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Statistical Notes for Agricultural Workers, No. 25. A Simplified Method of Analysis of Quasi-Factorial Experiments in Square Lattice with a Preliminary Note on Joint Analysis of Yield of Paddy and Straw

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NO. 25. A SIMPLIFIED METHOD OF ANALYSIS OF QUASI-FACTORIAL EXPERIMENTS IN SQUARE LATTICE WITH A PRELIMINARY NOTE ON JOINT ANALYSIS OF YIELD OF PADDY AND STRAW

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(With one text-figure)

INTRODUCTION

FISHER'S Randomized Block and Latin Square designs have proved very useful in agricultural field experiments, but can be adopted only when a limited number of plots, usually not greater than ten or eleven, is included in a block or row and column. This limitation on the number of treatments which can be tested simultaneously is a serious handicap in the case of factorial experiments in which the number of treatment combinations keeps on increasing with the number of factors and with the number of the levels at which each factor is tested. The devices of confounding main effects in split-plot designs and interactions among sub-blocks are used to overcome this difficulty.

Recently Yates [1936, 1] has developed another method for keeping the block size within efficient limits when the number of treatments of a non-factorial experiment becomes large. The need of such single factor experiment occurs very frequently in practice when we have to make preliminary selection from a large number of new strains of a crop. Since only a single factor (e.g. variety, in this case) is involved, the several comparisons among the treatments cannot be classified under sets of main effects and interactions. For this reason the principle of straightforward confounding is apparently inapplicable here. Yates however used the ingenious idea of using a factorizable number (say, $v = p.\ q.\ r.......$) for the number of varieties, and then viewing the varieties as all combinations of several factors (non-existent though) at levels $p, q, r, ...$, etc., for each group. We can then divide the varieties in v/p groups of p in each group, v/q groups of q in each group, v/r groups of r in each group, and so on. The comparisons among the v/p groups in the first type of division will correspond to the main effects and interactions of all factors excluding the first pseudo-factor ; the comparisons among the v/q groups in the second type of division will correspond to the main effects and interactions of all factors excluding the second pseudo-factor ; the comparisons among the v/r groups in the third type of division will correspond to the main effects and interactions of all factors excluding the third pseudo-factor and so on. We can then confound these main effects and interactions of the various pseudo-factors, by assigning to a block only those varieties which occur together in the same group.

If there is sufficient land each group may be replicated a number of times, which should be the same (n, say) for each group. Thus the number of blocks in the experiment will be $vn\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\ldots\right)$. The block size will not be the same if $p, q, r, \ldots$ are not equal. Thus there will be vn/p blocks of size p, vn/q blocks of size q, vn/r blocks of size r and so on. But instead of doing the experiment in blocks with v plots in each, we are thus able to reduce the block size to the comparatively smaller sizes of $p, q, r, \ldots$ plots. The designs which Yates got by this artifice of confounding of main effects and interactions of certain fictitious factors, super-imposed on the varieties, were given by him the name quasi-factorial designs.

The simpler and more efficient cases of this design occur when $p = q = r = \ldots$ Then v becomes some power of the number p. If, in addition, p is a prime number or power of a prime number, it is possible to confound all main effects and all interactions of the pseudo-factors. This will ensure the losing of equal amount of information from all varietal comparisons and thus give symmetry to the design with respect to every variety. This special design was called the 'symmetrical' quasi-factorial design.

When v is not a factorizable number the idea of introducing pseudo-factors fails completely and to meet this case Yates [1936, 2] has developed the brilliant idea of balanced incomplete randomized blocks. In these designs it is possible to secure equal accuracy for comparisons between every pair of varieties, and the symmetrical quasi-factorial design of $v = (p)^m$ occurs as a special case.

In the Calcutta Statistical Laboratory, Bose and Nair [1939] have developed a general class of designs called partially balanced incomplete block designs of which Yates' quasi-factorial designs for $(p)^m$ varieties in blocks of $(p)^{m-1}$ plots, and his balanced incomplete designs, happen to be special cases. While developing the method of analysis for this general class of designs, it was found that the method given by Yates for analysing the data from quasi-factorial experiments with p^2 varieties in blocks of p plots may be replaced by a simpler method. One of the objects of this note is to illustrate this new procedure of analysis with the help of data from a quasi-factorial experiment on rice.

In the season of 1937-38 two quasi-factorial experiments with 49 and 100 varieties of paddy were laid out by Mr S. C. Chakravarty at the Chinsurah Farm, Bengal, in 28 randomized blocks of 7 plots and 40 randomized blocks of 10 plots respectively. The designs were prepared at the Statistical Laboratory. In the experiment with 100 varieties it is impossible to achieve symmetry between every pair of varieties as 10 is not a power of a prime number. In the other experiment symmetry could have been achieved if the shape of the experimental piece of land was such as to accommodate four 7 × 7 Latin Squares. As this was not possible the symmetry was sacrificed, and in both experiments main effects only of the pseudo-factors were confounded. We shall use the second experiment for our illustration.

Besides the usual analyses of variance for grain and straw separately, the analysis of covariance also has been worked out. In an exploratory experiment with large number of varieties like the present one, it is not wise to limit the criterion of selection of strains to one character, namely, yield of grain alone. It is desirable to take into consideration the yield of both grain and straw (and also of other characters of economic importance as necessary); and for this purpose it is essential to include in the analysis the covariance between characters. Unfortunately adequate tests of significance and necessary tables for this purpose are not yet available. The problem is, however, receiving increasing attention [Lawley, 1938, 1939; Roy, 1939, 1, 2], and we may expect that necessary tables of significant levels will be available for this purpose in the near future. In the meantime we are taking this opportunity of explaining the procedure for calculating the various sums of products which will be needed in covariance analysis.

As a preliminary step, we have used the covariance analysis for a brief discussion of the method of selecting the varieties when the yields of grain and straw are both taken into consideration.

Details of lay-out

One hundred *aman* strains (other than *patnais*), a list of which is given in the appendix were selected for this experiment. These strains were assigned, at random, one hundred serial numbers 00, 01, 02,................09, 10, 11, ...19, ... 90, 91,............99, which are noted in column (1) of the appendix. These numbers were then written in the form of 10 × 10 square lattice of the following pattern :—

00	10	20	30	40	50	60	70	80	90
01	11	21	31	41	51	61	71	81	91
02	12	22	32	42	52	62	72	82	92
03	13	23	33	43	53	63	73	83	93
04	14	24	34	44	54	64	74	84	94
05	15	25	35	45	55	65	75	85	95
06	16	26	36	46	56	66	76	86	96
07	17	27	37	47	57	67	77	8	97
08	18	28	38	48	58	68	78	88	98
09	19	29	39	49	59	69	79	89	99

The variety occurring in the i-th column and j-th row of this square is denoted as variety [ij] in which both i and j vary from 0 to 9.

The varieties bearing numbers occurring in the same row or in the same column constitute a set. It is clear that there are only 20 such sets, of which 10 sets correspond to the 10 columns and the other 10 sets correspond to the 10 rows of the above square. The first 10 sets will be said to constitute group I, and the second 10 sets to constitute group II.

Each set must be replicated in the same number of randomized blocks according to the availability of the land. In this experiment only two replications were used, so that we had 40 randomized blocks giving four replications of each variety. The size of each plot was 8 ft. 3 in. × 8 ft. 3 in. Leaving a border of 9 in. all around, the net size came to 7 ft. 6 in. ×7 ft. 6 in.

The two replicated blocks of each of the 20 sets were first assigned at random among the total of 40 blocks. The 10 varieties of a set were allotted in a serial order in a random direction in one of the blocks assigned to that set, and in the other block they were randomized. The actual field layout is shown on the next page. The blocks have been numbered 1 to 40; and Table II gives, besides other things, the serial numbers of the blocks in which each set of the two groups was replicated.

Season and nature of crop

The season was quite favourable for rice; and transplanting for this experiment was done on 28th July 1937. The rainfall was slightly above normal, and the distribution was quite regular; and from the agricultural point of view the crop was considered to be normal. The normal rainfall and the rainfall during 1937-38 as recorded at the Chinsurah Farm are shown in Table I.

TABLE I

Rainfall at Chinsurah Farm during the season 1937-38

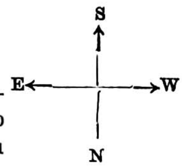
Month	Normal rainfall	Rainfall during the year 1937-38	Difference	Number of rainy days
April 1937	2·46	1·76	—0·70	2
May 1937	5·85	3·24	—2·61	10
June 1937	10·56	14·55	+3·99	14
July 1937	11·28	9·43	—1·85	25
August 1937	11·64	12·66	+1·02	21
September 1937	8·40	11·49	+3·09	21
October 1937	4·09	4·40	+0·31	6
November 1937	0·66	*Nil*	—0·66	*Nil*
December 1937	0·19	*Nil*	—0·19	*Nil*
January 1938	0·38	0·25	—0·13	1
February 1938	1·20	1·81	+0·61	3
March 1938	1·58	0·32	—1·26	1
Total	58·29	59·91	+1·62	104

Primary data

The primary data consist of yields of grain and straw expressed in ounces per plot. These are fully set out in Table II (*a*, *b*). The yield of straw was determined with less precision than the yield of grain. The total sum of squares with 399 degrees of freedom, and the block sum of squares with 39 degrees of freedom were calculated directly from Table II (*a*, *b*) for yield of grain as well as yield of straw, and also the corresponding sum of products. It is not necessary to give details about the calculation of these quantities.

Lay-out plan of the experiment

											Block No.		
	94	**84**	**74**	**64**	**34**	**14**	**04**	**54**	**44**	**24**	**40**		
	65	20	98	05	91	70	20	53	78	30	63	76	40
	35	21	94	95	41	71	10	73	08	31	66	66	41
	85	22	90	75	21	72	30	93	68	32	60	56	42
	15	23	97	45	11	73	60	03	58	33	61	26	43
	55	24	92	15	31	74	80	83	18	34	68	16	44
	95	25	96	35	01	75	50	23	98	35	69	06	45
	45	26	99	55	71	76	00	13	28	36	64	96	46
	25	27	91	25	81	77	70	63	38	37	67	46	47
	75	28	93	85	51	78	40	43	88	38	62	86	48
	05	29	95	65	61	79	90	33	48	39	65	36	49
Block No.	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**
	47	80	00	89	60	46	39	97	54	10	11	50	50
	42	81	07	99	10	26	34	67	84	11	18	57	51
	41	82	04	49	70	76	31	27	44	12	15	51	52
	43	83	01	79	20	66	37	77	64	13	17	55	53
	49	84	03	69	80	06	32	07	04	14	13	53	54
	48	85	08	09	30	16	35	17	14	15	14	54	55
	45	86	02	19	00	36	36	37	94	16	19	56	56
	44	87	06	29	50	56	30	87	74	17	12	58	57
	46	88	09	39	90	86	33	57	24	18	10	52	58
	40	89	05	59	40	96	38	47	34	19	16	59	59
Block No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**
	71	57	60	03	00	79	02	29	90	86	01	38	82
	78	37	61	13	01	59	52	23	91	81	71	58	32
	73	87	62	83	02	39	92	22	92	80	41	88	02
	76	77	63	43	03	99	32	20	93	88	11	28	22
	79	67	64	53	04	49	72	28	94	83	31	78	92
	72	17	65	93	05	69	62	27	95	82	81	08	42
	70	97	66	33	06	89	82	21	96	89	21	98	52
	74	07	67	73	07	09	12	24	97	85	61	68	12
	77	47	68	63	08	29	22	26	98	87	91	18	62
	75	27	69	23	09	19	42	25	99	84	51	48	72
Block No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**



TABLE

Yield of grain and of straw in each plot (upper figures

Group

Set 1	Bl. 5	Bl. 16	Set 2	Bl. 23	Bl. 24	Set 3	Bl. 8	Bl. 28	Set 4	Bl. 20	Bl. 36	Set 5	Bl. 14	Bl. 39
00	46·5	64·0	10	52·5	43·0	20	42·0	51·5	30	40·0	63·0	40	50·0	52·0
	40	58		68	52		48	84		46	80		56	80
01	49·0	54·5	11	34·0	42·0	21	40·0	45·5	31	56·5	59·0	41	56·0	41·5
	66	68		58	82		52	80		56	86		72	80
02	41·0	49·5	12	51·0	42·5	22	42·5	59·0	32	41·0	48·5	42	49·5	46·0
	48	60		66	56		56	84		64	82		78	68
03	42·5	38·5	13	51·5	50·0	23	40·0	55·0	33	38·5	57·5	43	56·5	47·5
	42	58		82	58		38	72		46	72		64	78
04	40·5	48·0	14	51·5	58·5	24	45·0	61·0	34	53·5	55·5	44	48·0	54·5
	56	80		60	62		58	94		62	66		64	70
05	44·0	43·5	15	48·5	48·0	25	49·0	59·5	35	41·5	57·5	45	58·0	67·5
	46	46		80	80		68	102		48	74		64	78
06	42·5	47·5	16	42·0	40·5	26	47·5	54·5	36	44·5	52·5	46	53·0	61·0
	40	52		70	60		64	74		48	60		66	76
07	43·0	51·5	17	40·0	49·5	27	43·5	61·0	37	47·5	60·0	47	55·0	56·5
	50	64		60	76		48	74		74	84		70	72
08	51·0	54·0	18	44·0	60·5	28	40·0	52·5	38	36·5	57·0	48	58·5	59·0
	52	60		68	98		54	72		38	64		64	68
09	48·0	44·0	19	36·0	45·0	29	44·0	60·0	39	51·0	51·0	49	53·5	56·5
	64	52		52	72		56	78		60	66		76	68

II (*a*)

indicate grain yield ; lower figures indicate straw yield)

I

Set 6	Bl. 25	Bl. 26	Set 7	Bl. 3	Bl. 37	Set 8	Bl. 1	Bl. 32	Set 9	Bl. 10	Bl. 15	Set 10	Bl. 9	Bl. 29
50	50·5	56·5	60	38·0	44·5	70	44·0	62·0	80	48·0	54·0	90	44·5	74·5
	58	68		56	74		68	110		52	54		54	106
51	48·0	56·5	61	41·0	60·0	71	55·0	61·0	81	46·0	53·5	91	44·5	53·0
	56	64		50	76		56	84		52	68		54	72
52	44·5	55·5	62	35·5	55·0	72	54·0	60·0	82	35·5	43·5	92	41·0	51·5
	48	56		40	78		72	92		38	50		48	82
53	49·0	51·5	63	46·5	61·0	73	51·5	57·0	83	42·0	52·5	93	39·0	55·5
	66	74		44	72		60	102		46	64		46	74
54	46·5	53·0	64	41·5	58·5	74	51·5	55·5	84	52·5	49·5	94	44·5	63·0
	70	80		60	82		64	72		74	78		48	106
55	50·0	53·5	65	45·5	54·0	75	54·5	55·5	85	40·0	42·0	95	35·5	44·0
	64	64		74	86		68	74		56	64		46	64
56	46·0	43·5	66	38·0	28·0	76	46·0	52·0	86	39·0	47·5	96	47·5	68·0
	64	54		56	84		68	80		54	58		52	86
57	56·0	51·0	67	48·5	57·0	77	48·5	50·5	87	45·0	41·0	97	47·0	76·0
	66	62		48	70		72	72		52	56		62	122
58	50·0	50·5	68	40·0	46·5	78	46·0	50·0	88	50·5	49·0	98	52·0	57·5
	58	60		68	80		82	82		50	54		66	84
59	36·5	42·0	69	43·5	45·0	79	48·5	51·0	89	43·5	40·0	99	54·0	58·0
	52	58		74	80		48	54		60	56		60	80

TABLE

Yield of grain and of straw in each plot (upper figures

Group

Set 1	Bl. 18	Bl. 33	Set 2	Bl. 11	Bl. 31	Set 3	Bl. 7	Bl. 13	Set 4	Bl. 4	Bl. 34	Set 5	Bl. 22	Bl. 40
00	52·5	61·0	01	48·0	63·0	02	37·5	50·0	03	41·0	57·0	04	46·0	57·5
	48	52		56	78		44	64		48	64		68	78
10	51·5	54·0	11	38·5	42·0	12	45·5	55·5	13	43·0	52·5	14	53·5	75·0
	68	76		46	72		62	74		50	62		60	90
20	51·5	50·0	21	38·0	48·0	22	49·5	50·0	23	45·0	48·0	24	41·5	64·0
	64	80		52	80		64	64		48	50		50	84
30	46·5	53·5	31	47·0	59·0	32	36·0	41·0	33	42·0	55·5	34	45·5	69·0
	56	82		40	72		48	72		50	64		50	96
40	44·0	55·5	41	43·5	60·0	42	43·5	40·5	43	37·0	50·0	44	47·5	57·0
	52	64		60	82		64	60		40	60		76	72
50	45·0	53·0	51	50·5	48·5	52	40·0	51·0	53	43·0	52·0	54	54·0	60·0
	62	66		54	60		40	56		76	112		96	84
60	49·0	49·5	61	47·0	56·0	62	35·5	58·5	63	46·0	56·5	64	53·5	64·0
	78	82		60	64		42	80		46	62		88	76
70	48·0	50·0	71	47·0	58·5	72	41·0	61·5	73	44·0	57·5	74	40·5	57·0
	66	72		48	62		58	82		56	84		52	80
80	55·5	56·0	81	42·0	52·5	82	37·0	38·5	83	40·0	54·0	84	52·0	62·0
	56	68		50	64		34	42		48	64		78	82
90	42·0	58·0	91	49·0	49·0	92	35·5	44·0	93	37·5	60·0	94	42·5	70·0
	48	64		56	82		38	58		48	94		58	116

II (b)

indicate grain yield ; lower figures indicate straw yield)

II

Set 6	Bl. 27	Bl. 30	Set 7	Bl. 19	Bl. 38	Set 8	Bl. 2	Bl. 21	Set 9	Bl. 12	Bl. 35	Set 10	Bl. 6	Bl. 17
05	60·0 62	66·5 82	06	46·5 52	56·0 66	07	45·0 58	47·5 64	08	51·5 52	62·0 74	09	41·0 42	47·0 62
15	59·0 98	53·5 80	16	45·5 64	55·5 82	17	38·0 62	44·0 74	18	56·5 102	62·5 102	19	45·0 56	44·0 68
25	60·0 82	54·0 80	26	51·0 72	57·5 82	27	57·0 64	58·0 66	28	45·0 56	49·0 60	29	52·0 68	49·5 68
35	70·5 100	62·5 82	36	47·0 52	50·5 62	37	37·0 62	39·0 60	38	41·5 48	52·0 62	39	51·0 60	39·5 48
45	64·5 68	65·0 76	46	58·5 66	63·5 78	47	45·0 52	41·0 46	48	51·5 60	54·5 56	49	42·0 52	50·5 66
55	60·0 98	49·5 66	56	42·0 60	46·5 76	57	38·5 50	39·5 58	58	43·5 58	60·0 110	59	35·5 50	34·5 48
65	65·5 82	52·5 80	66	41·5 76	32·0 86	67	45·5 48	55·0 62	68	45·5 78	49·5 82	69	32·5 54	39·0 78
75	61·0 70	65·0 82	76	51·5 82	31·0 82	77	36·5 50	47·5 64	78	37·0 72	56·0 92	79	38·5 34	49·5 50
85	53·5 80	48·5 66	86	42·5 52	54·0 68	87	39·0 44	38·5 44	88	53·5 56	64·5 68	89	38·5 50	51·5 72
95	49·5 74	48·5 90	96	43·0 48	63·0 76	97	45·0 60	58·5 72	98	47·5 64	53·5 68	99	43·5 52	52·5 64

Sum of squares and products due to varieties

It is in the calculation of the sum of squares and of products for variety, with 99 degrees of freedom, that all the complications of analysis set in. But once these sums are obtained, the residual sum of squares may be obtained by subtraction : Total *minus* Blocks *minus* Varieties.

The table of analysis of variance given by Yates [1936] mentions eleven sources of variation, of which five go to make up the variation among blocks, three go to make up the variation among varieties and three go to uncontrolled variation (namely, residual). Speaking generally, we need not take the trouble of calculating the sums of squares due to each of these eleven sources of variation. We are mostly interested to find out the value of the sums of squares for the three items : Blocks, Varieties and Residual ; and this is what we shall consider in the present paper.

The sums of squares and sum of products due to blocks are easily obtained from the totals of the 40 blocks. In getting the sums of squares and sum of products due to varieties, we are using a new procedure which simplifies considerably the computational work.

New procedure

We have 40 blocks with 10 plots each. We first calculate the mean yield of grain of each block. We next subtract from every plot yield the mean yield of the block in which the plot is located ; and call these the corrected values of the yield. There are four plots for each variety ; we next add the four corrected values of yield of each variety, and write this quantity as Q_{ij} for variety [ij]. It is obvious that the sum of the 100 values of Q_{ij} is zero. It is also clear that Q_{ij} can be calculated more easily by taking 10 times the total yield of variety [ij] and subtracting from it the sum of the total yields of the four blocks in which it has occurred, and then dividing the result by 10.

We then arrange Q_{ij} in a two-way table as shown in Table III ; and obtain the marginal means $\bar{Q}_i$ and $\bar{Q}_j$. If V_{ij} be the estimate of the effect of the variety [ij] on yield of grain, as measured from the general mean, we have

$$V_{ij} = \tfrac{1}{4}\,(Q_{ij} + \bar{Q}_i + \bar{Q}_j) \qquad (1)$$

which is obtained from Bose and Nair's general formula, after suitable substitution and simplification*.

It is easy to calculate V_{ij} from Table III. The mean of all V_{ij} will be zero. As it is usual to present varietal means instead of varietal effects, we may add to each V_{ij} the general mean for the whole experiment, and get the varietal means shown in Table IV. It should be noted that the varietal mean shown here is not the 'crude' mean of the observed yields of the four plots under a given variety, but is the mean yield per plot of the given variety after adjusting for block effects. These adjusted varietal means are, therefore, comparable among themselves. In Table IV we thus obtain the summary of the results of the experiment before calculating the sum of squares. This is only logical, as estimation should precede tests of significance.

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* For Yates' quasifactorial designs of p^2 varieties in blocks of p plots forming l groups of p sets each, and with r replications of each variety,

$$V_{ijk} = \frac{1}{r\,(l-1)}\left\{(l-1)\,Q_{ijk\ldots} + \bar{Q}_{i\ldots.} + \bar{Q}_{.i\ldots.} + \bar{Q}_{..k\ldots} + \ldots\right\}$$

In our case, $l=2$, $=4$.

Table III

Values of Q_{ij} (yield of grain)

j	0	1	2	3	4	5	6	7	8	9	Total	Mean ($\bar{Q}_j$)
0	27·10	5·35	—6·90	—0·80	—9·10	3·35	—14·35	—4·00	19·45	11·35	31·45	3·145
1	21·50	—35·25	—26·50	21·60	—5·70	5·75	12·55	17·40	3·85	—8·25	6·95	0·695
2	—5·45	12·30	12·55	—23·85	—17·65	2·80	2·60	21·95	—26·10	—22·20	—43·05	—4·305
3	—11·45	7·80	—7·45	—3·85	—13·15	0·30	21·10	8·45	0·90	—9·20	—6·55	—0·655
4	—13·50	34·25	1·00	11·10	—12·20	3·25	13·55	—12·10	13·35	3·75	42·45	4·245
5	2·80	—0·95	6·30	13·90	30·10	—2·95	7·85	13·70	—24·35	—44·45	1·95	0·195
6	0·35	—7·40	13·35	—4·55	30·15	—18·90	—51·10	—22·75	—6·30	18·60	—48·55	—4·855
7	3·20	—11·05	30·70	—7·20	0·00	—3·55	23·75	—11·90	—17·45	31·95	38·45	3·845
8	20·55	26·80	—16·45	—17·85	11·85	1·30	—14·90	—20·05	22·40	1·80	15·45	1·545
9	—2·00	—10·75	18·50	3·60	6·80	—38·25	—20·45	—5·60	—5·65	15·25	—38·55	—3·855
Total	43·10	21·10	25·10	—7·90	21·10	—46·90	—19·40	—14·90	—19·90	—1·40	0	0
Mean ($\bar{Q}_i$)	4·31	2·11	2·51	—0·79	2·11	—4·69	—1·94	—1·49	—1·99	—0·14	0	

TABLE IV

Estimated varietal effects on yield of grain (V_{ij} + *g. m.**)

i	0	1	2	3	4	5	6	7	8	9
0	58·2125	52·2250	49·2625	49·9625	48·6125	50·0250	46·2875	48·9875	54·7250	53·1625
1	56·2000	41·4625	43·7500	54·9500	48·8500	50·0125	52·4000	53·7250	50·2125	47·6500
2	48·2125	52·1000	52·2625	42·3375	44·6125	48·0250	48·6625	53·6125	41·4750	42·9125
3	47·6250	51·8875	48·1750	48·2500	46·6500	48·3125	54·2000	51·1500	49·1375	47·0750
4	48·3375	59·7250	51·5125	53·2125	48·1125	50·2750	53·5375	47·2375	53·4750	51·5375
5	51·4000	49·9125	51·8250	52·9000	57·6750	47·7125	51·1000	52·6750	43·0375	38·4750
6	49·5250	47·0375	52·3250	47·0250	56·4250	42·4625	35·1000	42·3000	46·2875	52·9750
7	52·4125	48·3000	58·8375	48·5375	51·0625	48·4750	55·9875	47·1875	45·6750	58·4875
8	56·1750	57·1875	46·4750	45·3000	53·4500	49·1125	45·7500	44·5750	55·0625	50·3750
9	49·1875	46·4500	53·8625	49·3125	50·8375	37·8750	43·0125	46·8375	46·7000	52·3875

*General mean (g. m.)=49·58.

TABLE V

Values of Q′ij (yield of straw)

j \ i	0	1	2	3	4	5	6	7	8	9	Total	Mean ($\bar{Q}'_j$)
0	—42·6	—2·4	10·0	6·0	—19·6	—0·6	24·4	37·6	—14·0	0·4	—0·8	—0·08
1	34·0	—1·8	4·6	2·6	29·0	—14·0	—9·0	—21·8	—3·4	—1·0	19·2	1·92
2	—8·8	7·4	17·8	23·8	14·2	—38·8	—9·8	41·4	—64·2	—29·8	—46·8	—4·68
3	—20·8	—6·6	—50·2	—18·2	—21·8	81·2	—33·8	31·4	—14·2	—1·8	—54·8	—5·48
4	18·4	—17·4	—3·0	—7·0	—12·6	52·4	17·4	—33·4	45·0	33·4	93·2	9·32
5	—34·0	42·2	36·6	16·6	—15·0	8·0	27·0	—13·8	—7·4	—27·0	33·2	3·32
6	—38·4	1·8	18·2	—43·8	6·6	—8·4	28·6	25·8	—19·8	—17·4	—46·8	—4·68
7	9·8	20·0	0·4	36·4	—17·2	—4·2	—23·2	—6·0	—33·6	58·8	41·2	4·12
8	—14·2	92·0	—35·6	—57·6	—35·2	19·8	30·8	38·0	—27·6	— 1·2	9·2	0·92
9	—4·4	—2·2	20·2	—7·8	6·6	—30·4	36·6	—76·2	10·2	0·6	—46·8	—4·68
Total .	—101·0	133·0	19·0	—49·0	—65·0	65·0	89·0	23·0	—129·0	15·0	0	0
Mean ($\bar{Q}'_i$) .	—10·10	13·30	1·90	—4·90	—6·50	6·50	8·90	2·30	—12·90	1·50	0	

TABLE VI

Estimated varietal effects on yield of straw (V'_{ij} + *g. m.**)

j \ i	0	1	2	3	4	5	6	7	8	9
0	52·50	68·40	68·65	65·95	59·15	67·15	74·00	75·65	58·95	66·15
1	72·15	69·05	67·80	65·60	71·80	64·30	66·15	61·30	62·10	66·30
2	59·80	69·70	69·45	69·25	66·45	56·45	64·30	75·45	45·25	57·45
3	56·60	66·00	52·25	58·55	57·25	86·25	58·10	72·75	57·55	64·25
4	70·10	67·00	67·75	65·05	63·25	82·75	74·60	60·25	76·05	76·75
5	55·50	80·40	76·15	69·45	61·15	70·15	75·50	63·65	61·45	60·15
6	52·40	68·30	69·55	52·35	64·55	64·05	73·90	71·55	56·35	60·55
7	66·65	75·05	67·30	74·60	60·80	67·30	63·15	65·80	55·10	81·80
8	59·85	92·25	57·50	50·30	55·50	72·50	75·85	76·00	55·80	66·00
9	60·90	67·30	70·05	61·35	64·55	58·55	75·90	46·05	63·85	65·05

*General mean (g. m.)=65·695

Similar calculations can be made for the yield of straw. Let Q'_{ij} and V'_{ij} correspond, in the case of straw, to Q_{ij} and V_{ij} defined in the case of yield of grain. Tables V and VI give the values of Q'_{ij} and V'_{ij} respectively.

Having calculated the values of these four quantities (Q_{ij}, V_{ij} and Q'_{ij}, V'_{ij}) we get the sum of squares due to varieties in the case of yield of grain with the help of the formula

$$\sum_{i=o}^{9} \sum_{j=o}^{9} V_{ij} Q_{ij} \tag{2}$$

and the sum of squares due to varieties in the case of yield of straw from the formula

$$\sum_{i=o}^{9} \sum_{j=o}^{9} V'_{ij} Q'_{ij} \tag{3}$$

The sum of products due to varieties for yield of grain and yield of straw can be obtained by either of the two expressions :

$$\sum_{i=o}^{9} \sum_{j=o}^{9} V_{ij} Q'_{ij} \text{ or } \sum_{i=o}^{9} \sum_{j=o}^{9} V'_{ij} Q_{ij} \tag{4}$$

which are identical. It is convenient, therefore, to calculate the product independently in both ways which furnishes a check on the whole set of calculations.

Tests of significance

Table VII gives the full analysis of variance and covariance.

Table VII

Analysis of variance and covariance

Variation due to	D. F.	Sum of product of grain and straw	Sum of squares		Coefficient of correlation
			Grain	Straw	
Blocks .	39	21917·10	12801·80	44680·79	+0·9164
Varieties .	99	1737·68	7694·07	23243·79	+0·1299
Error .	261	7230·22	5765·70	21074·21	+0·6559
Total .	399	30885·00	26261·57	88998·79	

Ratios of variances due to varieties and residual show that there are significant differences among the varieties with respect to yield of grain as well as of straw. These are shown in Table VIII.

TABLE VIII

Test of significance of varietal effects

Variation due to	D. F.	Variance		Ratio of variances		Expected R. V.	
		Grain	Straw	Grain	Straw	5 per cent	1 per cent
Blocks . .	39	328·26	1145·66				
Varieties .	99	77·71	234·79	3·52	2·91	$<$1·57	$<$1·88
Error . .	261	22·09	80·74				
Total .	399						

The standard error per plot for yield of grain is 4·70 or 9·48 per cent of mean. The standard error per plot for yield of straw is 8·98 or 13·67 per cent of mean. These compare well with the precision of ordinary randomized block experiments.

Tables IV and VI supply the summary of results from which detailed tests of significance can be used for differences between any pair of varieties. The varietal means were calculated correct to four decimal places for grain and two decimal places for straw in order to maintain a high order of precision in the sums of squares and of products.

The standard error of the differences between two varieties occurring in the same row or column is $\sqrt{(\frac{2}{4}\times\frac{11}{10})}$ times the standard error per plot, which works out to be 3·49 for grain and 6·66 for straw. The corresponding critical differences, at 5 per cent and 1 per cent levels, are respectively 6·87 and 9·05 for grain and 13·12 and 17·29 for straw.

The standard error of the difference between two varieties not occurring in the same row or column is $\sqrt{(\frac{2}{4}\times\frac{12}{10})}$ times the standard error per plot and works out to be 3·64 for grain and 6·95 for straw. The corresponding critical differences, at 5 per cent and 1 per cent levels, are respectively 7·16 and 9·44 for grain and 13·69 and 18·04 for straw.

There are 100 C_2 or 4950 comparisons between all pairs of varieties. Of these, 900 belong to pairs occurring in the same row or column; and 4050 to pairs not having a row or column in common with them. Thus, for example, of the three varieties numbered 42, 45 and 82, the pair formed with 42 and 45 and the pair formed with 42 and 82 belong to the first kind of comparisons; and the pair formed with 45 and 82 belongs to the second kind of comparisons. It will be noted that comparisons of the second kind have a larger error than the comparisons of the first kind. This is due to the fact that varietal pairs of the second kind do not occur together in the same block. Comparisons among them are thus affected by a greater amount of block variation than comparisons among pairs of the first kind which occur together in the same block.

Two more tables may be constructed rearranging the values of Tables IV and VI according to their decreasing magnitude, to show at a glance which varieties form classes of higher or lower yielders of grain and of straw.

This has been done in another way in Table IX, which shows against each variety its rank for yield of grain as well as for yield of straw.

TABLE IX

Ranked position of the varieties (upper figures indicate the rank for yield of grain ; lower figures indicate the rank for yield of straw)

i	0	1	2	3	4	5	6	7	8	9
	4	31	51	47	58	45	83	55	13	22
0	94	35	34	51	78	42	18	12	79	47
	8	97	89	12	56	46	27	16	44	70
1	22	33	37	53	23	58	48	69	66	46
	65	32	30	94	87	68	57	17	96	92
2	77	28	31	32	45	88	59	14	100	85
	71	33	66	64	79	62	14	38	53	74
3	87	49	97	81	86	2	82	20	83	60
	61	1	36	21	67	43	18	72	19	35
4	26	43	38	54	64	3	17	74	8	6
	37	48	34	24	5	69	39	25	90	98
5	91	5	7	30	70	25	13	63	67	75
	49	75	29	76	7	93	100	95	82	23
6	95	36	29	96	56	61	19	24	89	73
	26	63	2	59	40	60	10	73	85	3
7	44	15	40	16	72	39	65	52	93	4
	9	6	80	86	20	54	84	88	11	42
8	76	1	84	98	92	21	11	9	90	50
	52	81	15	50	41	99	91	77	78	28
9	71	41	27	68	57	80	10	99	62	55

CORRELATION BETWEEN GRAIN AND PADDY

The last column of Table VII gives the coefficient of correlation between yield of grain and yield of straw for variations due to blocks, varieties and residual (error).

The high correlation of $+0\cdot9164$ due to blocks indicates that blocks with large yields of grain also have large yields of straw, and blocks with low yields of grain have low yields of straw. This shows that the influence of soil fertility on the yield of both grain and straw is working in the same direction. That is, the yield of both grain and straw is greater in better type of soil which is just what is to be expected.

The inter-varietal correlation or the correlation between the mean yields of grain and of straw for the varieties is only +0·1299 which is insignificant at the 5 per cent level. It will be seen from the accompanying scatter diagram (Fig. 1) that there is little or no tendency towards clustering in the points. This shows that a variety having a high yield of grain does not necessarily have a high yield of straw. In fact, on an average among

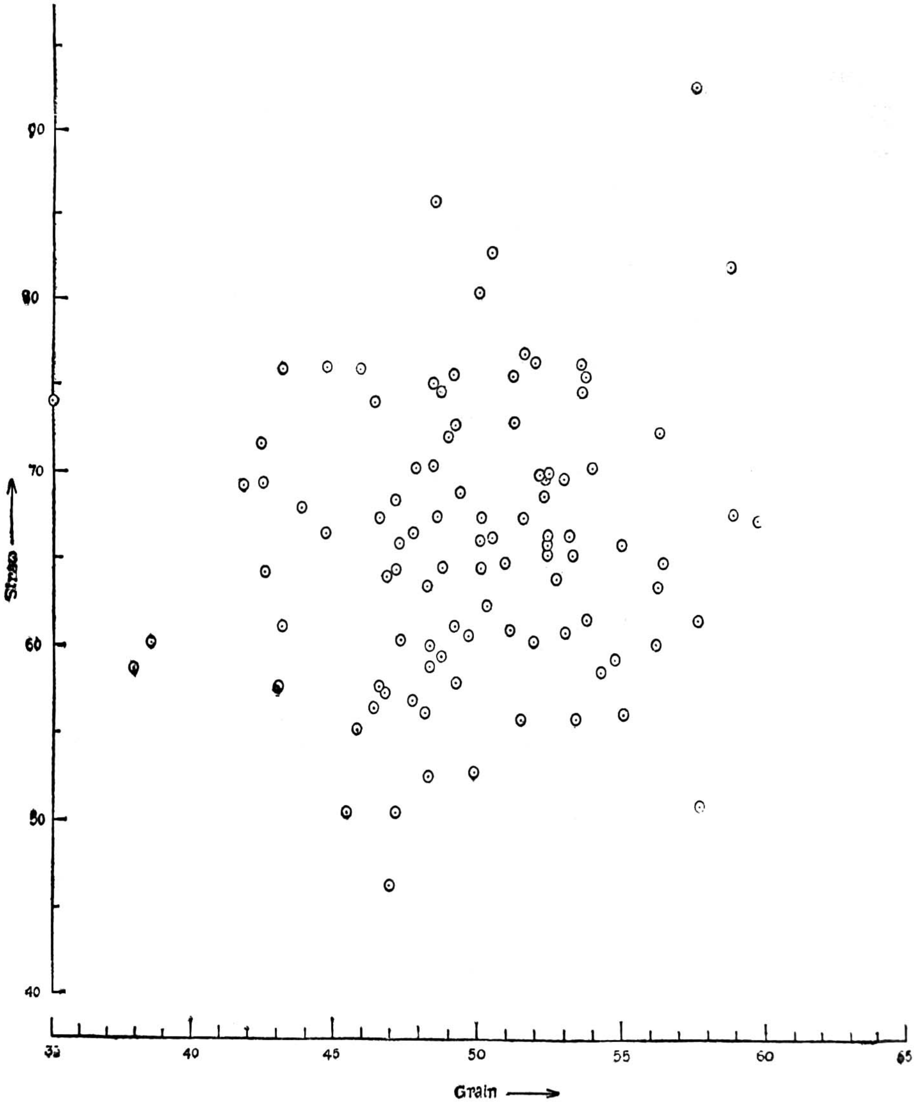

FIG. 1. Scatter diagram of the mean yields of grain and straw of the one hundred varieties

varieties giving high yields of grain there will be all kinds of yields of straw, high, medium and low. This clearly brings out the danger of confining our attention to a single character, say, the yield of grain, in pre-selection field trials.

The residual (error) correlation is $+0{\cdot}6559$ which is significant at the one per cent level. This gives the correlation between yield of grain and of straw of individual plots after correcting for effects of both blocks and varieties. The significant correlation shows that the uncontrolled factors, such as differences in fertility from plot to plot, exert their influence in the same direction on both grain and straw.

JOINT COMPARISON OF YIELDS

Before concluding we shall make a few preliminary observations regarding a comparison of the varieties jointly on the basis of yields of both grain and straw.

If we attach the same importance to both grain and straw (which, of course, is not really justified from the agricultural or economic point of view), a rough way of assessing the relative value of the crop as from both the two characters, will be to add the two ranks, and arrange the varieties in accordance with their combined ranks. If there had been perfect correlation between the two characters, the combined ranks would have taken the values 2, 4, 6 ....200. In our case the combined rank shown in Table X starts from 7 and ends with 196, with repetitions at some of the values. Thus rank 7 is shared by the varieties 18 and 97. Variety 18 stood first also in the ranking for straw yield; but variety 14, which stood first in the ranking for grain yield, stands low in the combined ranking, due to its poor straw yield.

TABLE X

Sum of the ranks of grain and straw yields

j \ i	0	1	2	3	4	5	6	7	8	
0	98	66	85	98	136	87	101	67	92	69
1	30	130	126	65	79	104	75	85	110	116
2	142	60	61	126	132	156	116	31	196	177
3	158	82	163	145	165	64	96	58	136	134
4	87	44	74	75	131	46	35	146	27	41
5	128	53	41	54	75	94	52	88	157	173
6	144	111	58	172	63	154	119	119	171	96
7	70	78	42	75	112	99	75	125	178	7
8	85	7	164	184	112	75	95	97	101	92
9	123	122	42	118	98	179	101	176	140	83

TABLE XI

Relative total money value of grain and straw (figures in parenthesis give the rankings)

j \ i	0	1	2	3	4	5	6	7	8	9
0	69·70 (11)	67·19 (30)	64·28 (52)	64·39 (50)	61·55 (69)	64·71 (48)	62·48 (62)	65·54 (42)	67·62 (22)	67·63 (21)
1	71·98 (5)	56·57 (92)	58·58 (86)	69·30 (12)	64·56 (49)	64·08 (53)	66·87 (35)	67·13 (32)	63·80 (54)	62·15 (64)
2	61·29 (70)	67·35 (28)	67·45 (26)	57·49 (90)	69·15 (15)	60·37 (79)	62·73 (60)	70·12 (7)	51·37 (98)	55·48 (96)
3	60·01 (80)	66·33 (39)	59·60 (82)	61·06 (74)	59·17 (83)	67·18 (31)	66·91 (34)	67·06 (33)	61·73 (67)	61·13 (73)
4	63·67 (55)	74·37 (3)	66·53 (38)	67·44 (27)	61·95 (66)	68·38 (18)	69·86 (9)	60·42 (78)	70·11 (8)	68·33 (19)
5	63·64 (56)	67·50 (25)	68·48 (17)	68·09 (20)	61·05 (75)	63·06 (58)	67·62 (23)	66·60 (37)	56·48 (93)	51·63 (97)
6	60·99 (76)	61·98 (65)	67·54 (24)	58·48 (87)	70·57 (6)	56·47 (94)	51·27 (99)	57·95 (88)	58·61 (85)	66·22 (40)
7	68·99 (16)	64·72 (47)	73·56 (4)	64·86 (45)	64·36 (51)	63·20 (57)	69·80 (10)	61·58 (68)	57·73 (89)	76·38 (2)
8	69·27 (13)	77·37 (1)	59·05 (84)	56·30 (95)	65·59 (41)	64·97 (43)	62·34 (63)	61·20 (71)	67·27 (29)	64·81 (46)
9	62·51 (61)	61·17 (72)	69·19 (14)	62·73 (59)	64·96 (44)	50·68 (100)	59·62 (81)	56·91 (91)	60·67 (77)	66·62 (36)

From the economic point of view the procedure of using the joint rank for grain and straw yields is unsatisfactory, for the money returns from equal weights of grain and straw are quite different. A better plan is to use the total money return of the crop for both paddy and straw taken together. For example, for the crop under consideration, we find that Rs. 2 and As. 7 may be taken as the average price of one maund of grain and of one maund of straw respectively ; and we can reduce the yield of straw to equivalent quantities of grain by using the multiplier 7/32.

The mean yield of straw measured on this new scale is added to the mean yield of grain of each variety to assess the total money value of the yields of grain and straw. These are given in Table XI in which the ranks are written in parenthesis. Varieties 18 and 97 now stand differentiated, as first and second, though there was a tie between them for the first place in the ranking given in Table X. Variety 14, which stood low in Table X in spite of having the highest yield in grain, stands third in importance according to the assessment made in Table XI.

While judging the superiority of one variety to another, ranks are, however, not quite satisfactory, and rigorous tests of significance should be used. We can do this for the values given in Table XI by using an analysis of variance of the ' money value ', namely, of the variable.

$$M = X + \tfrac{7}{32} Y \qquad (5)$$

where X = yield of grain and Y = yield of straw. The relevant data are given in Table XII. The sum of squares in any line of this table is obtained by multiplying by $(7/32)^2$, $(7/16)$ and (1) respectively the sum of squares of straw, sum of products, and sum of squares of grain, of the corresponding line of Table VII.

TABLE XII

Analysis of variance of total money value of grain and straw

——	D. F.	Sum of squares	Variance	Ratio of variances	
				Observed	Expected 1 per cent
Blocks	39	24528·5769	628·94		
Varieties . . .	99	9566·5567	96·63	2·54	<1·88
Error	261	9937·3552	38·07		
Total .	399	44032·4888			

The ratio of variances for M is highly significant, showing the high variation in the money value of the different varieties. The standard error per plot of the money value is 6·1705. The standard error of the difference between any two values occurring in the same row or column of Table XI is 4·576 and the critical differences at 5 per cent and 1 per cent levels are respectively 9·01 and

11·90. The standard error of the difference between any two values of Table XI not occurring in the same row or column is 4·780 and the critical differences at the 5 per cent and 1 per cent levels are respectively, 9·42 and 12·43. With the help of these critical values it is now possible to use tests of significance for comparing the money values of any two varieties.

SUMMARY

A new method of analyses of variance and covariance has been discussed, with the help of actual experimental data of a quasi-factorial experiment on paddy with one hundred varieties arranged in a square lattice design ; and it has been shown that the new procedure will considerably reduce the computational labour.

From the analysis of covariance it has been found that there is little or no inter-varietal correlation between mean yield of grain and mean yield of straw of the different varieties. This shows the need of taking into consideration more than one economic character of the plant, in this case, for example, the yield of both grain and straw in pre-selection trials.

For the purpose of grading the varieties it is clearly desirable to use a scale which will take into consideration both grain and straw. Two methods, namely, (1) the sum of the separate ranks of the two characters, and (2) the money return from both grain and straw have been briefly discussed for purposes of illustration. The varieties were found to be significantly differentiated with regard to the money return.

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We are grateful to Rao Bahadur M. Vaidyanathan for having drawn our attention to a short paragraph on page 125 of the revised edition (1937) of C. H. Goulden's *Methods of Statistical Analysis* in which the author gives an expression for the sum of squares of varieties alternative to the one given by Yates. The expression used by us is entirely different and applicable to the wider class of ' Partially Balance Incomplete Block Designs ', dealt with in this paper.

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Appendix

Random No.	Name of strain		Random No.	Name of strain	
00	Kalma	107/36	50	Harimai	1180/34
01	Maula	1332/36	51	Bankumari	1031/36
02	Jatakalma	389/33	52	Uttarekalma	386/33
03	Baktulsi	928/36	53	Karticsail	1350/36
04	Rupsail	849/36	54	Rangi	475/32
05	Ahamsail	1129/36	55	Kalakartic	135/32
06	Madhumalati	1515/36	56	Tengra	1541/36
07	Kanakchur	1221/36	57	Nona	251/32
08	Dudkalma	144/36	58	Jhingasail	220/36
09	Ramsail	1283/36	59	Kamalbhog	1127/36
10	Ailsail	1092/36	60	Kamalbhog	1532/36
11	Gopalbhog	916/36	61	Sundarsail	1346/36
12	Latamagurasail	1357/36	62	Peswari	1083/36
13	Baskamalbhog	1342/36	63	Jhingasail	221/36
14	Dudkalma	191/36	64	Kalma	94/36
15	Kartikbalam	1597/36	65	Luchai 15	945/36
16	Mugaibalam	1547/36	66	Tengrasylhet	940/36
17	Agniswar	1712/36	67	Dudkalma	366/33
18	Lalkalma	158/36	68	Kamanisail	938/36
19	Bankumari	1355/36	69	Kamalbhog	1607/36
20	Sarunagra	284	70	Karticbalam	1594/36
21	Kalma	112/36	71	Dudkalma	85/36
22	Peswari	1139/36	72	Jhingasail	275/36
23	Kalamkati	47/35	73	Seetasail	833/36
24	Mota	1690/36	74	Brindabansail	1219/36
25	Jhingasail	215/36	75	Tengrasylhet	1324/36
26	Gangajal	1709/36	76	Mahipal	1194/34
27	Harimai	1207/36	77	C-O 1	(1171/36)
28	Seetasail	496/32	78	Mausal 239	..
29	Lalkalma	176/32	79	Nagra 40	322/33
30	Chamarmani	874/36	80	Bachaibalam	1173/36
31	Kalma	95/36	81	Chamarmani	876/36
32	Harimai	1211/36	82	Baktulsi	915/36
33	Localnagra	..	83	Algorasail	1118/36
34	Kalma	182/36	84	Rupsail	196/33
35	T 31 (Pusa)	..	85	Ramsail	1503/36
36	Bakchur	..	86	Sitasail	830/36
37	C-O 1	..	87	Baktulsi	914/36
38	Nagra 100	266/32	88	Dudkalma	133/36
39	Jatakalma	401/33	89	GEB 24	..
40	SC 54/10	(1326/36)	90	Bhasamanik	..
41	Peswari	1096/36	91	Metekalma	183/32
42	Kamalbhog	1523/36	92	Dudkalma	166/36
43	Rudin	1531/36	93	Baktulshi	960/36
44	T 24 (Pusa)	..	94	Magurasail	1519/36
45	Nagra	126 (308/33)	95	T 52 Pusa	..
46	Patharkuchi	1273/36	96	Seetasail	834/36
47	Baskamalbhog	1036/36	97	Auspapri	1172/36
48	Nagra	65/5 (262/32)	98	Pubebalam	1599/36
49	Sitasail	815/36	99	Harimai local	1110/36

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