QUASI-LATIN SQUARES IN EXPERIMENTAL ARRANGEMENTS

In the thesis submitted by the author in 1943 to the Calcutta University, it was shown how the Design of Experiments consists of the Fundamental problem together with the problem of Balancing and the Construction of designs. Every experimental design is a mathematical solution to a combinatorial problem directly deducible from the mathematical model set up by the necessary balance and other requirements such as the block size, number of replications, cost of experimentation, etc., required by the experimenter. Two types of designs called as 'The partially balanced design' and 'The Intra- and Inter-group balanced design', gave a variety of designs in the case of incomplete block designs. These two systems, also, gave balanced or partially balanced confounded designs in the case of general factorial experiments, asymmetrical or symmetrical. It is proposed to extend these ideas to Quasi-Latin squares which are suggested by Fisher's requirement of the 'local control' by which the efficiency of an experimental design may be enhanced by minimising the error without increasing the number of replications. These were first introduced by Yates[1] in the case of incomplete block designs.

The Fundamental Problem.—The most general set up consists of v treatments denoted by $T_1, T_2, T_3 \ldots T_v$ tested in n lattice square arrangements of s^2 cells each, such that the i-th treatment is replicated r_i times and the treatments T_i and T_j appear together in columns and rows combined λ_{ij} times and in squares, on the total μ_{ij} times. The parameters are connected by the following relationships:—

$$\lambda_{ii}=0$$

$$\lambda_{ij}=\lambda_{ji},\ \mu_{ij}=\mu_{ji}$$

$$\Sigma r_i=n\,s^2$$

$$2\,(s-1)\,r_i=\sum_j \lambda_{ij}$$

$$r_i\,(s^2-1)=\sum_j \mu_{ij}$$

Analysis of the General Design.—Using the methods of analysis developed in the above thesis we get the estimating equation as,

$$Q_i=r_i\,(s-1)^2\,t_i-s\sum_j \lambda_{ij}\,t_j+\Sigma\,\mu_{ij}\,t_j$$

where t_i estimates the effect of the treatment T_i, and

$Q_i = s^2$ (sum of observations of the i-th treatment)
  $-\,s$(sum of row and column totals in which the i-th treatment occurs)
  $+$ the weighted sum of square totals the weights being number of times the i-th treatment occurs in a square.

From these set of equations, we can test the significance of any estimable linear expression of the treatment effects by comparing it against its estimated standard error. The estimates of linear expressions being linear functions of Q_i, their standard errors, ultimately, depend on the following results. Let Q_i be gathered from m_i lattice squares, the contribution from the j-th square being q_{ij}.

$$Q_i=\Sigma\,q_{ij}$$

$$V\,(Q_i)=\sum_j V\,(q_{ij})$$

$$V\,(q_{ij})=r\,(s-2\,s+r)^2+(s-r)^2\,2r+(s-r)^2 + r\,(r-1)\,(r-2s),^2$$

when the i-th treatment occurs r times in j-th lattice square.

Cov $(Q_i Q_m) = \Sigma$ Cov $(q_{ij} q_{mj})$.

The most general expression for the covariance has been found out but is not given here due to lack of space. There are two types of designs which appear to be fruitful.

Type 1.—Let $s\lambda_{ij} - \mu_{ij} = \nu_{ij}$. We impose restrictions on ν_{ij} similar to that of the partially balanced design (it may be noted that the second system of parameters introduced by Bose and Nair[2] are restrictions on the parameters ν_{ij}) or the Intra- and Inter-group balanced design of the incomplete blocks. Thus we get two subtypes in type 1.

Type 2.—$\mu_{ij} = \mu$ $i \neq j$, independently of i and j. In this case we impose restrictions on the parameters λ_{ij}. We get two subtypes by allowing the parameters to satisfy the conditions of the partially balanced or the Intra- and Intergroup balanced design of the incomplete block designs.

Various methods of construction of the designs such as the geometrical and the method of differences developed by Bose[3] and used by the author have been found out and a full list of practically useful designs will be given in an elaborate paper to be published shortly. Also the Quasi-Latin squares used for double confounding in factorial experiments, the necessary and the sufficient condition for which has been given by the author in the work referred to, come out as special cases.

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1. Yates, *Technical Communication* 35, *Imp. Bureau Soil Science*. 2. Bose and Nair, *Sankhya*, 4, 337–72. 3. —, *Ann. of Eugen*, 9, 358–99.