On the Evaluation of the Probability Integral of the D^2-Statistics.

The exact distribution of the D^2-statistics, constructed by P. C. Mahalanobis[1] in an attempt to estimate the divergence between two populations, was found by one of the authors in the form which after the substitutions

$$\lambda^2 = \tfrac{1}{2}\overline{n}\,P\,\Delta^2, \qquad L^2 = \tfrac{1}{2}\overline{n}\,P\,D_1^2$$

can be written as

$$F_p(L, \lambda)\ dL = \left(\frac{L}{\lambda}\right)^{\frac{P}{2}-1} L\, e^{-\frac{1}{2}(L^2+\lambda^2)} \times I_{\frac{p}{2}-1}(L\lambda)\, dL$$

where D_1^2 is the uncorrected sample value, and Δ^2 the population of D^2, and I is the Bessel function of pure imaginary argument.

In a previous letter, one of us invited the attention of mathematicians to the problem of the numerical evaluation of the incomplete integral

$$\Phi_p(L, \lambda) = \int_0^L F_p(L, \lambda)\, dL$$

which till then had baffled our attempts to tackle it [2]. Since then we have however overcome the difficulty in the following manner.

It is proved that

$$\Phi_p(L, \lambda) = \Phi_{p-2}(L, \lambda) - f_{p-2}(L, \lambda)$$

where $f_{p-2}(L, \lambda) = \left(\frac{L}{\lambda}\right)^{\frac{P}{2}-1} e^{-\frac{1}{2}(L^2+\lambda^2)} \times I_{\frac{p}{2}-1}(L\lambda)$

The function f obeys the recurrence formula

$$f_p(L, \lambda) = -\frac{p-2}{\lambda^2} f_{p-2}(L, \lambda) + \frac{L^2}{\lambda^2} f_{p-4}(L, \lambda)$$

This enables us to make $\Phi_p(L, \lambda)$ depend upon $\Phi_2(L, \lambda)$ or $\Phi_1(L, \lambda)$ according as P is even or odd. We show

$$\Phi_1(L, \lambda) = \frac{1}{\sqrt{2\pi}} \int_{\lambda - L}^{\lambda + L} e^{-\frac{1}{2}t^2} dt$$

which can be found from the tables of the probability integral. Also $\Phi_2(L, \lambda)$ is obtained in the form of the following convergent series.

$$\Phi_2(L, \lambda) = 1 - e^{-x} \sum_{n=0}^{\infty}{}' \frac{x^n}{n!} \left\{ 1 - e^{-\xi} \sum_{m=0}^{n-1} \frac{\xi^m}{m!} \right\}$$

where $x = \frac{1}{2} L^2$, $\xi = \frac{1}{2} \lambda^2$.

The actual numerical computation is proceeding in the Statistical Laboratory, Presidency College, Calcutta, and will be published in *Sankhya :* The Indian Journal of Statistics.

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24.9.35.

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1. *Journal of the Asiatic Society of Bengal* (1930).
2. SCIENCE AND CULTURE, *1*, 205, 1935.