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CORRELATION BETWEEN ANTHROPOMETRIC CHARACTERS IN SOME BENGAL CASTES AND TRIBES

By P. C. MAHALANOBIS and Mrs. CHAMELI BOSE

Statistical Laboratory, Calcutta.

Introduction

Extensive anthropometric data were collected by Sir H. H. Risley in 1886-88, and were published by him [11, 1891] in 1891. Conclusions based on these data have been incorporated in official publications by the Government of India and also in books of reference and text-books. A critical review of Risley's work on a scientific basis is therefore of fundamental importance for progress in Indian anthropometry. This question was considered by one of us [7, 1933; 8, 1934] in two earlier papers in which the following points were noted:

(*a*) Risley's measurements were taken on one definite system and under the supervision of one single individual.

(*b*) The material consists of measurements of 5784 males between ages 25 and 45 years belonging to 87 different castes and tribes distributed over practically the whole of North India. In the case of 73 samples the number of individuals was greater than 20, and in the case of 60 samples the number was greater than 40.

(*c*) The number of characters is also fairly large. Eleven measurements (comprising stature, weight, and nine measurements on the head) and eight indices are available for practically all the castes and tribes; two additional measurements on the head and two additional indices are available for 53 samples.

2. Risley's data thus represent the most important anthropometric material relating to Indian castes and tribes for which individual measurements are so far available. Extensive series of individual measurements were taken ten years ago at the time of the Indian Census of 1931; but these have not yet been published or made available for scientific examination and analysis.

3. The instructions in Risley's survey, however, included sentences recommending the rejection of persons of very black complexion and with very broad and depressed noses in the case of higher castes, and of men of very fair complexion among lower castes owing to the suspicion of intermixture. This may exaggerate the difference between high and low castes. But how far, if at all, the samples have been actually vitiated cannot be determined on *a priori* grounds and without comparison with unbiassed measurements. Fresh material is necessary for this purpose. Even when such fresh material becomes available it will be necessary to work out the statistical constants for purposes of critical comparison. A detailed analysis of Risley's material is thus essential even in order to settle the question whether the material may be accepted for scientific purposes or has to be rejected.

4. A second and more serious objection to Risley's data was pointed out long ago by Karl Pearson [3, 1902—03]. The numerical figures given in Risley's volume showed serious discrepancies. These were examined critically by one of us in great detail in two earlier papers [7, 1933; 8, 1934]. In the case of the material relating to tribes and castes of Bengal and Chittagong Hills it was found that the real defect in Risley's data

creeped in during the calculations of average values. Most of the discrepancies in the primary material could be reconciled without ambiguity. In the case of the Bengal material, out of 142 discrepancies not less than 133 could be corrected and confirmed by cross-checks with complete certainty; in 8 other cases the corrections were plausible. Only one single measurement was really doubtful and had to be rejected. In the case of the Chittagong Hills material there were only 7 discrepancies in 3080 individual measurements all of which could be traced to obvious printing mistakes, mistakes in using the index tables, etc., and could be corrected without ambiguity. The primary measurements (with the rejection of one single measurement in the case of Bengal) after correction can therefore be safely used for scientific purposes. The averages were recalculated and were published in the earlier papers.

5. This completed the first phase of the scientific review of Risley's work. The next important question is that of statistical methodology. The object of taking anthropometric measurements is to investigate the degree of resemblance (or lack of resemblance) in the physical appearance of different castes and tribes. It is convenient in this connexion to think in terms of what may be called the caste-distance a concept which was introduced by one of us [4, 1925]. Two castes which differ very much in physical appearance may be considered anthropometrically to be far apart, *i.e.*, to have a large caste-distance. In the same way, two castes which are nearly alike in physical appearance have a small caste-distance. Since 1925, considerable progress has been made in the statistical theory of the subject which has been summarised by the senior author elsewhere [6, 1930; 9, 1940; 10, 1940]. Professor R. A. Fisher recently reviewed the whole question in *Annals of Eugenics* [1, 1938] in which he gave a connected account of the work done on the discriminant function by himself and other workers in London; tests of significance in the case of multiple characters by Professor H. Hotelling in the United States; and on what Prof. Fisher called Mahalanobis's Generalized Distance (D^2-statistic) by workers in the Statistical Laboratory in Calcutta.

6. The generalized distance (D^2-statistic) may be now considered to have been accepted by statisticians as a convenient single measure of the degree of divergence in the mean values of different characters between different samples or groups. Further work by Fisher himself in London [2, 1939] and by S. N. Roy in Calcutta [12, 1939; (13, 14), 1940] deals with the question of differences in variances and correlations [15, 1941] between different characters.

7. The point which requires to be emphasized is that the coefficients of correlation explicitly occur in the formula for the generalized distance as well as in the more recent work of R. A. Fisher, S. N. Roy and others. *It is now established beyond any dispute that for a valid and scientific comparison of anthropometric measurements of more than one character it is not sufficient to restrict the work to the mean values and standard deviations. It is essential to take into consideration the coefficients of correlation in measuring either the generalized distance or in measuring what Fisher has called the direction or deviation from co-planarity of different castes and tribes or anthropometric groups.*

8. A detailed study of the coefficients of correlation between different characters is thus of basic importance in anthropometry. With the exception of an earlier study by one of us [9, 1940] of material relating to persons of mixed European and Indian parentage in Calcutta, the question of inter-correlation between anthropometric measurements does not appear to have received adequate attention in India so far. In the present paper a detailed discussion is given of the correlations for one Bengal caste, Brahmins, based on the revised version of Risley's material.

CORRELATION BETWEEN ANTHROPOMETRIC CHARACTERS

Nature of Material

In this paper only the measurements on the head have been analysed. Measurements of the following nine characters are available:

(1) Nasal height; (2) Nasal width; (5) Cephalic length; (6) Cephalic breadth; (7) Minimum frontal breadth; (8) Maximum bizygomatic breadth; (9) Height vertex to inter-superciliary point; (10) Height vertex to tragus; (11) Height vertex to chin.

The serial numbers in the above list show the standard order adopted throughout this paper. (Two measurements, namely (3) Bimalar breadth and (4) Nasomalar breadth have been omitted from the above list but occur in the case of a number of other castes.) Thus r_{12} will invariably mean the coefficient of correlation between Nasal height and Nasal width; and so on. We have already stated that the measurements were taken on adult males between the ages of 25 and 45 when physical growth had already taken place.

General Procedure

The coefficients of correlaton between different characters were calculated directly by grouping the material into a suitable number of classes in the form of two-way tables; and corresponding regression equation were worked out. The significance of the regression was tested in each case by an analysis of variance "between groups" and "within groups" with appropriate degrees of freedom. The adequacy or otherwise of the linearity of the regression was tested by further splitting up of the "between group" variation into two parts: (1) deviations due to regression, and (2) deviations from the fitted regression line.

Table 1. Bengal Brahmins (N = 100): Coefficients of Correlation†

Characters	(1) Nasal height	(2) Nasal width	(5) Cephalic length	(6) Cephalic breadth	(7) Minimum frontal breadth	(8) Maximum bizygomatic breadth	(9) Height vertex to inter-superciliary point	(10) Height vertex to tragus	(11) Height vertex to chin
(1) Nasal height	...	0·3286**	0·3042**	0·1274	−0·1924	0·3543**	−0·0782	0·0649	0·2596**
(2) Nasal width	0·3286**	..	0·2731**	0·1713	−0·1690	0·3238**	0·1357	0·2485*	0·3454**
(5) Cephalic length	0·3042**	0·2731**	...	−0·0419	−0·0712	0·2175*	−0·0268	0·1137	0·0587
(6) Cephalic breadth	0·1274	0·1713	−0·0419	...	0·4314**	0·4432**	0·0982	0·1750	0·2041*
(7) Minimum frontal breadth	−0·1924	−0·1690	−0·0712	0·4314**		0·4070**	0·2453*	0·1780	0·0786
(8) Maximum bizygomatic breadth	0·3543**	0·3238**	0·2175*	0·4432**	0·4070**	...	0·1000	0·1692	0·2149*
(9) Height vertex to intersuperciliary point	−0·0782	0·1357	−0·0268	0·0982	0·2453*	0·1000	...	0·2649**	0·6037**
(10) Height vertex to tragus	0·0649	0·2485*	0·1137	0·1750	0·1780	0·1692	0·2649**	...	0·4640**
(11) Height vertex to chin	0·2596**	0·3454**	0·0587	0·2041*	0·0786	0·2149*	0·6037**	0·4640**	...

† Measurements of serial no. (3) Bimalar breadth and serial no. (4) Nasomolar breadth are not available in the Brahmin Caste.

CORRELATION BETWEEN DIFFERENT CHARACTERS

In Table (1) the figures give the coefficient of correlation between pairs of characters in the corresponding intersecting cells. The size of the sample, that is, the number of individuals for whom measurements are available, is one hundred in each case.

For nine characters we have 36 correlations altogether. Coefficients which are significant on the 5 per cent level are marked with one star; and those significant on the one per cent level are marked throughout with two stars.

It will be seen that out of 36 coefficients 30 are positive and 6 negative; 18 coefficients, all of them positive, are significant at the five per cent level, and out of these 18 no less than 12 are significant at the one per cent level.

The highest correlation (+0·6037) is between 'Height vertex to chin' and 'Height vertex to inter-superciliary point' which is just what is to be expected, as 'Height vertex to inter-superciliary point' forms a part of 'Height vertex to chin'. After this we have four coefficients lying between 0·50 and 0·40, five between 0·40 and 0·30; and finally 8 coefficients between 0·30 and 0·20; the remaining 18 coefficients less than 0·20 are not significant.

For purposes of comparison, significant coefficients have been shown below in order of their magnitude.

Name of Characters	*Correlation coefficients.*
(9) Height vertex to inter-superciliary point × (11) Height vertex to chin	·6037**
(10) Height vertex to tragus × (11) Height vertex to chin	·4640**
(6) Cephalic breadth × (8) Maximum bizygomatic breadth	·4432**
(6) Cephalic breadth × (1) Minimum frontal breadth	·4314**
(7) Minimum frontal breadth × (8) Maximum bizygomatic breadth ...	·4070**
(1) Nasal height × (8) Maximum bizygomatic breadth	·3543**
(2) Nasal width × (14) Height vertex to chin	·3454**
(1) Nasal height × (2) Nasal width	·3286**
(2) Nasal width × (8) Maximum bizygomatic breadth	·3238**
(1) Nasal height × (5) Cephalic length	·3042**
(2) Nasal width × (5) Cephalic length	·2731**
(9) Height vertex to inter-superciliary point × (10) Height vertex to tragus	·2649**
(1) Nasal height × (11) Height vertex to chin	·2596**
(2) Nasal width × (10) Height vertex to tragus	·2485*
(7) Minimum frontal breadth × (9) Height vertex to inter-superciliary point	·2453*
(5) Cephalic length × (8) Maximum bizygomatic breadth	·2175*
(8) Maximum bizygomatic breadth × (11) Height vertex to chin ...	·2149*
(6) Cephalic breadth × (11) Height vertex to chin	·2041*

We find then that the characters are not independent. Most of them are positively correlated. The actual magnitude of correlation, however, is not high; and in most cases is probably of the order of +0·4 or less. We may mention here that similar results were obtained in the case of measurements on 200 Anglo-Indians of Calcutta. There was moderate or small positive correlation less than +0·4 in the case of Anglo-Indians [5, 1940].

TABLE 2. BENGAL BRAHMINS (N = 100): SIGNIFICANCE OF REGRESSIONS

Characters	r	r^2	η^2	Variance due to: Linear Regression	D. F.	Devn. from Linearity	D. F.	Between Groups	D. F.	Within Groups	D. F.	Total	D. F.	Ratio of Variances: Linear Regression	Devn. from Linearity	Between Groups
(1)	(2)	(3)	(4)	(5·1)	(5·2)	(6·1)	(6·2)	(7·1)	(7·2)	(8·1)	(8·2)	(9·1)	(9·2)	(10)	(11)	(12)
Nasal height on:																
Nasal width	·3286	·1080	·2672	46·6186	1	5·7297	12	8·8750	13	3·6792	86	4·3615	99	12·67**	1·55	2·41**
Cephalic length	·3042	·0925	·1761	39·9604	1	5·1552	7	9·5058	8	3·9093	91	,,	,,	10·22**	1·32	2·43*
Cephalic breadth	·1274	·0162	·0649	7·0080	1	2·1029	10	2·5488	11	4·5881	88	,,	,,	1·53	0·46	0·56
Minimum frontal breadth	−·1924	·0370	·1630	15·9767	1	6·8016	8	7·8210	9	4·0156	90	,,	,,	3·98*	1·69	1·95
Maximum bizygomatic breadth ...	·3543	·1255	·2141	54·2099	1	4·2490	9	9·2451	10	3·8128	89	,,	,,	14·22**	1·11	2·42*
Height vertex to inter superciliary pt.	−·0783	·0061	·2131	2·6434	1	6·8750	13	6·5727	14	3·9973	85	,,	,,	0·66	1·72	1·64
Height vertex to tragus	·0649	·0042	·0915	1·8165	1	3·4247	11	3·2907	12	4·5092	87	,,	,,	0·40	0·76	0·73
Height vertex to chin	·2596	·0674	·1355	29·0906	1	2·4519	12	4·5010	13	4·3404	86	,,	,,	6·70*	0·56	1·04
Nasal width on:																
Nasal height	·3286	·1080	·2266	80·1313	1	9·7818	9	16·8168	10	6·4497	89	7·4969	99	12·42**	1·52	2·61**
Cephalic length	·2731	·0746	·1197	55·3473	1	4·7801	7	11·1010	8	7·1800	91	,,	,,	7·71**	0·67	1·55
Cephalic breadth	·1713	·0293	·0884	21·7707	1	4·3822	10	5·9630	11	7·6886	88	,,	,,	2·83	0·57	0·78
Minimum frontal breadth ...	−·1690	·0286	·0704	21·1910	1	3·8852	8	5·8081	9	7·6658	90	,,	,,	2·76	0·51	0·76
Maximum bizygomatic breadth ...	·3238	·1048	·1367	77·8090	1	2·6295	9	10·1475	10	7·1990	89	,,	,,	10·81**	0·37	1·41
Height vertex to inter superciliary pt.	·1357	·0184	·0932	13·6652	1	4·2677	13	4·9390	14	7·9182	85	,,	,,	1·73	0·54	0·62
Height vertex to tragus	·2485	·0617	·1519	45·8228	1	6·0839	11	9·3955	12	7·2350	87	,,	,,	6·33*	0·84	1·30
Height vertex to chin	·3454	·1193	·1686	88·5232	1	3·0527	12	9·6274	13	7·1748	86	,,	,,	12·34**	0·43	1·34
Cephalic length on:																
Nasal height	·3042	·0925	·1661	36·7445	1	3·2453	9	6·5953	10	3·7201	89	4·0105	99	9·88**	0·87	1·77
Nasal width	·2731	·0746	·1611	29·6085	1	2·8644	12	4·9216	13	3·8728	86	,,	,,	7·65**	0·74	1·27
Cephalic breadth	−·0419	·0018	·0956	0·6956	1	3·7247	10	3·4493	11	4·0807	88	,,	,,	0·17	0·91	0·85
Minimum frontal breadth ...	−·0712	·0051	·1026	2·0146	1	4·8393	8	4·5254	9	3·9590	90	,,	,,	0·51	1·22	1·44
Maximum bizygomatic breadth ...	·2175	·0473	·1600	18·7705	1	4·9707	9	6·3507	10	3·7476	89	,,	,,	5·01*	1·33	1·69
Height vertex to inter superciliary pt.	−·0268	·0007	·1472	0·2851	1	4·4752	13	4·1759	14	3·9833	85	,,	,,	0·07	1·12	1·05
Height vertex to tragus	·1137	·0129	·0673	5·1341	1	1·9615	11	2·2259	12	4·2567	87	,,	,,	1·21	0·46	0·52
Height vertex to chin	·0587	·0034	·1467	1·3678	1	4·7382	12	4·4790	13	3·9397	86	,,	,,	0·35	1·20	1·14

TABLE 2—(*Contd.*) BENGAL BRAHMINS (N = 100) : SIGNIFICANCE OF REGRESSIONS

Characters	r	r^2	η^2	Variance due to : Linear Regression	D. F.	Devn. from Linearity	D. F.	Between Groups	D. F.	Within Groups	D. F.	Total	D. F.	Ratio of Variances : Linear Regression	Devn. from Linearity	Between Groups
(1)	(2)	(3)	(4)	(5·1)	(5·2)	(6·1)	(6·2)	(7·1)	(7·2)	(8·1)	(8·2)	(9·1)	(9·2)	(10)	(11)	(12)
Cephalic breadth on :																
Nasal height	·1274	·0162	·0753	8·7084	1	3·5193	9	4·0382	10	5·5750	89	5·4198	99	1·56	0·63	0·72
Nasal width	·1713	·0293	·2571	15·7389	1	10·1863	12	10·6135	13	4·6347	86	,,	,,	3·40	2·20*	2·29*
Cephalic length	-·0419	·0018	·1135	0·9401	1	8·5680	7	7·6145	8	5·2269	91	,,	,,	0·18	1·64	1·46
Minimum frontal breadth	·4314	·1861	·2390	99·8318	1	3·5490	8	14·2471	9	4·5371	90	,,	,,	22·00**	0·78	3·14**
Maximum bizygomatic breadth ...	·4432	·1964	·2859	105·4013	1	5·3325	9	15·3394	10	4·3052	89	,,	,,	24·48**	1·24	3·56**
Height vertex to inter-superciliary pt.	·0982	·0096	·1415	5·1778	1	5·4426	13	5·4237	14	5·4192	85	,,	,,	0·96	1·00	1·00
Height vertex to tragus	·1750	·0306	·1699	16·4364	1	6·7921	11	7·5958	12	5·1197	87	,,	,,	3·21	1·33	1·48
Height vertex to chin	·2041	·0416	·1978	22·3440	1	6·9833	12	8·1649	13	5·0048	86	,,	,,	4·46*	1·40	1·63
Minimum frontal breadth on :																
Nasal height	-·1924	·0370	·1545	16·7522	1	5·9117	9	6·9958	10	4·3010	89	4·5732	99	3·89	1·37	1·63
Nasal width	-·1690	·0286	·1687	12·9269	1	5·2865	12	5·8742	13	4·3766	86	,,	,,	2·95	1·21	1·34
Cephalic length	-·0712	·0051	·0937	2·2973	1	5·7323	7	5·3029	8	4·5091	91	,,	,,	0·51	1·27	1·18
Cephalic breadth	·4314	·1861	·2878	84·2382	1	4·6078	10	11·8470	11	3·6640	88	,,	,,	22·99**	1·26	3·23**
Maximum bizygomatic breadth ...	·4070	·1657	·2327	75·0094	1	3·3724	9	10·5361	10	3·9032	89	,,	,,	19·22**	0·86	2·70**
Height vertex to inter-superciliary pt.	·2453	·0602	·1423	27·2402	1	2·8611	13	4·6024	14	4·5684	85	,,	,,	5·96*	0·63	1·01
Height vertex to tragus	·1780	·0317	·1749	14·3430	1	5·8959	11	6·5998	12	4·2937	87	,,	,,	3·34	1·37	1·54
Height vertex to chin	·0786	·0062	·0628	2·7966	1	2·1376	12	2·1883	13	4·9337	86	,,	,,	0·57	0·43	0·44
Maximum bizygomatic breadth :																
Nasal height	·3543	·1255	·2142	70·6830	1	5·5466	9	12·0603	10	4·9708	89	5·6869	99	14·22**	1·12	2·43*
Nasal width	·3238	·1048	·2658	59·0232	1	7·5503	12	11·5097	13	4·8067	86	,,	,,	12·28**	1·57	2·39**
Cephalic length	·2175	·0473	·1561	26·6164	1	8·7516	7	10·9847	8	5·2211	91	,,	,,	5·10*	1·68	2·10*
Cephalic breadth	·4432	·1964	·3317	110·5952	1	7·6143	10	16·9762	11	4·2757	88	,,	,,	25·87**	1·78	3·97*
Minimum frontal breadth ...	·4070	·1657	·2211	93·2750	1	3·9020	8	13·8323	9	4·8723	90	,,	,,	19·14**	0·80	2·84**
Height vertex to inter-superciliary pt.	·1000	·0100	·0797	5·6328	1	3·0181	13	3·2049	14	6·0957	85	,,	,,	0·92	0·50	0·53
Height vertex to tragus	·1692	·0286	·2176	16·12·9	1	9·6714	11	10·2088	12	5·0632	87	,,	,,	3·18	1·91*	2·02*
Height vertexto chin	·2149	·0462	·2336	25·9830	1	8·7930	12	10·1153	13	5·0175	86	,,	,,	5·18*	1·75	2·02*

TABLE 2—(*Contd.*) BENGAL BRAHMINS (N = 100) : SIGNIFICANCE OF REGRESSIONS

Characters	r	r^2	η^2	Variance due to : Linear Regression	D. F.	Devn. from Linearity	D. F.	Between Groups	D. F.	Within Groups	D. F.	Total	D. F.	Ratio of Variances : Linear Regression	Devn. from Linearity	Between Groups
(1)	(2)	(3)	(4)	(5·1)	(5·2)	(6·1)	(6.2)	(7·1)	(7·2)	(8·1)	(8·2)	(9.1)	(9·2)	(10)	(11)	(12)
Height vertex to intersuperciliary pt. on :																
Nasal height	−·0782	·0061	·1934	6·2941	1	21·3976	9	19·8873	10	9·3173	89	10·3849	99	0·68	2·30*	2·13*
Nasal width	·1357	·0184	·2143	18·9296	1	16·7857	12	16·9507	13	9·3925	86	,,	,,	2·02	1·79	1·80
Cephalic length	−·0268	·0007	·0572	0·7382	1	8·2908	7	7·3467	8	10·6520	91	,,	,,	0·07	0·78	0·69
Cephalic breadth	·0982	·0096	·0882	9·9213	1	8·0809	10	8·2482	11	10·6521	88	,,	,,	0·93	0·76	0·77
Minimum frontal breadth	·2453	·0602	·1189	61·8573	1	7·5518	8	13·5857	9	10·0649	90	,,	,,	6·15*	0·75	1·35
Maximum bizygomatic breadth ...	·1000	·0100	·1080	10·2862	1	11·1970	9	11·1059	10	10·3039	89	,,	,,	1·00	1·09	1·08
Height vertex to tragus	·2649	·0702	·1816	72·1260	1	10·4167	11	15·5592	12	9·6712	87	,,	,,	7·46**	1·08	1·61
Height vertex to chin	·6037	·3644	·4639	374·6906	1	8·5185	12	36·6856	13	6·4093	86	,,	,,	58·46**	1·33	5·72**
Height vertex to tragus on :																
Nasal height	·0649	·0042	·1501	2·2329	1	8·6012	9	7·9643	10	5·0686	89	5·3611	99	0·44	1·70	1·57
Nasal width	·2485	·0617	·1706	32·7685	1	4·8160	12	6·9662	13	5·1185	86	,,	,,	6·40*	0·94	1·36
Cephalic length	·1137	·0129	·0493	6·8631	1	2·7551	7	3·2686	8	5·5451	91	,,	,,	1·24	0·50	0·59
Cephalic breadth	·1750	·0306	·2333	16·2585	1	10·7546	10	11·2550	11	4·6244	88	,,	,,	3·52	2·33*	2·43*
Minimum frontal breadth ...	·1780	·0317	·1110	16·8142	1	5·2628	8	6·5463	9	5·2426	90	,,	,,	3·21	1·00	1·25
Maximum bizygomatic breadth ...	·1692	·0286	·1398	15·1975	1	6·5584	9	7·4223	10	5·1295	89	,,	,,	2·96	1·28	1·45
Height vertex to intersuperciliary pt.	·2649	·0702	·1538	37·2342	1	3·4148	13	5·8304	14	5·2838	85	,,	,,	7·05**	0·65	1·10
Height vertex to chin	·4640	·2152	·3682	114·2413	1	6·7660	12	15·0333	13	3·8990	86	,,	,,	29·30**	1·74	3·86**
Height vertex to chin on :																
Nasal height	·2596	·0674	·1867	46·0043	1	9·0531	9	12·7483	10	6·2400	89	6·8974	99	7·37**	1·45	2·04*
Nasal width	·3454	·1193	·2·26	81·4444	1	7·0159	12	12·7412	13	6·0140	86	,,	,,	13·54**	1·17	2·12*
Cephalic length	·0587	·0034	·0426	2·3524	1	3·8233	7	3·6395	8	7·1838	91	,,	,,	0·33	0·53	0·51
Cephalic breadth	·2041	·0416	·1330	28·4355	1	6·2380	10	8·2559	11	6·7276	88	,,	,,	4·23*	0·93	1·23
Minimum frontal breadth	·0786	·0062	·0441	4·2179	1	3·2377	8	3·3466	9	7·2525	90	,,	,,	0·58	0·45	0·46
Maximum bizygomatic breadth ...	·2149	·0462	·1435	31·5137	1	7·3844	9	9·7974	10	6·5715	89	,,	,,	4·80*	1·12	1·49
Height vertex to intersuperciliary pt.	·6037	·3644	·4228	248·8583	1	3·0656	13	20·6223	14	4·6368	85	,,	,,	53·67**	0·66	4·45**
Height vertex to tragus	·4640	·2152	·2893	146·9779	1	4·5945	11	16·4597	12	5·5784	87	,,	,,	26·35**	0·82	2·95**

TABLE 3. BENGAL BRAHMINS (N = 100) : GRADUATION BY NORMAL CURVES

Nasal height			Nasal width			Cephalic length			Cephalic breadth			Minimum frontal breadth		
Class Intervals (in mms.)	Frequency		Class Intervals (in mms.)	Frequency		Class Intervals (in mms.)	Frequency		Class Intervals (in mms.)	Frequency		Class Interval (in mms.)	Frequency	
	Observed	Expected		Observed	Expected		Observed	Expected		Observed	Expected		Observed	Expected
(1·1)	(1·2)	(1·3)	(2·1)	(2·2)	(2·3)	(3·1)	(3·2)	(3·3)	(4·1)	(4·2)	(4·3)	(5·1)	(5·2)	(5·3)
39—40	2	1·25	28	1	0·75	170—172	7	5·59	131—132	1	0·96	96— 97	6	2·74
41—42	2	2·76	29	1	1·22	173—175	10	8·20	133—134	3	1·85	98— 99	...	4·61
43—44	8	6·19	30	...	2·58	176—178	12	14·31	135—136	5	4·27	100—101	8	9·25
45—46	8	11·57	31	7	4·63	179 - 181	19	19·11	137 - 138	5	7·84	102—103	19	14·25
47—48	18	16·44	32	8	7·67	182—184	14	19·79	139—140	15	12·51	104—105	15	18·35
49—50	18	19·32	33	13	10·91	185—187	20	15·64	141—142	12	15·83	106—107	20	18·16
51—52	21	17·64	34	16	13·53	188—190	11	10·01	143—144	16	17·38	108—109	15	15·02
53—54	10	12·53	35	11	14·67	191—193	5	4·95	145—146	19	15·16	110—111	9	9·54
55—56	7	7·35	36	11	13·89	194—196	2	1·82	147—148	12	11·49	112—113	4	5·01
57—58	5	3·29	37	11	11·48	197 and above	...	0·08	149—150	7	7·00	114—115	3	2·13
59—60	1	1·22	38	8	8·29				151—152	4	3·49	116—117	1	0·70
61 and above	0	·44	39	9	5·22				153 - 154	...	1·51	118 and above	...	0·24
			40	2	2·88				155—156	1	0·51			
			41	1	1·39				157 and above	..	0·20			
			42	1	0·58									
			43		0·31									
Total	100	100·00	.....	100	100·00	...	100	100·00	.....	100·00	100·00		100	100·00
$\chi^2 = 3·11$ D. F. = 4 P = 0·5411			D. F. = 6 $\chi^2 = 3·07$ P = 0·7996			D. F. = 5 $\chi^2 = 4·16$ P = 0·5280			D. F. = 6 $\chi^2 = 4·68$ P = 0·5863			D. F. = 5 $\chi^2 = 2·88$ P = 0·7253		

TABLE 3—(*Contd.*) BENGAL BRAHMINS (N = 100): GRADUATION BY NORMAL CURVES

Maximum bizygomatic breadth			Height vertex to inter-superciliary point			Height vertex to tragus			Height vertex to chin		
Class Intervals (in mms.)	Frequency		Class Intervals (in mms.)	Frequency		Class Intervals (in mms.)	Frequency		Class Intervals (in mms.)	Frequency	
	Observed	Expected		Observed	Expected		Observed	Expected		Observed	Expected
(6·1)	(6·2)	(6·3)	(7·1)	(7·2)	(7·3)	(8·1)	(8·2)	(8·3)	(9·1)	(9·2)	(9·3)
115—116	1	0·69	60— 62	1	0·45	110—112	1	0·12	181—184	1	0·11
117—118	...	1·38	63— 65	1	0·62	113—115	...	0·35	185—188	...	0·26
119—120	4	3·30	66— 68	1	1·32	116—118	...	1·03	189—192	1	0·73
121—122	7	6·33	69— 71	...	2·36	119—121	2	2·68	193—196	2	1·71
123—124	13	10·66	72— 74	3	3·94	122—124	5	5·67	197—200	1	3·62
125—126	10	14·33	75— 77	10	6·23	125—127	10	9·92	201—204	5	6·28
127—128	21	16·50	78— 80	7	8·35	128—130	20	13·95	205—208	12	9·95
129—130	13	16·31	81— 83	16	10·45	131—133	16	17·08	209—212	11	12·91
131—132	11	12·88	84— 86	12	12·30	134—136	10	16·92	213—216	20	15·23
133—134	10	8·93	87— 89	6	12·30	137—139	16	13·87	217—220	14	14·74
135—136	8	4·94	90— 92	10	11·53	140—142	13	9·23	221—224	11	12·98
137—138	...	2·39	93— 95	6	10·10	143—145	4	5·34	225—228	9	9·38
139—140	2	0·93	96— 98	11	7·54	146—148	1	2·48	229—232	6	6·16
141 and above	...	0·43	99—101	9	5·30	149—151	1	0·96	233—236	6	3·32
			102—104	6	3·46	152—154	1	0·30	237—240	...	1·63
			105—107	1	1·92	155 and above	...	0·10	241—244	1	0·65
			108 and above	...	1·83				245 and above	...	0·34
Total ...	100	100·00	...	100	100·00	...	100	100·00	..	100	100·00
$\chi^2 = 5·02$ D. F. = 6 P = 0·5414			D. F. = 8 $\chi^2 = 15·56$ P = 0·0498			D. F. = 5 $\chi^2 = 8·26$ P = 0·1440			D. F. = 7 $\chi^2 = 3·32$ P = 0·8513		

LINEARITY OF THE REGRESSIONS

In Table (2) the regressions of one character on another are shown in detail. The first column gives the names of characters; columns (2), (3) and (4) give respectively the correlation coefficients, squares of the correlation coefficients, and squares of the correlation ratio (Pearsonian Eta). The next two columns (5·1) and (5·2) give the variances due to the linearity of the regression and corresponding degrees of freedom; columns (6·1) and (6·2) the variances due to deviation from linearity and corresponding degrees of freedom. Column (7·1) gives the sum of columns (5·1) and (6·1), *i.e.*, the variance "between group", and column (7·2) the corresponding degrees of freedom (*i.e.*, total of columns (5·2) and (6·2)). Columns (8·1) and (8·2) give the variance and degrees of freedom of "within group" variation; while columns (9·1) and (9·2) give the total variance and total degrees of freedom, respectively. Columns (10), (11) and (12) show the ratio of variances due to linear regression, deviation from linearity, and "between group" variation in terms of "within group" variance, respectively.

In most cases the deviations from linearity are statistically insignificant. Out of 72 cases the results appear to deviate significantly from linearity at the five per cent. level of significance only in 4 cases:—

(1) Cephalic breadth on Nasal width.

(2) Maximum bizygomatic breadth on Height vertex to tragus.

(3) Height vertex to inter-superciliary point on Nasal height.

(4) Height vertex to tragus on Cephalic breadth.

Such deviations may, however, be considered to have arisen from sampling fluctuations.

Linear regressions thus appear to be quite adequate for describing the present material. Although larger samples may show significant deviations from linearity it is likely that a linear connexion between the characters may be assumed in dealing with samples of the size usually used in anthropometric work.

FREQUENCY DISTRIBUTION OF INDIVIDUAL CHARACTERS

The normal curve was fitted to each character and the goodness of fit was tested by the Chi-square method. The results are given in Table (3) in which the class interval is shown in col. (1), and the observed and expected frequencies in columns (2) and (3) for each character. The observed value of Chi-square is shown at the bottom in each case together with the available degrees of freedom. The probability of occurrence of deviations equal to or larger than the observed deviations from expected values (P) is also given for each character. For example, in the case of the Nasal height the observed and expected frequencies were grouped into seven cells. Allowing for the constant size of the sample and the two fitted constants used in graduating with the normal curve four residual degrees of freedom are left for the Chi-square test. The actual value of Chi-square came out as 3·11. The probability of occurrence of such a value of Chi-square with four degrees of freedom is 0·5411 This shows that in the case of 54 per cent of similar samples the observed deviations would be worse than the present set of deviations. The graduation may therefore be considered quite satisfactory. The value of (P) is usually high; the lowest value is practically 0·05, and the average value for all nine characters is 0·53. This shows excellent agreement between expected and observed frequencies. All the graduations thus appear to be quite satisfactory in the case of Bengali Brahmins.

So. far as the frequency distribution of individual characters is concerned we find then that the normal curve is quite adequate for the present material. Significant deviations from normality may be shown by samples of much larger size, but in working with samples of the size ordinarily used in Anthropometry the normal curve is likely to be adequate for all practical purposes.

We may conclude therefore that the nine characters discussed in this paper may be considered to conform, within the limits of errors of sampling, to a normal multivariate linearly correlated system. This is a result of great practical value as it enables the theory of sampling from normal populations being used in anthropometric work.

Summary of Results

Recent advances in statistical theory have established beyond doubt that the comparison of anthropometric measurements cannot be based merely on mean values and standard deviations but must take into consideration the coefficients of correlation between different anthropometric characters. In the present paper a detailed correlational analysis has been given of the revised version of Risley's measurements of 9 characters of the head for 100 adult male Brahmins of Bengal. The chief results are given below in a summary form :—

(1) All the nine characters can be satisfactorily graduated by normal curves within the limits of errors of sampling.

(2) The characters are not statistically independent ; 18 out of 36 coefficients are significant at the five per cent level.

(3) Significant correlations are all positive and moderately large in magnitude and lie between 0·2 and 0·6.

(4) The regressions are essentially linear in character. Four significant deviations from linearity (at the five per cent level) out of 72 regressions may be considered to have arisen from sampling fluctuations.

(5) The whole system may be thus described as a random sample from a multivariate linearly correlated normal population.

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