

SOME OBSERVATIONS ON THE PROCESS OF GROWTH OF NATIONAL INCOME

By P. C. MAHALANOBIS

1.1. In recent years I have had occasion to come into contact with the work on national income and also with some of the problems of economic planning in India. I am not an economist; I have been mostly concerned with analytical statistics and my thoughts naturally turned to the possibility of using simple models on the lines of the physical sciences to study some of these problems.

1.2. In the industrially developed countries in West Europe a high standard of living has been already attained, and the aim is to maintain it at its high level without too violent fluctuations. The position is similar in the U.S.A. and Canada with this difference that in these countries there has also been a large increase in economic activities in recent years. It is natural that West European and North American countries should be greatly concerned with problems of business cycles and economic oscillations (with some increasing interest in recent years in problems of growth in the U.S.A.).

1.3. In India, on the other hand, the present scale of living is extremely low and the most urgent problem is to attain a much higher level which is socially desirable. It may be useful, therefore, to examine how the national income may increase rapidly, in fact, so rapidly that business cycles may be neglected in the first approximation or treated as secondary oscillations superposed on a steeply rising trend. Secondly, in order to make the task manageable, questions of distribution and patterns of consumption expenditure, although of great social significance, may be left out of consideration at first. In the present approach it is assumed that the economic development would take place under central planning.

1.4. It is under the above assumptions that I am studying some very simple models (and am deliberately refraining from considering the detailed mechanism on lines somewhat analogous to a thermodynamic study, as distinguished from a dynamic theory, of the behaviour of gases in physics). Some over-all economic quantities such as the total national income or some of its components are considered as 'variables' and an attempt is made to study the variables in terms of certain

'parameters' which can be treated as constants in comparison with the variables. The variables must be measurable and the parameters estimable from available data, and the relevant concepts must be operational in the sense Bridgeman used this word. Also, both variables and parameters are, of course, subject to errors of observation and estimation but such errors are considered to be dimensionally smaller for parameters in comparison to variables; furthermore such errors are neglected in the present note. I shall conclude this long introduction by stating that I took up those studies primarily as an aid to clear thinking on my own part; and I am publishing the present note in the hope that it may be of some help to others who like me may desire to approach the subjects from the statistical side.

2.1. In a lecture delivered at the National Institute of Sciences of India on 4 October 1952, I used a model to represent economic growth as a first approximation. According to this model, the ratio of net investment to net national income at factor cost in any period of time is taken to be a constant α . Secondly, in any time period the increment of income divided by the investment is also supposed to be a constant depicted by β . On these assumptions, the rate of growth of the economy is given by $\alpha\beta$. If we suppose that the population of the country increases at the constant rate ρ then the rate of growth of the per capita income is given by $\alpha\beta - \rho$.

2.2. When I was studying about this model, I did not know that considerable work had been previously done on models of the same type by Harrod, Domar and others because I was not familiar with economic literature. Now that I have come to know about the previous work I wish to acknowledge their priority.

2.3. The model presented earlier gives a very simplified picture of the economic process; and yet, when data are available over a long time, I noticed that the observed rates of growth in several countries corresponded fairly satisfactorily to the expected rates calculated from this model, the parameters having been estimated from the available data. I have discussed these points in my lecture already mentioned.

3.1. One elaboration of this model is separating the investment in investment goods industries from the investment in consumer goods industries. We assume that the increment of consumption is a constant proportion β_0 of the investment in consumer goods industries, and the increment in investment is a constant proportion β_1 of the investment in investment goods industries. I have not considered the industry producing intermediate products separately. Conceptually, industries producing raw materials for consumer goods industries are included in the consumer goods industries. Similarly, industries producing raw materials for investment goods industries are included in investment goods industries. It may be mentioned that the division of industries between investment goods industries and consumer goods industries adopted here is somewhat different from Marx's division between Department 1 and Department 2; according to Marx, Department 1 includes *all* raw material producing industries. Secondly, we suppose that

a constant percentage λ_c of investment goes to consumer goods industries, and the complementary percentage λ_i going to the investment goods industries ($\lambda_c + \lambda_i = 1$). Under this hypothesis the national income (at constant prices) in year t is given by:

$$Y_t = Y_0 \left[1 + \alpha_0 \left(\frac{\lambda_i \beta_i + \lambda_c \beta_c}{\lambda_i \beta_i} \right) \{ (1 + \lambda_i \beta_i)^t - 1 \} \right] \quad \dots (1)$$

where α_0 is the initial investment rate, and Y_0 the initial national income.

3.2. This model has got several interesting features. As is to be expected, with β_c greater than β_i , in the initial stage of development, the larger the percentage, investment on consumer goods industries, the larger will be the income generated. But there is a critical range of time and as soon as this is passed, the larger the investment in investment goods industries the larger will be the income generated. Hence, it would be desirable to invest relatively more on the consumer goods industries provided we are interested in the immediate future. If, on the other hand, we are interested in the more distant future, relatively larger investment on investment goods industries would give distinctly better results. The following table, which gives the income (at constant price) at different points of time, illustrates the point for some arbitrary values of the parameters:

$$Y_0 = 1000, \quad \alpha_0 = 5 \text{ p.c.} \quad \beta_c = 30 \text{ p.c.} \quad \beta_i = 10 \text{ p.c.}$$

year	value of λ_c		
	90 p.c.	80 p.c.	70 p.c.
0	1000	1000	1000
5	1071	1068	1064
10	1148	1142	1138
15	1226	1225	1223
20	1308	1316	1322
25	1397	1416	1438
30	1487	1527	1571
35	1583	1650	1726
40	1684	1785	1905
45	1791	1935	2113
50	1904	2100	2354

It will be seen that up to 15 years the highest income is obtained when 90 p.c. of

investment goes to consumer goods industries but after this a smaller percentage of investment on consumer goods industries begins to give a better result.

4.1. It may be noted that in the above model the over-all ratio β of increment of income to investment is given by :

$$\beta = \lambda_c \beta_c + \lambda_i \beta_i \quad \dots (2.1)$$

The rate of investment

$$\alpha_t = \frac{I_t}{Y_t} \quad \dots (2.2)$$

depends on how the capital stock was originally distributed between the two departments. If the ratio of investment in the investment goods industries (λ_i) is larger than the corresponding ratio of the original capital stocks, then the rate of investment will gradually rise until it reaches a limiting value given by $\lambda_i \beta_i / \beta$. If on the other hand λ_i is smaller than the corresponding ratio of the original capital stocks, then the ratio will gradually fall until it reaches the same limiting value. If the two ratios are identical, then the rate of investment is a constant and the rate of growth of national income also is a constant. In an under-developed country the current rates of investment in investment goods is likely to be small, and, therefore, if we increase this ratio then the value of α would also gradually increase.

4.2. If we consider a case in which investment rate is rising, the initial rise of consumption will be more rapid in a variant of a plan in which λ_c is larger. If, however, we consider the position attained after the lapse of a critical period of time, a smaller value of λ_c will lead to a large rise in consumption. For example, if we have two alternative variants of the investment plan with $\lambda_c = 70$ p.c. and $\lambda_c = 60$ p.c. (other constants having the same values as in the table) the consumption according to the second variant will surpass that according to the first variant in the 29th year.

4.3. Another numerical example with $\alpha_0 = 10$ p.c., $\beta_c = 36$ p.c., $\beta_i = 24$ p.c. may be given. The national income becomes higher in the 5th year and the production of consumer goods greater in the 11th year for the variant $\lambda_c = 80$ p.c. compared with the results produced by a variant with $\lambda_c = 90$ p.c. A lower value of λ_c thus gives distinctly better results when the ratio of β_c to β_i is lower. (The value of λ_c cannot, however, be fixed entirely arbitrarily, and can be changed only within certain technical limits).

4.4. We made some attempt to apply this model to empirical data but so far results have not been satisfactory.

5.1. In the above model β is assumed to be a constant. Certain recent studies have indicated that this is not always true. In the U.S.A., for example, during the period 1897-1950, β exhibited a kind of a parabolic trend. On the other hand, in a pre-industrial economy, I believe it may be possible to secure a steadily

increasing β by deliberate planning. We have considered below a model in which β is supposed to be a function of time. Thus, we may assume that

$$\frac{dY}{dt} = \alpha\beta_t Y \quad \dots (3)$$

where β_t is equal to $k+lt$, or $k+lt+mt^2$; k, l, m , being constants.

5.2. The solution of the differential equation in the first case is:

$$Y_t = Y_0 e^{\alpha \left(kt + \frac{lt^2}{2} \right)} \quad \dots (3.1)$$

where Y_t is the national income in the year t , and Y_0 the original income. However, this model also did not give a good fit to data in respect of the U.S.A. probably because the assumption that α is constant was not valid over the period considered.

5.3. The constants for the equation for β_t in the second case were estimated from the material given by Goldsmith in his recent paper in *Income and Wealth, Series II*; and the equation is

$$\beta_t = 0.33729 - 0.00661t + 0.00016t^2 \quad \dots (3.2)$$

Also, α was calculated from data given by S. Kuznets. The solution for U.S.A. is then given by the following equation:

$$Y_t = Y_0 e^{\frac{11}{100} \left\{ 0.33729t - 0.00661 \frac{t^2}{2} + 0.00016 \frac{t^3}{3} \right\}} \quad \dots (3.3)$$

6.1. In a planned economy, it may be possible to go on increasing α up to a certain level deliberately. The earlier model considered in this note may, therefore, be realistic under certain conditions of planning with the restriction that the ratios of increment of net outputs to investment are taken as constants. It should be also possible to increase the ratios (β_c, β_i) through deliberate planning. Suppose that in the earlier stage of development, a product of an industry has got practically no market, and therefore is of negligible value. At a later stage of development it may, however, be possible to use this waste product as raw material in some other industry which is established by planning. In an economy which is developing harmoniously, a number of such by-products may gradually become useful. Hence the ratio of the value of net output to the value of raw materials would increase because by-products of the type considered above would continue to be sold at negligible prices. An increase in the ability of machines or of labour to convert the same amount of raw materials into a larger quantity of final product will produce the same effect, price situation remaining the same. A steady increase of β may be, therefore, a characteristic of harmonious development. (Similar effects may also arise at a stage of very high and diversified industrialization and may therefore explain the increase in β in recent years in U.S.A., as observed by Goldsmith.)

6.2. For rapid development it would be desirable to aim at increasing both α and β . The first model is concerned only with an increment of α while the second model is concerned with an increment of β . It will be of interest to consider a model which allows simultaneous variations of α and β . It will be also noticed that in the second model β has been assumed to vary simply with time; it would be more satisfactory if a model can be constructed in which the variation of β would depend on some economic factors. I and some of my young colleagues are engaged in studying problems of this type, and we hope something of practical interest would emerge in time.

Calcutta, 27 July, 1953.