REPRINTED FROM

THE INDIAN JOURNAL OF AGRICULTURAL SCIENCE.

Vol. II, Part II, April, 1932.

A STATISTICAL NOTE ON CERTAIN RICE-BREEDING EXPERIMENTS IN THE CENTRAL PROVINCES.

BY

P. C. MAHALANOBIS, M.A. (CANTAB.), B.Sc. (CAL.).
Professor of Physics, Presidency College, Calcutta.

(Received for publication on the 9th October 1931.)

1. In the June (1931) issue of this *Journal*, Messrs. D. N. Mahta and B. B. Dave gave a most interesting account of "Rice-breeding in the Central Provinces"*. The experimental work done by the authors and the results achieved by them are likely to be of far-reaching importance to Indian Agriculture. I may, therefore, be excused for offering a few remarks on the statistical methods used by the authors to test the significance of their results.

The authors raised a number of new varieties by hybridization. These hybrids (which possessed the desirable characters) were "tested for yield among themselves and against their parents. The testing of yield was carried out by the Latin Square method." The yield of 13 varieties replicated 13 times in plots (4′×4′) were recorded (page 361). The Table I giving the primary data (yield in ounces of each strain noted below the varietal number) is reproduced here (from page 361) for convenience of reference.

---

* *Ind. J. Agric. Sci.*, Vol. I, Part III, June 1931, pp. 351—371. Subsequent page references refer to this paper.

TABLE I.

(*Yield. Data from the* Indian Journal of Agricultural Science, *Vol. I, Part III, Page 361.*)

B. P. 22/1 19·5	Parewa 20·0	B. S. 23 28·0	B. P. 15 30·75	B. S. 30 25·5	Bhondu 31·0	B. S. 24 29·5	Surmatia 22·75	B. P. 19 33·0	B. P. 22/2 27·75	B. P. 12 26·5	B. P. 20 28·75	B. P. 11 29·5
Bhondu 19·5	20 16·0	19 21·0	23 18·25	11 19·0	22/2 18·25	Parewa 21·5	24 25·0	30 13·25	22/1 10·0	15 22·5	Surmatia 18·25	12 24·5
12 25·5	22/2 21·0	Surmatia 22·0	30 25·5	Parewa 23·0	24 27·75	22/1 30·25	19 25·5	20 27·5	11 21·75	23 27·0	Bhondu 29·25	15 32·0
24 23·0	30 21·75	12 26·0	11 27·0	23 25·75	15 25·5	Surmatia 21·75	22/1 27·0	Bhondu 27·75	19 25·25	Parewa 24·0	22/2 24·5	20 27·25
19 26·5	Surmatia 17·25	24 28·5	Bhondu 28·0	22/1 27·5	30 25·5	15 25·75	22/2 23·25	Parewa 26·0	12 25·0	20 28·5	11 28·25	23 28·25
22/2 25·75	24 24·0	Bhondu 25·5	Parewa 21·0	Surmatia 20·3	22/1 27·25	23 26·25	12 24·25	11 19·0	20 25·0	19 26·5	15 25·25	30 21·0
Parewa 22·75	23 25·5	22/1 7·0	Surmatia 20·0	20 23·5	12 23·0	30 20·5	Bhondu 21·25	22/2 23·25	15 27·0	11 24·0	19 26·0	24 29·25
20 26·5	11 18·0	30 15·5	24 25·5	22/2 19·5	Parewa 21·0	19 23·0	15 19·25	12 23·5	23 23·75	Surmatia 21·0	22/1 27·5	Bhondu 29·75
15 21·5	Bhondu 22·75	11 19·0	22/2 20·75	12 22·75	19 17·5	20 20·5	23 26·0	22/1 24·75	Parewa 20·5	30 21·25	24 30·0	Surmatia 21·5
23 25·5	22/1 29·0	15 16·5	20 23·0	19 18·25	11 24·0	12 26·0	Parewa 22·0	Surmatia 22·5	24 31·25	Bhondu 29·0	30 25·5	22/2 21·5
11 28·25	19 20·75	Parewa 15·0	12 25·5	24 25·0	20 24·75	Bhondu 27·25	30 21·75	15 29·0	Surmatia 27·5	22/2 26·0	23 31·5	22/1 31·5
30 20·75	12 20·75	20 21·0	22/1 25·75	15 25·75	Surmatia 21·25	22/2 25·75	11 26·5	23 29·5	Bhondu 28·5	24 29·0	Parewa 24·5	19 22·25
Surmatia 16·75	15 24·75	22/2 21·5	19 13·0	Bhondu 22·0	23 30·5	11 22·5	20 21·5	24 26·75	30 20·0	22/1 22·25	12 24·5	Parewa 19·25

In discussing the significance of the results the authors calculated the mean and standard deviations (based on samples of 13). and used the classical theory of errors to test the differences in yield.

2. A more recent statistical procedure can, however, be adopted with advantage and will lead to a greater precision in the quantitative interpretation of the results. This is attained in two ways. First by the use of the appropriate theory of small samples.* Secondly, by eliminating the effect due to soil heterogeneity by using Fisher's method of "analysis of variance", a procedure which, very fortunately, can be adopted in the present case owing to the use of the Latin Square arrangement. It is desirable that agricultural experimenters in India should make themselves familiar with Fisher's method. Full details of the numerical calculations are, therefore given below.†

* The application of the classical theory of errors (which was developed on the assumption of large samples) will not yield absolutely correct results in the case of small samples. This difficulty can be met by using Fisher's t-test. I have discussed this question in a recent note of the *Indian Journal of Agricultural Science* (February, 1932) which may be referred to for full details.

† I am intentionally confining my remarks to the explanation of the actual procedure of numerical calculations. Excellent descriptions of the method will be found in R. A. Fisher's 'Statistical Methods for Research Workers' (3rd edition, 1930, Chap. VIII). J. O. Irwin has recently given a resume of the underlying theory in an article on "Mathematical Theorems Involved in the Analysis of Variance" in the *Journal of the Royal Statistical Society*, Vol. XCIV, 1931, Part II, p. 284.

There are three Arithmetical slips in the numerical calculations in Mahta and Dave's paper. There is a serious mistake in 'B.×S. No. 24', Serial No. 13 (p. 367). The yield is printed as 19·50; the correct figure (from Table I) is 29·50. The corresponding square should be changed from 380·2 to 870·25.

For 'Bhondu' (p. 368) serial number 4, the yield is printed as 27·75. The correct value is 27·25. The total yield of Bhondu has, however, been evidently calculated with the figure 27·25). The square 742·7 should be corrected to 742·56.

For 'Surmatia', serial No. 8, the printed figure is 20·50; the correct figure should be 20·33. The square should also be changed from 420·4 to 413·31.

Table II.

Yield in ounces minus 24 oz.

(Derived from Table I.)

		1	2	3	4	5	6	7	8	9	10	11	12	13	Total for rows.
	1	—4·50	—4·00	+4·00	+6·75	+1·50	+7·00	+5·50	—1.25	+9·00	+3·75	+2·50	+4·75	+5·50	+40·50
	2	—4·50	—8·00	—3·00	—5·75	—5·00	—5·75	—2·50	+1·00	—10·75	—5·00	—1·50	—5·75	+0·50	—56·00
	3	+1·50	—3·00	—2·00	+1·50	—1·00	+3·75	+6·25	+1·50	+3·50	—2·25	+3·00	+5·25	+8·00	+26·00
	4	—1·00	—2·25	+2·00	+3·00	+1·75	+1·50	—2·25	+3·00	+3·75	+1·25	..	+0·50	+3·25	+14·50
	5	+2·50	—6·75	+4·50	+4·00	+3·50	+1·50	+1·75	—0·75	+2·00	+1·00	+4·50	+4·25	+4·25	+26·25
	6	+1·75	0·0	+1·50	—3·00	—3.67	+3·25	+2·25	+0·25	—5·00	+1·00	+2·50	+1·25	—3·00	—0·92
	7	—1·25	+1·50	+3·00	—4·00	—0·50	—1·00	—3·50	—2·75	—0·75	+3·00	..	+2·00	+5·25	+1·00
	8	+2·50	—6·00	—8·50	+1·50	—4·50	—3·00	—1·00	—4·75	—0·50	—0·25	—3·00	+3·50	+5·75	—18·25
	9	—2·50	—1·25	—5·00	—3·25	—1·25	—0·50	—3·50	+2·00	+0·75	—3·50	—2·75	+6·00	—2·50	—23·25
	10	+1·50	+5·00	—7·50	—1·00	—5·75	..	+2·00	—2·00	—1·50	+7·25	+5·00	+1·50	—2·50	+2·00
	11	—0·75	—3·25	—9·00	+1·50	+1·00	+0·75	+3·25	—2·25	+5·00	+3·50	+2·00	+7·50	+7·50	+16·75
	12	—3·25	—3·25	—3·00	+1·75	+1·75	—2·75	+1·75	+2·50	+5·50	+4·50	+5·00	+0·50	—1·75	+9·25
	13	—7·25	+0·75	—2·50	—11·00	—2·00	+6·50	—1·50	—2·50	+2·75	—4·00	—1·75	+0·50	—4·75	—26·75
Sum of columns	..	—15·25	—30·50	—25·50	—8·00	—14·17	+5·25	+8·50	—6·00	+13·75	+10·25	+15·50	+31·75	+25·50	+11·08

3. In reducing any considerable body of field data, it is usually convenient to adopt a suitable base number, and subtract this base number from each individual figure of the original data. In the present example, 24 oz. will be a convenient number, as this happens to be the approximate value of the general mean.* We now subtract 24 from each of the yield figures in Table I, and enter the deviation (plus or minus) from 24 in Table II. For convenience of reference, I shall use the following serial numbers for the different varieties.

No.	Variety
1.	B. × P. No. 11
2.	B. × P. No. 12
3.	B. × P. No. 15
4.	B. × P. No. 19

No.	Variety
5.	B. × P. No. 20
6.	B. × P. No. 22/1
7.	B. × P. No. 22/2
8.	B. × S. No. 23

No.	Variety
9.	B. × S. No. 24
10.	B. × S. No. 30
11.	Bhondu
12.	Parewa
13.	Surmatia

The figures in each column and in each row are next added and the sums entered as shown in Table II. (For example, the sum of the figures in row 1 is +10·50, of column 5 is —14·17, of row 6 is —0·92, and so on). Adding the marginal totals of columns we get +11·03. (This is checked by adding the marginal totals of rows.) The general mean is then given by 24+11·08/169=24·07 approximately. (This may, and should be checked by direct addition of all the yields.)

The squares of individual deviations are then taken from a table of squares (like Barlow's Tables) and entered in Table III Rows and columns are added. and then the marginal totals as before. The gross total of all (169) deviations from 24 is found to be 2590·1564.

* It does not, however, really matter what particular number is selected. For ease of subtraction. 20 might have been selected as the base number.

TABLE III.

Squares of deviations given in Table II.

	1	2	3	4	5	6	7	8	9	10	11	12	13	Total of rows
1	20·2500	16·0000	16·0000	45·5625	2·2500	49·0000	30·2500	1·5625	81·0000	14·0625	6·2500	22·5625	30·2500	335·0000
2	20·2500	64·0000	9·0000	33·0625	25·0000	33·0625	6·2500	1·0000	115·5625	25·0000	2·2500	33·0625	0·2500	367·7500
3	2·2500	9·0000	4·0000	2·2500	1·0000	14·0625	39·0625	2·2500	12·2500	5·0625	9·0000	27·5625	64·0000	191·7500
4	1·0000	5·0625	4·0000	9·0000	3·0625	2·2500	5·0625	9·0000	14·0625	1·5625	—	0·2500	10·5625	64·8750
5	6·2500	45·5625	20·2500	16·0000	12·2500	2·2500	3·0625	0·5625	4·0000	1·0000	20·2500	18·0625	18·0625	167·5625
6	3·0625	—	2·2500	9·0000	13·4689	10·5625	5·0625	0·0625	25·0000	1·0000	6·2500	1·5625	9·0000	86·2814
7	1·5625	2·2500	9·0000	16·0000	0·2500	1·0000	12·2500	7·5625	0·5625	9·0000	—	4·0000	27·5625	91·0000
8	6·2500	36·0000	72·2500	2·2500	20·2500	9·0000	1·0000	22·5625	0·2500	0·0625	9·0000	12·2500	33·0625	224·1875
9	6·2500	1·5625	25·0000	10·5625	1·5625	42·2500	12·2500	4·0000	0·5625	12·2500	7·5625	36·0000	6·2500	166·0625
10	2·2500	25·0000	56·2500	1·0000	33·0625	—	4·0000	4·0000	2·2500	52·5625	25·0000	2·2500	6·2500	213·8750
11	0·5625	10·5625	81·0000	2·2500	1·0000	0·5625	10·5625	5·0625	25·0000	12·2500	4·0000	56·2500	56·2500	265·3125
12	10·5625	10·5625	9·0000	3·0625	3·0625	7·5625	3·0625	6·2500	30·2500	20·2500	25·0000	0·2500	3·0625	131·9375
13	52·5625	0·5625	6·2500	121·0000	4·0000	42·2500	2·2500	6·2500	7·5625	16·0000	3·0625	0·2500	22·5625	284·5625
Total of columns.	133·0625	226·1250	314·2500	271·0000	120·2189	213·8125	134·1250	70·1250	318·3125	170·0625	117·6250	214·3125	287·1250	2590·1564

Gross (uncorrected) sum of squares = 2590·1564.

A separate table (Table IV) is now formed for the deviations for each variety, and sums of deviations obtained by addition. (For example, the sums of deviations for variety No. 9 is + 42·50). The squares of these deviations are also independently written down, and added. The total again comes to 2590·1564, thus furnishing an absolute check on the arithmetic so far.

Table IV.

Deviations for each variety.

(Reference numbers of varieties on page 1.)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
1	−1·50	+0·50	+0·75	−11·00	−2·50	−1·75	−2·50	+6·50	+2·75	−4·00	−2·00	−4·75	−7·25
2	+2·50	−3·25	+1·75	−1·75	−3·00	+1·75	+1·75	+5·50	+5·00	−3·25	+4·50	+0·50	−2·75
3	0·0	+2·00	−7·50	−5·75	−1·00	+5·00	−2·50	+1·50	+7·25	+1·50	+5·00	−2·00	−1·50
4	−0·75	+1·50	+5·00	−3·25	+0·75	+7·50	+2·00	+7·50	+1·00	−2·25	+3·75	−9·00	+3·50
5	−5·00	−1·25	−2·50	−6·50	−3·50	+0·75	−3·25	+2·00	+6·00	−2·75	−1·25	−3·50	−2·50
6	−6·00	−0·50	−4·75	−1·00	+2·50	+3·50	−4·50	−0·25	+1·50	−8·50	+5·75	−3·00	−3·00
7	0·0	−1·00	+3·00	+2·00	−0·50	+3·00	−0·75	+1·50	+5·25	−3·50	−2·75	−1·25	−4·00
8	−5·00	+0·25	+1·25	+2·50	+1·00	+3·25	+1·75	+2·25	0·0	−3·00	+1·50	−3·00	−3·67
9	+4·25	+1·00	+1·75	+2·50	+4·50	+3·50	−0·75	+4·25	+4·50	+1·50	+4·00	+2·00	−6·75
10	+3·00	+2·00	+1·50	+1·25	+3·25	+3·00	+0·50	+1·75	−1·00	−2·25	+3·75	0·0	−2·25
11	−2·25	+1·50	+8·00	+1·50	+3·50	+6·25	−3·00	+3·00	+3·75	−1·50	+5·25	−1·00	−2·00
12	−5·00	+0·50	−1·50	−3·00	−8·00	−5·00	−5·75	−5·75	+1·00	−10·75	−4·50	+2·50	−5·75
13	+5·50	+2·50	+6·75	+9·00	+4·75	−4·50	+3·75	+4·00	+5·50	+1·50	+7·00	+4·00	−1·25
Total .	−10·25	+5·75	+13·50	−13·50	+1·75	+26·25	−13·25	+33·75	+42·50	−34·25	+30·00	−31·50	−39·17

The sums of deviations for rows, columns, and varieties are next written in Table V, columns 2, 4, and 6 respectively. (It will be noticed that the sum of deviation of row 3 is + 26·00 which is taken from Table II, or the sum of deviation of variety No. 9 is + 42·50 which is taken from Table IV, and so on). Squares of these sums of deviations are then entered in columns 3, 5, and 7 respectively. Adding the figures in columns 3, 5, and 7 we find the sums of squared deviations to be 8312·7214, 4406·3514, and 8885·4764 respectively.

TABLE V.

Sums of deviations for rows, columns, and varieties.

No.	Rows		Columns		Varieties	
	(1) Deviation	(2) Squares	(3) Deviation	(4) Squares	(5) Deviation	(6) Squares
1	+40·50	1,640·2500	—15·25	232·5625	—10·25	105·0625
2	—56·00	3,136·0000	—30·50	930·2500	+ 5·75	33·0625
3	+26·00	676·0000	—25·50	650·2500	+13·50	182·2500
4	+14·50	210·2500	— 8·00	64·0000	—13·50	182·2500
5	+26·25	689·0625	—14·17	200·7889	+ 1·75	3·0625
6	— 0·92	0·8464	+ 5·25	27·5625	+26·25	689·0625
7	+ 1·00	1·0000	+ 8·50	72·2500	—13·25	175·5625
8	—18·25	333·0625	— 6·00	36·0000	+33·75	1,139·0625
9	—23·25	540·5625	+13·75	189·0625	+42·50	1,806·2500
10	+ 2·00	4·0000	+10·25	105·0625	—34·25	1,173·0625
11	+16·75	280·5625	+15·50	240·2500	+29·50	870·2500
12	+ 9·25	85·5625	+31·75	1,008·0625	—31·50	992·2500
13	—26·75	715·5625	+25·50	650·2500	—39·17	1,534·2889

As we have worked with sums of 13 individual figures, in the case of rows, columns and varieties these three sums are divided by 13, giving the following results.

GROSS SUM OF SQUARES.

Rows	639·4401
Columns	338·9501
Varieties	683·4982

So far the sum of squares have been found for the deviations of each plot yield from the arbitrary base number 24. We require however the sums of squares for deviations from the true mean (24·07). These can be obtained by applying a correction to the sum of squares obtained above.

The sum of all deviations with respect to base number 24 is + 11·08. Squaring 11·08 we get 122·7664. Dividing this number by 169 (the total number of all deviations) we get —0·7264, the required correction. This must be subtracted, from the above uncorrected sums of squares of deviations.*

Hence, we derive the corrected sum of squares.

	Sum of squares
Rows	638·7137
Columns	338·2237
Varieties	682·7718

4. We are now in a position to form Fisher's Table of Analysis of Variance (Table VI). In column 1 is given the nature of variation. In column 2 the corresponding number of degrees of freedom. These represent the total possible number of independent comparisons in each case. For example, for the 13 rows only 12 independent comparisons are possible, and similarly for the columns and varieties. For the whole sample of 169 plots the total number of comparisons possible is 168. (In fact in field trials, the number of degrees of freedom is usually obtained by subtracting 1 from the size of the sample).

The sums of squares of deviations for rows, columns and varieties 638·7137, 338·2237 and 682·7718 are written down in column 3, and added giving a total of 1659·7092. This represents the contribution of 36 degrees of freedom (usually written as D. F.) due to rows *plus* columns *plus* varieties (each of which absorbs 12 degrees of freedom). The total sums of squares of deviations, 2589·4300, is obtained by applying the correction (— 0·7264) to 2590·1564, the gross total already found in Table III. This represents 168 degrees of freedom. Subtracting 1659·7092 from 2589·4300, we get finally 929·7208 as the sums of squares of deviations due to the residual causes of variation (usually called random errors) with 168—32=136 degrees of freedom. For purposes of comparison it is this residual variation which gives the probable margin of experimental errors (including residual effects of soil heterogeneity). The enormous advantage of eliminating the effects of other factors of variation is obvious.

5. Dividing the sums of squares (column 3) by the corresponding degrees of freedom (column 2), we get the quantities known as "variances" given in column 4. We notice that the variance due to differences of varieties is 56·8976, while the random variance is 7·0433. (For purposes of comparison only these two variances

* This correction is always negative, *i.e.*, must always be subtracted.

are required in practice). We may now use Fisher's 'z-test' to find whether the varietal differences may be considered significant in comparison with the random variance, *i.e.*, whether 56·8976 may be considered to be significantly greater than 7·0433. The natural logarithms (that is logarithms to be base 'e', and not to the base "10") of the variances are required for this purpose. They are entered (from mathematical tables) in column 5 of Table VI.

TABLE VI.

Analysis of variance.

Variance due to	D. F.	Sum of squares	Mean square	$\log_e (\sigma^2)$.
Varieties . .	12	682·7718	56·8976	4·04112
Columns . .	12	338·2237	28·1853	..
Rows . .	12	638·7137	53·2261	..
Error . .	132	929·7208	7·0433	1·95203
Total .	168	2589·4300	..	2·08909

The z-test may be now applied in the following way. Let v_1 and v_2 be the variances for varietal differences and random errors respectively.

Then $z=\frac{1}{2}(\log_e v_1-\log_e v_2)=1·0445$ with n_1 (the D. F. corresponding to the larger variance)=12, and $n_2=132$. We may now use Fisher's Table VI (Statistical Methods, page 215). We find that the one per cent. point (that is, for probability of 1 in 100), with $n_1=12$, and $n_2=60$, $z=0·4574$, while for $n_1=12$, and $n_2=\infty$, $z=0·3908$. The observed value of $z=1·0445$. It is clear that the odds are much greater than 100 to 1 against such a value of z (=1·0445) occurring by chance. We conclude that the observed value of $z=1·0445$ indicates a significant difference between 56·8976 and 7·0433. That is, we conclude that the varietal differences are statistically significant in comparison with the random error of the experiment.

6. We may now proceed to test the individual differences in yield between the different varieties. The residual variance is 7·0433. The variance for mean values based on samples of 13 will be given by 7·0433/13. The variance of differences between any two such mean values (each based on samples of 13) is given by $2\times 7·0433/13=1·08374$. Extracting the square root of this quantity we obtain 1·041 as the value of the standard error for comparison of mean yields based on 13 replications each.

The differences in mean yield are tabulated systematically in Table VII.

TABLE VII.

Differences in yield.

No.	Variety.	B.×P. No. 11.	B.×P. No. 12.	B.×P. No. 15.	B.×P. No. 19.	B.×P. No. 20.	B.×P. No. 22/1.	B.×P. No. 22/2.	B.×S. No. 23.	B.×S. No. 24.	B.×S. No. 30	Bhondu	Parewa.	Surmatia.	(16) Difference for (24·07).	(17) Mean yield of each variety Mean yield.
1	B.×P. No. 11	—	+1·23	+1·83	—0·25	+0·92	+2·81	—0·23	+3·38	+4·06	—1·85	+3·05	—1·63	—2·22	—0·86	23·21
2	B.×P. „ 12	—1·23	—	+0·60	—1·48	—0·31	+1·58	—1·46	+2·15	+2·83	—3·08	+1·82	—2·86	—3·45	+0·37	24·44
3	B.×P. „ 15	—1·83	—0·60	—	—2·08	—0·91	+0·98	—2·06	+1·55	+2·23	—3·68	+1·22	—3·46	—4·05	+0·97	25·04
4	B.×P. „ 19	+0·25	+1·48	+2·08	—	+1·17	+3·06	+0·02	+3·63	+4·31	—1·60	+3·30	—1·38	—1·97	—1·11	22·96
5	B.×P. „ 20	—0·92	+0·31	+0·91	—1·17	—	+1·89	—1·15	+2·46	+3·14	—2·77	+2·13	—2·55	—3·14	+0·06	24·13
6	B.×P. „ 22/1	—2·81	—1·58	—0·98	—3·06	—1·89	—	—3·04	+0·57	+1·25	—4·66	+0·24	—4·44	—5·03	+1·95	26·02
7	B.×P. „ 22/2	+0·23	+1·46	+2·06	—0·02	+1·15	+3·04	—	+3·61	+4·29	—1·62	+3·28	—1·40	—1·99	—1·09	22·98
8	B.×S. „ 23	—3·38	—2·15	—1·55	—3·63	—2·46	—0·57	—3·61	—	+0·08	—5·23	—0·33	—5·01	—5·60	+2·52	26·59
9	B.×S. „ 24	—4·06	—2·83	—2·23	—4·31	—3·14	—1·25	—4·29	—0·08	—	—5·91	—1·01	—5·69	—6·28	+3·20	27·27
10	B.×S. „ 30	+1·85	+3·08	+3·68	+1·60	+2·77	+4·66	+1·62	+5·23	+5·91	—	+4·91	+0·22	—0·37	—2·71	21·36
11	Bhondu .	—3·05	—1·82	—1·22	—3·30	—2·13	—0·24	—3·28	+0·33	+1·01	—4·90	—	—4·68	—5·27	+2·19	26·26
12	Parewa .	+1·63	+2·86	+3·46	+1·38	+2·55	+4·44	+1·40	+5·01	+5·69	—0·22	+4·68	—	—0·59	—2·49	21·58
13	Surmatia .	+2·22	+3·45	+4·05	+1·97	+3·14	+5·03	+1·99	+5·60	+6·28	+0·37	+5·27	+0·59	—	—3·08	20·90
	Mean yield .	23·21	24·44	25·04	22·96	24·13	26·02	22·98	26·59	27·27	21·36	26·26	21·58	20·90	—	—

N. B.—Column (16) gives the difference in yield of each variety from the general mean yield (24·07 oz.)
Column (17) gives the mean yield of each variety. These figures are also repeated in the last row.

In order to appreciate the significance of these differences we proceed in the following way. We have seen that the standard error of the mean difference is 1·041. We may use Fisher's t-table (Table IV, p. 139), although in this case the classical theory will give practically correct results. Adopting a level of significance of 1 per cent. (P=·01, or odds of 100 to 1), we find that for n=132, the critical value of "t" is 2·536. Since "t" is expressed in terms of the standard error, we must multiply it by 1·041, and obtain the critical value of the difference to be 2·640 in the present case. In other words the odds are 100 to 1 that any observed difference (in mean yields given in Table VII) of this magnitude is definitely significant *. For any pair of varieties we can therefore find the significance of the comparison by a mere inspection of Table VII. For example for variety No. 9 (*i.e.*, B. × S. No. 24) we notice that it gives a significantly better yield than No. 1 (B. × P. No. 11), No. 4 (B. × P. No. 19), No. 5 (B. × P. No. 20) No. 7 (B. × P. No. 22/2), No. 10 (B. × S. No. 30), No. 12 (Parewa) and No. 13 (Surmatia). Or let us compare No. 2 (B. × P. No. 12) and No. 6 (B. × P. No. 22/1). The observed difference is 1·58, and is not significant. We cannot assert that on a repitition of the trial 'B. × P. No. 22/1' is definitely more likely to produce a better yield than 'B. × P. No. 12'.

Expressing the critical difference 2·911 as a percentage of the general mean yield (24·07), we get 11·0 per cent. approximately. The order of accuracy attained in the present experiment is therefore such that differences in yield of the order of 11 per cent. may be detected with a certainty of 100 to 1. With a lower level of significance (of 20 to 1), the critical difference is 8·4 or 8 per cent. approximately.

7. If we compare the results given by the author (pp. 370—371) with my results, we find several discrepancies. The authors state that the difference between 'B. × P. No. 11' and 'Bhondu' is insignificant, my analysis shows that the observed difference of 3·05 may be considered to be significant. In the same way I find that the difference between 'B. × P. No. 19' and 'Bhondu' (3·30) is also significant. Adopting the 5 per cent. level of significance (for which the critical difference is 2·022), we notice, that two other differences ('B. × P. No. 20' and 'Bhondu', 'B. × P. No. 20' and 'Parewa') rejected as insignificant by the authors may also be considered significant.

The authors give 20 comparisons, out of which 10 are insignificant. Against this in Table VII we have figures for all possible comparisons, in this case 68 in all. With 1 per cent. probability, (critical difference 2·64), no less than 34 differences out of 68 are significant. With 5 per cent. probability (critical difference 2·02)

* If we use a lower level of significances of 5 per cent., the critical value of 't' is 1·942, and the critical difference is 2·022.

8 more differences may be considered significant. We find, therefore, that on the basis of the present experiment 42 differences are significant, while 26 must be considered insignificant.

The above discussion shows that the present experiments are actually more valuable than one would gather from the analysis originally given by the authors.

The advantages of the systematic procedure described above is then two-fold :—

(*a*) There is a substantial gain in the precision of the comparison, and

(*b*) the process is exhaustive, so that not a single significant difference is likely to be missed.

Further, as already pointed out, the statistical theory used is one which was specially developed to meet the requirements of small samples.

The numerical calculations were completed under the supervision of my computing assistant Babu Sudhir Kumar Banerjee, and were rendered possible by a research grant from the Imperial Council of Agricultural Research.