Weighted broadcast in linear radio networks ☆

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Abstract

The *weighted* version of the broadcast range assignment problem in ad hoc wireless network is studied. Efficient algorithms are presented for the *unbounded* and *bounded-hop* broadcast problems for the linear radio network, where radio stations are placed on a straight line. For the *unbounded* case of the problem, the proposed algorithm runs in $O(n^2)$ time and using $O(n)$ space, where n is the number of radio stations in the network. For the h-hop broadcast problem, the time and space complexities of our algorithm are $O(hn^2 \log n)$ and $O(hn)$, respectively. This improves time complexity of the existing results for the same two problems by a factor of n and $n^2/\log n$, respectively [C. Ambuhl, A.E.F. Clementi, M.D. Ianni, G. Rossi, A. Monti, R. Silvestri, The range assignment problem in non-homogeneous static ad hoc networks, in: Proc. 18th Int. Parallel and Distributed Precessing Symposium, 2004].

Keywords: Analysis of algorithms; Weighted broadcast; Linear network

1. Introduction

In ad hoc wireless network, the *range assignment* problem is studied extensively in the context of information broadcast, accumulation and all-to-all communication [11]. Here, a set of radio stations $S = \{s_1, s_2, \ldots, s_n\}$ is assumed to be placed in $\mathbb{R}^d$, $d \geqslant 1$. If a radio station s_i is assigned a range ρ, it can communicate with any other radio station(s) located in the hyper-sphere of radius ρ centered at s_i. The cost (power consumption) of assigning a range ρ_i to a radio station s_i is $w_i \times \rho_i^{\mu}$, where μ is a fixed constant, and is called the distance power gradient; w_i is also a constant attached with s_i, and is referred to as the positional parameter of s_i. Though μ can take any value from 1 to 6, for all practical applications μ is assumed to be 2. In the *broadcast* problem, a source radio station $s^* \in S$ is specified, and we need to assign ranges to the radio stations in S such that s^* can broadcast message to all other radio stations in S using at most h hops ($1 \leqslant h \leqslant n-1$). Here, the objective is to minimize the total cost $\sum_{i=1}^{n} w_i \rho_i^2$ of the entire network. If the restriction on the number of hops h is not mentioned (i.e., $h = n-1$ in the worst case), then it is referred to as the *unbounded* version of the problem.

Most of the existing literatures study the *unweighted* version of the broadcast range assignment problem, i.e., $w_i = 1$ for $i = 1, 2, \ldots, n$. It is shown that the broadcast range assignment problem (with $h > 2$) in $\mathbb{R}^d$ is NP-hard even for $d = 2$ and in the *unweighted* case [5].

A dynamic programming based $O(n^7)$ time algorithm is proposed in [2] for the *unweighted* 2-hop broadcast range assignment in $\mathbb{R}^2$. Approximation algorithms are available for the *unbounded* version of that problem, with approximation factor 6 [1]. For the linear radio network, an $O(hn^2)$ time algorithm is proposed for the *unweighted h*-hop broadcast range assignment problem [7]. Later, the time complexity of this problem is improved to $O(n^2)$ [8]. For a detailed survey on the broadcast range assignment problem, see [6,10].

Though there is a long history of the *unweighted* broadcast range assignment problem, the *weighted* version is studied very little. The first work on this problem appeared in [3]. For the *unbounded* case, the proposed algorithm runs in $O(n^3)$ time and $O(n^2)$ space. For h-hop broadcast, the time and space complexities are $O(hn^4)$ and $O(hn^2)$, respectively. Several other variations of this problem are studied in [3].

We will use graph-theoretic formulation to design algorithms for the *weighted* version of the broadcast problem in linear radio network. Our algorithms for the *unbounded* and *bounded-hop* broadcast range assignment problems run in $O(n^2)$ and $O(hn^2 \log n)$ times, respectively. Thus our proposed algorithms improve the existing results on the time complexity of the same two problems by factors of n and $n^2/\log n$, respectively. The space complexity for both the problems are also improved by a factor of n.

In spite of the fact that the model considered for the broadcast problem is simple, it is very much useful in studying road traffic information system where the vehicles follow roads and messages are broadcasted along lanes. Typically, the curvature of the road is small in comparison to the transmission range, and thus we may consider that the road is a straight line [4]. Linear radio networks have been observed to be important in several recent studies [4,7,9].

2. Preliminaries

Let $S = \{s_1, s_2, \ldots, s_n\}$ be the set of n radio stations on a straight line from left to right. An weight $w_i > 0$ is assigned with each s_i, $i = 1, 2, \ldots, n$. Let $s^* = s_\theta \in S$ be the source radio station where from the message needs to be broadcast. Let $\mathcal{R} = \{\rho(s_1), \rho(s_2), \ldots, \rho(s_n)\}$ be a range assignment, where $\rho(s_i)$ is the range assigned to s_i. The directed graph $G = (V, E)$ with $V = S$ and $E = \{(s_i, s_j), d(s_i, s_j) \leqslant \rho(s_i)\}$ is referred to as the *communication graph* for the range assignment $\mathcal{R}$.

Fig. 1 demonstrates an instance of the unbounded weighted broadcast range assignment problem with five radio stations, along with the optimum solution.

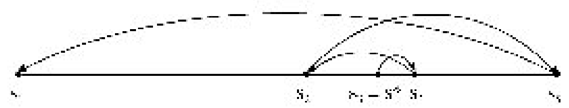

Fig. 1. An example of linear weighted broadcast problem.

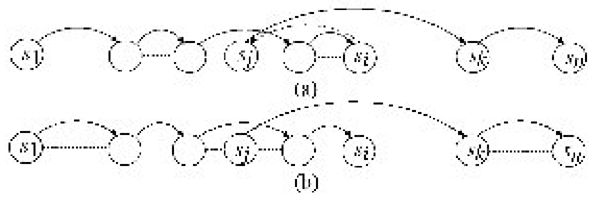

Fig. 2. Proof of Lemma 2.

Here, $d(s_1, s_2) = 8$, $d(s_2, s_3) = 2$, $d(s_3, s_4) = 1$ and $d(s_4, s_5) = 4$. The weight of the radio stations $s_1, s_2, \ldots, s_5$ are 10, 1, 10000, 100 and 0.01, respectively, source is s_3 ($= s^*$), and the cost of the optimum range assignment is 10,951.25 units.

Lemma 1. *(See [8].) For any given integer h, if $\mathcal{R} = \{\rho(s_1), \rho(s_2), \ldots, \rho(s_n)\}$ denotes the minimum cost range assignment of $\{s_1, s_2, \ldots, s_n\}$ for the h-hop weighted broadcast, then $\rho(s_i) \in \{d(s_i, s_j) \mid j = 1, 2, \ldots, n\}$ for all $i = 1, 2, \ldots, n$.*

Definition 1. A directed edge $e = (s_i, s_j)$ is said to be *functional* in the communication graph G corresponding to a h-hop broadcast range assignment $\mathcal{R}$, if the removal of the edge e from G indicates that there exists a radio station $s_k \in S$ which is not reachable from s^* (source) using a h-hop path in G.

Consider a path $\Pi = \{s_\theta = s_{i_1}, s_{i_2}, \ldots, s_{i_k} = s_j\}$ from the source $s^* = s_\theta$ to a radio station s_j ($j > \theta$) in the broadcast communication graph G corresponding to a range assignment $\mathcal{R}$. An edge $(s_{i_a}, s_{i_{a+1}})$ is said to be a *right back edge* if $i_{a-1} < i_a$ and $i_{a+1} < i_a$. Similarly, on a path from s_θ to a radio station s_j ($j < \theta$) a *left back edge* can be defined.

Lemma 2. *If $\theta = 1$ (resp., $\theta = n$), then in the minimum cost h-hop weighted broadcast range assignment, there is no functional right (resp., left) back edge.*

Proof. Let $\theta = 1$. Suppose there exists a functional right back edge (s_i, s_j) on the path Π in the communication graph G corresponding to the optimum broadcast range assignment (see the dashed edge in Fig. 2(a)). Note that, $j < i$, and there are paths from s_θ to both s_i and s_j without using that back edge. There also exists path from s_j to s_k for all $k > i$. Thus, broadcast is still

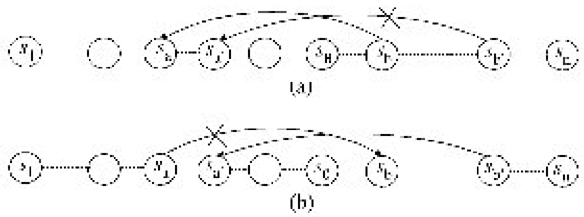

Fig. 3. Illustrations of (a) Lemma 3 and (b) Lemma 4.

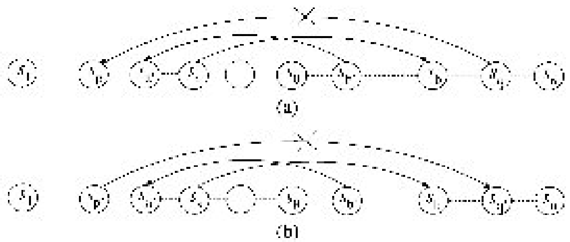

Fig. 4. Illustration of Lemma 5.

possible from s_θ to all the members in S if we remove the edge (s_i, s_j) from graph G by setting $\rho(s_i) = 0$ (see Fig. 2(b)). Thus, we have a contradiction as the total cost gets reduced. □

Definition 2. In a h-hop broadcast range assignment $\mathcal{R}$, a *right-bridge* $\overleftarrow{s_a s_b}$ corresponds to a pair of radio stations (s_a, s_b) such that s_a is to the left of s^*, s_b is to the right of s^*, and s_b can reach s_a in 1 hop due to its assigned range, but it cannot reach s_{a-1} in 1 hop.

A right-bridge $\overleftarrow{s_a s_b}$ (if exists) is said to be *functional*, if the directed edge (s_b, s_a) in the communication graph G is functional.

Similarly, one can define a *left-bridge* $\overrightarrow{s_a s_b}$ and a *functional left-bridge* in a h-hop broadcast range assignment, where s_a and s_b are, respectively, in the left and right sides of s^*.

Lemma 3. *If $\overleftarrow{s_a s_b}$ and $\overleftarrow{s_{a'} s_{b'}}$ are two functional right-bridges present in a h-hop weighted broadcast range assignment $\mathcal{R}$, then* (i) $b \neq b'$, (ii) $a \neq a'$, *and* (iii) *if $b < b'$, then $a' < a$.*

Proof. (i) If $b = b'$ then trivially one of these two bridges will not remain functional. Same argument holds for part (ii) also.

(iii) On the contrary, let $a' \geqslant a$. Now any path from source s_θ to $s_{b'}$ implies that there is also a path from s_θ to s_b as $b' > b > \theta$. Since $a' \geqslant a$, all the radio stations s_k ($a' \leqslant k < \theta$) are reachable using the right-bridge $\overleftarrow{s_a s_b}$. Thus, the right-bridge $\overleftarrow{s_{a'} s_{b'}}$ will no longer remain functional (see Fig. 3(a)). □

Lemma 4. *Let $\overrightarrow{s_a s_b}$ be a functional left-bridge and $\overleftarrow{s_{a'} s_{b'}}$ be a functional right-bridge in a h-hop weighted broadcast range assignment $\mathcal{R}$. Now,* (i) *if $a \leqslant a'$ then $b' < b$, and* (ii) *if $a > a'$ then $b' \geqslant b$.*

Proof. (i) On the contrary, let $b' \geqslant b$. Now, the path from the source s_θ to s_a can use the right-bridge $\overleftarrow{s_{a'} s_{b'}}$ or not. If that path uses the right-bridge $\overleftarrow{s_{a'} s_{b'}}$, then obviously left-bridge $\overrightarrow{s_a s_b}$ will not be functional (see Fig. 3(b)). Again, if the path does not use right-bridge $\overleftarrow{s_{a'} s_{b'}}$, then obviously it is not functional.

(ii) Proof of this part is similar to part (i). □

Lemma 5. *Let $\overrightarrow{s_a s_b}$ be a left-bridge and $\overleftarrow{s_{a'} s_{b'}}$ be a right-bridge such that $a' < a < \theta < b' < b$. Now, if both the bridges $\overrightarrow{s_a s_b}$ and $\overleftarrow{s_{a'} s_{b'}}$ are functional in a h-hop weighted broadcast range assignment, then* (i) *there is no functional right-bridge $\overleftarrow{s_p s_q}$ such that $p \leqslant a'$ and $q \geqslant b$ (see Fig.* 4(a)) *and* (ii) *there is no functional left-bridge $\overrightarrow{s_{p'} s_{q'}}$ such that $p' \leqslant a'$ and $q' \geqslant b$ (see Fig.* 4(b)).

Proof. (i) On the contrary, let $\overleftarrow{s_p s_q}$ be a functional right-bridge. Assume that there is no functional left-bridge $\overrightarrow{s_e s_f}$ such that $p < e \leqslant a'$ and $b < f \leqslant q$; otherwise it contradicts the part (ii) of this lemma. Note that, both $e = p$ and $f = q$ are not possible since $\overleftarrow{s_p s_q}$ and $\overrightarrow{s_e s_f}$ are both functional.

Since there is no 1 hop path from any radio station in $\{s_{p+1}, s_{p+2}, \ldots, s_{a'}\}$ to any radio station in $\{s_{b+1}, s_{b+2}, \ldots, s_q\}$, there exists a path from s_θ to s_q without using the bridge $\overleftarrow{s_{a'} s_{b'}}$. Again, a radio station, which is covered by the bridge $\overleftarrow{s_{a'} s_{b'}}$, is also covered by $\overleftarrow{s_p s_q}$. Thus, both the left-bridges $\overleftarrow{s_{a'} s_{b'}}$ and $\overleftarrow{s_p s_q}$ cannot be functional. Thus, we have a contradiction.

(ii) By similar argument, if $\overrightarrow{s_{p'} s_{q'}}$ is a functional left-bridge, then $\overrightarrow{s_a s_b}$ is not functional. □

The optimum h-hop weighted broadcast range assignment may consist of many left-bridges and/or many right-bridges (see Fig. 1). Feasible configurations of overlapping bridges are shown in Fig. 5. We now introduce the concept of *right-most functional right-bridge* and *left-most functional left-bridge* as follows. These help us in designing our algorithms.

Definition 3. A functional right-bridge $\overleftarrow{s_{a^*} s_{b^*}}$ in a range assignment is said to be *right-most functional right-bridge* if there exists no other functional right-bridges $\overleftarrow{s_a s_b}$ in that range assignment satisfying $b \geqslant b^*$.

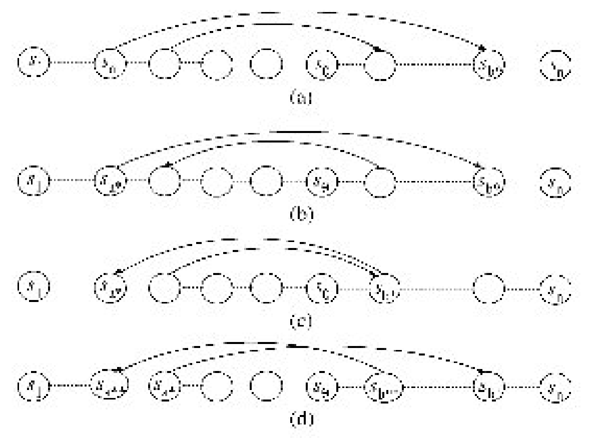

Fig. 5. Illustration of left-most functional left-bridge and right-most functional right-bridge.

Lemma 3 says that, if more than one right-bridges exist in a range assignment, and $\overleftarrow{s_{a^*}s_{b^*}}$ is the right-most functional right-bridge, then for all other functional right-bridge $\overleftarrow{s_a s_b}$, $a > a^*$.

Similarly, one can define a *left-most functional left-bridge*. In Fig. 5(a) and (b), $\overrightarrow{s_{a^*}s_{b^*}}$ is the left-most functional left-bridge; in Fig. 5(c), $\overleftarrow{s_{a^*}s_{b^*}}$ is the right-most functional right-bridge; in Fig. 5(d), $\overrightarrow{s_{a^*}s_{b^*}}$ and $\overleftarrow{s_{a^{**}}s_{b^{**}}}$ are left-most and right-most functional bridges, respectively.

We now use Lemmas 4 and 5 to describe our strategy for computing the optimal solution for the unbounded and h-hop broadcast range assignment with no functional bridge and a right-most functional right-bridge. The method of handling the case with a left-most functional left-bridge is similar.

3. Unbounded weighted broadcast problem

In this section, we consider unbounded version ($h = n - 1$) of the weighted broadcast problem. In the preprocessing phase, we use dynamic programming to create three arrays M, N and P, each of size n. Each entry of these array is a tuple (χ, γ) as described below.

$M[i].\chi$ stores the cost of the optimum broadcast from s_i to all the radio stations in the set $S_i^+ = \{s_i, s_{i+1}, \ldots, s_n\}$, and $M[i].\gamma$ stores the index of the farthest radio station to the right of s_i which can be reached from s_i in 1 hop due to the assigned range of s_i.

$N[i].\chi$ contains the cost of optimum broadcast range assignment from s_i to all the nodes in $S_i^- = \{s_1, s_2, \ldots, s_i\}$, and $N[i].\gamma$ contains the index of the farthest radio station to the left of s_i which can be reached from s_i in 1 hop due to the assigned range of s_i.

In order to explain the fields attached the array P, let us consider a complete digraph G with the vertices corresponding to the radio stations in S, and the weight of a directed edge (s_a, s_b) is $w_a \times (d(s_a, s_b))^2$. $P[i].\chi$ contains the cost of the shortest path from s_θ to s_i in the graph G, and $P[i].\gamma$ contains the index of the predecessor of s_i in the shortest path from s_θ to s_i.

3.1. Preprocessing

We will describe an algorithm for computing the arrays M and P. The array N can be computed using the technique for computing M.

Lemma 6. *If $i < n$ then $M[i].\chi = \min_{k=i}^{n}(w_i \times (d(s_i, s_k))^2 + M[k].\chi)$, and if minimum is obtained for $k = k^*$, then $M[i].\gamma = k^*$. If $i = n$ then $M[i].\chi = 0$ and $M[i].\gamma = n$.*

Proof. The case for $i = n$ is trivial. So, we consider the case where $i < n$. It is clear that, if there is a path from s_i to s_n in the communication graph corresponding to a range assignment, then that range assignment is feasible for the broadcast from s_i to all the members in S_i^+. By Lemma 2, in the optimum range assignment for broadcasting from s_i to the members in S_i^+, there is no functional back edge. Thus, there exists some $s_k \in S_i^+$, where s_i first reaches s_k in 1 hop, and then reaches s_n from s_k in a minimum cost path. This proves the lemma. □

Thus, Lemma 6 gives a dynamic programming based algorithm for computing M and N that runs in $O(n^2)$ time.

In order to compute the entries of the array P, we may use Dijkstra's single source shortest path algorithm to compute the cost of the shortest path from s_θ to each radio station $s_i \in S$. This needs $O(n^2)$ time. Note that, the weight of an edge in G can be computed from the positional information and weight attached to its end-vertices. Thus, P can be computed without explicitly constructing the graph G.

By Lemma 2, $M[1].\chi$ and $N[1].\chi$ give the cost of range assignment for the weighted unbounded broadcast when $\theta = 1$ and $\theta = n$, respectively. We now consider the case where $s^* = s_\theta$, and $\theta \neq 1$ or n. The above discussions say that the optimum unbounded weighted broadcast range assignment will either be (i) bridge-free or (ii) there is a left-most functional left-bridge or (iii) there is a right-most functional right-bridge. We compute the optimum solution in each case separately. Finally, the one having minimum cost is reported.

3.2. Bridge-free solution

The algorithm for the optimal solution with no functional bridge uses two variables *opt_cost* and *opt_range*, where *opt_cost* stores the cost of optimum range assignment, and *opt_range* stores the index α such that $\rho(s_\theta) = d(s_\theta, s_\alpha)$ in the range assignment corresponding to *opt_cost*. The stepwise description of the algorithm is as follows:

Step 1: $opt\_cost = \infty$.

Step 2: Consider each element $s_i \in S \setminus \{s_\theta\}$ in order of their distances from s_θ.
Compute the optimum range assignment with $\rho(s_\theta) = d(s_\theta, s_i)$ as follows:
Let s_α and s_β be, respectively, the left-most and right-most radio-station satisfying $d(s_\theta, s_\alpha) \leqslant \rho(s_\theta)$, and $d(s_\theta, s_\beta) \leqslant \rho(s_\theta)$.
If $i = \alpha$, then
$C = N[\alpha].\chi + w_\theta \times (\rho(s_\theta))^2 + \min_{j=\theta+1}^{\beta} M[j].\chi$,
and if $i = \beta$, then
$C = \min_{j=\alpha}^{\theta-1} N[j].\chi + w_\theta \times (\rho(s_\theta))^2 + M[\beta].\chi$.
If the value of C is less than *opt_cost*, then update *opt_cost* and *opt_range*.

Step 3: (* Range assignment *)
Set $\rho(s_\theta) = d(s_\theta, s_{\alpha^*})$, where α^* is stored in *opt_range*. The range of the other radio stations are computed as follows:
Let $\alpha^* < \theta$. We assign $\alpha = \alpha^*$. Let s_β $(\beta > \theta)$ be the right-most radio station such that s_θ can reach s_β in 1 hop.
We identify an index k such that
$M[k].\chi = \min_{j=\theta+1}^{\beta} M[j].\chi$.
Next, we assign
$\rho(s_\alpha) = d(s_\alpha, s_{N[\alpha].\gamma})$ and $\rho(s_k) = d(s_k, s_{M[k].\gamma})$.
We proceed further in both left and right direction separately. At each move towards left (resp., right) we update $\alpha = N[\alpha].\gamma$ (resp., $k = M[k].\gamma$), and assign the range of s_α (resp., s_k) as mentioned above, until $\alpha = 1$ (resp., $k = n$) is reached. Range of the other radio stations are assigned to zero.
The case, where α^* $(= opt\_range) > \theta$ is similarly handled.

3.3. Solution having right-most functional right-bridge

In this subsection, we describe the algorithm for computing the optimal solution having right-most functional right-bridge. It considers each possible right-bridge as a right-most functional right-bridge and computes the cost of the corresponding range assignment. Finally, we choose the one having minimum cost.

Let $\overleftarrow{s_p s_q}$ denote the right-most functional right-bridge that produces optimum cost. We compute the optimum cost range assignment for reaching s_q from s_θ using the array P. Let s_r be the right-most radio station where s_q can reach in 1 hop due to its assigned range. Thus, all the radio stations $S_{pq} = \{s_p, s_{p+1}, \ldots, s_q, s_{q+1}, \ldots, s_r\}$ are reached from s_θ. Now, we need to consider the path for reaching s_1 to the left and s_n to the right.

For reaching s_1, we will only consider s_p, and compute the minimum cost path. The reasons are stated below.

(i) If we choose some element $s_j \in S_{pq}$, $j < \theta$ then the cost of reaching s_1 can further be reduced by choosing the bridge $\overleftarrow{s_j s_q}$, which we have separately considered.
(ii) If we choose some element $s_j \in S_{pq}$, $j > \theta$ then such a pair of overlapping functional right-bridges cannot exist in the optimum solution (see Lemma 4).
(iii) If we choose some element s_j, $j > q$, for reaching s_1 and it hops at $s_{p'}$ then $p' < p$ (by Lemma 4). In this situation, the right-bridge $\overleftarrow{s_{p'} s_j}$ is another right-bridge, which will be separately considered as the right-most functional right-bridge.

But, one may choose any radio-station s_t $(p < t \leqslant r)$ for reaching s_n due to the weight constraint. Here it needs to be mentioned that, if we choose a radio station $s_{t'}$, $t' \leqslant p$ for reaching s_n, then the optimum path from $s_{t'}$ to s_n will take its first hop at a node $s_{r'}$, where $r' > q$ (by Lemma 4). But, this situation results the same cost while considering $\overrightarrow{s_{t'} s_{r'}}$ as the left-most functional left-bridge. Thus, such a choice of $s_{t'}$ $(t' \leqslant p)$ is not required.

We maintain *opt_cost* to store the optimum cost. We also maintain a tuple of 5 integer variables (p, q, r, t, f); p, q, r and t are as explained earlier, and f is a flag bit. The arrays M and N are used for computing the range assignments for reaching s_n from s_t, and s_1 from s_p respectively. The stepwise algorithm is stated below.

Step 1: Initialize $opt\_cost = \infty$.

Step 2: (* Compute the cost of optimum solution with a right-most functional right-bridge *)
Consider each radio-station s_b, $\theta < b \leqslant n$.
We maintain four variables ℓ, ℓ', k and k'.
While considering a bridge $\overleftarrow{s_a s_b}$, (i) s_ℓ denotes the right-most radio station such that $d(s_b, s_\ell) \leqslant d(s_b, s_a)$, (ii) the index k is such that $M[k].\chi =$

$\min_{j=a+1}^{\ell} M[j].\chi$, and (iii) ℓ' and k' are used as temporary variables.

Step 2.0: Initialize $\ell = b$ and the index k is such that $M[k].\chi = \min_{j=\theta}^{b} M[j].\chi$.
Consider each right-bridge $\overleftarrow{s_a s_b}$, $a = \theta - 1, \theta - 2, \ldots, 1$ in order, and execute Steps 2.1–2.5.

Step 2.1: Set $\rho(s_b) = d(s_a, s_b)$, and $\ell' = \ell$.
Repeatedly increment ℓ by 1 until $d(s_b, s_\ell) > d(s_a, s_b)$. Finally decrement ℓ by 1.

Step 2.2: Compute k' such that
$M[k'].\chi = \min_{j=\ell'+1}^{\ell} M[j].\chi$.
Finally set k by comparing $M[k'].\chi$, $M[k].\chi$ and $M[a+1].\chi$, and choosing the minimum one.

Step 2.3: Thus the optimum cost for considering the right-bridge $\overleftarrow{s_a s_b}$ as the right-most functional right-bridge is

$$C = P[b].\chi + w_b \times \left(d(s_a, s_b)\right)^2 + N[a].\chi + M[k].\chi.$$

Step 2.4: If $C < opt\_cost$, then set $opt\_cost = C$, and set $(p, q, r, t, f) = (a, b, \ell, k, 0)$.

Step 2.5: If $d(s_b, s_{\ell+1}) < d(s_{a-1}, s_b)$ then $\rho(s_b) = d(s_b, s_{\ell+1})$ also serves the role of right-bridge $\overleftarrow{s_a s_b}$, and we compute the optimum cost for this assignment of $\rho(s_b)$. If the corresponding cost is less than *opt_cost*, then store it in *opt_cost*, set $f = 1$, and the set the other fields of the 5-tuple appropriately.

Step 3: Finally, the range assignment with right-bridge $\overleftarrow{s_p s_q}$, where (p, q, r, t, f) is the 5-tuple corresponding to the optimum solution, is done as follows:

Assign $\rho(s_q) = d(s_p, s_q)$ or $d(s_q, s_r)$ depending on whether f bit is 0 or 1. The range assignment of all the radio stations on the path from s_θ to s_q are obtained from the array P. The range assignment from s_p to s_1 are obtained from the array N and the range of the radio stations from s_t to s_n are obtained from the array M as described in Section 3.2.

3.4. Correctness and complexity

The correctness of the algorithm for computing optimum solution without any functional bridge follows from the fact that we have considered all possible range of the source s_θ. For each choice of the range s_θ, we have computed the optimum range assignment of the other radio stations for reaching s_1 and s_n. Lemmas 3, 4 and 5 justifies the correctness of the algorithm for computing the optimum range assignment with a right-most functional right-bridge. Exactly same method as in Section 3.3 works for computing the optimum range assignment with a left-most functional left-bridge. The following theorem says the time and space complexity results of our algorithm.

Theorem 1. *The worst case time and space complexities of computing the weighted unbounded broadcast range assignment are* $O(n^2)$ *and* $O(n)$, *respectively.*

Proof. The space complexity follows from the size of the arrays M, N and P. The time required for computing these arrays is $O(n^2)$. After computing M and N, the time required for computing the optimum solution with no functional bridge is $O(n)$ time. In Step 2 of the algorithm, we fix a radio station b and consider the right-bridges $\overleftarrow{s_a s_b}$ for all $a = \theta - 1, \theta - 2, \ldots, 1$ in order. Note that, the computation of ℓ and k in each iteration of Steps 2.1–2.5 (i.e., for a particular index a) is incremental. Step 2.5 may also need some incremental cost, and this reduces the time requirement of the next iteration. Thus, for a particular radio station s_b, $b > \theta$, the total number of distance computations in Step 2.1 and finding minimum value in the M array in Step 2.2 is $O(n)$. This indicates that the total time complexity of Step 2 is $O(n^2)$. The range assignment in Step 3 needs $O(n)$ time. □

4. Weighted *h*-hop broadcast problem

If the number of hops is restricted to a specified integer h $(1 \leqslant h \leqslant n-1)$, the graph-theoretic approach, described above, does not work. We apply dynamic programming based approach for solving this problem. We first compute three $n \times h$ matrices, namely A, B and C, whose each entry is a tuple (χ, γ) as mentioned below.

(a) $A[i, j].\chi$ = minimum cost for sending message from s_i to the radio stations in $S_{i+1}^{+} = \{s_{i+1}, s_{i+2}, \ldots, s_n\}$ using at most j hops; $A[i, j].\gamma$ = index k, such that in the minimum cost j-hop path from s_i to s_n, the first hop takes place at s_k.

(b) $B[i, j].\chi$ = minimum cost for sending message from s_i to the radio stations in $S_{i-1}^{-} = \{s_1, s_2, \ldots, s_{i-1}\}$ using at most j hops; $B[i, j].\gamma$ = index k, such that in the minimum cost j-hop path from s_i to s_1, the first hop takes place at s_k.

(c) $C[i, j].\chi$ = minimum cost of sending message from s_θ (source) to s_i using at most j hops; $C[i, j].\gamma$ = index k, such that in the minimum cost j-hop path from s_θ to s_i, the last hop takes place from s_k to s_i.

We explain the computation of matrices A and C. The computation of the matrix B is similar to that of A.

The elements of the first column of matrix A are $A[i,1].\chi = w_i \times (d(s_i, s_n))^2$ and $A[i,1].\gamma = n$ for $i = 1, 2, \ldots, n$. After computing the $(j-1)$th column, the computation of the jth column is as follows: $A[i,j].\chi = \min_{k=i}^{n}(w_i \times (d(s_i, s_k))^2 + A[k, j-1].\chi)$. If the minimum is achieved for $k = k'$, then we set $A[i,j].\gamma = k'$.

The elements of the first column of matrix C are $C[i,1].\chi = w_\theta \times (d(s_\theta, s_i))^2$ and $C[i,1].\gamma = \theta$, for $i = 1, 2, \ldots, n$. After computing the $(j-1)$th column, the computation of the jth column is as follows: $C[i,j].\chi = \min_{k=1}^{n}(C[k, j-1].\chi + w_k \times (d(s_k, s_i))^2)$. If the minimum is achieved for $k = k'$, then we set $C[i,j].\gamma = k'$.

It is clear from the above discussion that the time required for computing the matrices A, B and C is $O(hn^2)$. In the following two subsections we describe the method of computing the optimum range assignment for the h-hop broadcast (i) with no functional bridge and (ii) with right-most functional right-bridge. The optimum solution with left-most functional left-bridge is similarly computed.

4.1. Bridge-free solution

The algorithm for the weighted h-hop broadcast range assignment problem having no functional bridge is similar to the algorithm for the unbounded version of the same problem described in Section 3.2. The only change is that, here we need to replace $M[i]$ and $N[i]$ by $A[i, h-1]$ and $B[i, h-1]$, respectively.

4.2. Solution with right-most functional right bridge

We consider each $\overleftarrow{s_a s_b}$ $(a < \theta < b)$, and compute the minimum cost of range assignment with $\overleftarrow{s_a s_b}$ as the right-most functional right-bridge. Finally, the one having the overall minimum cost, is identified.

Here also, we will initialize *opt_cost* by ∞, and will use the 5-tuple (p, q, r, t, f) as described in the algorithm of Section 3.3.

For each right-bridge $\overleftarrow{s_a s_b}$ $(1 \leqslant a < \theta < b \leqslant n)$ and for each k $(1 \leqslant k < h)$ we perform the following steps.

Step 1: Assign $\rho(s_b) = d(s_a, s_b)$. Let ℓ be the maximum index such that $d(s_b, s_\ell) \leqslant d(s_a, s_b)$.

Step 2: Use the matrix C to compute the range assignment from s_θ to s_b in k hops. Due to this range assignment, the following three sets of vertices are already reached from s_θ.
- (i) In the k hops path, $S^1_{ab} = \{s_i, s_{i+1}, \ldots, s_b\}$ are reached from s_θ in $h_i, h_{i+1}, \ldots, h_b$ hops, respectively, where each $h_j \leqslant k$,
- (ii) the bridge $\overleftarrow{s_a s_b}$ enables $S^2_{ab} = \{s_a, s_{a+1}, \ldots, s_{i-1}\}$ to be reached from s_θ in $k+1$ hops (where $a < i$), and
- (iii) due to the assigned range of s_b, the radio stations in $S^3_{ab} = \{s_{b+1}, s_{b+2}, \ldots, s_\ell\}$ are all reached from s_θ in $k+1$ hops (where $\ell > b$).

Step 3: As described in Section 3.2, here also we reach s_1 from s_a only, and for reaching s_n, we need to choose an appropriate radio stations in S_{ab} for which the cost is minimum.
Compute $A^* = \min(A^*_1, A^*_2, A^*_3)$, where $A^*_1 = \min_{j=i}^{b} A[j, h-h_j]$ (* the cost of reaching s_n from any one of the vertices in S^1_{ab} *),
$A^*_2 = \min_{j=a}^{i-1} A[j, h-k-1]$ (* cost of reaching s_n from any one of the vertices in S^2_{ab} *), and
$A^*_3 = \min_{j=b+1}^{\ell} A[j, h-k-1]$ (* cost of reaching s_n from any one of the vertices in S^3_{ab} *).

Step 4: Compute

$$cost = C[b,k] + B[a, h-k-1] + A^*.$$

Step 5: If $cost < opt\_cost$ then
set $opt\_cost = cost$, and the 5-tuple $(p, q, r, t, j) = (a, b, \ell, j^*, 0)$, where j^* is the index of the radio station for which the minimum occurs in the expression of A^*.

Step 6: If $d(s_b, s_{\ell+1}) < d(s_{a-1}, s_b)$ then
assign $\rho(s_b) = d(s_b, s_{\ell+1})$ (* This also serves the role of right bridge $\overleftarrow{s_a s_b}$ *)

We repeat Steps 2 to 5 for computing the optimum cost for assigning this range of $\rho(s_b)$. If the observed cost is less than *opt_cost*, then update *opt_cost* and the 5-tuple appropriately with the flag bit f set to 1.

The optimal range assignments of the radio stations can be done using the 5-tuple, and using the same technique as stated in Section 3.2.

In order to analyze the time complexity of our algorithm, let us consider a right-bridge $\overleftarrow{s_a s_b}$, and an integer k $(1 \leqslant k < h)$, where k is the number of hops needed to reach s_b from s_θ. Note that, the minimum cost of reaching from s_θ to s_b using k hops, and from s_a to s_1 using $(h-k-1)$ hops can be obtained in $O(1)$ time from $C[i,k].\chi$ and $A[a, h-k-1].\chi$, respectively. The time for computing the minimum cost for reaching s_n from any of the nodes s_i $(a+1 \leqslant i \leqslant \ell)$ is at most $O(n)$. Thus, we have the following theorem.

Theorem 2. *The worst case time and space complexities of the weighted h-hop broadcast problem is* $O(hn^3)$ *and* $O(nh)$, *respectively.*

Proof. The minimum cost range assignment without any functional bridge can be computed in $O(n)$ time using the matrices A and B. In order to compute the solution with a functional bridge, we may need to consider $O(n^2)$ pairs of nodes in S as the possible right-most functional right-bridge (resp., left-most functional left-bridge), and for each such pair, the possible choices of k is at most $h-1$. As mentioned earlier, the time required for considering each triple (s_a, s_b, k) is $O(n)$ (where $\overleftarrow{s_a s_b}$ is the bridge, and k is the number of hops to reach from s_θ to s_b). Thus, the time complexity result follows. The space complexity result follows from the storage requirement of the matrices A, B and C. □

4.3. Further refinement

The time complexity of Step 3 of the algorithm can be reduced if we can get the three quantities A_1^*, A_2^* and A_3^* on demand. In order to achieve this, we store each column (k) of the matrix A in a height balanced binary tree T_k whose each node is a tuple (s, c). The s fields of the leaf nodes contain $s_1, s_2, \ldots, s_n$ in left to right order, and their corresponding c fields contain the elements of the kth column of matrix A. The discriminant value stored at each internal node (say v) of T_k is the average of the s values stored in its inorder predecessor and successor nodes. The corresponding c field stores the minimum value of the c fields attached to all the nodes rooted at v in T_k.

Let us now fix a node s_b, $b > \theta$, and consider the minimum cost of range assignment with all possible right bridges $\overleftarrow{s_a s_b}$, where $a < \theta$. Let s_b be reached from s_θ in k hops. While computing the k hop path from s_θ to s_b, we can get the set S_{ab}^1, and the number of hops needed for each node to reach from s_θ. Thus, we can also compute A_1^* in $O(|S_{ab}^1|)$ time using the matrix A as mentioned in Step 3 of the algorithm.

Now, assign different ranges to s_b for generating right bridges $\overleftarrow{s_a s_b}$ for different $a < \theta$. This determines the set S_{ab}^2 and S_{ab}^3. Now we can get A_2^* and A_3^* on demand, by searching in the tree T_k in $O(\log n)$ time. Note that, for a given k and b, A_1^* does not change. So, we can compute the cost of the range assignment for all the bridges initiated from b in $O(n \log n)$ time. Thus, we have the following theorem:

Theorem 3. *The worst case time and space complexities of the weighted h-hop broadcast problem is* $O(hn^2 \log n)$ *and* $O(nh)$, *respectively.*

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