An application of the Technique of Analysis of Variance in an experiment in the field of Educational Measurement

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Introduction

Analysis of variance is a statistical technique developed by R. A. Fisher in connection with agricultural experiments, but the technique can be widely applied in many other branches of experimental work. The application of this method to a concrete case in the field of educational measurement has been discussed here.

Suppose, there are two characteristics x and y and there are mn normal populations classified into two ways—in m classes in respect to characteristic x and n classes in respect to characteristic y. The mean of the (i, j)th population is Z_{ij} (i= ...1,..., m; j = 1,......, n) and these mn population have the common variance σ^2.

The mean Z_{ij} can be written as

$$Z_{ij} = p + A_i + B_j + C_{ij} \qquad i = 1 \ldots\ldots m, \quad j = 1 \ldots\ldots n$$

where p = mean of all Z_{ij}'s

$$= \sum_{j=1}^{n} \sum_{i=1}^{m} Z_{ij}/mn$$

A_i = the effect due to i th x characteristic.

$$= \left(\sum_{j=1}^{n} Z_{ij}/n \right) - p$$

B_j = the effect due to j th y characteristic.

$$= \left(\sum_{i=1}^{m} Z_{ij}/m \right) - p$$

C_{ij} = the interaction between i th x and jth y

$$= (Z_{ij} - A_i - B_j - p).$$

Hence from the above it follows

$$\sum_{i=1}^{n} A_i = 0, \qquad \sum_{j=1}^{n} B_j = 0, \qquad \sum_{i=1}^{m} C_{ij} = 0 \text{ for } j = 1 \ldots\ldots\ldots\ldots n$$

$$\sum_{j=1}^{n} C_{ij} = 0 \text{ for } i = 1 \ldots\ldots\ldots\ldots m$$

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Sometime it is necessary to test the following hypotheses on the basis of samples drawn from these mn population.

Hypothesis I : $A_1 = A_2 = \ldots = A_m = 0$ i.e., the main effect due to x is zero.

Hypothesis II : $B_1 = B_2 \ldots = B_n = 0$ i.e., the main effect due to y is zero.

Hypothesis III : $C_{ij} = 0$ for $i - 1, 2 \ldots\ldots m; j = 1, 2 \ldots\ldots n$ i.e., the interaction between x and y is zero.

Now consider the educational experiment where m tests are administered upon the students reading in n different schools. The number of students is different in different schools and all the students take m tests. The purpose is to test the hypotheses.

(a) Means of the tests for different school groups are not different from each other with respect to the abilities measured by the tests.

(b) The tests are not different from each other.

(c) The interaction between schools and tests is zero.

The difference among individuals is the major source of variation in psychological experiments. So the most desirable situation would be one in which every individual is subjected to all the treatments. But in many occasions this is not feasible and the present experiment is one of such type. In this experiment, one particular student cannot be taught in different schools at the same time. Hence if school's teaching has got any effect on students and if this effect varies from school to school it is not possible to control the individual difference in order to compare and find out whether there is any school difference at all. But as one student can take all the tests, the individual differences can be controlled in comparing the test effect.

An experimental design in which each subject takes all tests but cannot take them under all the conditions due to other factor, is called as 'mixed design' by Lindquist (1). A mixed design is the design in which some of the treatment comparisons are inter-subject and some are intra-subject comparisons, and the inter-subject comparisons are generally less precise than the intra-subject comparison. Lindquist describes different types of mixed designs and also the process of analysing the variances in such designs. According to him the present experiment is Type I mixed design. Here there are two factors x and y and each of the x treatments is administered to the same subject in combination with *any one* of y treatments but with each y treatment administered to different groups of subjects.

Here the total sum of square can be broken into five components as follows.

Total sum of square = between components + within components = school sum of square + error (between) + test sum of square + (school × test) sum of square + error (within).

To compute the sums of square the procedure is as follows. Let X_{ijk} = the score made by the kth student in the ith test reading in the jth school.

There are m tests, *n schools* and r_j students in the jth school.

Let N = the total number of scores obtained in the experiment $= m \sum_{j=1}^{n} r_j$,

$$\text{Let } T_{ij} = \sum_{k=1}^{r_j} X_{ijk}$$

$$T_{i.} = \sum_{j=1}^{n} \sum_{k=1}^{r_j} X_{ijk}$$

$$T_{.j} = \sum_{i=1}^{m} \sum_{k=1}^{r_j} X_{ijk}$$

$$T = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r_j} X_{ijk}$$

$$\text{Total sum of square} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r_j} X_{ij}{}^2k - T^2/N$$

$$\text{Between subject sum of square} = \sum_{j=1}^{n} \sum_{k=1}^{r_j} \Big(\sum_{i=1}^{m} X_{ijk}\Big)^2/m - T^2/N$$

Within subject sum of square = (Total sum of square)—(between subject sum of square).

$$\text{School sum of square} = \sum_{j=1}^{n} \frac{(T.j)^2}{mr_j} - T^2/N.$$

$$\text{Test sum of square} = \sum_{i=1}^{m} \frac{(Ti.)^2}{\Sigma r_j} - T^2/N.$$

Interaction i.e. (Test X School) sum of square

$$= \left(\sum_{j} \sum_{i} \frac{T_{ij}{}^2}{r_j} - T^2/N \right) \text{—(School sum of square)—(Test sum of square)}$$

Error (between) sum of square = (Between subject sum of square)—(school sum of square).

Error (within) sum of square = (Within subject sum of square)—(Test sum of square)—(Test X School sum of square).

In the present experiment there are 9 aptitude tests (objective in nature) and these are administered on 6 different schools, i.e., $m = 9$ and $n = 6$. Total number of students is 327 and the numbers of students in the six schools are

$$r_1 = 100, r_2 = 61, r_3 = 37, r_4 = 35, r_5 = 39, r_6 = 55.$$

Following the procedure outlined above, the table for **analysis of variance** is computed and this is presented below.

TABLE I

Showing the results obtained from Analysis of Variance

Source	Degrees of freedom	Sum of square	Mean sum of square	F-value
Between subject	326	50462.50	154.79	
School	5	9571.85	1914.37	15.02**
Error (between)	321	40890.74	127.38	
Within subject	2616	321345.89	122.84	
Test	8	277583.54	34697.94	2717.14**
Test X School	40	10958.12	273.95	21.45**
Errors (within)	2568	32804.23	12.77	
Total	2942	371808.48		

From Table 1 it follows that School effect, Test effect and the interaction Test X School are highly significant. This implies that

(1) Average performances of the school groups are significantly different from each other with respect to the abilities measured.

(2) Tests means are significantly different from each other.

(3) As the interaction between schools and tests is significant it can be concluded that the students in different schools do well in different tests and vice versa.

Now there are several advantages of applying this technique and these are as follows:

(1) The differences among the schools are simultaneously tested, and hence separate comparison is not necessary.

(2) It reduces the number of comparisons between the different mean values of the tests.

(3) The interaction between tests and schools can be tested by this technique which cannot be done otherwise.

REFERENCES

1. LINDQUIST, E. F. (1953)—*Design and Analysis of Experiments in Psychology and Education.* Houghton Mifflin Company. Boston.

2. MUKERJEE, M. (1965)—Construction and standardisation of a Differential Aptitude Test Battery—Unpublished D. Phil. Thesis, Calcutta University.