

APPENDIX I.

A STATISTICAL REPORT ON THE RUPEE CENSUS.

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Introduction.

1. The following questions were referred to us by the Reserve Bank of India early in 1940 :

(a) Whether the Findlay Shirras formula for estimating the amount of Rupee coins in actual circulation in India is reasonably accurate or whether it would be possible to improve on it ;

(b) Whether the data collected in the manner prescribed in the Resource Manual are sufficient as a basis for applying the formulæ ; and

(c) Whether methods could be devised to ensure that these data are correct.

Available Material.

2. For a thorough investigation of these questions it is necessary to examine the results of the sample census in detail for each individual issue of the rupee coins. Unfortunately these primary records down to 1934 had been destroyed. Even in recent years the material was incomplete in many ways, as can be seen from Table (1) which shows the number (and distribution of treasuries by circles) for which individual records were available.

Table 1. Number of Treasuries from which Individual Records were available.

Circle (1)	1935 (2)	1936 (3)	1937 (4)	1938 (5)	1939 (6)	1940 (7)	Total (8)
Bombay	7	5	5	17
Calcutta	30	30
Cawnpore	36	..	20	18	74
Lahore	33	2	6	7	48
Madras	..	29	29	25	1	1	86
Rangoon	3	2	5
Total	..	29	65	58	10	35	260

3. Besides this we were supplied with the sample proportions of all coins of each reign grouped together for all treasuries for the period 1911-1940. Similar grouped data for each reign were available for the period 1935-40 for a number of individual treasuries the distribution of which is shown in Table (2).

Table 2. Number of Treasuries from which grouped data were available.

Circle (1)	1935 (2)	1936 (3)	1937 (4)	1938 (5)	1939 (6)	1940 (7)	Total (8)
Bombay	40	43	39	122
Calcutta	66	66	132
Cawnpore	49	47	96
Karachi	1	4	3	8
Lahore	36	36	40	41	193
Madras	..	28	28	25	26	27	161
Rangoon	37	37	74
Total	..	28	64	61	108	260	786

4. We also had Mr. Adie's notes on Rupee Census from which it was possible to get the pooled proportions from 1875 to 1899 of coins of different years of issue, that is, the proportions of each issue for all samples in India taken together.

History of the Rupee Census in India.

5. The Rupee Census was started in 1876. Originally 2,000 coins used to be sorted at each treasury; but from 1895 the number was increased to 10,000. Between 1894 and 1898 various improvements were introduced by Mr. Harrison, an account of which, given by himself, can be found in the Report of the Fowler Committee on Indian Currency, 1898. Mr. Adie in 1899-1900 used a slightly different method, based on a more sound mathematical treatment, for estimating the volume of coins in circulation. Mr. Findlay Shirras in 1919 gave a clarified description of Adie's method with some improvements and checks in computational procedure in "Indian Finance and Banking." In all these methods it was assumed that the rate of absorption was proportional to the total number of coins in circulation which may be conveniently described as a compound interest law with negative rate of interest. Since 1919 the question does not appear to have been further examined.

The Present Review.

6. We have used substantially the same law of absorption as used by Adie. Our fundamental assumption is, to a first approximation, that the number of rupees absorbed from circulation is proportional to the total number in circulation at any given instant. Secondly, we have assumed that the rate of absorption is the same for all coins in all years. We shall call this the simple theory to distinguish it from other possible sets of more complicated assumptions. Using recent advances in the theory of statistical estimation we have calculated the absorption coefficient and the volume of circulation of rupee coins for each year based on available material relating to the Rupee Census.*

Revised Estimates of Circulation.

7. The results are given in Tables (3) and (4). In Table (3), col. (1) gives the year of the Census on which our calculations are based; col. (2) the absorption coefficient; col. (3) the estimated circulation based on the corresponding coefficient of absorption; and col. (4) the official estimates made from time to time, for comparison. There is a gap from 1900 to 1911 as the results of the Rupee Census were not available for these years. From 1922 only our own estimates are given as apparently no official estimates were prepared.

8. The coefficients of absorption estimated on the basis of the simple theory reveal some interesting features. In the earlier period from 1881 to 1899 the average rate of absorption was 0.0256; and fluctuations from year to year were more or less random. This may be interpreted as an empirical corroboration or justification of the simple theory showing that the rate of absorption was substantially constant in this period. During the second period, from 1912 to 1920, the average rate of absorption was slightly lower (0.0244); and there was some evidence of a decrease in the rate of absorption from 1916. During the period 1921-28 the average value was distinctly lower (0.0209), but fluctuations were probably random. Since 1929 there has been a steady and rapid increase in the coefficient of absorption. This shows that either the simple theory has become increasingly inadequate, or there has been a deterioration in the reliability of the Rupee Census in recent years.

Large Fluctuations in Recent Years.

9. A glance at Table (4) shows that results based on individual treasuries in recent years are not satisfactory. The estimated coefficient of absorption differs appreciably from circle to circle and from year to year. For example, the coefficient of absorption was so low as 0.0319 in Cawnpore Circle in 1936 and so high as 0.0649 in Rangoon Circle in 1940. These are extreme cases; but a careful examination of the other values shows large and irregular variations.

10. The fluctuation in the coefficient of absorption in recent years is too large to admit of any reliable estimate being made of the total circulation of Rupee coins on the basis of the simple theory. We must now consider possible sources of variation. These may be separated into two broad groups:

- (i) lack of homogeneity in the primary material, *i.e.*, in the results of the Rupee Census; and
- (ii) inadequacy of the simple theory.

Heterogeneity of the Material.

11. Whatever be the particular formula used for determining the volume of Rupee coins in circulation, no valid estimates can be made unless and until the results of the Rupee Census for different places in different years are statistically homogeneous. It is inevitable that the observed proportion of coins of a particular issue should change from sample to sample due to chance fluctuations. But such fluctuations must lie within the limits of random variation (which can be calculated from the theory of probability) in order that valid estimates may be obtained.

* Relevant mathematical formulæ are given concisely in the form of an Appendix.

12. The available data for individual coins for the years 1935-40 were subjected to appropriate statistical tests. The results are concisely given in Table (5) in which col. (1) gives the names of circles; col. (2) the number of years for which comparisons are made; and col. (6) number of Treasuries compared.

Table 5. Significance of the Variation in the Proportion of Coins between "Years" and between "Treasuries."

Circle	No. of years	Variation between years			No. of Treasuries	Variation between Treasuries		
		V. E.	Ed. VII	G. V.		V. E.	Ed. VII	G. V.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Bombay	3	S*	S	S	35	S*	S*	S*
Calcutta	2	S	..	S	61	S*	S*	S*
Cawnpore	2	46	S
Lahore	5	S*	S*	..	35	S*	S*	S*
Madras	6	S*	S*	S*	23	S*	S*	S*
Rangoon	2	37	S*	S*	S*

V. E. = Victoria Empress
 Ed. VII = Edward VII
 G. V. = George V

13. The results of comparison are given in cols. (3)—(5) for "years", and in cols. (7)—(9) for "treasuries." Differences considered to be real on the five per cent. level are marked by the symbol 'S'. This means that the odds are 19 to 1 against the observed difference having occurred by pure chance. In other words, the betting is 19 to 1 in favour of some systematic cause of variation having come into operation at the time of the sample census. When the difference appears to be even more marked (using one per cent. level of significance) a star is attached to the symbol 'S'. Thus 'S*' indicates that the odds are 99 to 1 in favour of the observed difference having been brought about by systematic causes of fluctuation. For example, in the first line in Bombay circle the comparison is based on 3 years, and the differences between years are considered to be significant on the five per cent. level (odds of 19 to 1) in Edwardian and Georgian coins, and significant on the one per cent. level (odds 99 to 1) in the case of the Victorian coins. A glance at the table is enough to show that, except in Cawnpore and a few other isolated cases, the differences are generally real. The material cannot therefore be treated as homogeneous in the statistical sense, and cannot be validly used in estimating the total circulation without further examination.

14. This heterogeneity of the material may be ascribed to one or several of these causes :—

- Local differences in the circulation between different regions commanded by different treasuries in the same year due to 'friction' in the circulation of coins;
- Differences in the local circulation in the same place from year to year;
- Seasonal differences in the same place during different parts of the same year;
- Gross errors of counting due to negligence or fudging; and
- Sampling fluctuations.

15. To get a valid estimate of the circulation we must eliminate (d) gross errors in counting, and study items (a) to (c) with a view to devising a method which will take these sources of fluctuation into consideration. Work on this line is still proceeding in the Statistical Laboratory, Calcutta.

Summary.

Methods for estimating the rupee circulation were concisely reviewed. On the assumption of a simple law of constant rate of absorption a new formula was developed to estimate this coefficient absorption. This coefficient was quite steady during the earliest period from 1881 to 1899 for which records were available. During the second period from 1912 to 1920 the value of the coefficient was lower but again fairly steady. From 1921 to 1928 there was a further decrease in the numerical value of the coefficient. Since 1929 there has been a rapid increase in its value, and fluctuations have also become irregular. This shows that either the simple theory has become increasingly inadequate or the results of the rupee census have become increasingly unreliable. The recent data were subjected to appropriate statistical tests for examining the homogeneity of the material; and it was found that they were markedly heterogeneous. Further investigations are therefore necessary in order to devise a reliable method for estimating the rupee circulation.

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Table (3). Estimates of Circulation of Rupee Coins.

Year	Estimated Coefficient of Absorption	Estimated Circulation in crores of Rupees	Official Estimates in crores of Rupees
(1)	(2)	(3)	(4)
1881	·0259	132
1882	·0223	147
1883	·0240	141
1884	·0249	139
1885	·0229	152
1886	·0282	138
1887	·0247	153
1888	·0257	154
1889	·0276	151
1890	·0247	168
1891	·0256	168
1892	·0239	180
1893	·0269	173
1894	·0284	164
1895	·0306	151
1896	·0267	160
1897	·0273	154
1898	·0234	166
1899	·0235	162
1912	·0257	230
1913	·0261	238
1914	·0274	231
1915	·0275	226
1916	·0228	267
1917	·0246	277
1918	·0205	338
1919	·0224	362
1920	·0224	364
1921	·0220	360
1922	·0215	356
1923	·0205	356
1924	·0204	350
1925	·0201	345
1926	·0212	329
1927	·0206	327
1928	·0212	316
1929	·0244	282
1930	·0266	260
1931	·0283	241
1932	·0207	226
1933	·0307	213
1934	·0354	180
1935	·0387	158
1936	·0358	166
1937	·0332	173
1938	·0409	131
1939	·0412	125
1940	·0455	105

} Not available.

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Table (4). Estimates of Circulation of Rupee Coins (from Ungrouped Data.)

Circle and year of sampling	Number of Treasuries	No. of coins in the sample per thousand	Estimated coefficient of Absorption	Estimated Circulation in Crores of Rupees					
				1935	1936	1937	1938	1939	1940
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Bombay—									
1938	7	70	-0487	121	115	110	104	100	95
1939	5	50	-0486	121	115	110	104	100	95
1940	5	50	-0569	121	93	88	83	79	74
Calcutta—									
1940	30	245	-0540	106	100	95	90	85	81
Cawnpore—									
1936	36	349	-0319	193	187	181	176	170	165
1939	20	200	-0361	171	165	160	153	148	142
1940	18	176	-0370	166	160	154	149	144	138
Lahore—									
1937	33	321	-0353	175	169	163	157	152	146
1938	2	20	-0425	142	136	131	125	120	115
1939	6	60	-0463	129	123	117	112	107	102
1940	7	70	-0637	84	79	74	69	65	61
Madras—									
1935	29	268	-0342	180	174	169	163	157	152
1936	29	266	-0338	182	176	170	165	159	154
1937	25	243	-0329	187	181	175	169	164	159
1938	1	10	-0426	128	122	116	111	106	101
1939	1	10	-0452	133	127	121	116	111	106
1940	1	10	-0452	133	127	121	116	111	106
Rangoon—									
1939	3	30	-0649	82	76	72	67	63	59
1940	2	20	-0573	98	92	87	82	78	73
All Circles—									
1935	29	268	-0342	180	174	168	163	157	152
1936	65	616	-0327	188	182	176	171	165	160
1937	58	564	-0343	180	174	168	162	157	152
1938	10	100	-0472	126	120	114	109	104	99
1939	35	350	-0428	126	121	116	111	106	102
1940	63	572	-0466	128	122	116	111	106	101

MATHEMATICAL APPENDIX.

(a) Let N_{it} be the number of coins minted in the i -th year which survive up to the t -th year (i and t being measured from any convenient base year). Assuming that the proportionate rate of absorption is constant and equal to r it can be easily shown that

$$(1) \dots\dots\dots N_{it} = N_{ii} e^{-r(t-i)}$$

where, of course, N_{ii} is the number of coins minted in i -th year. Our problem is to estimate this rate of absorption r .

The proportion of the i -th year coins in circulation in the t -th year is $N_{it}/\sum_i N_{it}$ where \sum_i represents a summation for all values of i . It easily follows that the probability of a sample having n_{it} coins of the i -th year is

$$(2) \dots\dots\dots \frac{\prod_i (n_{it})!}{\prod_i (n_{it}!)^{n_{it}}} \cdot \frac{\prod_i (N_{it})^{n_{it}}}{\left[\sum_i N_{it} \right]^{\sum_i n_{it}}}$$

(multinomial theorem)

where \prod_i represents the product for all values of i .

This probability is a maximum when

$$(3) \dots\dots\dots \sum_i (i P_{it}) = \frac{\sum_i (i N_{it} e^{-ri})}{\sum_i (N_{it} e^{-ri})} \text{ where}$$

$$P_{it} = \frac{n_{it}}{\sum_i n_{it}}$$

Of this the left side is known from the actual sample. The right side is a function of r . Thus r can be easily calculated from this equation.

The corresponding equation for the grouped proportions is easily found to be

$$(4) \dots\dots\dots \sum_k P_{kt} \left\{ \frac{\sum_{i=k}^{k'} i N_{it} e^{-ri}}{\sum_{i=k}^{k'} N_{it} e^{-ri}} \right\} = \frac{\sum_i i N_{it} e^{-ri}}{\sum_i N_{it} e^{-ri}}$$

Having calculated r we can easily get the total circulation from equation (1).

(b) To test the homogeneity of the material we used the analysis of variance after applying Bartlett's transformation of $\text{Sin}^{-1} \sqrt{p}$ to the observed proportions of different coins in the sample.

APPENDIX II.

LIST OF SCHEDULED BANKS.

1. Ajodhia Bank
2. Allahabad Bank
3. American Express Co. Inc.
4. Banco Nacional Ultramarino
5. Bank of Baroda
6. Bank of Behar
7. Bank of China
8. Bank of Communications
9. Bank of Hindustan
10. Bank of India
11. Bank of Taiwan
12. Bank of Upper Burma
13. Bengal Central Bank
14. Calcutta Commercial Bank
15. Calcutta National Bank
16. Canara Bank
17. Canara Banking Corporation
18. Canara Industrial & Banking Syndicate
19. Central Bank of India
20. Chartered Bank of India, Australia & China
21. Comilla Banking Corporation
22. Comilla Union Bank
23. Comptoir National D'Escompte de Paris
24. Devkaran Nanjee Banking Co.
25. Dinajpore Bank
26. Eastern Bank
27. Grindlay & Co.
28. Hongkong and Shanghai Banking Corporation
29. Imperial Bank of India
30. Indian Bank
31. Indian Overseas Bank
32. Indo-Commercial Bank
33. Industrial Bank of Western India
34. International Bank of India
35. Jwala Bank
36. Karmani Industrial Bank
37. Laxmi Bank
38. Lloyds Bank
39. Mercantile Bank of India
40. Mitsui Bank
41. Mohaluxmi Bank
42. Nadar Bank
43. Nath Bank
44. National Bank of India
45. National City Bank of New York
46. Nedungadi Bank
47. Netherlands India Commercial Bank N. V.
48. Netherlands Trading Society
49. New Citizen Bank of India
50. New Standard Bank
51. Noakhali Union Bank
52. Oudh Commercial Bank
53. Oversea Chinese Banking Corporation
54. Palai Central Bank
55. Pioneer Bank
56. Punjab and Sind Bank
57. Punjab Co-operative Bank
58. Punjab National Bank
59. Simla Banking & Industrial Co.
60. Thos. Cook & Son (Bankers)
61. Union Bank of India
62. United Industrial Bank
63. U. Rai Gyaw Thoo & Co.
64. Yokohama Specie Bank.