, comprising Jämtland, Härjedalen, Dalarne, Värmland, Västmanland, Närke, Dalsland, Bohuslän and Västergötland.

(C) *East Sweden*, comprising Medelpad, Hälsingland, Gästrikland, Uppland, Södermanland, Östergötland, Småland and Öland Island.

(D) *South Sweden*, comprising Halland, Skåne, Blekinge and Gotland Island.

(E) The four biggest *Cities*: Stockholm, Göteborg, Malmö and Norrköping.

* I am much indebted to Professor H. Lundborg for kindly sending me a copy of this book immediately after its publication.

† In *Biometrika* it has been applied to racial characters in silkworms and to those of Macedonian local groups.

The material from each territory was further classified into four groups on an occupational basis:

(α) *Agricultural* communities in which, according to the 1910 Census, more than 60 % of the inhabitants earned their livelihood through agriculture, forestry and fishing.

(β) *Mixed* communities in which the corresponding percentage was under 60 but over 30.

(γ) *Industrial* communities in which the percentage was less than 30.

(δ) A fourth group, the *Urban* communities, consisted of the inhabitants of the cities, towns and market towns (exclusive of (E)).

We thus get the scheme (shown in Table I) for the whole of Sweden divided into 17 sections.

TABLE I.

Divisions of the Population of Sweden.

Territory	Occupational Group	Section Number	Number of persons examined
(A) North	Agricultural	1	2993
	Mixed ...	2	1059
	Industrial	3	406
	Urban ...	4	337
(B) West	Agricultural	5	7054
	Mixed ...	6	3200
	Industrial	7	1245
	Urban ...	8	1723
(C) East	Agricultural	9	6496
	Mixed ...	10	4642
	Industrial	11	1894
	Urban ...	12	2465
(D) South	Agricultural	13	3687
	Mixed ...	14	2665
	Industrial	15	625
	Urban ...	16	1737
(E) Four Largest Cities		17	4755
		Total	46,983

Mean values of the different characters for each section are shown in Table III on the following page.

Pooling certain of the above sections we obtain the geographical territories and occupational classes shown in Table II.

Several of the mean values for the occupational classes given in Table III were calculated by me; the other figures were taken from the published volume.

I may note here that after a careful comparison with the Census figures for the whole of Sweden the authors came to the conclusion that the geographical as well

as the occupational and social distributions of the persons measured were representative of the whole population. In other words, the present material may be considered to be a fair sample of the male Swedish population for the age-group 20—22 years*.

The values of the general means and standard deviations for the total sample are given in Table IV. The standard deviations for Bi-acromial Index, Supra-

TABLE II.

Divisions of the Population of Sweden (continued).

Territory.	Number examined	Occupational Class	Number examined
(A) North	4,795	(α) Agricultural	20,230
(B) West	13,222	(β) Mixed ...	11,566
(C) East	15,497	(γ) Industrial	4,170
(D) South	8,714	(δ) Urban ...	6,262
(E) Cities	4,755	(E) Cities ...	4,755
Total	46,983	Total	46,983

TABLE IV.

General Means, Standard Deviations and Coefficients of Variation, with their Probable Errors†, based on the total Population of 46,983.

(Body measurements are in cms. and head measurements in mms.)

Character	Mean	Standard Deviation	Coefficient of Variation
1. Stature	172·23 ± ·018	5·93 ± ·013	3·44 ± ·008
2. Supra-sternal Height ...	140·89 ± ·013	5·29 ± ·012	3·75 ± ·008
3. Trunk Length	52·37 ± ·007	2·41 ± ·005	4·60 ± ·010
4. Arm Length	78·46 ± ·010	3·34 ± ·007	4·26 ± ·009
5. Leg Length	92·02 ± ·013	4·30 ± ·009	4·67 ± ·010
6. Bi-acromial Diameter ...	39·23 ± ·005	1·67 ± ·004	4·26 ± ·009
7. Inter-iliocristal Breadth ...	28·80 ± ·005	1·52 ± ·003	5·27 ± ·012
8. Trunk Length Index ...	30·49 ± ·004	1·18 ± ·003	3·88 ± ·009
9. Leg Length Index	53·43 ± ·004	1·29 ± ·003	2·41 ± ·005
10. Bi-acromial Index	22·80 ± ·003	0·92 ± ·002	4·04 ± ·009
11. Head Length	193·84 ± ·019	6·19 ± ·014	3·20 ± ·007
12. Head Breadth	150·44 ± ·016	5·10 ± ·011	3·39 ± ·007
13. Face Breadth	136·02 ± ·015	4·84 ± ·011	3·56 ± ·008
14. Morphological Face Height	126·57 ± ·022	6·92 ± ·015	5·46 ± ·012
15. Minimum Frontal Diameter	104·57 ± ·013	4·33 ± ·010	4·14 ± ·009
16. Cephalic Index	77·69 ± ·010	3·14 ± ·007	4·04 ± ·009
17. Morphological Face Index...	93·14 ± ·017	5·61 ± ·012	6·02 ± ·013

* Lundborg and Linders say: "the geographical distribution of the primary material must be regarded as satisfactory" (*op. cit.* p. 18), and again: "the agreement (with Census figures) must be regarded as good and the primary material fully representative from the social standpoint" (p. 20).

† Standard Errors are given throughout *The Racial Characters of the Swedish Nation*, and the probable errors in this table were found as those in Table III.

sternal Height, and Leg Length Index were not given by the authors. They were obtained indirectly in the way explained below. The Bi-acromial Index is defined as the ratio of the Bi-acromial Diameter to the Stature, i.e.,

$$\text{Bi-acromial Index } (z) = 100\,\frac{\text{Bi-acromial diameter } (x)}{\text{Stature } (y)}.$$

Therefore writing r_{xy} as the correlation between Bi-acromial Diameter (x) and Stature (y), we have approximately:

$$v_z^2 = v_x^2 + v_y^2 - 2r_{xy}\,v_x v_y,$$

where v_z, v_x, v_y are the coefficients of variation ($100\,\sigma/M$) for the Bi-acromial Index, Bi-acromial Diameter and Stature respectively. Substituting the constants for the total sample (Table IV):

$$M_y = 172{\cdot}23, \quad \sigma_y = 5{\cdot}93, \quad v_y = 3{\cdot}44, \quad r_{xy} = +0{\cdot}47,$$
$$M_x = 39{\cdot}23, \quad \sigma_x = 1{\cdot}67, \quad v_x = 4{\cdot}26, \quad M_z = 22{\cdot}80,$$

we obtain $\sigma_z = 0{\cdot}92$ approximately.

Again the authors define* (p. 73)

Supra-sternal Height (z) = Trunk Length (x) + Leg Length (y) − 3·5 cm.

so that $$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2r_{xy}\sigma_x\sigma_y.$$

Since $$\sigma_x = 2{\cdot}41, \quad \sigma_y = 4{\cdot}30, \quad \text{and} \quad r_{xy} = \cdot\cdot 0{\cdot}18$$

we have $\sigma_z^2 = 28{\cdot}0288$, and $\sigma_z = 5{\cdot}29$ approximately.

Finally, the standard deviation for Leg Length Index was directly calculated from the frequency distribution given in the Swedish work, Table VIII (Supplement, p. 34).

The authors state that the measurements were taken to the nearest millimetre with an "Anthropometer" (compass callipers or Tasterzirkel), and sliding callipers (Gleitzirkel) supplied by P. Hermann of Zürich.

The following notes on measurements are given (pp 10—11):

"*Bi-acromial Diameter*, defined as the distance between the acromial points, is measured, in departure from the instructions given in Martin†, from the back, the immediate reason for this being to control the posture during measurement."

"*Morphological Face Height* must be regarded as less exactly determined than the other measurements of the head, since the examiner often cannot locate the nasion (sutura naso-frontalis) with certainty."

"*Trunk Length* was calculated as the difference between the supra-sternal height and the height of symphysion."

"*Height of Symphysion* (upper border of symphysis pubis in the middle line). Measurement is rendered difficult in rare cases of excessive corpulence."

"*Arm Length* is the difference between the height of acromion and the height of dactylion, and *Leg Length* is obtained by adding 35 mm. to the height of symphysion (all according to Martin†)."

* [An arbitrary definition, which does not allow for personal or racial variation. ED.]

† It is clear from the remarks under "Bi-acromial Diameter" and "Leg Length" that Martin's directions (presumably those given in his *Lehrbuch der Anthropologie*, 1st edition, 1914) were followed in all cases unless otherwise mentioned.

"*Height of Acromion* could not always be determined with accuracy, since the processus acromialis sometimes showed malformation or at least considerable deviations from the normal form."

"*Height of Dactylion* presented difficulties of measurement in certain cases when the subject could not fully extend the right arm, also when malformations existed in the fingers of the right hand. In such cases this measurement was made from the left."

The *Bi-acromial, Leg Length, Trunk Length* and *Arm Length Indices* all have the stature as the denominator.

2. *Comparisons by the Method of the Coefficient of Racial Likeness.* The main object of the present paper, as I have said, is to present the results of comparisons between the various groups of the Swedish material, described above, made by Professor Karl Pearson's method of the Coefficient of Racial Likeness. In *The Racial Characters of the Swedish Nation* detailed comparisons are made between the means and standard deviations of the characters considered singly, and between the correlations for some pairs of characters calculated for different divisions of the total population. These correlations are shown to be remarkably constant and few significant differences in variability are observed. The means are less constant, and it was felt that a far clearer conception of the anthropological significance of these differences would be given by a generalised criterion, which takes into account a number of characters at the same time, than by the more usual method which deals with individual characters. The coefficient of racial likeness has been extensively used in craniometric work, but little has yet been done in applying it to measurements on the living. One of the principal objections against its use in this case has been the fact that the technique of measurement has not been standardised satisfactorily, and thus the data provided by different observers can seldom be compared with safety*. Such an objection does not apply to the material now under consideration; it constitutes the most complete and most valuable description of the population of a single country which has hitherto been provided. Numbers of individuals large enough to form statistically adequate samples are dealt with, which unfortunately can seldom be the case for cranial series. The number of characters determined is less satisfactory as we can only use 17, and intra-racial correlations between some pairs of these are known to be high. The problem of determining a sufficient number of head and body measurements which are all uncorrelated, or at least lowly correlated, with one another is yet unsolved, and the characters which are customarily determined have certainly not been chosen with this object in view.

If m_s is the mean of the sth character in the first group, σ_s its standard deviation and n the size of the sample, while m_s', σ_s' and n' are the corresponding

* See P. C. Mahalanobis: "On the Need for Standardisation in Measurements on the Living," *Biometrika*, Vol. xx^A. (1928), pp. 1—31.

quantities for the second sample, then Professor Pearson's coefficient of racial likeness is defined to be

$$S\left\{\frac{1}{M}\frac{(m_s-m_s')^2}{\frac{\sigma_s^2}{n}+\frac{\sigma_s'^2}{n'}}\right\}-1+\frac{1}{M}\pm\cdot67449\sqrt{\frac{2}{M}}\quad\ldots\ldots\ldots\ldots\ldots(1),$$

where there are M characters compared*. If pairs of samples are drawn from the same population the coefficients between them will vary round zero with the probable error shown. In the present investigation the number of characters used (17) is the same in every comparison and the term $\frac{1}{M}(=\cdot06)$ has been neglected. The standard deviations are those of the samples and when these are small, as in craniometric work, it has been customary to suppose that they are equal to one another and to the values available for the longest related racial series. The constants have been provided for the Swedish data and they are practically identical for different sections in the case of a particular character and also equal to the general standard deviations calculated for the total sample of 46,983. If the last be denoted by $\bar{\sigma}_s$, then the calculation is greatly simplified by assuming that $\sigma_s=\sigma_s'=\bar{\sigma}_s$. The coefficient becomes

$$S\left(\frac{1}{M}\frac{nn'}{n+n'}\frac{(m_s-m_s')^2}{\bar{\sigma}_s^2}\right)-1\pm\cdot67449\sqrt{\frac{2}{M}}\quad\ldots\ldots\ldots\ldots\ldots(2),$$

when the term $\frac{1}{M}$ is neglected. This is the form which has been used. Values of $\bar{\sigma}$ for the 17 characters are given in Table IV. The characters used should theoretically be uncorrelated with one another, but this condition is far from being satisfied. We are dealing with five indices and the two component lengths from which each is derived are used in addition. The spurious correlations in such cases are probably all greater than 0·5. A number of the absolute measurements also cover one another. Several of the correlations are given in *The Racial Characters of the Swedish Nation*, and in the case of stature and leg length the values for five groups and for the total sample are between 0·86 and 0·88. If the condition were made that no pairs of the measurements used should have correlations greater than 0·5 with one another, then all except three or four of the 17 would have to be rejected. The inclusion of highly correlated measurements is necessitated if the Swedish material is to be dealt with by the method of the coefficient of racial likeness, although these high correlations are far from satisfactory. The procedure is partly justified, perhaps, by the fact that precisely the same group of characters is used in every case. The comparison of these coefficients of racial likeness with others calculated for a different group of measurements would not be justified.

The coefficient provides a measure of the probability that the two samples compared were drawn from the same population. This probability will depend on the sizes of the samples available. It has been suggested that comparable measures of

* Karl Pearson: "On the Coefficient of Racial Likeness," *Biometrika*, Vol. XVIII. (1926), pp. 105—117.

the absolute divergences of the populations represented by the samples may be obtained by reducing each coefficient to the value it would have if each sample were of a standard size*. In the present paper the coefficients have been reduced to values they would have had if each series in the comparison had contained 100 individuals. These values are given by

$$50 \times \frac{\bar{n}+\bar{n}'}{\bar{n}\bar{n}'}\left\{S\left(\frac{1}{M}\frac{nn'}{n+n'}\frac{(m_s-m_s')^2}{\bar{\sigma}_s^2}\right)-1\right\} \pm \cdot 67449 \times 50 \times \frac{\bar{n}+\bar{n}'}{\bar{n}\bar{n}'}\sqrt{\frac{2}{M}} \ldots (3).$$

Crude coefficients of racial likeness, calculated from formula (2), were first found for the 17 sections defined in Table I and for the territories and occupational classes defined in Table II. The occupational samples for the whole country were made up by pooling the relevant sub-groups of the four major territorial divisions and hence some of the larger groups are not independent samples. No comparisons were made in such cases. Every crude coefficient differs significantly from zero. The values for the sections range from $1{\cdot}04 \pm {\cdot}23$ to $142{\cdot}54 \pm {\cdot}23$ and the mean of the 136 coefficients is 24·85. The values between the territories and occupational classes range from $3{\cdot}99 \pm {\cdot}23$ to $169{\cdot}34 \pm {\cdot}23$ and the mean of the 20 coefficients is 68·22. The difference between these means must be attributed to the fact that the samples are larger in the one case than in the other. All the samples are large and hence it is not surprising to find that the majority of the coefficients are of an order which would indicate marked racial divergence if found for short cranial series. The coefficients clearly increase with the sizes of the series compared and no direct comparison can be made between them until correction is made for this varying factor.

Reduced coefficients of racial likeness calculated from formula (3) are given in Table V*a* for the 17 sections and in Table V*b* for the five territories and four occupational classes. The reduction when all the means are supposed to be based on 100 individuals only is very great in all cases, and values as low as many shown have seldom been found for cranial comparisons. All differ significantly from zero. The 136 coefficients between the sections range from $0{\cdot}05 \pm {\cdot}005$ to $5{\cdot}98 \pm {\cdot}030$ and their mean is 1·38: the 20 coefficients between the territories and occupational classes range from $0{\cdot}08 \pm {\cdot}005$ to $3{\cdot}13 \pm {\cdot}005$ and their mean is 0·97. All the lowest reduced coefficients between the sections are indicated in Fig. I. These measures of relationship suggest an arrangement of the territories (Table V*b*) which is almost linear. The north and west divisions occupy extreme positions, with the east very close to the west and the south closest to the east and rather closer to the west than to the north†. A comparison of the sections of the territories representing any one particular occupational class leads to almost precisely the same geographical arrangement. The urban sections Nos. 4, 16, 12 and 8 are of the north, south, east and west territories respectively, and their reduced coefficients of racial likeness (Table V*a*) give the same arrangement as the total samples for the territories, except

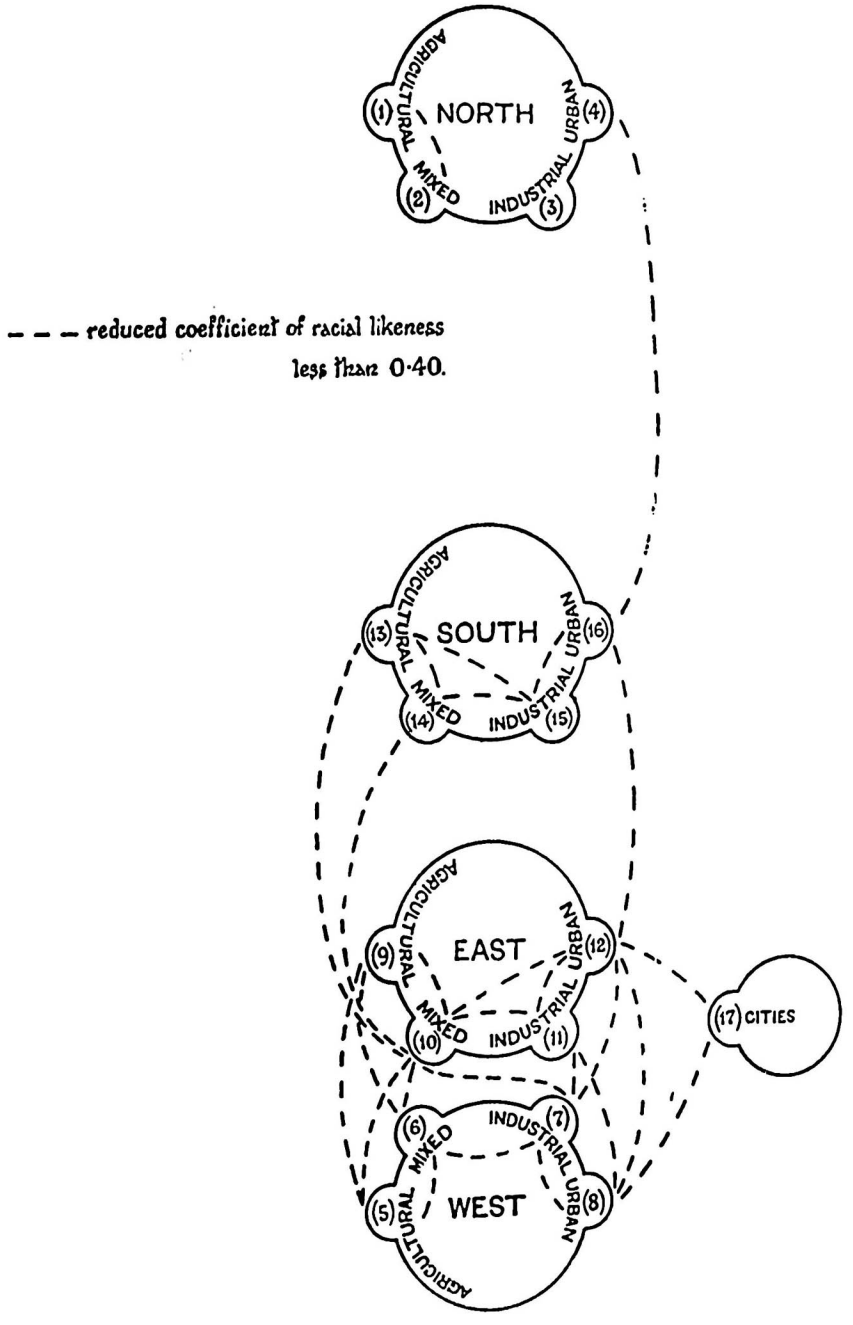
* Karl Pearson: "Note on Standardisation of Method of using the Coefficient of Racial Likeness," *Biometrika*, Vol. xx^B. (1928), pp. 376—378.

† To correspond more exactly to this arrangement the distance between the circles representing the north and south territories in Fig. I should be increased considerably.

that the south is now rather nearer to the north than to the west. The same is found for the industrial sections except that the south is rather nearer to the west than to the east. For the mixed sections the south is again rather nearer to the north than to the west, and the single inversion in the case of the agricultural sections is the slightly closer approach of the north to the west than to the east. In spite of these

FIG. I

INTER-RELATIONSHIPS OF VARIOUS GROUPS OF THE POPULATION OF SWEDEN.



small discrepancies, it is true to say that the best linear arrangement of the territories is precisely the same whether we consider their total samples, or the samples for any single one of the four occupational classes. The underlying geographical, or racial, differences can be appreciated nearly as well by considering one particular class only as by considering the total populations irrespective of class. It does not follow from this fact, of course, that the relationships between the territories are precisely the same for one class as for another and, indeed, the contrary can be easily demonstrated. In the comparison of any pair of the four geographical divisions the reduced coefficient between the urban sections is always less than the coefficient between the sections representing any other occupational class. The mixed sections have the next closest degree of relationship in four out of the six possible comparisons, and in five cases the coefficients between the industrial sections are the greatest found. The classes can thus be arranged fairly definitely in the sequence: urban—mixed—agricultural—industrial, with the first on the average showing the minimum and the last the maximum racial differences. All the most intimate connections between the sections are shown in Fig. I, the upper limit being fixed arbitrarily as a reduced coefficient of 0·40. The territories were arranged by considering their total samples and these closest links are now only found between the sections of contiguous territories, and there are far more of them between the east and west sections than between those of the east and south, or south and north territories. A comparison of the total occupational samples for the whole of Sweden (Table V*b*, facing p. 97) gives the definite sequence: agricultural—mixed—industrial—urban, and precisely the same order is given by the sections within any one of the territories west, east or south. The connections between any two classes are approximately the same for all these three. The sections of the north territory have different relationships. The agricultural and mixed sections are still as closely connected as for the other territories, but the resemblances of all other pairs of sections are far less close. The same two also stand nearer to the urban than to the industrial sections and the urban stands nearer to the mixed than to the industrial. It is clear that the occupational arrangement which applies uniformly to the south, east and west territories is different in the case of the north owing to the less homogeneous racial constitution of the last territory. Its agricultural and mixed sections are closely linked to one another and they are distinct from all other samples and must therefore be supposed to contain a peculiarly large proportion of a racial element which is foreign to the bulk of the Swedish population. The industrial community of the north territory also stands apart but the urban is not distinguished in this way (see Fig. I). The racial significance of the observed relationships will be considered later.

3. *Comparisons of Individual Characters.* In making comparisons by the method of the coefficient of racial likeness it has been constantly observed that on the average the differences between the various characters vary greatly in significance. The values of the a's* have been used in examining this point, but one

* $a=\frac{nn'}{n+n'}\left(\frac{m_s-m_s'}{\sigma_s}\right)^2$.

objection to their use is that they are influenced, like the coefficients, by the sizes of the samples compared. Since all characters for any one of our samples are based on the same number of individuals it was not necessary to calculate the individual a's in the present investigation. A more direct method of grading the characters can be employed, however. In Table VI are given the inter-group standard deviations (Σ) for the 17 sections of the Swedish material. The co-group standard deviations ($\bar{\sigma}$) in the same table are the general values given for the total sample

TABLE VI.

Inter- and Co-Group Standard Deviations with their Probable Errors.

Character	Inter-group Standard Deviation (Σ)*	Co-group Standard Deviation ($\bar{\sigma}$)†	$\frac{\Sigma}{\bar{\sigma}}$
Cephalic Index	·565 ± ·065	3·14 ± ·007	·180
Head Breadth	·804 ± ·093	5·10 ± ·011	·158
Arm Length	·509 ± ·059	3·34 ± ·007	·153
Bi-acromial Index	·132 ± ·015	0·92 ± ·002	·143
Face Breadth	·655 ± ·076	4·84 ± ·011	·135
Inter-iliocristal Breadth ...	·205 ± ·024	1·52 ± ·003	·135
Bi-acromial Diameter ...	·214 ± ·025	1·67 ± ·004	·128
Stature	·680 ± ·079	5·93 ± ·013	·115
Leg Length	·478 ± ·055	4·30 ± ·009	·111
Head Length	·686 ± ·079	6·19 ± ·014	·111
Supra-sternal Height ...	·569 ± ·066	5·29 ± ·012	·108
Morphological Face Height	·688 ± ·080	6·92 ± ·015	·099
Trunk Length Index ...	·106 ± ·012	1·18 ± ·003	·091
Minimum Frontal Diameter	·372 ± ·043	4·33 ± ·010	·086
Leg Length Index	·106 ± ·012	1·29 ± ·003	·082
Trunk Length	·175 ± ·020	2·41 ± ·005	·072
Morphological Face Index...	·368 ± ·043	5·61 ± ·012	·066

of 46,983 individuals and these are almost precisely the same as the values found for any one of the 17 sections. The $\bar{\sigma}$'s are the ones which were used in computing the coefficients of racial likeness. It is clear that the ratio of Σ to $\bar{\sigma}$ will give a measure of the average significance of the differences found for the various characters. The inter-group variability is small compared with the intra-group variability in every case, but there are still marked differences between the measurements in this respect. The cephalic index tends to show more significant differences than any other character and this has been confirmed in the case of several other comparisons of measurements made either on the living or on the skull. It has been usual to find, too, that the head breadth varies more significantly than the head length and much more significantly than the minimum frontal diameter. The stature is less capable of differentiating the groups than several of the other characters. An index, such as the cephalic or bi-acromial, may vary more significantly than either of its component lengths, or the reverse may hold, as for the leg length and morphological face indices.

* These standard deviations are for the means of the 17 sections given in Table III.

† These standard deviations are for the total sample of 46,983 individuals.

The 17 characters may now be considered individually with regard to both geographical position and occupational class. They can be divided into a number of groups by considering whether the order in which each arranges the 17 sections is controlled more by one of these factors than by the other. The cephalic index is extreme in this respect. The four lowest means are for the sections of the west territory, the sections of the east and the urban section (No. 17) follow next and then the four of the south, while the cephalic indices for the sections of the north territory are greater than any others. There is thus a clear distinction between the territories, and they are arranged in the order shown in Fig. I. The maximum difference between the sections of the same territory with extreme cephalic indices is only 4·0 times its probable error (south and east territories) and no significance whatever can be attached to the orders in which the occupational classes are arranged within the territories. This character is clearly controlled by geographical position and there is no evidence of any significant association with occupational class. The bi-acromial index affords an example of a measurement which is affected by conditions entirely different from these. The order in which the means arrange the 17 sections appears to have no geographical significance whatever, but the three highest indices are for agricultural sections, the lowest is for the sample from the four largest cities (No. 17) and the next four lowest are for the other urban sections. For each territory the highest index is for the agricultural section, the second highest for the mixed, the next for the industrial and the lowest for the urban section. The differences between the agricultural and urban sections of the same territory are very significant in every case, being 8·4, 12·2, 16·9 and 15·1 times their probable errors for the north, south, east and west divisions respectively. The bi-acromial index is thus clearly controlled by the occupational class, and there is no evidence of any significant association with geographical position. These two characters are at opposite extremes in so far as they are controlled by one or other of the factors on the basis of which the groupings were made, but in most other cases there is a definite tendency for a measurement to approach one extreme in this respect rather than the other. The orders in which the sections are arranged may be supposed to have been influenced by both factors in the majority of cases. Whenever there is a clear territorial sequence, or the suggestion of such a sequence, it is always: north, south, east and west. Whenever there is a clear occupational sequence common to all the territories, or the suggestion of such a sequence, it is always: agricultural, mixed, industrial and urban. Paying due regard to the significance of the differences, the following classification of the characters can be made:

(*a*) Characters showing markedly significant territorial differences, but no occupational sequence within the territories—cephalic index, stature, supra-sternal height.

(*b*) Characters showing significant territorial differences and a significant occupational sequence within the territories—head breadth, head length, inter-iliocristal breadth.

(*c*) Characters showing a suggestion of territorial differences, but no occupational sequence within the territories—minimum frontal diameter, leg length.

(*d*) Characters showing a faint suggestion of territorial differences and a markedly significant occupational sequence within the territories—face breadth, arm length, bi-acromial diameter.

(*e*) Characters showing a faint suggestion of territorial differences and a significant occupational sequence within the territories—leg length index, trunk length index.

(*f*) Characters showing no territorial differences and a markedly significant occupational sequence within the territories—morphological face height, bi-acromial index.

(*g*) Characters showing no territorial differences and no occupational sequence within the territories—morphological face index, trunk length.

The comparison of individual characters has confirmed in a very satisfactory way the scheme of relationships suggested by the coefficients of racial likeness. It can now be seen that there are marked differences between the characters not only in their average effect on the coefficients, but also according as they are more or less capable of discriminating between regional or occupational samples. The two in group (*g*) above are the only ones which appear to be quite incapable of serving either purpose and these are the two with the lowest values of $\Sigma/\bar{\sigma}$ (see Table VI). By making a suitable selection from the other 15 it would clearly be possible to obtain coefficients which would emphasise the geographical differences and obscure the occupational, while the reverse effect could be obtained by making a different selection. The characters which show territorial differences will be considered again in the next section. There are seven absolute and three indicial measurements which furnish either a significant, or a markedly significant occupational sequence, and for all except one the agricultural section tends to have the greatest mean and the urban the least. The trunk length index is greater for the urban than for the rural populations, but the reverse is found for the bi-acromial diameter and index, the arm length, the head and face breadths, the head length, the morphological face height, the inter-iliocristal breadth and the leg length index. The fact that the first three of the last nine measurements are greater for rural than for urban samples was to be expected. The relations observed in the case of the others suggest that the agricultural workers have skeletons which are broader in all ways and with relatively longer limbs than town dwellers, though no differences between the statures of the groups can be detected. The differences between the extreme means are all very small, however. The bi-acromial diameter provides a more definite occupational sequence than any other absolute measurement, but the largest mean for a section only exceeds the smallest by 7·4 mm. Whether any of the differences between occupational classes are due to use, or whether they are occasioned by selection, cannot be decided from these Swedish data. With smaller samples, or in the case of a more racially heterogeneous population, it would probably be impossible to prove their existence.

4. *The Racial Constitution of the Swedish Population.* The present study is restricted, on the regional side, to a comparison of the four territorial divisions into

which the whole of Sweden was divided, and some important facts may be overlooked by taking such large areas. All the individuals examined were born in Sweden. The remarkable constancy of the coefficients of variation and correlation, provided in *The Racial Characters of the Swedish Nation*, suggests that the populations are now thoroughly hybridised if they once had diverse racial origins. The coefficients of racial likeness suggested the simple linear arrangement shown in Fig. I and the reasonableness of this order was emphasised by finding that all the characters which are capable of making definite distinctions between the territories show the same sequence from the north at one extreme to the west at the other. Of the 17 measurements there are only six which give significant or markedly significant regional differences when a single occupational class is considered. The total means for the territories are given for these in the table below and this comparison is now not quite so convincing since the relative proportions of the different classes are not the same for all the territories.

Territories	Cephalic index	Stature	Supra-sternal height	Head breadth	Head length	Inter-iliocristal breadth
North	78·84	171·23	140·13	152·27	193·33	28·43
South	78·02	171·88	140·62	150·69	193·33	28·79
East	77·53	172·32	140·96	150·14	193·85	28·86
West	77·31	172·43	141·04	150·23	194·52	28·95

Some pairs of these six measurements are lowly correlated with one another and the fact that they provide the same sequence is all the more significant on that account. The coefficients between the sections are so low that it can only be assumed that all divisions of the total population of Sweden belong predominantly to the same racial type. The observed relationships can be explained on the hypothesis that this basic type has been modified slightly, but in different degrees in different territories, by admixture with another race. The north territory was more modified than any other by this means, the south considerably less, the east still less and the west territory may have been unaffected, or modified to a less extent than any other. The racial crossing seems to have resulted in a perfect blending of all the characters for which data are available and those which show no territorial differences may be assumed to have been the same for the two racial types. These are the conclusions suggested by a purely statistical analysis of the material and we may attempt to reconcile them with what is known of the ethnic history of the country. The following particulars are taken from the section written by Rolf Nordenstreng in *The Racial Characters of the Swedish Nation**.

"The Swedes have inhabited their country since later neolithic times. The main body of the prehistoric population seems to have been of rather distinctly Nordic

* "Origin, Growth and Racial Components of the Swedish Nation," *op. cit.*, pp. 41—49 and summary on p. 174.

race, though other types also occur ...The finds from the Bronze Age and the Iron Age do not present any new types, but agree with those from the Stone Age....The early Swedish kingdom did not consist of more than the present central territories about Lake Mälaren; but gradually other parts of the present kingdom were conquered, the people of the Gauts south of the Swedish settlements between the Baltic Sea and the North Sea being the most important of those incorporated into the nation. All these peoples on the Scandinavian Peninsula were Teutons like the Swedes, of much the same race, and using similar languages. Only in the northernmost part of the country lived Lapps, roving since prehistoric times. The Swedish dominion was early extended to territories east of the Baltic, whence in the course of time came an influx of the East Baltic race, especially in a Finnish immigration in the last years of the 16th, and the first half of the 17th century....That the Nordic race has been the chief stock of Sweden's ancient population, as of the present, is beyond all doubt. But as to what extent it was mixed and with which races, we can venture nothing more than a guess....It is not impossible that the East Baltic race is very ancient in this country, more ancient even than the Nordic, but this cannot be proved and is hardly very likely; the possibility should not, however, be wholly dismissed. The most noteworthy support is given by the type demonstrated by Arbo in South Norway and often called 'the blond brachycephal,' a type which reminds one not a little of the Finnish....According to Lönborg's calculations Sweden (except Norrbotten) and parts of East Norway had at the close of the 17th century a Finnish population of between twelve and thirteen thousand persons. This figure is very likely too low, but nevertheless is highly appreciable, considering that Sweden's entire population then amounted to hardly 1½ millions, and that the parts of the country in which the Finns were living were certainly very sparsely populated. As these immigrants were unusually prolific, their offspring undoubtedly increased at a proportionately higher rate than those of the real Swedes....There is also a Lappic race-admixture in the Finnish population of North Sweden....The number of Finnish-speaking persons in Northern Sweden probably amounts at present to about 30,000....How strong the race-mixture with East Baltic blood has been in Sweden is at present impossible to state. But it would hardly be an exaggeration to assert that at least some hundred thousand present-day Swedes and perhaps many more evince more or less East Baltic characters."

The only foreign races which are known to have influenced the population of Sweden to any marked extent since neolithic times are thus the Finns and, to a lesser extent, the inhabitants of the East Baltic states; the Lapps, as far as is known, have lived in the north as long as the country was inhabited at all. All these alien races are closely allied to one another, and, where they differ from the Swedish type, they apparently do so in the same direction as, for example, in possessing higher cephalic indices and shorter statures. The miscegenation with the so-called nordic population must have been extremely thorough, since the variabilities for all sections are almost identically the same. Even in the north, the bulk of the population must be of "nordic" origin, and it is not surprising to find that the effects of slight differences between the types with which admixture was made in

different regions cannot be detected at all by considering large groups of the existing population. Comparisons made in that way only suggest that there was a crossing with a single racial type resulting in a perfect blending of all the characters considered. The alien element is far more evident in the north than in any other territory; it produced a greater effect in the south than in the central regions of Sweden, and the east was slightly more affected than the west.

Note added in proof.

This paper was originally written as an integral part of an empirical study of certain alternative formulae for the measurement of racial divergence. Very extensive and substantial editing of the text was therefore necessary in publishing it in the present form. I am deeply indebted to Dr G. M. Morant for having carried out the editing work much more satisfactorily than I could have done myself. I also wish to acknowledge the help I received from my assistant Mr Sudhir Kumar Banerjee in reducing the statistical material for the paper. P. C. Mahalanobis, Calcutta, 22nd July, 1930.

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