Fuzzy Grammars in Syntactic Recognition of Skeletal Maturity from X-Rays

AMITA PATHAK AND SANKAR K. PAL, SENIOR MEMBER, IEEE

Abstract—A hierarchical three-stage syntactic recognition algorithm using six-tuple fuzzy and seven-tuple fractionally fuzzy grammars is described for identifying different stages of maturity of bones from X-rays of hand and wrist. The primitives considered are "dot," "straight line," and "arc" as obtained elsewhere. For each arc, its memberships in the sets of "sharp," "fair," and "gentle" arcs have been considered in order to describe and interpret the structural development of epiphysis and metaphysis with growth of a child. The two algorithms are illustrated with the help of the radiograph of a 10–12-year old boy along with some "noisy" versions of the radiograph, which was artificially generated by taking into account possible variations in shape of the relevent contours in the radiograph. Relative merits of the two algorithms with respect to each other and as well as the existing nonfuzzy approach are also discussed.

I. Introduction

THE PRESENT WORK is a continuation of the previous correspondence on image description and primitive extraction using fuzzy sets [1] and is an attempt at syntactic recognition of different stages of maturity of bones from X-rays of hand and wrist using fuzzy grammar and the fuzzy primitives obtained from [1]. The ultimate aim is to be able to make computer-diagnosis of diseases and effects of malnutrition on the skeletal growth of a child.

During the growth of a child, each of the bones of the hand and wrist, as shown in Fig. 1, provides us with an invariant sequence of events that invariably occur in the same order in all individuals and cover the developmental age-span evenly and completely. These sequences therefore provide us with some basis for defining different stages of maturity (age) of the bones. The radius, ulna, metacarpals, and phalanges of the hand and wrist provide us with 28 such sequences, with events in one or another sequence occurring at almost all stages of development [2].

The problem of recognition therefore involves four major parts, namely,

1) study of the radiograph and detection of the specific bones and their location,
2) preprocessing of X-ray images with a view to extracting the edges of the different regions of bones and tissues,
3) primitive extraction of the edge-detected images, and
4) syntactic classification into one of the possible stages of skeletal maturity.

We are concerned here with the last part. The results of the previous parts have already been reported [1], [3]–[5].

Formal language theory has been applied to syntactic recognition of patterns which are rich in structural information i.e., the patterns contain most of their information in their structure rather than in numeric values [6]–[12]. To increase the generative power of a grammar for pattern recognition problems, the concept of phrase-structure grammars has been extended to stochastic grammars [7], [13] and fuzzy grammars, respectively [8], [11], [15]–[19] by randomizing and fuzzifying the use of the production rules. A fuzzy grammar produces a language that is a fuzzy set of strings with each string's membership value (lying in the interval [0, 1]), denoting the degree of belonging of the string in that language. These languages have shown some promise in dealing with patterns which possess ill-defined (fuzzy) boundaries [8], [11], [15], [16].

A three-stage hierarchical syntactic approach [9] is presented here for automatic recognition of the ages of different bone. The classifier accepts strings of primitives [1] defining approximated versions of contours in radiographs representing the epiphyses[1] and metaphyses[2] including palmar and dorsal[3] surfaces [2] as input. Two algorithms based on six-tuple fuzzy grammars and seven-tuple fractionally fuzzy grammars [14] have been used separately for classification at each stage. The primitives considered are a line segment of unit length, clockwise and counter-clockwise curves and a "dot." (By a curve here we mean a simple curve and not a curve obtained by the concatenation of simple curves.) For any such curve we have also defined its membership values corresponding to fuzzy sets of "sharp," "fair," and "gentle" curves.

The two algorithms are illustrated with the help of an X-ray image of the radius of a 10–12-year old boy. Some other distorted versions (artificially generated) of the input string are also considered for their implementation.

Manuscript received August 27, 1984; revised December 13, 1985 and June 1, 1986. This work was supported by the Medical Research Council, UK.

The authors are with the Electronics and Communication Sciences Unit, Indian Statistical Institute, Calcutta 700 035, India.

IEEE Log Number 8610217.

[1]An epiphysis, in some bones, is a separate terminal ossification which only becomes united with the main bone at the attainment of maturity.

[2]A metaphysis of a long bone is the end of the shaft where it joins the epiphysis.

[3]The palmar surface of any bone in the hand and wrist is that surface which is towards the palm of the hand. Likewise, the dorsal surface is the diametrically opposite one.

0018-9472/86/0900-0657$01.00 ©1986 IEEE

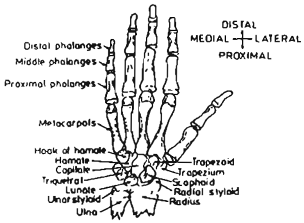

Fig. 1. Bones of hand and wrist [2].

II. Different Stages of Maturity

Fig. 2 shows different stages of skeletal maturity of radius of hand and wrist. This is considered as a typical illustration since radius contributes mostly in determining maturity score [2]. Its structural development with growth is explained below.

In the beginning, the epiphysis is totally absent (Stage A). It gradually appears above the metaphysis as a single (or, rarely, as multiple) deposit(s) of calcium with irregular outline (Stage B). After that, it gradually assumes a well-defined oval shape as seen in the radiograph (Stage C), its maximum diameter being less than half the width of the metaphysis. It continues to grow in size but becomes slightly tapering at its medial end, being more rounded at the lateral end (Stage D). Its maximum diameter now exceeds half the width of the metaphysis. In Stage E, its shape is more or less the same though it becomes larger, and a thickened white line representing the edge of the palmar surface appears within it at the distal border. In Stage F, the palmar surface of the proximal border also develops and becomes visible as a thickened white line at the proximal edge of the epiphysis. At Stage G, the palmar surface of the medial border also becomes apparent as a white line so that the three visible palmar surfaces combine to appear as a single continuous, thickened ⊂-shaped contour. The epiphysis continues to grow larger, and by Stage H it caps the metaphysis almost entirely (at one end or both). The styloid process is also much developed. Finally, at Stage I, fusion of the epiphysis and the metaphysis begins.

The features of the structural development of the radius therefore include the contour, shape, and orientation of the metaphysis and epiphysis including palmar and dorsal surfaces as appearing on the epiphysis and metaphysis with growth, and styloid process. A similar sequence of stages of structural development is also observed [2] in the other bones, namely, ulna, metacarpals, and phalanges (Fig. 1).

III. Definitions

Definition 1a): A fuzzy grammar (FG) is a six-tuple

$$FG = (V_N, V_T, P, S, J, f)$$

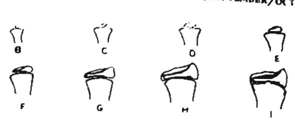

Fig. 2. Different stages of skeletal maturity of radius (Stage A, in which epiphysis is totally absent, is not shown here.)

where

- V_N set of nonterminals, i.e., labels of certain fuzzy sets on V_T^* called fuzzy syntactic categories,
- V_T set of terminals such that $V_N \cap V_T = \phi$,
- V_T^* set of finite strings constructed by concatenation of elements of V_T,
- P set of production rules,
- S starting symbol ($\in V_N$),
- J $\{r_i | i = 1, 2, \cdots, n, n = \#(P)\}$, a set of distinct labels for all productions in P, where $\#(P)$ is the number of elements in the set P,
- f mapping $f: J \rightarrow [0,1]$, $f(r_i)$ denoting the fuzzy membership in P of the rule labelled r_i.

Definition 1b): For any string x having m (≥ 1) derivation(s) in the language L(FG) generated by FG, its membership in L(FG) is given by

$$\mu_{L(FG)}(x) = \max_{1 \leq k \leq m} \left[\min_{1 \leq i \leq l_k} f(r_i^k) \right],$$

where

- k index of a derivation chain leading to x,
- l_k length of the kth derivation chain,
- r_i^k label of the ith production rule in the kth derivation.

Definition 2a): A fractionally fuzzy grammar (FFG) is a seven-tuple $FFG = (V_N, V_T, P, S, J, g, h)$ where V_N, V_T, P, S are as above, and g and h are mappings from J into the set of nonnegative integers such that

$$g(r_k) \leq h(r_k), \quad \forall r_k \in J.$$

Definition 2b): The membership of any string x having $m (\geq 1)$ derivation(s) in the language L(FFG) generated by FFG is

$$\mu_{L(FFG)}(x) = \sup_{1 \leq k \leq m} \frac{\sum_{j=1}^{l_k} g(r_j^k)}{\sum_{j=1}^{l_k} h(r_j^k)}$$

where 0/0 is taken to be zero by convention.

IV. Classification Algorithm

Remarks: In this section, G has been used to denote a specific type of grammar. For the fuzzy grammar approach, G denotes a fuzzy grammar, while for the fractionally fuzzy grammar approach it denotes a fractionally fuzzy grammar (FFG).

The symbol $\in_C$ used denotes "is classified into." (This binary relation is defined later on.)

We have defined $\mu_S(b)$, $\mu_F(b)$, and $\mu_G(b)$, the degrees of membership of a curve b in the set of sharp, fair, and gentle curves, respectively, as follows.

A. Determination of μ_S, μ_F, and μ_G Values

For any curve b, the degree of arcness $\mu_{arc}(b)$ has been defined in the primitive extraction algorithm [1] as

$$\mu_{arc}(b) = \left(1 - \frac{l}{p}\right)^{F_e}$$

where l is the length of the line segment joining the two extreme points of the arc b, p is the length of the arc b and F_e is a suitably chosen exponential fuzzifier with $F_e > 0$. Clearly, when the arc b is a line segment, we have $p = l$ so that $\mu_{arc}(b) = 0$, whatever F_e may be. Also, $\mu_{arc}(b)$ can never attain the value 1 although it does approach that value as the sharpness of b increases, so that $\mu_{arc} \in [0, 1)$.

For any curve b for which $\mu_{arc}(b) > 0$, its degrees of membership $\mu_S(b)$, $\mu_F(b)$, and $\mu_G(b)$ to the fuzzy sets of sharp, fair, and gentle curves, respectively, may be defined as

$$\mu_S(b) = f_S(\mu_{arc}(b)) \tag{2a}$$

$$\mu_F(b) = f_F\left(|\mu_{arc}(b) - \frac{1}{2}|\right) \tag{2b}$$

and

$$\mu_G(b) = f_G(\mu_{arc}(b)) \tag{2c}$$

such that

a) $f_G(\cdot)$ and $f_F(\cdot)$ are monotonically decreasing functions over [0, 1] and [0, 1/2], respectively;
b) $f_S(\cdot)$ is a monotonically increasing function over [0, 1]; and
c) $\mu_S(b)$, $\mu_F(b)$, and $\mu_G(b)$ all take values in [0, 1] only.

For example, we can take

$$\mu_S(b) = S\left(\mu_{arc}(b); 0, \frac{1}{2}, 1\right) \tag{3}$$

$$\mu_F(b) = \begin{cases} S\left(\mu_{arc}(b); 0, \frac{1}{4}, \frac{1}{2}\right), & \text{if } 0 \leq \mu_{arc}(b) \leq \frac{1}{2} \quad (4a) \\ 1 - S\left(\mu_{arc}(b); \frac{1}{2}, \frac{3}{4}, 1\right) & \text{if } \frac{1}{2} \leq \mu_{arc}(b) \leq 1 \quad (4b) \end{cases}$$

$$\mu_G(b) = 1 - S\left(\mu_{arc}(b); 0, \frac{1}{2}, 1\right) \qquad \mu_{arc}(b) \in (0, 1) \tag{5}$$

where S denotes standard S function [20] such that

$$S(x; \alpha, \beta, \gamma) = \begin{cases} 0, & x \leq \alpha \\ 2\left(\frac{x - \alpha}{\gamma - \alpha}\right)^2, & \alpha \leq x \leq \beta \\ 1 - 2\left(\frac{\gamma - x}{\gamma - \alpha}\right)^2, & \beta \leq x \leq \gamma \\ 1, & x \geq \gamma \end{cases} \tag{6}$$

with $\beta = (\alpha + \gamma)/2$.

As mentioned earlier, $\mu_{arc} = 0$ (which corresponds to a straight line) is not included in computing μ_S, μ_F, and μ_G values. However, even if we put $\mu_{arc} = 0$ in (3)–(5), the values we get namely, $\mu_G = 1$, $\mu_F = \mu_S = 0$ do not contradict our intuition, since a straight line can be looked upon as the *most* gentle curve. Again, since the boundaries among the fuzzy sets sharp, fair, and gentle are not hard, any curve may have nonzero membership values for all three sets.

As an example, we consider $\mu_{arc}(b) = 0.22$, 0.52, and 0.82. Then from the following table, the degree of membership of these b, as expected, is found to be maximum for the sets gentle, fair, and sharp, respectively.

	$\mu_{arc}(b)$		
	0.22	0.52	0.82
μ_S	0.097	0.539	0.935
μ_F	0.387	0.997	0.181
μ_G	0.903	0.461	0.065

Besides using the standard functions (3)–(5), one can also use a function

$$f(\mu_{arc}) = \left[1 + \left(\frac{|\mu_{arc} - \mu|}{F_d}\right)^{F_e}\right]^{-1} \tag{7}$$

(where μ is some reference constant) that approximates the standard functions. For $\mu = 1$, 0.5, and 0, (7) represents the membership function corresponding to the sets sharp, fair, and gentle, respectively. Positive constants F_e and F_d are the fuzzifiers which control the amount of fuzziness in a set.

B. Algorithm

The structure of the three-stage hierarchical procedure is depicted in Fig. 3. At each stage context-free grammars with $V_T = \{a, b, \bar{b}, c\}$ have been used. The a, b, $\bar{b}$, and c denote a line segment of unit length, a clockwise curve, an anticlockwise curve, and a dot, respectively.

Let x denote the string representing the contour of the epiphysis and y the string representing the interior of the epiphysis contour, i.e., the boundaries of the image of the palmar surface of the epiphysis.

Stage I (primary classification): We define five classes C_i, $i = 1, 2, \cdots, 5$ as $C_1 = \{A\}$, $C_2 = \{B\}$, $C_3 = \{C, D, E\}$, $C_4 = \{F, G, H\}$, $C_5 = \{I\}$.

Let G_i denote the grammar corresponding to class C_i, $i = 1, 2, \cdots, 5$ and $L(G_i)$ the language generated by G_i.

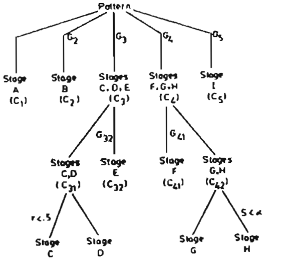

Fig. 3. Three-stage hierarchical classification scheme.

If x is found to be the empty string (λ), we infer the class C_1 (Stage A). If not, and if x is parsed by the first stage grammar, then

$$x \in_C L(G_k), \text{if } \mu_{L(G_k)}(x) = \max_{2 \le i \le 5} \mu_{L(G_i)}(x), \qquad k = 2,3,4,5.$$

Ties, if present, can be dealt with in a number of ways. A statistical approach is to use randomization techniques whereby the final decision is based on the outcome of a random experiment, usually simulated with the help of random numbers.

The reasons for adopting this particular form of clustering of events A–I are rather obvious from Fig. 2. For example, each of the stages A, B, and I is unique in itself and hence is put in a separate class. Again, the forms of x in Stages C, D, and E bear greater similarity to each other than to strings from other classes. These are, therefore, put together in C_3. The same reasoning applies to F, G, and H. Of course, there is the possibility that C_3 and C_4 will overlap, mainly because of the similarities in E and F in respect to x. Provisions have been made at the next stage for minimizing the error resulting from this.

If $x \in_C L(G_i)$, $i = 1,2,5$, then stop; otherwise, go to the second stage.

Stage 2: We come here if in the first stage $x \in_C L(G_3)$ or $x \in_C L(G_4)$. We now bring y into the picture. If $x \in_C L(G_3)$, go to step 2.1, and if $x \in_C L(G_4)$, go to step 2.2.

Step 2.1: a) If y can be parsed by means of the second-stage grammar and if

$$\mu_{L(G_{32})}(y) = \max\left[\mu_{L(G_{32})}(y), \mu_{L(G_{41})}(y)\right],$$

i.e.,

$$y \in_C L(G_{32})$$

then decide on Stage E. If not, go to step 2.3. b) If y can not be parsed by means of the second stage grammar, go to step 3.1.

Step 2.2: a) If y can be parsed by means of the second stage grammar and

$$\mu_{L(G_{41})}(y) = \max\left[\mu_{L(G_{32})}(y), \mu_{L(G_{41})}(y)\right],$$

TABLE I
LIST OF STRINGS FROM EACH CLASS

Class	Strings
C_2	$X = c$
C_3	$X = a^m b a^n b$, $m, n \geq 0$ and b is fair or sharp
C_4	$X = P b a^q b Q$, b is sharp or fair, $q \geq 0$ with $P = a^r$ or $a^r b a^s$ (b 'gentle'); $Q = a^t R$ or $a^t R^2$; and $R = b$ (sharp or fair), $r, s, t > 0$.
C_5	$X = L'\bar{b}E\bar{b}L''$, b sharp or fair, with $L = a^x$, $a^x M a^y M a^z$ or $a^x M M a^z$; $M = b$ or $\bar{b}$ (gentle), $x, y, z > 0$; $L' = L$, Lb, or LbL; $L'' = L$, bL, or LbL, b 'sharp' or 'fair'; $E = GF$, G^2F, GFa^y or G^2Fa^y, $x, y > 0$; $F = LbLb$; $G = a^x b$
C_{32}	$Y = L^* b L^* b$, b 'not gentle' with $L^* = L$, ML, LM, $a^x M a^y$, $M a^x M a^y M$, $a^x M a^y M$ or $M a^x M a^y$; L, M are as above and $x, y > 0$
C_{41}	$Y = L^* b L^* \bar{b} L^* b L^* b L^* \bar{b} L^* b$, b 'not gentle'. L^* is as above.

i.e., if $y \in_C L(G_{41})$, decide on Stage F. If not, go to step 2.3. b) If y can not be parsed by the second stage grammar go to step 3.2.

Step 2.3: We come here if there are contradictory decisions in the first two stages, that is, either i) $x \in_C L(G_3)$ but $y \in_C L(G_{41})$ or ii) $x \in_C L(G_4)$ but $y \in_C L(G_{32})$. We can tackle this situation in either of two ways.

1) We can completely ignore the first-stage information and take the second-stage decision to be final. However, such decisionmaking is not sound.

2) We can combine the information obtained at both stages and then come to a final decision. This can be done in a number of ways. For instance, writing

$$a_3 = \mu_{L(G_3)}(x), \quad a_4 = \mu_{L(G_4)}(x),$$
$$b_3 = \mu_{L(G_{32})}(y), \quad b_4 = \mu_{L(G_{41})}(y),$$

we decide on the class C_{32} (Stage E), if $\phi_3 = \max[\phi_3, \phi_4]$ and on the class C_{41} (Stage F), otherwise, where ϕ_i, $i = 3,4$, can be defined in one of the following ways (using collective or connective property):

1) $\phi_i = (a_i + b_i)/2$
2) $\phi_i = (a_i^2 + b_i^2)^{1/2}$
3) $\phi_i = \min(a_i, b_i)$
4) $\phi_i = \max(a_i, b_i)$.

It can be observed from Fig. 2 that the interior of the epiphysis contour is empty in Classes C and D but not in classes E, F, G, and H. It is this additional information that we utilize at this stage. The forms of y in $C_{31} = \{C, D\}$, $C_{32} = \{E\}$, $C_{41} = \{F\}$, and $C_{42} = \{G, H\}$ are distinct enough to facilitate differentiation by syntactic means.

Stage 3:

Step 3.1: Determine D_E (the maximum diameter of the epiphysis) and W_M (the width of the metaphysis). If $r = D_E/W_M \leq 0.5$, decide on event C; otherwise, decide on D.

Step 3.2: Determine S_E (the slope of the proximal edge of the epiphysis at the medial end) and S_M (the slope of the distal edge of metaphysis at the medial end). There are numerous algorithms available in the literature for this purpose [21]. If $S = S_E - S_M$ is less than some predetermined α, suitably small, then decide on event H; otherwise, decide on event G.

In practice, S_E and S_M are reflected by the degree of arcness of the curve at the medial end of the epiphysis contour.

In this stage, the classification is not, strictly speaking, syntactic in nature. We have merely made use of some differences between C and D, and between G and H, as described before, to facilitate classification.

The sample strings from each class used for constructing the grammars is given in Table I. The grammars for the first stage are given in Table II while those for the second stage are given in Table III.

It is not difficult to verify that every one of the representatives of each class, given in Table I, has by our grammars, a maximum membership for the language corresponding to its own class, its membership in all other languages (at the same stage) being less or at most as large. The classification algorithm is described in the structured format of Listing 1.

Listing 1. Classification Algorithm.

```
Procedure CLASSIFY:
    BEGIN;
    IF x ∈ L(G1) THEN decide on A;
      ELSE IF x ∈ L(G2) THEN decide on B;
            ELSE IF x ∈ L(G3) THEN
                  DO;
                    IF y can be parsed by second stage grammar THEN
                      DO;
                        IF μL(G31)(y) =
                          max[μL(G31)(y), μL(G41)(y)]
                          THEN decide on E;
                        ELSE DO;
                          a3 ← μL(G3)(x);
                          a4 ← μL(G4)(x);
                          b3 ← μL(G31)(y);
                          b4 ← μL(G41)(y);
                          compute φ3;
                          compute φ4;
                          /* Definitions of
                          φ3, φ4 given in
                          text */
                          IF φ3 = max(φ3, φ4)
                          THEN decide on E;
                          ELSE decide on F;
                          END;
                      END;
                        ELSE DO;
                          DE ← maximum diameter of the epiphysis;
                          WM ← width of the metaphysis;
                          r ← DE/WM;
                          IF r ≤ 0.5 THEN decide on C;
                          ELSE decide on D;
                          END;
                  END;
                  ELSE IF x ∈ L(G4) THEN
                    DO;
                        IF y can be parsed by second stage grammar
                          THEN
                            DO;
                              IF μL(G41)(y) = max[μL(G31)(y), μL(G41)(y)] THEN decide on F;
                                ELSE DO;
                                  a3 ← μL(G3)(x);
                                  a4 ← μL(G4)(x);
                                  b3 ← μL(G31)(y);
                                  b4 ← μL(G41)(y);
                                  compute φ3;
                                  compute φ4;
                                  IF φ4 = max (φ3, φ4) THEN decide on F; ELSE decide on E;
                                  END;
                            END;
                        ELSE DO;
                              SE ← slope of the proximal edge of the epiphysis at the medial end;
                              SM ← slope of the distal edge of the epiphysis at the medial end;
                              S ← SE − SM;
                              IF S < α/* α predetermined */ THEN decide on H;
                              ELSE decide on G;
                          END;
                    END;
                    ELSE IF x ∈ L(G5) THEN decide on I;
    END.
```

TABLE II
PRODUCTION RULES FOR THE FIRST STAGE

Srl.	Production Rules	Membership Values $(FG)_3$	$(FG)_4$	$(FG)_5$	g_i/h_i Values $(FFG)_3$	$(FFG)_4$	$(FFG)_5$
1	$S \to AA$	1	0	0	10/10	0/10	0/10
2	$S \to AAA$	0	1	0	0/10	10/10	0/10
3.	$S \to ACA$	0	1	0	0/10	10/10	0/10
4	$S \to DS$	0	1	0	0/5	5/5	0/5
5	$S \to SC$	0	1	0	0/5	5/5	0/5
6	$S \to LFM$	0	0	1	0/10	0/10	10/10
7	$A \to BC$	1	1	0	0/0	0/0	0/5
8	$A \to C$	1	0	0	1/1	0/1	0/1
9	$B \to aB$	1	1	1	0/0	0/0	0/0
10	$B \to a$	1	1	1	0/0	0/0	0/0
11	$C \to b$	$\mu_F(b)$	$\mu_F(b)$	1	$g_F(b)/5$	$g_F(b)/5$	0/5
12	$C \to b$	$\mu_S(b)$	$\mu_S(b)$	1	$g_S(b)/5$	$g_S(b)/5$	0/5
13	$D \to BE$	0	1	1	0/1	1/1	1/1
14	$E \to b$	0	$\mu_G(b)$	$\mu_G(b)$	0/5	$g_G(b)/5$	$g_G(b)/5$
15	$F \to \bar{b}G\bar{b}$	0	0	1	0/5	0/5	5/5
16	$G \to AHH$	0	0	1	0/1	0/1	1/1
17	$G \to AHHB$	0	0	1	0/1	0/1	1/1
18	$G \to AG$	0	0	1	0/1	0/1	1/1
19	$H \to IC$	0	0	1	0/1	0/1	1/1
20	$I \to B$	0	0	1	0/1	0/1	1/1
21	$I \to BKB$	0	0	1	0/1	0/1	1/1
22	$I \to KB$	0	0	1	0/1	0/1	1/1
23	$I \to BK$	0	0	1	0/1	0/1	1/1
24	$K \to \bar{b}$	0	0	$\mu_G(\bar{b})$	0/5	0/5	$g_G(\bar{b})/5$
25	$K \to b$	0	0	$\mu_G(b)$	0/5	0/5	$g_G(b)/5$
26	$L \to ICI$	0	0	1	0/2	0/2	2/2
27	$L \to IC$	0	0	1	0/2	0/2	2/2
28	$L \to I$	0	0	1	0/2	0/2	2/2
29	$M \to ICI$	0	0	1	0/2	0/2	2/2
30	$M \to CI$	0	0	1	0/2	0/2	2/2
31	$M \to I$	0	0	1	0/2	0/2	2/2

TABLE III
PRODUCTION RULES FOR THE SECOND STAGE

i	Production Rules	Membership Values $(FG)_{32}$	$(FG)_{41}$	g_i/h_i Values $(FFG)_{32}$	$(FFG)_{41}$
1	$S \to A$	1	0	10/10	0/10
2	$S \to BB$	0	1	0/10	10/10
3	$A \to DD$	1	1	0/0	0/0
4	$B \to DFD$	0	1	0/2	2/2
5	$D \to Eb$	$1-\mu_G(b)$	$1-\mu_G(b)$	$\bar{g}_G(b)/2$	$\bar{g}_G(b)/2$
6	$D \to b$	0	$1-\mu_G(b)$	0/2	$\bar{g}_G(b)/2$
7	$E \to H$	1	1	0/0	0/0
8	$E \to HJH$	1	1	0/0	0/0
9	$E \to JE$	1	1	0/0	0/0
10	$E \to HL$	1	1	0/0	0/0
11	$E \to HJHL$	1	1	0/0	0/0
12	$E \to JEL$	1	1	0/0	0/0
13	$E \to J$	0	1	0/0	0/0
14	$F \to \bar{b}$	0	$1-\mu_G(\bar{b})$	0/2	$\bar{g}_G(\bar{b})/2$
15	$F \to E\bar{b}$	0	$1-\mu_G(\bar{b})$	0/2	$\bar{g}_G(\bar{b})/2$
16	$H \to aH$	1	1	0/0	0/0
17	$H \to a$	1	1	0/0	0/0
18	$J \to K$	1	1	0/0	0/0
19	$J \to KK$	1	1	0/0	0/0
20	$J \to KHK$	1	1	0/0	0/0
21	$K \to \bar{b}$	$\mu_G(\bar{b})$	$\mu_G(\bar{b})$	$g_G(\bar{b})/2$	$g_G(\bar{b})/2$
22	$K \to b$	$\mu_G(b)$	$\mu_G(b)$	$g_G(b)/2$	$g_G(\bar{b})/2$
23	$L \to J$	1	1	0/0	0/0
24	$L \to JL$	1	1	0/0	0/0

C. *Some Guidelines and Observations*

It should be noted that for all the grammars used in the first stage (as also in the second stage), we have used the same production rules but of course, with different values of $\mu_i(g_i/h_i)$. This is because of the basic similarity between the patterns of the different classes in the first stage (second stage).

A brief discussion of the manner in which the weights of the production rules are assigned for the two approaches is in order.

The Fuzzy Grammar Approach: At either stage, some of the rules have weights of either zero or one for the different classes. The interpretation is obvious; a rule has membership 0 for the grammar of a class if it plays no part in the generation of the language corresponding to that class. On the other hand, if a rule plays with certainty, a role in the generation, it has membership 1 for that class. Some rules have weights of a third type—they depend on the values of μ_S, μ_F, or μ_G for the corresponding curves. For example, rule numbered 25 (Table II) has a weight $\mu_G(b)$ for the grammar $(FG)_5$. This means that its weight is dependent on the gentleness of the curve in the sense that the gentler the curve, the greater the weight of the rule.

Fractionally Fuzzy Grammar Approach: For assigning g_i/h_i values to different production rules we have been guided by the criteria laid down by DePalma and Yau [14] which are as follows:

First, a rule which cannot help to distinguish one class from another can be given the value 0/0, and would then have no effect on the final membership assuming some rule i, for which $h_i \neq 0$, is also applied.

Second, a rule for which h_i is small has little effect on the final membership of any string generated by that rule

Third, any rule for which h_i is large has a large effect on the final membership of any string generated by using that rule.

Fourth, if rule i is used, the fuzzy membership of the string is changed in the direction towards the value g_i/h_i by the application of rule i. Thus if g_i/h_i is close to zero, it is decreased.

Finally, a rule which is used in all the strings can be given a membership value which could serve as a starting point from which we could subtract by rules with $g_i/h_i = 0$ and to which we could add by rules with $g_i/h_i = 1$.

For some of the rules, the g-values have been made dependent on μ_S or μ_F or μ_G-values by means of nondecreasing integer-valued functions $g_i(b)$ defined, for $i = S, F, G$ as

$$g_i(b) = \text{int}\,[h \times \mu_i(b)], \quad \text{if } h\mu_i(b) - \text{int}\,[h \times \mu_i(b)] \leq \frac{1}{2}$$
$$= 1 + \text{int}\,[h \times \mu_i(b)], \quad \text{otherwise} \tag{8}$$

where $\text{int}\,[x]$ = integer part of x, x being any real number, and h is the corresponding h_i-value.

Another issue of concern to us is whether the absolute dimensions of the subject and hence those of the epiphysis and the metaphysis may affect the results of the classification. The magnification or reduction of a given image of the epiphysis will cause two types of changes:

1) the straight line segments in the image will increase/decrease in length; and
2) the curves in the image will become gentler and sharper, or both.

The first is taken care of in the grammars by means of production rules of the type given by rules 9 and 10 for Stage 1 and 16 and 17 for Stage 2. Changes of the second type will, in general, change the weights of those rules which depend on μ_S, μ_F, or μ_G values. However, the relative values of the weights remain unchanged, and hence the final outcome is not affected.

Finally, we would also like to point out that although in Tables II and III the membership values of certain productions are taken to be zero and one, we consider this to be an oversimplification of the situation. It would be more realistic to have for such rules membership values which are close to zero or one. This entails that they be estimated with the help of a large number of samples with known classification, however. In other words, supervised learning is required.

D. Some Practical Considerations

The classification algorithm as described before has been developed on the basis of the description given in Section II [2] for the different stages. As far as possible, the minor variations in pattern that are quite likely to occur have been accounted for in the grammars.

However, in practice, due to the limitations of the pre-processing (digitization, thresholding, enhancement, and contour extraction) algorithms, it is quite likely that we may encounter situations in which the above algorithm will need some modification for machine identification of different stages. For instance, in the cases of C_3 and C_4 (though it is very unlikely for C_3), we may obtain an edge-detected image in which the contours representing the epiphysis and the metaphysis are partly joined. In such a case, we skip the method of primary classification in Step 1 and proceed from Step 2 for final classification.

Fig. 4. Input image.

V. Implementation and Results

Fig. 4 shows an edge-detected version of an 128 × 145 dimensional image of radius of 10–12-year old boy [4]. These contours are extracted using contrast intensification operator along with S and π membership functions. The computer-based description of the relevant contours (with a 90° clockwise rotation of the image) after a) octal code representation, b) smoothing to remove the spurious wiggles, and c) segmentation [1], is as follows.

1) *Starting Point of Contour*; (22, 1)
   End of Contour: (129, 1)
   Description of the Contour:
   $L_3\ A_{0.465}\ L_2\ \bar{A}_{0.541}\ L\ A_{0.272}\ L\ A_{0.272}$
   $L_{15}\ A_{0.533}\ L_7\ A_{0.272}\ L_3\ A_{0.465}\ L_2\ A_{0.541}$
   $\bar{A}_{0.272}\ L\ A_{0.272}$;
2) *Starting Point of Contour*: (24, 1)
   End of Contour: (119, 1)
   Description of the Contour:
   $\bar{A}_{0.272}\ L_2\ A_{0.377}\ L_3\ \bar{A}_{0.644}\ L_2\ A_{0.488}\ L_{12}$
   $A_{0.757}\ L_4\ \bar{A}_{0.348}\ L_4\ A_{0.644}\ \bar{A}_{0.272}\ A_{0.272}$
   $L_8\ \bar{A}_{0.702}\ L_3\ A_{0.816}\ L_2\ A_{0.644}\ A_{0.541}\ L_4$
   $\bar{A}_{0.765}\ L_3\ A_{0.465}\ L\ \bar{A}_{0.541}\ A_{0.645}\ A_{0.587}$
   $A_{0.559}\ \bar{A}_{0.816}\ A_{0.707}\ A_{0.272}\ \bar{A}_{0.47}\ L_3\ A_{0.429}$
   $L_5\ A_{0.816}\ L_6\ A_{0.272}\ L\ A_{0.377}\ L_2\ A_{0.272}\ L_2$;
3) *Starting Point of Contour*: (22, 64)
   The Contour is Closed
   Description of the Contour:
   $L_{11}\ A_{0.86}\ L_4\ A_{0.272}\ L\ \bar{A}_{0.662}\ L_4\ A_{0.598}\ L_7$
   $\bar{A}_{0.272}\ A_{0.765}\ \bar{A}_{0.816}\ \bar{A}_{0.272}\ L\ A_{0.765}$.

Here, L, A, and $\bar{A}$ denote the straight line, clockwise arc, and counterclockwise arc, respectively. (This was the notation used in [1] to denote what we have called a, b, and $\bar{b}$ in this work.) Suffices of L and A represent the number of

TABLE IV
μ_{arc}- AND μ_G-VALUES OF ARCS IN STRINGS y, y_1, y_2, y_3, y_4

String		μ_{arc} AND μ_G-VALUES OF ARCS IN THE STRINGS								
y	μ_{arc}	0.860	0.272	0.662	0.598	0.272	0.765	0.816	0.272	0.765
	μ_G	0.039	0.852	0.228	0.323	0.852	0.110	0.068	0.852	0.110
y_1	μ_{arc}	0.272	0.816	0.765	0.662	0.272	0.765	0.860	0.816	
	μ_G	0.852	0.068	0.110	0.228	0.852	0.110	0.039	0.068	
y_2	μ_{arc}	0.860	0.765	0.662	0.662	0.816	0.860			
	μ_G	0.039	0.110	0.228	0.228	0.068	0.039			
y_3	μ_{arc}	0.816	0.598	0.598	0.272	0.272	0.765	0.272	0.765	0.816
	μ_G	0.068	0.323	0.323	0.852	0.852	0.110	0.852	0.110	0.068
y_4	μ_{arc}	0.272	0.765	0.598	0.662	0.816	0.765	0.765		
	μ_G	0.852	0.110	0.323	0.228	0.068	0.110	0.110		

TABLE V
LIST OF LEFT-MOST DERIVATIONS OF STRING y AND THEIR EVALUATIONS

Leftmost Derivation of String	Evaluation of Derivation				Membership of String in			
	$(FG)_{32}$	$(FG)_{41}$	$(FFG)_{32}$	$(FFG)_{41}$	$L(FG)_{32}$	$L(FG)_{41}$	$L(FFG)_{32}$	$L(FFG)_{41}$
a) $(2)(4)(5)(7)(16)^{10}(17)(15)(8)(16)^3(17)(18)(22)(17)(5)(7)(16)^3(17)(4)(5)(10)(16)^6(17)(23)(18)(21)(14)(5)(9)(18)(21)(7)(17)$	0	0.110	0.156	0.594				
b) $(2)(4)(5)(7)(16)^{10}(17)(15)(8)(16)^3(17)(18)(22)(17)(5)(7)(16)^3(17)(4)(5)(10)(16)^6(17)(23)(18)(21)(15)(13)(18)(21)(5)(7)(17)$	0	0.110	0.281	0.719				
c) $(2)(4)(5)(8)(16)^{10}(17)(18)(22)(16)^3(17)(15)(7)(17)(5)(7)(16)^3(17)(4)(5)(10)(16)^6(17)(23)(18)(21)(14)(5)(9)(18)(21)(7)(17)$	0	0.110	0.219	0.656	0	0.110	0.281	0.719
d) $(2)(4)(5)(8)(16)^{10}(17)(18)(22)(16)^3(17)(15)(7)(17)(5)(7)(16)^3(17)(4)(5)(10)(16)^6(17)(23)(18)(21)(15)(13)(18)(21)(5)(7)(17)$	0	0.110	0.281	0.719				

line units and the degree of arcness μ_{arc} of the arc A, respectively. To explain the meaning of the descriptions given above, let us consider for example, the contour number (3).

The starting point is given as (22, 64), which means that the location of the point at which the scan of the contour begins with respect to the coordinate (m, n)-axis shown in Fig. 4, is (22, 64). As the contour is closed, the end-point of the contour is the same, i.e., (22, 64). The contour starts with a line segment of eleven units followed by a clockwise curve whose degree of arcness is 0.86 and so on, and finally it terminates with a clockwise curve having $\mu_{arc} = 0.765$. Since we are interested only in the epiphysis and metaphysis, other contours of the image (Fig. 4) are not considered.

From this image pattern we find that the contours representing the epiphysis and the metaphysis are partly joined. So we proceed directly from Step 2 of the algorithm. Here we have the string corresponding to the palmar and dorsal surface

$$y = a^{11}ba^4ba\bar{b}a^4ba^7\bar{b}b\bar{b}\bar{b}ab. \tag{9}$$

The values of μ_{arc} and μ_G for the sequence of arcs in this string are given in Table IV. The values of μ_G and g_G for these arcs are computed, with (5) and (8).

The different derivations for the string y given in (9) as well as their corresponding evaluations are given in Table V. As is evident from the table, the string is classified into C_{41}; that is, the input image (Fig. 4) is identified by both approaches as being in stage F as far as maturity of the radius is concerned.

Let us consider again the contours of different regions in Fig. 4. These are seen to have some staircase lines, wiggles and minor arcs of two to three pixels which have been generated during its edge-detection process [4]. To extract primitives, four different smoothers were used before hand whose purpose was to make the contours as straight as

TABLE VI
EVALUATION OF DERIVATIONS OF y_1, y_2, y_3, y_4 AND THEIR MEMBERSHIP VALUES

String	Derivation	Evaluation of Derivation $(FG)_{32}$	$(FG)_{41}$	$(FFG)_{32}$	$(FFG)_{41}$	Membership of String in $L(FG)_{32}$	$L(FG)_{41}$	$L(FFG)_{32}$	$L(FFG)_{41}$
y_1	1	0	0.110	0.267	0.733	0	0.110	0.267	0.7 33
	2	0	0.068	0.267	0.733				
y_2	1	0.039	0	0.636	0.182	0.039	0.772	0.636	1
	2	0	0.772	0.462	1				
y_3	1	0.068	0	0.714	0.357	0.68	0.148	0.714	0.813
	2	0	0.068	0.188	0.625				
	3	0	0.068	0.313	0.750				
	4	0	0.110	0.250	0.688				
	5	0	0.148	0.375	0.813				
	6	0	0.110	0.250	0.688				
	7	0	0.110	0.375	0.813				
	8	0	0.110	0.375	0.813				
	9	0.068	0	0.607	0.250				
	10	0	0.110	0.375	0.813				
	11	0	0.068	0.250	0.688				
	12	0	0.068	0.250	0.688				
	13	0.068	0	0.643	0.286				
y_4	1	0.068	0	0.458	0.042	0.885	0.068	0.110	0.4580.885
	2	0	0	0.110	0.346				
	3	0.068	0	0.458	0.042				

possible by eliminating such undesirable elements. The string y (9), in fact, corresponds to such a smoothed (approximated) version of the contour of the palmar–dorsal surface.

Therefore, if there is any such variation in contour pattern that might occur because of the inherent variability of the classes, these can either be removed or be reduced greatly leaving behind some gentle curves (i.e., some gentle curve may remain in the straighter part even after smoothing) during their primitive extraction operation. Such possibilities have also been accounted for in the grammars. For example, the string y may take one of the following typical forms (artificially generated), among others, for Stage F.

1) $y_1 = a^4ba^3ba^6\bar{b}a^5ba^8\bar{b}aba\bar{b}ab$
2) $y_2 = a^{11}ba^5\bar{b}a^5ba^9ba\bar{b}a^2b$
3) $y_3 = a^{11}ba^6\bar{b}a^4ba^2ba^2\bar{b}bba\bar{b}b$
4) $y_4 = a^8\bar{b}ba^4\bar{b}a^5ba^{10}b\bar{b}b$.

For these strings also, the values of μ_{arc} and μ_G for the sequence of arcs are given in Table IV. The evaluations of their different derivations as well as the corresponding memberships are shown in Table VI. To limit the size of the paper, the details of their parses are not shown. In each case, the string is identified as undergoing Stage F by both approaches.

VI. Discussion

Two different syntactic recognition algorithms based on fuzzy and fractionally fuzzy grammars are developed here for identifying stages of bone maturity from X-ray images using the primitives extracted in the earlier work [1]. Of the two approaches, the fractionally fuzzy one has a slight edge over the other because of the following reasons [14].

With a parsing algorithm that requires backtracking, it is not just sufficient to keep track of the derivation tree alone when a fuzzy grammar is being used. The fuzzy value at each step must also be remembered at each node, so that the memory requirements are greatly increased for many practical problems. (This, incidentally, places a fuzzy grammar at a disadvantage with respect to a non-fuzzy grammar too.) With a fractionally fuzzy grammar, however, backtracking poses no problems, as we only need to subtract the g and h values for the rule being eliminated from the respective running totals.

A second drawback of fuzzy grammar in pattern recognition is the fact that all strings in L(FG) can be classified into a finite number of subsets by their membership in the language. The number of such subsets is strictly limited by the number of productions in the grammar. With a fractionally fuzzy grammar, this problem does not arise.

An algorithm for recognizing maturity using ordinary grammars had also been reported [22] by the authors. In that approach, the sets of sharp, fair, and gentle curves were sharply defined by means of thresholds on the μ_{arc} values. Separate grammars were defined for the different classes using the same three-stage hierarchical procedures. In the present algorithms, the sets of sharp, fair, and gentle curves have been treated as fuzzy subsets so that, in general, any arc can have nonzero (but not equal) memberships in all three. The incorporation of the element of fuzziness in defining sharp, fair, and gentle curves in the present algorithms has enabled us to work with a smaller number of primitives. By introducing fuzziness in the physical relations among the primitives, it has also been possible to use the same set of production rules and nonterminals at each stage.

However, for a given stage, the different production rules of the single grammar used therein are given different weights for the classes considered at that stage, to reflect the characteristics peculiar to that class. The grammars are, in general, ambiguous, but different parses of a single string may have distinct weights generally, depending upon the weights of the rules in the parse lists. The degree of belonging to the language corresponding to a given class is taken to be equal to the largest of the weights, for that class, of its different parses. The string is finally assigned to a class to which its degree of membership (belonging) is maximum. Therefore, we may need to parse an input string with only one grammar at each stage, unlike the case of the nonfuzzy approach [22] where we may have to parse each string by more than one grammar in general, at each stage. However, this has to be balanced against the fact that the grammars used here are not as simple as the corresponding nonfuzzy grammars [22]. Furthermore, these grammars need not be unambiguous, whereas non-ambiguity is an absolutely necessary requirement for the nonfuzzy approach.

In this connection mention must be made of the attributed grammars [23] to tackle similar situations where the patterns are having shapes slightly differing in details for different classes. The local shape information of the palmar and dorsal surfaces of X-ray image was used in extracting primitives [1] and in the present work the global structural information is incorporated by the weighted production rules. These two steps are combined into one in case of attributed grammar, i.e., the production rule is used to guide the primitive extraction. In attributed grammars, semantic information about the shape of a curve is borne by the attributes, namely direction, curve length, total angular change, and degree of declination. Since the information carried by primitives is of a high order, the production rules can be made simple. In our method, semantic information is carried in the μ_S, μ_F, and μ_G values of a curve and in the length of a line segment.

It is to be mentioned here that the descriptions of the different stages of maturity are standard and are taken from the book of Tanner *et al.* [2]. They have emphasized the point that samples from the same stage may exhibit a great deal of variation. In developing the grammars, we have taken into account all such variations. In fact, the noisy versions (Section V) of the input string generated artificially also takes into account those considerations. The robustness of the algorithm has been exhibited by the correct classification of the noisy inputs. Furthermore, the recognition ambiguity (as seen from Figs. 2 and 3) lies mostly between classes E and F, and we have considered patterns from Stage F to demonstrate the robustness of the algorithm.

Acknowledgment

The authors wish to acknowledge Professor D. Dutta Majumder and Dr. R. A. King for their interest in this work and Mrs. S. De Bhowmick and Mr. N. Chatterjee for typing the manuscript. Authors' thanks are also due to Dr. A. A. Hashim and Prof. J. M. Tanner for their assistance in initiating the project at Imperial College, London.

References

[1] S. K. Pal, R. A. King, and A. A. Hashim, "Image description and primitive extraction using fuzzy sets," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-13, no. 1, pp. 94–100, 1983.

[2] J. M. Tanner, R. H. Whitehouse, W. A. Marshall, M. J. R. Healy, and H. Goldstein, *Assessment of Skeletal Maturity and Prediction of Adult Height (TW2 Method)*. New York: Academic, 1975.

[3] S. A. Kwabwe, S. K. Pal, and R. A. King, "Recognition of bones from X-rays of the hand and wrist," *Int. J. Syst. Sci.*, vol. 16, no. 4, pp. 403–413, 1985.

[4] S. K. Pal and R. A. King, "On edge detection of X-ray images using fuzzy sets," *IEEE Trans. Patt. Anal. Machine Intell.*, vol. PAMI-5, no. 1, pp. 69–77, 1983.

[5] S. K. Pal, R. A. King, and A. A. Hashim, "Automatic grey level thresholding using index of fuzziness and entropy," *Patt. Recog. Lett.*, vol. 1, pp. 141–146, Mar. 1983.

[6] K. S. Fu and P. H. Swain, "On syntactic-pattern recognition," *Software Engineering*, J. T. Tou, Ed. New York, Academic, 1971, pp. 155–182.

[7] P. H. Swain and K. S. Fu, "Stochastic programmed grammars for syntactic pattern recognition," *Patt. Recog.*, vol. 4, pp. 83–100, 1972.

[8] W. G. Wee, "A formulation of fuzzy automata and its application as a model of learning systems," *IEEE Trans. Syst., Sci., Cybern.*, vol. SSC-5, pp. 215–223, July 1969.

[9] K. S. Fu, *Syntactic Pattern Recognition and Applications*. Englewood Cliffs, NJ: Prentice-Hall, 1982.

[10] E. T. Lee, "Proximity measures for the classification of geometric figures," *J. Cybern*, vol. 2, pp. 43–59, 1972.

[11] E. T. Lee and L. A. Zadeh, "Note on Fuzzy Languages," *Inform. Sci.*, vol. 1, pp. 421–434, 1969.

[12] T. G. Evans, "Grammatical inference technique in pattern analysis," in *Software Engineering*, vol. 2, J. T. Tou, Ed. New York: Academic, 1971.

[13] K. S. Fu and T. J. Li, "On stochastic automata and languages," *Inform. Sci.*, vol. 1, pp. 403–419, 1969.

[14] G. F. DePalma and S. S. Yau, "Fractionally fuzzy grammars with application to pattern recognition," in *Fuzzy Sets and their Applications to Cognitive and Decision Processes*, L. A. Zadeh, K. S. Fu, and M. Shimura, Eds. New York: Academic, 1975.

[15] M. G. Thomason, "Finite fuzzy automata, regular fuzzy languages and pattern recognition," *Patt. Recog.*, vol. 5, pp. 383–390, 1973.

[16] S. Tamura and K. Tanaka, "Learning of fuzzy formal language," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-3, pp. 98–102, 1973.

[17] D. D. Majumder and S. K. Pal, "On Fuzzification, fuzzy language and multicategory fuzzy classifier," in *Proc. IEEE Seventh Int Conf. Cybern. Soc.*, 1977, pp. 591–595.

[18] A. Kandel, *Fuzzy Techniques in Pattern Recognition*. New York: Wiley Interscience, 1982.

[19] S. K. Pal and D. D. majumder, *fuzzy Mathematical Approach to Pattern Recognition*. New York: Wiley, 1986.

[20] L. A. Zadeh, "Calculus of fuzzy restrictions," in *Fuzzy Sets and Their Applications to Cognitive and Decision Processes*, L. A. Zadeh, K. S. Fu, K. Tanaka, and M. Shimura, Eds. New York: Academic, 1975, pp. 1–26.

[21] G. T. Toussaint, "Computational geometric problems in pattern recognition," in *Pattern Recognition Theory and Applications*. J. Kittler, K. S. Fu, and L. F. Pau, Eds. Dortrecht, FRG: D. Reidel, 1982, pp. 73–91.

[22] A. Pathak, S. K. Pal, and R. A. King, "Syntactic recognition of skeletal maturity," *Patt. Recogn. Lett.*, vol. 2, pp. 193–197, 1984.

[23] K. C. You and K. S. Fu, "A syntactic approach to shape recognition using attributed grammars," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-9, pp. 334–345, 1979.

Amita Pathak received the B.Sc. degree from Presidency College, Calcutta, India, in 1979, and the M.Sc. degree in statistics from the University of Calcutta in 1981.

Presently, she is a Senior Research Fellow in the Electronics and Communication Sciences Unit, Indian Statistical Institute, working towards the Ph.D. degree. Her research interest includes pattern recognition and machine learning.

Sankar K. Pal (M'80–SM'84) received the M. Tech. and Ph.D. degrees in radiophysics and electronics from Calcutta University, India, in 1974 and 1979, respectively. In 1982 he received the Ph.D. degree along with DIC in electrical engineering from Imperial College, London University, England. Currently, working as an Associate Professor in Electronics and Communication Sciences Unit, Indian Statistical Institute, Calcutta. He is also a guest teacher in Computer Science, Calcutta University. His research interest includes Pattern Recognition, Image Processing, Artificial Intelligence, and Fuzzy Sets and Systems.

Dr. Pal is the recipient of Commonwealth Scholarship and MRC (UK) post-doctoral Fellowship for study at Imperial College, London, and the Fulbright Post-doctoral Fellowship for study at the University of California, Berkeley, and the University of Maryland, USA. He is the author of the book *Fuzzy Mathematical Approach to Pattern Recognition* (John Wiley 1986), which has been awarded the best prize in the Seventh World Book Fair, and has more than 60 research papers to his credit. He is one of the reviewers of the *Mathematical Reviews* (American mathematical Society), a Fellow of the IETE, and Treasurer of the Indian Society for Fuzzy Mathematics and Information Processing (ISFUMIP).