59. Extension of X-table (corresponding to Fisher's Z-table) for testing the significance of two observed variances.

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R. A. Fisher's method of comparing two observed variances depend upon the calculation of his z-statistics defined by

$$Z=\frac{1}{2}\log_e\frac{s_1^2}{s_2^2}$$

where s_1^2, s_2^2 are two variances based on n_1 and n_2 degrees of freedom.

The exact distribution of Z, which was given by Fisher, can be reduced to the following form :—

$$df=\frac{\Gamma\left(\frac{n_1+n_2}{2}\right)}{\Gamma\left(\frac{n_1}{2}\right)\Gamma\left(\frac{n_2}{2}\right)}\cdot t^{\frac{n_1}{2}-1}(1-t)^{\frac{n_2}{2}-1}dt$$

where

$$x=e^{2z}=\left(\frac{n_2}{n_1}\cdot\frac{t}{1-t}\right).$$

The distribution may therefore be expressed in terms of the incomplete Beta-function :

$$B_t\left(\frac{n_1}{2},\frac{n_2}{2}\right).$$

Fisher has published a table of one per cent. and five per cent. values of z for certain values of n_1 and n_2. In actual practice it is more convenient to use

$$x=e^{2z}=\frac{s_1^2}{s_2^2}$$

instead of z, and P. C. Mahalanobis in 1932 published a table of x based on Fisher's table of z.

In view of the importance of the test more extensive tables of 'x' have now been directly calculated by trivariate interpolation in Karl Pearson's Tables of Beta-functions.