Some Models for Assessment of Intelligence and Scholastic Attainment*

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Introduction

Comparative and longitudinal studies of intelligence and scholastic attainment call for measurement models which permit a broad assessment of intellectual abilities and provide a succinct summary statement of an individual's intelligence and attainment. Two models are proposed to satisfy these criteria; one is based on statistical sampling theory and the other utilizes multivariate statistical methods. In the statistical sampling theory model, samples of questions drawn by stratified sampling from a universe of questions constitute tests of intelligence or attainment. Individual students answer several samples of questions, and from their performance it is possible to estimate individual mean performance, the associated confidence interval, and the probability with which the student is classified into categories of intelligence or into degree classes. The second model employs multivariate statistical analysis to determine the linear function of intellectual abilities with maximum variance. That linear function is the first principal factor of a set of abilities and from the regression coefficients of the several abilities on that factor, individual scores on the first factor can be obtained. The sampling theory model may be usefully employed in comparative studies of intelligence and scholastic attainment in different human populations and environments, and in longitudinal studies requiring successive measurements on the same individuals over a period of time. The multivariate model can provide a single measure of intellectual performance for correlation with environmental, physical, and physiological data in comparative studies and cross-sectional studies of growth and development. In subsequent portions of this paper, the two models will be briefly described and some illustrative results presented.

A Statistical Sampling Theory Model

The term *unit* is used to refer to an objective multiple-choice attainment question or intelligence test item, and is designated as U_j. The *universe of questions* (8) is conceptualized as the population of units, *i.e.*, the aggregate of objective multiple-choice attainment questions or intelligence test items. For each area of scholastic attainment and for each factor or aspect of intelligence, there will be a corresponding universe of questions. All questions belonging to the universe, listed with proper identification, comprise the *sampling frame* and for

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each universe of questions there is a separate sampling frame. The set of questions or items that makes up a test is a *sample* of questions which has been selected from the universe of questions, as listed in the sampling frame, according to the laws of probability. Homogeneous groupings of questions within the universe of questions are referred to as *strata*, and if the universe is stratified, the sampling frame will list all units according to strata. Table 1 gives the sampling notation for stratified sampling of the universe of questions.

TABLE 1

Notation for Stratified Sampling

Sl. No.	Concept	Level *	General Element	Number	Subscript
(1)	(2)	(3)	(4)	(5)	(6)
1	A question	P	U_j	M	$j = 1, 2, \ldots\ldots, M$
		s	u_j	m	$j = 1, 2, \ldots\ldots, m$
2	A Stratum	P	S_i	K	$i = 1, 2, \ldots\ldots, K$
		s	S_i	K	$i = 1, 2, \ldots\ldots, K$
3	A student's answer	P	Y_{ij}	M_i	$j = 1, 2, \ldots\ldots, M_i$; $i = 1, 2, \ldots\ldots, K$
		s	y_{ij}	m_i	$j = 1, 2, \ldots\ldots, m_i$; $i = 1, 2, \ldots\ldots, K$

* P = population. s = sample.

For every student there is a hypothetical population of answers corresponding to the universe of questions (3). Consider only the population of answers for one student. The characteristic of a sampling unit U_j is the student's answer. Its value is denoted Y_j; if the answer is correct, the value is 1, and if it is wrong, the value is 0. When stratified sampling is employed, the value is denoted Y_{ij} to indicate stratum and unit within the stratum. The answers to a sample of questions are the sample observations for the population of answers. The objective of the model is to measure the intelligence or scholastic attainment of individual students. For each student, his true knowledge may be defined as his performance on the universe of questions. This true value is unknown, and must be estimated from observations on the student's answers to a sample of questions.

The attributes of a student's knowledge may be regarded as the parameters of the population of answers. The *parameters* are functions of the population, defined by the mathematical expressions of these functions. The corresponding functions of the sample are termed *statistics*. The rule or method for estimating the value of a parameter from a sample is termed an *estimator*, and the value which the estimator takes for a given sample is termed an *estimate*. Four parameters will be considered in the present context, and their definitions and estimators are given in Table 2. *Total* is the total number of correct answers obtained by a given student. *Mean* is a proportion which may be interpreted as the probability that any question in the universe would be correctly answered by the student. If the mean is multiplied by 100, it becomes the familiar percentage correct. The modified *variance* is the sum of the squared deviations around the mean divided by the number of questions. Stratified sampling permits a more refined variance measure, the variance within strata. The *sampling variance* of the mean indicates the precision of the mean estimator, and for stratified sampling, is based on the modified variance within strata (3). The values of these parameters are estimated from sample observations by the estimators in Table 2, using a stratified sampling design.

TABLE 2

Notation for Estimation in Stratified Sampling Without Replacement

Sl. No.	Function	Level *	Symbol	Definition or Estimator
(1)	(2)	(3)	(4)	(5)
1	Total	P	Y	$\sum_{i=1}^{K} \sum_{j=1}^{M_i} Y_{ij}$
		s	y	$\sum_{i=1}^{K} \frac{M_i}{m_i} \sum_{j=1}^{m_i} y_{ij}$

* P = population. s = sample.

TABLE 2—(*contd.*)

Sl. No.	Function	Level	Symbol	Definition or Estimator
(1)	(2)	(3)	(4)	(5)
2	Mean	P	Y	$\frac{1}{M} \sum_{i=1}^{K} \sum_{j=1}^{M_i} Y_{ij} = \sum_{i=1}^{K} \frac{M_i}{M} Y_i$ where $Y_i = \frac{1}{M_i} \sum_{j=1}^{M_i} Y_{ij}$
		s	y	$\sum_{i=1}^{K} \frac{M_i}{M} y_i$ where $y_i = \frac{1}{m_i} \sum_{j=1}^{m_i} y_{ij}$
3	Modified variance within strata	P	S_i^2	$\frac{1}{(M_i - 1)} \sum_{j=1}^{M_i} (Y_{ij} - Y_i)^2$
		s	s_i^2	$\frac{1}{(m^i - 1)} \sum_{j=1}^{m_i} (y_{ij} - y_i)^2$
4	Sampling variance of sample mean	P	$V(y)$	$\sum_{i=1}^{K} \left(\frac{M^i}{M}\right)^2 \left(\frac{1}{m^i} - \frac{1}{M_i}\right) S_i^2$
		s	$V'(y)$	$\sum_{i=1}^{K} \left(\frac{M_i}{M}\right)^2 \left(\frac{1}{m_i} - \frac{1}{M^i}\right) s_i^2$

* P = Population. s = Sample.

It is well known among psychometricians and educators that assessments of intelligence and scholastic attainment lack perfect precision. Yet the assessments are reported as if they were perfectly accurate. A parallel to this situation exists in censuses and complete enumerations, which are assumed to yield completely accurate results but can, in fact, contain unassessed and unassessable error (9). Sample surveys, in contrast, can provide an estimate of the error in their results because the data collection schemes are based on the laws of probability. Sample surveys are also more economical than complete enumerations. Similarly, by sampling from universes of questions, an estimation of the error in the individual assessments of intelligence and attainment can be made. The estimates of the student's capability can also be economically obtained. The sampling model proposed here requires students to answer three or more independent samples of questions from the desired universe of questions. Under simple random sampling and under stratified sampling with probability proportional to stratum size, the sample mean for a student is

$$y = \frac{1}{m} \sum_{j=1}^{m} y_j \qquad [1]$$

which gives the proportion of correct answers of that student to the sample of questions. This can be expressed as a percentage,

$$p = 100\, y \qquad [2]$$

or the sample,

$$P = 100\, Y \qquad [3]$$

being the parameter or true percentage of correct answers to the universe of questions. Let there be r independent samples of the same size, and let the u^{th} sample estimate of P be denoted by p_u ($u = 1, \ldots, r$). The combined estimate of P is

$$= \frac{1}{r} \sum_{u=1}^{r} p_u \qquad [4]$$

and the sample estimate of its sampling variance is obtained from

$$V'(p) = \frac{1}{r(r-1)} \sum_{u=1}^{r} (p_u - p)^2 = \frac{1}{r} s^2 \qquad [5]$$

10, page 227). Assuming that p is normally distributed with mean P and variance V (p), the 95% confidence limits are given by

$$p - 1.96 \sqrt{V'(p)},\ p + 1.96 \sqrt{V'(p)} \qquad [6]$$

and the confidence interval is given by

$$p \pm 1.96 \sqrt{V'(p)}, \qquad [7]$$

Similarly, the 99% confidence limits are given by

$$p - 2.58 \sqrt{V'(p)},\ p + 2.58 \sqrt{V'(p)} \qquad [8]$$

and the confidence interval is given by

$$p \pm 2.58 \sqrt{V'(p)}. \qquad [9]$$

The student's true intelligence or scholastic attainment is expected to lie, with 95% or 99% confidence, within these limits. Table 3 presents the sample estimates and confidence limits for five students to illustrate this approach. In this way, the accuracy of each student's assessment can be measured. It can be contrasted with the standard error of measurement commonly employed in psychometry and educational evaluation which imposes an estimate of the group's error on the individual student (4).

Independent samples of questions also permit estimation of the probabilities with which a student is classified as " superior ", " average ", or " poor ", or as " first class ", " second class "; " third class " or " failure ". The mean of the sample estimates of P can be classified as first, second, or third class, or as failure, in terms of three constants which will be called d_1, d_2 and d_3. Four individual probabilities are defined :

$$\xi_1 = \text{prob. } (p \geqslant d_1), \text{ first class ;} \quad [10]$$

$$\xi_2 = \text{prob. } (d_1 > p \geqslant d_2), \text{ second class ;} \quad [11]$$

$$\xi_3 = \text{prob. } (d_2 \geqslant p > d_3) \text{ third class ; and} \quad [12]$$

$$\xi_4 = \text{prob. } (0 \leqslant p < d_3), \text{ failure.} \quad [13]$$

These individual probabilities are estimated by :

$$\xi_1' = \phi\left(\frac{d_1 - p}{\sqrt{V'(p)}}\right), \quad [14]$$

$$\xi_2' = \phi\left(\frac{d_2 - p}{\sqrt{V'(p)}}\right) - \phi\left(\frac{d_1 - p}{\sqrt{V'(p)}}\right), \quad [15]$$

$$\xi_3' = \phi\left(\frac{d_3 - p}{\sqrt{V'(p)}}\right) - \phi\left(\frac{d_2 - p}{\sqrt{V'(p)}}\right), \quad [16]$$

and

$$\xi_4' = 1 - \phi\left(\frac{d_3 - p}{\sqrt{V'(p)}}\right). \quad [17]$$

The sum of these probabilities is unity :

$$\xi_1' + \xi_2' + \xi_3' + \xi_4' = 1. \quad [18]$$

Using as constant values $d_1 = 59.5$, $d_2 = 47.5$, and $d_3 = 29.5$ for a standard university examination curve with $\mu = 45$ and $\sigma = 15$ (12, page 103), illustrative data for individual probabilities are presented in Table 4.

The sampling theory model which has been proposed can now be briefly summarized and its potential usefulness indicated. For any intellectual ability or attainment a population of questions s prepared which is designed to cover all aspects of that ability or attainment. The population or universe of questions is divided into several strata, each of which represents a homogeneous aspect

TABLE 3

Confidence Limits for Individual Assessments

Sl. No	Function: Name	Function: Symbol	Equation Number	Student 1	Student 2	Student 3	Student 4	Student 5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Estimate of P, sample 1	p_1	[2]	63.33	40.00	50.00	53.33	33.33
2	Estimate of P, sample 2	p_2	[2]	46.67	46.67	83.33	56.67	26.67
3	Estimate of P, sample 3	p_3	[2]	46.67	56.67	60.00	56.67	13.33
4	Estimate of P, sample 4	p_4	[2]	46.67	23.33	73.33	40.00	36.67
5	Mean of sample estimates	p	[4]	50.84	41.67	66.66	51.67	27.50
6	Square root of estimate of sampling variance of p	$\sqrt{V'(p)}$	[5]	4.17	7.01	7.33	3.97	5.16
7	1/2 length of 95% confidence interval of P	$1.96 \sqrt{V'(p)}$	[7]	8.16	13.73	14.36	7.78	10.11
8	95% confidence limits of P	$p \pm 1.96 \sqrt{V'(p)}$	[7]	42.68, 59.00	27.94, 55.40	52.30, 81.02	43.89, 59.45	17.39, 37.61
9	1/2 length of 99% confidence interval of P	$2.58 \sqrt{V'(p)}$	[9]	10.75	18.08	18.90	10.24	13.31
10	99% confidence limits of P	$p \pm 2.58 \sqrt{V'(p)}$	[9]	40.09, 61.59	23.59, 59.75	41.76, 85.56	41.43, 61.91	14.19, 40.81

TABLE 4

Individual Probabilities Associated with Classification of a Student

Sl. No	Function: Name	Function: Symbol	Equation Number	Student 1	Student 2	Student 3	Student 4	Student 5
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Mean of the sample estimates	p	[4]	50.84	41.67	66.66	51.67	27.50
2	Square root of the estimate of sampling variance of p	$\sqrt{V'(p)}$	[5]	4.17	7.01	7.33	3.97	5.16
3	Individual probability, first class, $d_1 = 59.5$	ξ_1'	[14]	.0188	.0055	.8365	.0244	.0000
4	Individual probability, second class, $d_2 = 47.5$	ξ_2'	[15]	.7693	.1978	.1590	.8287	.0000
5	Individual probability, third class, $d_3 = 29.5$	ξ_3'	[16]	.2118	.7558	.0044	.1468	.3483
6	Individual probability, failure	ξ_4'	[17]	.0001	.0409	.0001	.0001	.6517
7	Sum of individual probabilities.		[18]	1.0000	1.0000	1.0000	1.0000	1.0000

of the ability or attainment being tested. It is possible to set up intelligence or attainment tests by stratified sampling, without replacement, of the population or universe of questions. To assess intelligence and scholastic attainment, several independent samples of questions are selected and administered to students. From the several sample estimates so obtained, the mean and the square root of its sampling variance are computed for each student. These two values serve as the basis of confidence intervals and individual probabilities for each student.

To compare intelligence and scholastic attainment in different human populations and environments, the measuring instruments or tests must be comprehensive and unbiased. The sampling approach described above provides an objective and statistically sound procedure for preparing such tests. In addition, the accuracy of individual assessments should be determined before comparing different groups of individuals. The sampling theory also permits a statement of the confidence interval attached to each student's assessment. If the number of individuals in different intelligence or attainment categories are to be compared, the accuracy of the classification decision should also be known. The individual probabilities of being classified as first, second or third class satisfy this requirement. It has also been suggested that the sampling theory model could be usefully applied in longitudinal studies requiring successive measurements on the same individuals over a period of time. A peculiar difficulty in measuring intelligence or attainment over and over in the same persons arises because of the human memory. The successive measurements are not independent. Independent random samples which are drawn from the same universe of questions obviate this difficulty. It is also possible to increase the complexity and difficulty of the samples, as required when children grow and undergo schooling, at the same time ensuring that the same aspect of intelligence or attainment is being measured.

A Model based on Multivariate Analysis

The principal factor solution of factor analysis is based upon the theory of principal components (5 & 7). Its objective is to reduce the complexity of multiple measurements by expressing them in terms of a coordinate system based on their internal relations. If the number of multiple measurements is denoted by p, then the vector

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_p \end{pmatrix} . \qquad [19]$$

having as its general element x_i ($i = 1, 2..., p$), represents the p variables. The intercorrelations between the p variables are represented by the matrix

$$R = \begin{pmatrix} 1 & . & . & . & r_{1p} \\ . & 1 & & & \\ . & & 1 & & \\ . & & & 1 & \\ r_{p1} & . & . & . & 1 \end{pmatrix} \qquad [20]$$

in which the general element r_{ij} ($i, j = 1, 2,..., p$) is the correlation between the x_i and x_j variables. The first task of present model is to represent R in reduced form as the first principal factor

$$A = \begin{pmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_p \end{pmatrix} \qquad [21]$$

which has as its general element a_i ($i = 1, 2,..., p$). The vector A comprises the first factor coefficients and is obtained by solving the characteristic equation

$$[R - \lambda I] = 0 \qquad [22]$$

which includes the correlation matrix R, the characteristic root λ and the identity matrix I. The characteristic equation is to be solved by maximizing the scalar quantity

$$A' A \qquad [23]$$

which means maximizing the contribution of the first factor to the total communality. The first characteristic vector is scaled so that sum of squares of its elements equals the first characteristic root (2 & 5).

The second task of the present model is to obtain a vector which permits estimation of the first factor for any individual or student for which measurements on the vector X are available (3). Equation [21] gives the vector A. The coefficients a_i can also be represented in a matrix D,

$$D = \begin{pmatrix} a_1 & & & & 0 \\ & a_2 & & & \\ . & & . & & \\ . & & & . & \\ . & & & & . \\ 0 & & & & a_p \end{pmatrix}. \qquad [24]$$

and the vector to be obtained is B

$$B = \begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_p \end{pmatrix} \qquad [25]$$

A numerically simple solution for B (11, p. 224). in matrix notation, is

$$B = \frac{1}{[1 + A'(I - DD')^{-1} A]} (I - DD')^{-1} A \qquad [26]$$

where I is the identity matrix. The general element b_i ($i = 1, 2, \ldots, p$) can be interpreted as a regression coefficient permitting estimation of the first factor. The factor score is defined as the sum of the products of the standard scores (means and standard deviations of all variables are equal) and the appropriate regression coefficients. If the standard score of a student on variable x_i is represented by z_i, then his factor score y is given by

$$y = \sum_{i=1}^{p} b_i z_i \, . \qquad [27]$$

The factor scores can be readily computed using electronic digital computers (6). As, however, the nature of the diagonal entries in R is an important issue in factor analysis, it must be considered in this context. Two basically different diagonal entries are possible, unities and communalities. If unities are employed, the total variance in the matrix is to be accounted for and the factor scores can be directly computed for each individual from his standard scores. If communalities are entered in the diagonal (*e.g.*, highest column correlation, reliability coefficient, squared multiple correlation), the total communality or common variance is to be accounted for and the factor scores can only be estimated (5 & 11). When communalities are used, the resulting factor scores will be referred to as y_1, and when unities are used, y_2 will symbolize the factor scores.

To illustrate this model for assessment of performance, two sets of tests, labelled A and B, have been studied. Each set comprised 7 variables or predictors, and both sets were designed as measures of intelligence. Verbal, non-verbal, and quantitative aspects of intelligence have been covered in both sets (1). Table 5 gives the correlation matrices for both sets ; the triangle below the diagonal gives the correlations for set A, and the triangle above it gives the correlations for set B. The first principal factor has been extracted for both sets using unities as well as communalities in the diagonal. Table 6 presents the vector A, equation [21], in columns (3) and (6) for matrices with communality and unity entries in the diagonal. The corresponding vector B, equation [25], is reported in columns (4) and (7). After obtaining the factor scores y_1 and y_2 from the appropriate B vectors, their correlations with the predictors have been computed. The correlation coefficients between the B vector scores and the standard scores are reported in columns (5) and (8) for the two types of correlation matrix. Finally Table 7 gives the standard scores of ten students on set B, and their y_1 and y_2 factor scores *.

TABLE 5

Correlation Matrices for Two Sets of Tests

Predictor		Set B						
		x_1	x_2	x_3	x_4	x_5	x_6	x_7
(1)		(2)	(3)	(4)	(5)	(6)	(7)	(8)
Set A	x_1	...	.5237	.3554	.5294	.5679	.5409	.4825
	x_2	.5114	...	.2195	.3373	.5114	.5420	.4554
	x_3	.3407	.2425	...	.4746	.4393	.2609	.2627
	x_4	.4224	.3223	.4242	...	.6511	.4410	.4069
	x_5	.5853	.4533	.4348	.5909	...	.5684	.5583
	x_6	.4326	.4198	.2483	.4797	.5601	...	.5241
	x_7	.5120	.4608	2651	.4794	.5892	.5378	...

TABLE 6

Coefficients and Correlations for the First Principal Factor of Two Sets of Tests

Set	Predictor	Communalities in Diagonal			Unities in Diagonal		
		Factor coefficients	Regression coefficients	Factor score correlations	Factor coefficients	Regression coefficients	Factor score correlations
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Set A	x_1	.7149	.1530	.7545	.7563	.1499	.7528
	x_2	.6161	.1039	.6436	.6667	.1019	.6410
	x_3	.4944	.0685	.5101	.5492	.0667	.5090
	x_4	.6962	.1414	.7328	.7351	.1357	.7295
	x_5	.7996	.2321	.8699	.8444	.2497	.8769
	x_6	.6890	.1372	.7339	.7336	.1348	.7320
	x_7	.7304	.1639	.7818	.7709	.1613	.7798
Set B	x_1	.7386	.1608	.7868	.7866	.1678	.7906
	x_2	.6525	.1124	.6864	.7007	.1119	.6884
	x_3	.5055	.0672	.5209	.5536	.0649	.5166
	x_4	.7218	.1491	.7533	.7491	.1388	.7464
	x_5	.8157	.2411	.8765	.8474	.2445	.8762
	x_6	.7176	.1463	.7608	.7641	.1493	.7635
	x_7	.6761	.1232	.7120	.7221	.1227	.7126

* Computations reported in Tables 5, 6 and 7 were carried out on the IBM 1401 Electronic Data Processing System, Data Processing Unit, Research and Training School, Indian Statistical Institute, Calcutta.

TABLE 7
Illustrative Predictor and Factor Scores

Student Number	Predictor Scores							Factor Scores	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	y_1	y_2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	38.00	53.00	35.00	36.00	26.00	42.00	34.00	36.38	36.38
2	62.00	56.00	44.00	57.00	54.00	60.00	56.00	56.41	56.48
3	42.00	58.00	32.00	38.00	40.00	48.00	32.00	41.69	41.77
4	62.00	58.00	59.00	65.00	59.00	57.00	49.00	58.74	58.70
5	42.00	47.00	44.00	42.00	45.00	29.00	39.00	41.15	41.11
6	50.00	33.00	39.00	35.00	32.00	31.00	39.00	36.64	36.71
7	46.00	39.00	61.00	44.00	45.00	49.00	57.00	47.48	47.47
8	42.00	33.00	42.00	49.00	58.00	44.00	42.00	46.18	46.17
9	60.00	70.00	51.00	55.00	67.00	57.00	42.00	58.81	58.90
10	56.00	64.00	63.00	57.00	67.00	61.00	71.00	62.75	62.76

The comparison between the results obtained for matrices with unity and communality diagonal entries provided by Tables 6 and 7 suggests that the factor scores obtained are remarkably similar, and in practical situations, either diagonal entry may be employed. However, investigations varying the size of the communalities would be required to confirm this observation. The high correlations in columns (5) and (8) of Table 6 for both sets of data indicate that the factor score adequately represents the multiple measurements. Columns (3) and (5) are very similar, as are columns (6) and (8), which is to be expected as the vector A is regarded as an estimate of the predictor-factor correlations. It should also be noted that columns (6) and (8), based on unities, are more similar to each other than columns (3) and (5), based on communalities. This result is also in conformity with the literature (5 & 11). Before concluding, it should be recalled that it is possible to obtain the vector B directly from the correlation matrix R (5, 6 & 11). Such an approach has not been utilized for the present model because it does not yield the vector A which is also of interest in investigations of intelligence and attainment, as it is the most frequently reported result of factor analysis.

The multivariate statistical model yields the first principal factor of a set of predictor variables or tests of intelligence or attainment, and the corresponding vector of regression coefficients from which an estimate of the first principal factor for each individual can be obtained. The estimate of the first principal factor provides a single measure of intelligence or attainment which can be usefully employed in studies of intelligence and attainment. The model may be particularly appropriate when a single measure is required for correlation with environmental, physical and physiological variables, and also for cross-sectional studies of growth and development. The statistical model is based on the principle that the best single score of an individual on a set of tests is the estimate of his first principal factor (11).

SUMMARY

Two statistical models have been proposed for the assessment of intelligence and attainment. The first model is based on statistical sampling theory and includes the following procedures : (i) independent samples are drawn from a universe of questions to form tests of intelligence or attainment ; (ii) from the answers of a student to several independent samples of questions, the mean of the sample estimates and the sampling variance of the mean are obtained ; (iii) a confidence interval and individual probabilities of being placed in different degree classes are calculated for each student.

The second model is based on multivariate analysis and calls for : (i) computation of the first principal factor of a set of tests or predictors ; (ii) determination of the regression coefficients for the first principal factor ; and (iii) estimation of the first principal factor for each student or individual being assessed.

For both models, illustrative data and comments on their applicability to the assessment of intelligence and attainment have been presented.

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