IMPROVED ALGORITHM FOR MINIMUM COST RANGE ASSIGNMENT PROBLEM FOR LINEAR RADIO NETWORKS*

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ABSTRACT

In the unbounded version of the range assignment problem for all-to-all communication in 1D, a set of n radio stations are placed arbitrarily on a line; the objective is to assign ranges to these radio-stations such that each of them can communicate with the others (using at most $n-1$ hops) and the total power consumption is minimum. A simple incremental algorithm for this problem is proposed which produces optimum solution in $O(n^3)$ time and $O(n^2)$ space. This is an improvement in the running time by a factor of n over the best known existing algorithm for the same problem.

Keywords: Range assignment; linear radio network; mobile communication; algorithm.

1. Introduction

A multi-hop mobile radio network, is a self-organized and rapidly deployable network in which neither a wired backbone nor a centralized control exists. The network nodes communicate with one another over scarce wireless channels in a multi-hop fashion. Its importance has been increased due to the fact that, there exists situations where the installation of traditional wired network is impossible, and in some cases, even if it is possible, it involves very high cost in comparison to radio networks. Several variations of routing, broadcasting and scheduling problems on radio networks are discussed in the literature [1, 2, 3, 4, 5].

A radio network is a finite set of *radio-stations* S located on a geographical region which can communicate each other by transmitting and receiving radio signals.

Each radio-station $s \in S$ is assigned a range $\rho(s)$ (a positive real number). A radio-station s can communicate (i.e., send a message) directly (i.e., in *1-hop*) to any other station t if the Euclidean distance between s and t is less than or equal to $\rho(s)$. If s can not communicate directly with t due to its assigned range, then communication between them can be achieved using *multi-hop transmissions*. The power $power(s)$ required by a radio-station s to transmit a message to another radio-station s' satisfies $\frac{power(s)}{d(s,s')^\beta} > \gamma$ [4], where $d(s, s')$ is the Euclidean distance between s and s', β is referred to as the distance-power gradient which may vary from 1 to 6 depending on various environmental factors; $\gamma(\geq 1)$ is the transmission quality of the message. We assume the ideal case, i.e., $\beta = 2$ and $\gamma = 1$. Thus, the total cost of a range assignment $\mathcal{R} = \{\rho(s) \mid s \in S\}$ is written as $cost(\mathcal{R}) = \sum_{s \in S} power(s) = \sum_{s \in S} (\rho(s))^2$. If the number of hops (h) is small, then communication between a pair of radio-stations happens very quickly, but the power consumption of the entire radio network becomes high. On the other hand, if h is large, the power consumption decreases, but communication delay takes place. The tradeoffs between power consumption of the radio network and maximum number of hops needed between a communicating pair of radio-stations are studied extensively [6, 7].

In 1D variation of this problem, the n radio-stations in set S are placed arbitrarily on a line. Several variations of the *1D range assignment problem for h-hop all-to-all communication* are studied by Kirousis et al. [6]. For the uniform chain case, i.e., where each pair of consecutive radio-stations on the line is at distance δ, the tight upper bounds on the minimum cost of range assignment is shown to be $OPT_h = \Theta(\delta^2 n^{\frac{2^{h+1}-1}{2^h-1}})$ for any fixed h. In particular, if $h = \Omega(\log n)$ in the uniform chain case, then $OPT_h = \Theta(\delta^2 \frac{n^2}{h})$. For the general problem in 1D, i.e., where the points are arbitrarily placed on a line, a 2-approximation algorithm for the range assignment problem for h-hop all-to-all communication is proposed by Clementi et al. [8]. The worst case running time of this algorithm is $O(hn^3)$. For the unbounded case, i.e., where $h = n - 1$, a dynamic programming based $O(n^4)$ time algorithm is available [6] which produces a range assignment achieving minimum cost. Efficient polynomial time algorithms for the optimal 1D range assignment for broadcasting from a single node are also available [9, 10].

We propose a simple algorithm for the unbounded version of 1D range assignment problem for all-to-all communication. It runs in $O(n^3)$ time using $O(n^2)$ space. This improves the existing time complexity result on this problem by a factor of n keeping the space complexity invariant [6]. In spite of the fact that the model considered in this paper is simple, it is useful in studying road traffic information system where the vehicles follow roads and messages are transmitted along lanes. Typically, the curvature of the road is small in comparison to the transmission range so that one may consider that the vehicles are moving on a line [8]. Several other vehicular technology applications of this problem can also be found in the literature [11, 12, 13, 7].

2. Preliminaries

Let $S = \{s_1, s_2, \ldots, s_n\}$ be a set of n radio-stations placed on a line. Without loss of generality, we name the elements of S as $\{s_1, s_2, \ldots, s_n\}$, ordered from left to right. We will use $d(s_i, s_j)$ to denote the distance between the radio-stations s_i and s_j. A *range assignment* for the set of radio-stations S is a vector $\mathcal{R} = \{\rho(s_1), \rho(s_2), \ldots, \rho(s_n)\}$, where $\rho(s_i)$ denotes the range assigned to radio-station $s_i \in S$. Given a range assignment $\mathcal{R}$, the corresponding *communication graph*, denoted by $G_\mathcal{R} = (S, E_\mathcal{R})$ is a directed graph whose set of vertices correspond to the radio-stations in S, and the edge set $E_\mathcal{R} = \{(s_i, s_j) | d(s_i, s_j) \leq \rho(s_i)\}$.

Definition 1 *A communication graph $G_\mathcal{R}$ corresponding to a range assignment $\mathcal{R}$ is said to be h-hop connected if from each vertex $s_i \in S$ there exists a directed path of length less than or equal to h to every other vertex $s_j \in S$.*

For each radio-station s_i, we maintain an array D_i which contains the set of distances $\{d(s_i, s_j), j = 1, \ldots, n\}$. Now we have the following lemma.

Lemma 1 *For any given h, if $\mathcal{R} = \{\rho_1, \rho_2, \ldots, \rho_n\}$ denotes the h-hop optimum range assignment of $\{s_1, s_2, \ldots, s_n\}$ for h-hop all-to-all communication then $\rho_i \in D_i$ for all $i = 1, 2, \ldots, n$.*

Proof. Let us assume that $\rho_i = r$ for some i, and $r \notin D_i$. Let $G_\mathcal{R}$ be the corresponding communication graph. Surely, $Min\{D_i\} \leq r \leq Max\{D_i\}$, since failing the left-hand terminal condition disables s_i to transmit its message to any member in $S \setminus \{s_i\}$, and the right-hand terminal condition ensures the *1-hop* reachability of s_i to all other vertices in S. Assume that the elements in D_i are sorted in increasing order, and there exist a pair of consecutive elements $\alpha, \beta \in D_i$ such that $\alpha < r < \beta$.

Consider a different range assignment $\mathcal{R}' = \{\rho_1, \rho_2, \ldots, \rho_{i-1}, \alpha, \rho_{i+1}, \ldots, \rho_n\}$, and its corresponding communication graph $G_{\mathcal{R}'}$. The difference of $\mathcal{R}'$ from $\mathcal{R}$ is the range of s_i (α instead of r) only. But, this change does not delete any edge from $G_\mathcal{R}$. Therefore, $G_\mathcal{R} \equiv G_{\mathcal{R}'}$. Thus, the *h-hop* connectivity of each vertex in S to all other vertices is maintained for the range assignment $\mathcal{R}'$. Again, $cost(\mathcal{R}') = cost(\mathcal{R}) - r^2 + \alpha^2 < cost(\mathcal{R})$. Hence we have the contradiction that $\mathcal{R}$ is the optimum range assignment. □

Note: The result stated in Lemma 1 is valid if the range assignment problem is considered in any arbitrary dimension.

From now onwards, we shall restrict ourselves to the unbounded version of the problem, i.e., $h = n-1$. Here the optimal solution corresponds to a range assignment such that the communication graph $G_\mathcal{R}$ is strongly connected, and the sum of powers of all the radio-stations is minimum. The following two lemmata indicate two important features of the optimum range assignment.

Lemma 2 *Let ρ be the range assigned to a vertex s_i; s_r and s_ℓ be respectively the rightmost and leftmost radio-stations such that $d(s_i, s_r) \leq \rho$ and $d(s_i, s_\ell) \leq \rho$. Now, if we consider the optimum range assignment of the radio-stations $\{s_\ell, s_{\ell+1}, \ldots, s_i, \ldots, s_{r-1}, s_r\}$ only subject to the condition that $\rho(s_i) = \rho$, then (i) the range assigned to the radio-station s_α is equal to $d(s_\alpha, s_{\alpha+1})$ for all $\alpha = \ell, \ell+1, \ldots, i-1$, and (ii) the range assigned to the radio-station s_β is equal to $d(s_\beta, s_{\beta-1})$ for all $\beta = i+1, i+2, \ldots, r$.*

Proof. Let $G_\mathcal{R}$ be the communication graph corresponding to a optimum range assignment $\mathcal{R}$. Clearly, in $G_\mathcal{R}$ there are directed paths from s_ℓ to s_i and s_r to s_i.

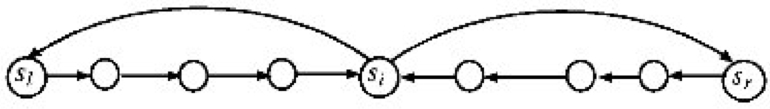

Fig. 1. Illustration of Lemma 2.

Since, for any two positive real numbers p and q, $(p+q)^2 > p^2+q^2$ and the objective is to minimize the power, the lemma follows (see Figure 1). □

Lemma 3 *In optimum range assignment* $\mathcal{R} = \{\rho_1, \rho_2, \ldots, \rho_n\}$, $\rho_1 = d(s_1, s_2)$ *and* $\rho_n = d(s_{n-1}, s_n)$.

Proof. On the contrary, let us assume that $\rho_1 = d(s_1, s_i)$, where $i > 2$. Now, we prove the lemma considering the following two cases : (i) $\rho_2 \leq d(s_2, s_i)$, and (ii) $\rho_2 > d(s_2, s_i)$. In Case (i), let us consider a modified assignment $\mathcal{R}' = \{\rho'_1, \rho'_2, \ldots, \rho'_n\}$, where $\rho'_1 = d(s_1, s_2), \rho'_2 = d(s_2, s_i), \rho'_3 = \rho_3, \rho'_4 = \rho_4 \ldots, \rho'_n = \rho_n$. Note that, the communication graph corresponding to the range assignment $\mathcal{R}'$ is still strongly connected, and $cost(\mathcal{R}') = cost(\mathcal{R}) - (d(s_1, s_i))^2 - \rho_2^2 + (d(s_1, s_2))^2 + (d(s_2, s_i))^2$ $= cost(\mathcal{R}) - 2d(s_1, s_2)d(s_2, s_i) - \rho_2^2 < cost(\mathcal{R})$. In Case (ii) also, let us consider a modified assignment $\mathcal{R}' = \{\rho'_1, \rho'_2, \ldots, \rho'_n\}$, where $\rho'_1 = d(s_1, s_2), \rho'_2 = \rho_2, \rho'_3 = \rho_3, \ldots, \rho'_n = \rho_n$. Note that, the communication graph corresponding to the new range assignment $\mathcal{R}'$ is still strongly connected, and $cost(\mathcal{R}') = cost(\mathcal{R}) - (d(s_1, s_i))^2 + (d(s_1, s_2))^2 = cost(\mathcal{R}) - 2d(s_1, s_2)d(s_2, s_i) - (d(s_2, s_i))^2 < cost(\mathcal{R})$. Therefore, in both the cases, there is another range assignment $\mathcal{R}'$ with $\rho_1 = d(s_1, s_2)$ whose cost is less than that of $\mathcal{R}$. The second part of the lemma can be proved in exactly similar manner. □

Our proposed algorithm is an incremental one. We denote the optimal range assignment of a subset $S_k = \{s_1, s_2, \ldots, s_k\}$ by $\mathcal{R}_k = \{\rho_1^k, \rho_2^k, \ldots, \rho_k^k\}$. Here the problem is: given $\mathcal{R}_j$ for all $j = 2, 3, \ldots, k$, obtain $\mathcal{R}_{k+1}$ by considering the next radio-station $s_{k+1} \in S$. An almost similar dynamic programming approach is used in [6] for solving the same problem in $O(n^4)$ time. Our approach is based on a detailed geometric analysis of the optimum solution, and it solves the problem in $O(n^3)$ time.

3. Method

We assume that for each $j = 2, 3, \ldots, k$, the optimal range assignment of $S_j = \{s_1, s_2, \ldots, s_j\}$ is stored in an array $\mathcal{R}_j$ of size j. The elements in $\mathcal{R}_j$ are $\{\rho_1^j, \rho_2^j, \ldots, \rho_j^j\}$, and $cost(\mathcal{R}_j) = \sum_{\alpha=1}^{j}(\rho_\alpha^j)^2$. The radio-station s_{k+1} is the next element under consideration. An obvious choice of $\mathcal{R}_{k+1}$ for making the communication graph $G_{\mathcal{R}_{k+1}}$ strongly connected is $\rho_{k+1}^{k+1} = d(s_k, s_{k+1})$ and $\rho_k^{k+1} = max(d(s_k, s_{k+1}), \rho_k^k)$. Lemma 4 says that this may not lead to an optimum result.

Lemma 4 $(d(s_k, s_{k+1}))^2 \leq cost(\mathcal{R}_{k+1}) - cost(\mathcal{R}_k) \leq (max(d(s_k, s_{k+1}), \rho_k^k))^2 - (\rho_k^k)^2 - (d(s_k, s_{k+1}))^2$.

Proof. In $\mathcal{R}_{k+1}$, s_{k+1} will receive range equal to $d(s_k, s_{k+1})$ for connecting it with its closest member $s_k \in S_k$ (see Lemma 3). Thus, the left hand side of the inequality

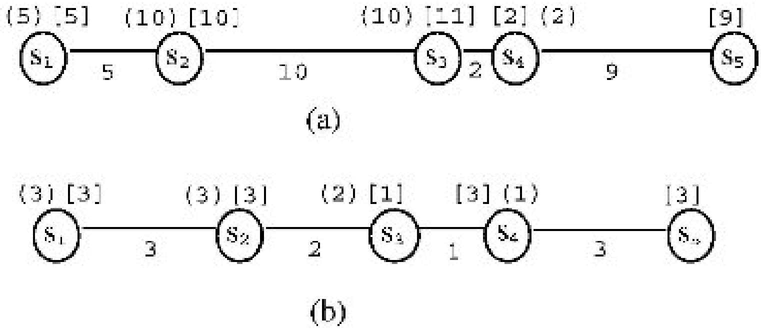

Fig. 2. Proof of Lemma 4.

follows. The equality takes place when s_{k+1} is reachable from some member in S_k with its existing range assignment in $\mathcal{R}_k$. If this situation does not take place, then one needs to extend the range of some member in S_k to reach s_{k+1}. The inequality in the right hand side follows from the obvious choice s_k for which the range $\rho_k^k < d(s_k, s_{k+1})$, and is extended to $d(s_k, s_{k+1})$. Here, the equality takes place if $\rho_k^k \geq d(s_k, s_{k+1})$. □

Illustrative examples are demonstrated in Figure 2, where the distance between each two consecutive nodes is shown along that edge; the range assignment for each node before and after inserting radio-station s_5 are shown in parenthesis and square bracket respectively. From the left hand inequality of Lemma 4, the range of s_{k+1} (i.e., ρ_{k+1}^{k+1}) needs to be assigned to $d(s_k, s_{k+1})$ (see the range assigned to s_5 in both the figures). Now we analyze the different cases that may be observed in $\mathcal{R}_k$, and the actions necessary for all those cases such that at least one member of S_k can communicate with s_{k+1} in 1-hop, and the total cost becomes minimum.

The simplest situation occurs if $d(s_i, s_{k+1}) \leq \rho_i^k$ for at least one $i = 1, 2, \ldots, k$. In this case, $\rho_i^{k+1} = \rho_i^k$ for all $i = 1, \ldots, k$. If $d(s_i, s_{k+1}) > \rho_i^k$ for all $i = 1, \ldots, k$, then we need to increase the range of some member in S_k for the communication from S_k to s_{k+1}. This may sometime need changes in different elements of $\mathcal{R}_k$ to achieve $\mathcal{R}_{k+1}$. We have demonstrated two examples in Figure 2, where the optimal range assignment of $\{s_1, s_2, s_3, s_4, s_5\}$ is obtained from that of $\{s_1, s_2, s_3, s_4\}$. The optimal range assignment in $\mathcal{R}_4$ and $\mathcal{R}_5$ are given in parenthesis and square bracket respectively. In Figure 2(a) the optimal range assignment is obtained by incrementing the range of s_3 only. But in Figure 2(b), in addition to incrementing the range of s_4, the range of s_3 is needed to be decremented to get the optimal assignment.

We use $\mathcal{R}_{k+1}^i$ to denote the optimum range assignment of the members in S_{k+1} subject to the condition that $\rho_i^{k+1} = d(s_i, s_{k+1})$. Now, $\mathcal{R}_{k+1}$ can be obtained by computing $\mathcal{R}_{k+1}^i$ for all $i = 1, 2, \ldots, k$, and then identifying an i^* such that $cost(\mathcal{R}_{k+1}^{i^*}) = Min_{i=1}^k\, cost(\mathcal{R}_{k+1}^i)$. We first describe a preprocessing step. Next, we describe in detail the computation of $\mathcal{R}_{k+1}^i$.

3.1. *Preprocessing*

In this step, we create the following two matrices using the given set of radio-stations $S = \{s_1, s_2, \ldots, s_n\}$.

$T1$: It is an $n \times n$ matrix. Its (i, j)-th entry contains the index α $(i \leq \alpha < j)$ such that $d(s_\alpha, s_{\alpha+1}) = Max_{\beta=i}^{j-1} d(s_\beta, s_{\beta+1})$.

$T2$: It is also an $n \times n$ matrix. Its (i, j)-th entry contains an index α such that if s_i is assigned the range $d(s_i, s_j)$ then s_i can communicate with s_α in 1-hop, but s_i can not communicate with $s_{\alpha-1}$ (resp. $s_{\alpha+1}$) in 1-hop depending on whether $i < j$ (resp. $i > j$).

Lemma 5 *Both the matrices $T1$ and $T2$ can be computed in $O(n^2)$ time.*

3.2. *Computation of R^i_{k+1}*

First, we introduce the notion of *left-cover* which will be used extensively in designing our algorithm.

Definition 2 *The* left-cover *of a radio-station s_α for its assigned range ρ is the left-most radio-station s_β which is reachable from s_α in 1-hop. Thus, s_β = left-cover(s_α, ρ), where $\beta < \alpha$ and $d(s_\alpha, s_\beta) \leq \rho < d(s_\alpha, s_{\beta-1})$. If $\beta = 1$ then the right-hand inequality condition is not required.*

Similarly, the *right-cover* of s_α for its assigned range ρ is defined as s_γ = *right-cover*(s_α, ρ), where $\gamma > \alpha$ and $d(s_\alpha, s_\gamma) \leq \rho < d(s_\alpha, s_{\gamma+1})$. If $\gamma = n$, the right-hand inequality condition is not required.

For notational convenience we will use ρ_j to denote ρ_j^{k+1}, for $j = 1, 2, \ldots, k+1$. We first assign $\rho_i = d(s_i, s_{k+1})$ and $\rho_{k+1} = d(s_k, s_{k+1})$. Let s_ℓ = *left-cover*(s_i, ρ_i). This implies, s_i can communicate with all the radio-stations $\{s_\ell, s_{\ell+1}, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_{k+1}\} = SS^i$ (say) in 1-hop, but s_i can not communicate with $s_{\ell-1}$ in 1-hop. Let us denote $SS^i_L = \{s_\ell, s_{\ell+1}, \ldots, s_{i-1}, s_i\}$, and $SS^i_R = \{s_{i+1}, s_{i+2}, \ldots, s_{k+1}\}$. Thus, we have $SS^i = SS^i_L \cup SS^i_R$.

By applying Lemma 2, we assign $\rho_j = d(s_j, s_{j-1})$ for all $s_j \in SS^i_R$, and $\rho_j = d(s_j, s_{j+1})$ for all $s_j \in SS^i_L \setminus \{s_i\}$. Due to this *changed range assignment*, none of the nodes in SS^i_R can communicate with a node to the left of s_i in 1-hop, but there may exist some member(s) in the set SS^i_L whose *left-cover* is in $S_{\ell-1}$. Let s_m be the leftmost radio-station such that s_m = *left-cover*(s_α, ρ_α) for some $s_\alpha \in SS^i_L$. We now need to consider the following two cases depending on whether (i) $m < \ell$ and (ii) $m = \ell$.

Case (i) $[m < \ell]$**:** Using the same argument as stated in Lemma 2, we further update the range of the radio-station s_j to $\rho_j = d(s_j, s_{j+1})$ for all $j = m, m+1, \ldots, \ell-1$ (see Figure 3(a)). This makes the communication subgraph with radio-stations $\{s_m, s_{m+1}, \ldots, s_{\ell-1}, s_\ell, \ldots, s_i, \ldots, s_{k+1}\}$ strongly connected. This new assignment of range may cause some one to the left of s_m to be reachable in 1-hop from $\{s_m, s_{m+1}, \ldots, s_{\ell-1}\}$. We update the set SS^i_L to $SS^i_L \cup \{s_m, s_{m+1}, \ldots, s_{\ell-1}\}$. As a result, SS^i is also being updated accordingly, and m is considered to be as ℓ. Again, we need to consider one among the cases (i) and (ii). Note that, while calculating

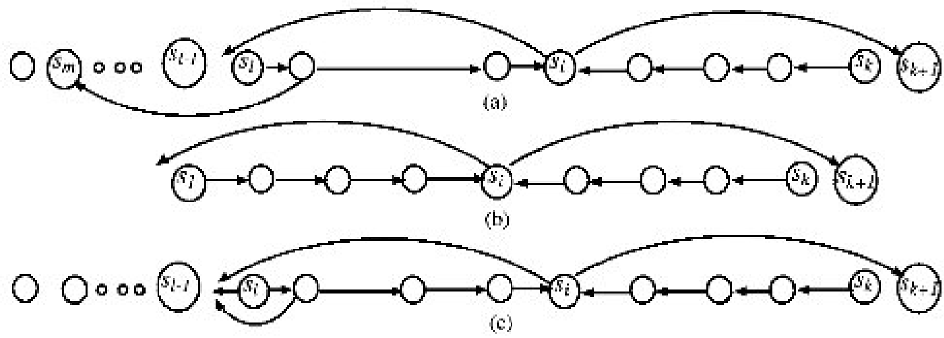

Fig. 3. Illustration of (a) Case (i), (b) Case (ii) with $m = \ell = 1$, and (c) Case (ii) with $m = \ell > 1$.

the *left-cover* of the updated set of nodes SS^i_L, we need to consider only the newly added nodes in SS^i_L.

Case (ii) $[m = \ell]$: Here several nodes in SS^i_L exist whose assigned range enables it to communicate with s_m in 1-hop but not with s_{m-1} (if exists). Thus, Case (i) fails to recur (see Figure 3(c)). Here $SS^i = \{s_m, s_{m-1}, \ldots, s_i, \ldots, s_k, s_{k+1}\}$, and m is referred to as the *maximal-left-cover*. The optimum range assignments for the radio-stations in SS^i are as follows:

- $\rho_i = d(s_i, s_{k+1})$ (as assumed),
- $\rho_j = d(s_{j-1}, s_j)$ for all $j = i+1, i+2, \ldots, k+1$, and
- $\rho_j = d(s_j, s_{j+1})$ for all $j = m, m+1, \ldots i-1$.

Observation 1 *The left-cover of every member in SS^i with respect to the above range assignment lies inside SS^i.*

Now, two cases may arise depending on whether $m = 1$ or $m > 1$. For $m = 1$, the optimum range assignment $\mathcal{R}^i_{k+1}$ is already computed (see Figure 3(b)). However, if $m > 1$, we need to compute the range assignments of the members in S_{m-1} and establish communication among SS^i and S_{m-1}.

Let us now consider $\mathcal{R}_m$, and set $\rho_j = \mathcal{R}_m[j]$ for $j = 1, 2, \ldots, m-1$. Since $\mathcal{R}_m$ supports strong connectivity among the members in S_m, at least one member in S_{m-1} directly (in 1 hop) communicates with a member in SS^i with the range assignment $\mathcal{R}_m$. Let s_μ be the rightmost member in SS^i which is directly (in 1 hop) reachable from a member $s_\nu \in S_{m-1}$. But, no element in SS^i can communicate with S_{m-1} with its presently assigned range. We now introduce the concept of *critical-gap* and use it to describe two procedures for restoring the strong connectivity in the entire S_{k+1}.

Definition 3 *Let $\{s_a, s_{a+1}, \ldots, s_b\}$ be a sequence of radio-stations such that $\Delta = Max_{j=a}^{b-1} d(s_j, s_{j+1}) = d(s_\tau, s_{\tau+1})$ (say). Here, Δ is said to be the* critical-gap *of the sequence of radio-stations $\{s_a, s_{a+1}, \ldots, s_b\}$.*

Lemma 6 *Let $(s_a, s_{a'})$ and $(s_b, s_{b'})$ be two pairs of radio-stations such that $a < b' < a' < b$, and the range assigned to s_a and s_b be $\rho_a = d(s_a, s_{a'})$ and $\rho_b =$*

$d(s_b, s_{b'})$ respectively (see Figure 4(a)). If the critical-gap in $\{s_{b'}, s_{b'-1}, \ldots, s_{a'}\}$ is $d(s_\tau, s_{\tau+1})$, where $b' \leq \tau < a'$, then in the optimum (cost) range assignment of the radio-stations $\{s_a, s_{a-1}, \ldots, s_{b'}, \ldots, s_{a'}, \ldots, s_b\}$, (i) $\rho_j = d(s_j, s_{j-1})$ for $j = a+1, a+2, \ldots, \tau$ and (ii) $\rho_j = d(s_j, s_{j+1})$ for $j = \tau+1, \tau+2, \ldots, b-1$.

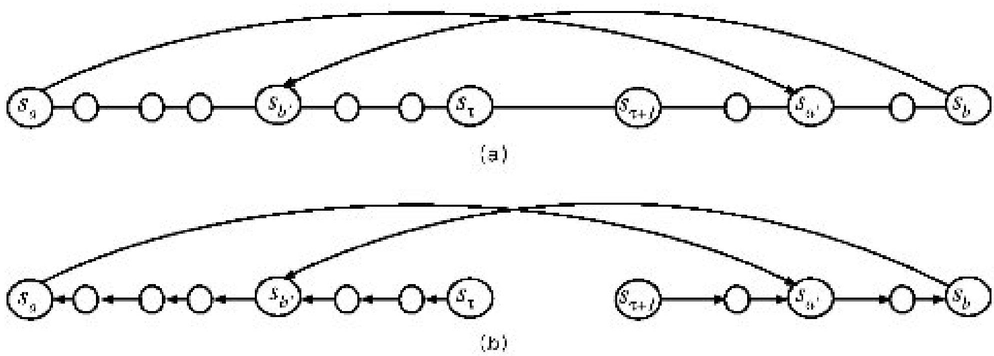

Fig. 4. Proof of Lemma 6.

Proof. Since $\rho_a = d(s_a, s_{a'})$, $\rho_b = d(s_b, s_{b'})$ and $a < b' < a' < b$, the communication graph among the nodes $\{s_a, s_{a+1}, \ldots, s_b\}$ remains strongly connected if we choose an index $t \in [b', a']$ and assign (i) ρ_j is equal to $d(s_j, s_{j-1})$ for $j = a+1, a+2, \ldots, t$ and (ii) ρ_j is equal to $d(s_j, s_{j+1})$ for $j = t+1, t+2, \ldots, b-1$ (see Figure 4(b) for the demonstration). Thus, the total cost becomes $(d(s_a, s_{a'}))^2 + (d(s_b, s_{b'}))^2 + \sum_{j=a+1}^{t}(d(s_j, s_{j-1}))^2 + \sum_{j=t+1}^{b-1}(d(s_j, s_{j+1}))^2$
$= (d(s_a, s_{a'}))^2 + (d(s_b, s_{b'}))^2 + \sum_{j=a}^{b-1}(d(s_j, s_{j+1}))^2 - (d(s_t, s_{t+1}))^2$. As $d(s_\tau, s_{\tau+1})$ is the *critical-gap*, the minimum cost is achieved for $t = \tau$. ☐

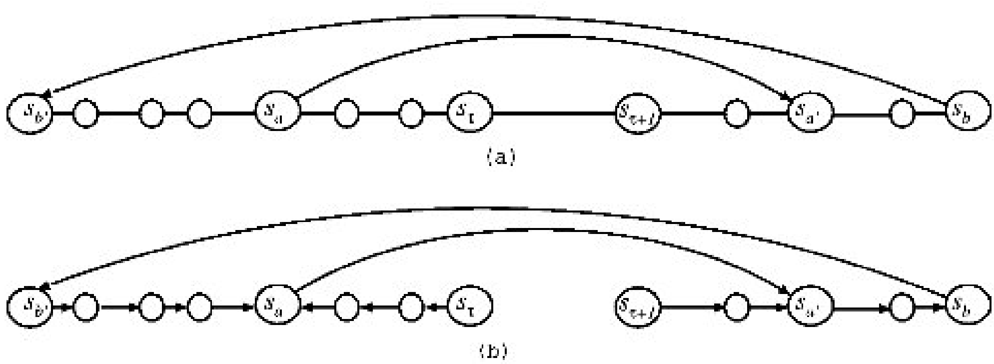

Fig. 5. Proof of Lemma 7.

Lemma 7 *Let $(s_a, s_{a'})$ and $(s_b, s_{b'})$ be two pairs of radio-stations such that $b' < a < a' < b$, and the range assigned to s_a and s_b be $\rho_a = d(s_a, s_{a'})$ and $\rho_b = d(s_b, s_{b'})$ respectively (see Figure 5(a)). If the critical-gap in $\{s_a, s_{a-1}, \ldots, s_{a'}\}$ is $d(s_\tau, s_{\tau+1})$, then in the optimum (cost) range assignment of the radio-stations $\{s_{b'}, s_{b'+1}, \ldots, s_b\}$, (i) $\rho_j = d(s_j, s_{j+1})$ for $j = b', b'+1, \ldots, a-1$, (ii) $\rho_j =$*

$d(s_j, s_{j-1})$ for $j = a+1, a+2, \ldots, \tau$, and (iii) $\rho_j = d(s_j, s_{j+1})$ for $j = \tau+1, \tau+2, \ldots, b-1$.

Proof. Proof is similar with Lemma 6. □

3.2.1. *Procedure-1*

This procedure is applicable if $\mu > m$. Since the members in SS^i are strongly connected with their existing range assignments and $\mu > m$, there exists some radio-station(s) to the right of s_μ whose assigned range enables it to reach a radio-station to the left of s_μ. We assume that s_θ is the leftmost one among such radio-stations, where $m < \theta < \mu$. Thus, a situation as in Figure 4(a) (ignoring the suffixes of the radio-stations) appears here. Let $\Delta = d(s_\tau, s_{\tau+1})$ be the *critical-gap* in $\{s_\theta, s_{\theta+1}, \ldots, s_\mu\}$. We apply Lemma 6 to update the range assignment as $\{\rho_j = d(s_j, s_{j-1}), j = \tau, \tau-1, \ldots, m\}$ (see Figure 4(b)). The range assignments of the other radio-stations remain unchanged, and the strong connectivity of the entire S_{k+1} is restored. The cost of the updated range assignment is then computed and stored in C^*. We also allocate a variable α^* and initialize it with 0. Here C^* and α^* are used respectively to store the optimum cost of $\mathcal{R}^i_{k+1}$ and the optimum choice of α whose range is to be increased for communication with S_{m-1}.

Note that, if $\mu = m$ then this procedure is not applicable. In that case, we initialize C^* by $\sum_{j=1}^{k+1} \rho_j^2$, where ρ_j is the presently assigned range of s_j; α^* is initialized with 0.

3.2.2. *Procedure-2*

This procedure is executed irrespective of whether $\mu = m$ or $\mu > m$. Here we restore the strong connectivity by increasing the range of a member $s_\alpha \in SS^i_L$ so that it can communicate with a member in S_{m-1}. We consider each member $s_\alpha \in SS^i_L$ separately, and increase its range to $\rho_\alpha = d(s_\alpha, s_{m-1})$. This needs updating the ranges of the radio-stations in SS^i_L to achieve the minimum cost. We use $(\mathcal{R}^i_{k+1} \mid \rho_\alpha = d(s_\alpha, s_j))$ to denote the optimum range assignments for maintaining strong connectivity among the members in S_{k+1} with $\rho_i = d(s_i, s_{k+1})$ and $\rho_\alpha = d(s_\alpha, s_j)$.

Consider the computation of $cost(\mathcal{R}^i_{k+1} \mid s_\alpha = d(s_\alpha, s_{m-1}))$. Here, the following two instances are created where we need to compute the *critical-gap* for updating the ranges of the radio-stations in SS^i_L.

The range assignments $\rho_\nu = d(s_\nu, s_\mu)$ $(\nu < m)$ and $\rho_\alpha = d(s_\alpha, s_{m-1})$ are such that, both s_ν and s_α can communicate with a non-empty subset of radio-stations, namely $\{s_{m-1}, s_m, \ldots, s_\phi\}$, where $\phi = min(\mu, \alpha)$. We compute the *critical-gap* $\Delta_1 = Max_{j=m-1}^{\phi-1} d(s_j, s_{j+1}) = d(s_\tau, s_{\tau+1})$ (say).

Let $s_{\alpha'} = $ *right-cover*(s_α, ρ_α), where $\rho_\alpha = d(s_\alpha, s_{m-1})$. As originally SS^i was strong connected, there must exist a radio-station s_β $(\beta \geq \alpha')$ which can communicate with a node $s_{\beta'}$ (say) to the left of $s_{\alpha'}$ in 1-hop. In other words, $s_{\beta'} = $ *left-cover*(s_β, ρ_β). Thus, s_α and s_β can communicate with a non-empty subset of radio-stations, namely $\{s_\psi, s_{\psi+1}, \ldots, s_{\alpha'}\}$, where $\psi = Max(\alpha, \beta')$.

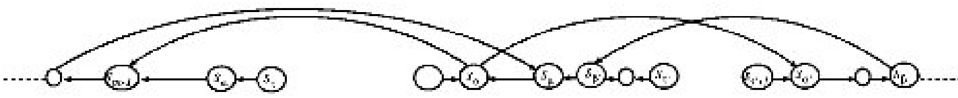

Fig. 6. Updating range assignment using *critical-gap*.

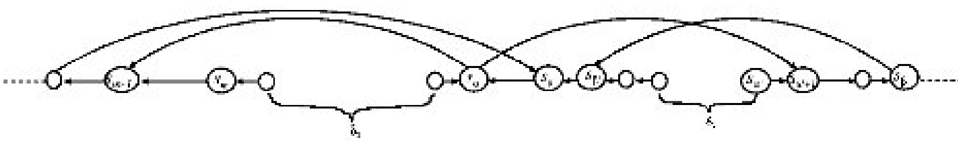

Fig. 7. Increasing the range of s_α, and the resulting two *critical-gaps*.

We compute the *critical-gap* $\Delta_2 = Max_{j=\psi}^{\alpha'-1} d(s_j, s_{j+1}) = d(s_{\tau'}, s_{\tau'+1})$ (say).

Next, we apply Lemma 6 and Lemma 7 adequately to revise the range assignments as follows (see Figure 6 for an illustration):

- $\rho_\alpha = d(s_\alpha, s_{m-1})$ (as assumed),
- $\rho_j = d(s_j, s_{j-1})$ for $j = \tau, \tau-1, \ldots, m$,
- $\rho_j = d(s_j, s_{j-1})$ for $j = \tau', \tau'-1, \ldots, \alpha+1$,
- The range of other radio-stations remain unchanged.

Given $\rho_i = d(s_i, s_{k+1})$ and $\rho_\alpha = d(s_\alpha, s_{m-1})$, $(\mathcal{R}^i_{k+1} \mid \rho_\alpha = d(s_\alpha, s_{m-1}))$ produces the minimum cost because (i) apart from ρ_i and ρ_α we need to assign the ranges of $(k-1)$ radio-stations, (ii) we have chosen minimum cost range assignments for the $m-1$ radio-stations $\{s_1, s_2, \ldots, s_{m-1}\}$ from $\mathcal{R}_m$, (iii) the range of each of the remaining $(k-m)$ radio-stations is equal to its distance from one among its two neighbors, and (iv) we have $(k-m+2)$ such pairwise distances among the radio-stations $\{s_m, s_{m+1}, \ldots, s_{k+1}\}$, and we have chosen $(k-m)$ such distances leaving the two *critical-gaps* Δ_1 and Δ_2.

Some times the range $\rho_\alpha = d(s_\alpha, s_{m-1})$ is such that a very small further increase of ρ_α enables s_α to communicate with $s_{\alpha'+1}$ directly, and thus a larger *critical-gap* $d(s_{\alpha'}, s_{\alpha'+1})$ can be reduced from the total cost of range assignment. The following two lemmata indicate that only one more range $\rho_\alpha = d(s_\alpha, s_{\alpha'+1})$ of s_α need to be considered, and the situation where such a choice of ρ_α may produce lower cost.

Lemma 8 *If* $d(s_\alpha, s_{m-1}) \leq d(s_\alpha, s_{\alpha'}) + C$, *where* $C = max\{d(s_j, s_{j-1}) \mid j = m-1, m, \ldots, \alpha'-1\}$, *then* $cost(\mathcal{R}^i_{k+1} \mid \rho_\alpha = d(s_\alpha, s_{\alpha'+1})) > cost(\mathcal{R}^i_{k+1} \mid \rho_\alpha = d(s_\alpha, s_{m-1}))$.

Proof. Let $d(s_\alpha, s_{m-1}) = d(s_\alpha, s_{\alpha'}) + C_1$, $d(s_\alpha, s_{\alpha'+1}) = d(s_\alpha, s_{m-1}) + C_2$, and $D = cost(\mathcal{R}^i_{k+1} \mid \rho_\alpha = d(s_\alpha, s_{\alpha'+1})) - cost(\mathcal{R}^i_{k+1} \mid \rho_\alpha = d(s_\alpha, s_{m-1}))$. To prove $D > 0$ if $C_1 < C$.

Consider Figure 7. Here $D = [(d(s_\alpha, s_{m-1}) + C_2)^2 - (\delta_1)^2 - (\delta_2)^2] - [(d(s_\alpha, s_{m-1}))^2 - (\Delta_1)^2 - (\Delta_2)^2]$, where δ_1 and δ_2 are the *critical-gaps* for assigning $\rho_\alpha = d(s_\alpha, s_{\alpha'+1})$ in the two sides of s_α.

Since m is the *maximal-left-cover*, we have $\delta_2 = C_1 + C_2 < 2d(s_\alpha, s_{m-1}) - C_1$. We also have $\delta_1 < C_2$, since we have increased the range of s_α by an amount C_2.

On simplification of the expression of D, we have
$D = 2C_2 \times d(s_\alpha, s_{m-1}) + (C_2)^2 + (\Delta_1)^2 + (\Delta_2)^2 - (\delta_1)^2 - (\delta_2)^2 \geq (\Delta_1)^2 + (\Delta_2)^2 - (C_1)^2$.
The claim of the lemma follows from the fact that $D > 0$ if $C_1 < max(\Delta_1, \Delta_2)$. □

Lemma 9 *If ρ_α is increased to communicate with S_{m-1}, then the possible values of ρ_α to be considered are $d(s_\alpha, s_{m-1})$ and $d(s_\alpha, s_{\alpha'+1})$.*

Proof. The first choice of the value of ρ_α is obvious. Let us now consider the second choice $\rho_\alpha = d(s_\alpha, s_{\alpha'+1})$. Let $s_\psi =$ *left-cover*$(s_\alpha, d(s_\alpha, s_{\alpha'+1}))$. Following the same convention as in Lemma 8, let $C' = max(\delta_1, \delta_2)$ and $d(s_\alpha, s_{\alpha'+1}) = d(s_\alpha, s_\psi) + C'_1$. From Lemma 8, we have $d(s_{\alpha'}, s_{\alpha'-1}) = C_1 + C_2$. Again since $\psi \leq m-1$, we have $d(s_\alpha, s_{\alpha'+1}) - d(s_\alpha, s_\psi) < C_2$. The lemma follows from the fact that $C'_1 < C_2 < d(s_{\alpha'}, s_{\alpha'+1}) < C'$, since $d(s_{\alpha'}, s_{\alpha'+1}) \leq \delta_2$. □

Lemma 9 says that, we need to compute $cost(\mathcal{R}^i_{k+1}|s_\alpha = d(s_\alpha, s_{m-1}))$ and $cost(\mathcal{R}^i_{k+1}|s_\alpha = d(s_\alpha, s_{\alpha'+1}))$ for each s_α, $\alpha = m, m+1, \ldots, i$. At each step, if the minimum of these two costs is less than C^*, then C^* is updated accordingly, and α is also stored in α^*. Finally, $cost(\mathcal{R}^i_{k+1}) = C^*$. If $\alpha^* = 0$, we need to run Procedure-1 once again to get the optimum range assignment $\mathcal{R}^i_{k+1}$. Otherwise, we run Procedure-2 with $\alpha = \alpha^*$ to get $\mathcal{R}^i_{k+1}$.

3.3. *Computation of* $\mathcal{R}_{k+1}$

The following two lemmata say that the computation of $\mathcal{R}_{k+1}$ can be made fast if $\mathcal{R}^i_{k+1}$ are executed for $i = k, k-1, \ldots, 1$ in order.

Lemma 10 *Let m and m' be the maximal-left-cover for $\mathcal{R}^i_{k+1}$ and $\mathcal{R}^j_{k+1}$ respectively. Now, if $i < j$ then $m \leq m'$. Furthermore, if s_ℓ=left-cover(s_i, ρ_i) and $\ell \geq m'$, then $m = m'$.*

Proof. The first part of the lemma trivially follows from the fact that if s_i is to the left of s_j and $d(s_i, s_{k+1}) > d(s_j, s_{k+1})$, then $SS^j \subseteq SS^i$. The second part follows from the fact that (a) in $\mathcal{R}^j_{k+1}$ the ranges assigned to each node $s_\beta \in SS^j_L (= \{s_m, s_{m+1}, \ldots, s_{j-1}\})$ is $d(s_\beta, s_{\beta-1})$, and (b) while computing $\mathcal{R}^i_{k+1}$, the range assigned to each node $s_\alpha \in \{s_\ell, s_{\ell+1}, \ldots, s_{i-1}\}$ is equal to $d(s_\alpha, s_{\alpha+1})$. Since $\ell \geq m'$, the repeated computation of *left-cover* will terminate after observing the *maximal-left-cover* $m = m'$. □

Lemma 11 *Let m' be the* maximal-left-cover *for $\mathcal{R}^j_{k+1}$. While computing $\mathcal{R}^i_{k+1}$ for some $i < j$, if $\ell \geq m'$, then $cost(\mathcal{R}^i_{k+1}) > cost(\mathcal{R}^j_{k+1})$.*

Proof. Let m be the *maximal-left-cover* for $\mathcal{R}^i_{k+1}$. As $i < j$, $\ell \geq m'$, we have $m = m'$. While increasing the range of s_α to ρ ($= d(s_\alpha, s_{m-1})$ or $d(s_\alpha, s_{\alpha'+1})$ as discussed in Lemma 9) to communicate with S_{m-1}, the *critical-gap* Δ_1 generated for both $\mathcal{R}^j_{k+1}$ and $\mathcal{R}^i_{k+1}$ become same. Let s_β = *right-cover*(s_α, ρ). If $\beta \leq i$, then Δ_2 value for computing both $\mathcal{R}^j_{k+1}$ and $\mathcal{R}^i_{k+1}$ become same. If $\beta > i$, then Δ_2 value for $\mathcal{R}^j_{k+1}$ is greater than Δ_2 value for $\mathcal{R}^i_{k+1}$ because in the former case Δ_2 is $Max\{d(s_\alpha, s_{\alpha+1}), d(s_{\alpha+1}, s_{\alpha+2}), \ldots, d(s_{\hat\beta-1}, s_{\hat\beta})\}$ (where $\hat\beta = Min(j, \beta)$) and in the latter case Δ_2 is $Max\{d(s_\alpha, s_{\alpha+1}), d(s_{\alpha+1}, s_{\alpha+2}), \ldots, d(s_{i-1}, s_i)\}$.

The lemma follows from the fact that $d(s_j, s_{k+1}) < d(s_i, s_{k+1})$ and Δ_2 value for $\mathcal{R}^j_{k+1}$ is greater than or equal to Δ_2 for $\mathcal{R}^i_{k+1}$. □

Lemmata 10 and 11 lead to the following conclusion towards accelerating the execution of the algorithm.

> While computing $\mathcal{R}^i_{k+1}$ if (i) $cost(\mathcal{R}^{j^*}_{k+1}) = Min^k_{j=i+1} cost(\mathcal{R}^j_{k+1})$ and the *maximal-left-cover* of s_{j^*} is s_{m^*} in the range assignment $\mathcal{R}^{j^*}_{k+1}$, (ii) the *left-cover* of s_i is s_ℓ for its range assignment $\rho'_i = d(s_i, s_{k+1})$, and (iii) $\ell > m^*$, then $cost(\mathcal{R}^i_{k+1}) > cost(\mathcal{R}^{j^*}_{k+1})$. So, we need not have to compute $cost(\mathcal{R}^i_{k+1})$ in such a case.

3.4. *Algorithm*

We are now in a position to present the stepwise description of our algorithm. In the preprocessing step, we create two matrices $T1$ and $T2$ of size $n \times n$ each as described in Subsection 3.1. Note that, if the (i, j)-th entry of the matrix $T2$ contains α and $i < j$ (resp. $i > j$) then $s_\alpha =$ *left-cover*$(s_i, d(s_i, s_j))$ (resp. $s_\alpha =$ *right-cover*$(s_i, d(s_i, s_j))$). The input for computing $\mathcal{R}_{k+1}$ is $\{\mathcal{R}_j, j = 2, 3, \ldots, k\}$; these are computed in the previous $(k-1)$ iterations. In addition, we need four scalar locations, namely *opt*, C^*, i^* and α^*, and two arrays R and LC, each of size n. The array R is used for generating $\mathcal{R}^i_{k+1}$, and the array LC contains the *left-cover* of some selected radio-stations after assigning their ranges. More specifically, each element of the array LC is a tuple (a, b), where $m < a \leq i$ and $s_b =$ *left-cover*(s_a, ρ_a). The first element of LC is with $a = i$, and the indices (a values) of only those radio-stations are to be stored in LC such that the corresponding b values are in strictly decreasing order.

Step 1 Check whether there exists any radio-station $s_i \in S_k$ whose range ρ_i ($\in \mathcal{R}_k$) is greater than or equal to $d(s_i, s_{k+1})$. If the check succeeds, then the algorithm terminates by copying the elements in $\mathcal{R}_k$ in first k elements of $\mathcal{R}_{k+1}$, and assigning $d(s_k, s_{k+1})$ to the $(k+1)$-th element of $\mathcal{R}_{k+1}$.

Step 2 If Step 1 fails, then (* run the algorithm for computing $\mathcal{R}_{k+1}$ *)

- Initialize $opt \longleftarrow \infty$, $m \longleftarrow k+1$ and
- For each $i = k, k-1, \ldots, 1$, execute the following sub-steps to compute $\mathcal{R}^i_{k+1}$. This identifies an i^* such that the cost of $\mathcal{R}^{i^*}_{k+1}$ is minimum. As mentioned above, at each iteration (corresponding to each value of i) the array R will be used to generate $\mathcal{R}^i_{k+1}$. For the sake of notational simplicity, we will use ρ_j to denote $R[j]$.

Step 2.1 Compute $\ell =$ *left-cover*$(s_i, d(s_i, s_{k+1})) = T2[i, k+1]$.

Step 2.2 Let m^* is the *maximal-left-cover* at the $(k - i^* - 1)$-th iteration, which has produced the optimum solution till the $(k-i)$-th iteration. Now, if $\ell < m^*$ then execute the following steps (* if $\ell \geq m^*$, we need not have to process s_i (by Lemma 11) *).

Step 2.3 Initialize the elements of R as follows. During this process, we also compute the *maximal-left-cover* m and the array LC.

Step 2.3.1 Assign $\rho_i = d(s_i, s_{k+1})$; $LC\_ptr = 1$; $LC[1] = (i, \ell)$; $m = \ell$ and $\alpha = i$.

Step 2.3.2 Assign $\rho_j = d(s_j, s_{j-1})$ for $j = k+1, k, k-1, \ldots, i+1$.

Step 2.3.3 *for* $j = \alpha - 1, \alpha - 2, \ldots, \ell$ *do*
$\rho_j = d(s_j, s_{j+1})$ and $m =$ *left-cover*(s_j, ρ_j).
if $m < LC[LC\_ptr].b$, *then* $LC\_ptr = LC\_ptr + 1$; $LC[LC\_ptr] = (j, m)$
endfor

Step 2.3.4 *if* $m < \ell$ *then* execute Step 2.3.3 with $\alpha = \ell$ and $\ell = m$.

Step 2.3.5 Assign $\rho_j = j$-th element of $\mathcal{R}_m$ for $j = 1, 2, \ldots, m-1$.

Step 2.4 *if* $m = 1$, *then*
Compute $C = cost(R)$.
if $C < opt$, *then* assign $opt = C$, $i^* = i$ and exit from Step 2.

Step 2.5 Set the *critical-gap* $\Delta_1 = 0$.
Compute $\mu = \text{Max}\{$*right-cover*(s_j, ρ_j), j=1, 2, ..., m-1$\}$.
(* Since S_m is strongly connected with range assignments $\mathcal{R}_m$, we have $\mu \geq m$ *)

Step 2.6 (* Procedure-1 - If $\mu > m$ then execute this step. *)

- (* Compute s_β, the left-most radio-station which is 1-hop reachable from the radio-stations to the right of s_μ including itself *)
  $TEMP = LC\_ptr$ (* LC_ptr will again be used in Procedure-2 (Step 2.7) *)
  while $LC[LC\_ptr].a < \mu$ do $LC\_ptr = LC\_ptr - 1$
  $\beta = LC[LC\_ptr].b$
  $LC\_ptr = TEMP$ (* Get back LC_ptr *)
- Assign $\delta = T1[\beta, \mu]$ and compute $\Delta_1 = d(s_\delta, s_{\delta+1}) =$ *critical-gap* in $\{s_\beta, s_{\beta+1}, \ldots, s_\mu\}$.
- Revise the range assignment using the *critical-gap* Δ_1 as described in Lemma 6.
- Compute $C = cost(R) - (\Delta_1)^2 + (d(s_m, s_{m-1}))^2$
- If $C < C^*$ then set $C^* = C$, $\alpha^* = 0$ and $i^* = i$.

Step 2.7 (* Procedure-2 *) Increase the range of each member in $\{s_m, s_{m+1}, \ldots, s_i\}$ one by one for communication with S_{m-1}. Let s_α be under consideration.

Step 2.7a Increase the range of s_α to $\rho'_\alpha = d(s_{m-1}, s_\alpha)$.
(* Compute Δ_1 *)

- Assign $\beta = Min(\mu, \alpha)$
- Assign $\theta = T1[m-1, \beta]$ and compute $\Delta_1 = d(s_\theta, s_{\theta+1})$

(* Compute Δ_2 *)
Let $\alpha' = T2[\alpha, m-1]$, (* $s_{\alpha'} =$ *right-cover*$(s_\alpha, d(s_\alpha, s_{m-1}))$ *)
(* Compute $s_{\beta'}$, the left-most radio-station which is 1-hop reachable from a radio-station to the right of $s_{\alpha'}$ including itself. *)

- While $LC[LC\_ptr].a < \alpha'$ do $LC\_ptr = LC\_ptr - 1$
- $\beta' = LC[LC\_ptr].b$.
- If $\beta' \geq \alpha$ then set $\theta = T1[\beta', \alpha']$.
  Otherwise set $\theta = T1[\alpha, \alpha']$
- Compute $\Delta_2 = d(s_\theta, s_{\theta+1})$.

- Compute $C = cost(R) - (\rho_\alpha)^2 + (d(s_\alpha, s_{m-1}))^2 - (\Delta_\_)^2 - (\Delta_2)^2$.
- If $C < C^*$, then set $C^* = C$, $\alpha^* = \alpha$ and $i^* = i$.

Step 2.7b Increase the range of s_α to $\rho'_\alpha - d(s_\alpha, s_{\alpha'+1})$.
(* Compute Δ'_1: Let s_δ be the leftmost radio-station which is 1-hop reachable from s_α *)
Compute $\delta' = Max(\nu, \delta)$
Assign $\theta' = T1(\delta', \beta)$ (* β is computed earlier *)
Compute $\Delta'_1 = d(s_{\theta'}, s_{\theta'+1})$

(* Compute Δ'_2: Let $s_{\beta'}$ be the left-most radio-station which is 1-hop reachable from a radio-station to the right of $s_{\alpha'+1}$ including itself. *)

- Compute $\Delta'_2 = Max(\Delta_2, d(s_{\alpha'}, s_{\alpha'+1}))$.
- Compute $C = cost(R) - (\rho_\alpha)^2 + (d(s_\alpha, s_{\alpha'+1}))^2 - (\Delta'_\_)^2 - (\Delta'_2)^2$.
- If $C < C^*$, then set $C^* = C$, $\alpha^* = \alpha$ and $i^* = i$.

Step 3: If $C^* < opt$ then assign $opt = C^*$, and
repeat Step 2.1 to 2.7 with $\alpha = \alpha^*$, and copy the values of R in $\mathcal{R}_{k+1}$.

3.5. *Correctness of the algorithm*

The following lemma is relevant in the context of the proof of correctness of the algorithm.

Lemma 12 *While computing the* maximal-left-cover *for the range assignment* $\mathcal{R}^i_{k+1}$*, it is enough to consider* $\rho(s_i) = d(s_i, s_{k-1})$ *as the range of* s_i.

Proof. Consider a typical situation where m is the *maximal-left-cover* with $\rho(s_i) = d(s_i, s_{k-1})$. Here s_i covers s_ℓ towards its left, but not $s_{\ell-1}$ for a very small (ϵ) shortage of range, i.e., $d(s_i, s_{\ell-1}) - \epsilon < \rho(s_i) < d(s_i, s_{\ell-1})$. We will show that if such a case arises, then also we need not have to consider $d(s_i, s_{\ell-1})$ as a choice for computation of m (the *maximal-left-cover*). Here two cases need to be considered, namely $m < \ell$ and $m = \ell$.

In the first case, the *maximal-left-cover* computed using $\rho(s_i) = d(s_i, s_{k+1})$ will be the same as the *maximal-left-cover* with $\rho'(s_i) = d(s_i, s_{\ell-1})$. Thus, the range assignment of radio-stations $S_{k+1} \setminus \{s_i\}$ using our algorithm will remain same for both the range assignments $\rho(s_i)$ and $\rho'(s_i)$. Thus, we will loose in terms of cost if we use $\rho'(s_i)$ instead of $\rho(s_i)$.

In the second case, for the assignment of $\rho'(s_i) = d(s_i, s_{\ell-1})$, we will surely get a *maximal-left-cover* m' where $m' \leq m$. Here the cost of the range assignments for the radio-stations $S_{m-1} = \{s_1, s_2, \ldots, s_m\}$ using $\rho'(s_i)$ is greater than that using

$\rho(s_i)$. The reason is that, in the former case we only use $\mathcal{R}_m^{m-1}$ (which in turn uses $\mathcal{R}_{m'}$), whereas in the latter case, we consider the optimal range assignment $\mathcal{R}_m$. In the part SS^i, surely, the range of one member in SS_L^i needs to be increased to reach $s_{m-1} - s_{\ell-1}$. Here the effect of increasing the range of s_i to $\rho'(s_i)$ will also be considered. Thus the lemma follows. □

Theorem 1 *Our proposed algorithm correctly computes* $\mathcal{R}_{k+1}^i$.

Proof. After assigning $\rho_i = d(s_i, s_{k+1})$ and computing the *maximal-left-cover* m, the cost of range assignments of SS^i is $\sum_{\alpha=m}^{i-1}(d(s_\alpha, s_{\alpha+1}))^2 + (d(s_i, s_{k+1}))^2 + \sum_{\alpha=i+1}^{k+1}(d(s_\alpha, s_{\alpha-1}))^2$, which is equal to $(d(s_i, s_{k+1}))^2$ + the sum of square of the length of the gap between each pair of consecutive radio-stations.

While assigning ranges of the members in S_{m-1}, we need to ensure (i) the communication between every pair of members in S_{m-1} and (ii) a communication from S_{m-1} to at least one member in SS^i. We have used $\mathcal{R}_m \setminus \{\rho_m\}$ as the range assignments of the members in S_{m-1}. This produces minimum cost satisfying (i) and (ii) stated above, due to the following facts:

- In $\mathcal{R}_m$, we have $\rho_m = d(s_m, s_{m-1})$ (see Lemma 3), and
- If there exists some other range assignments $\mathcal{R}'_{m-1}$ for the members in S_{m-1} with cost less than that of $\mathcal{R}_m \setminus \{\rho_m\}$, then $\mathcal{R}'_{m-1} \cup \{\rho_m\}$ would have cost less than $\mathcal{R}_m$.

Next, we have established the communication from SS^i to S_{m-1} by increasing the range of only one radio-station. Each element $s_\alpha \in SS_L^i$ is considered for this purpose. For each s_α, only two feasible choices of range (see Lemma 9) is considered, and the cost is computed by increasing its range and doing necessary modifications of the range of other radio-stations considering two *critical-gaps* Δ_1 and Δ_2. Thus, the correctness of the algorithm follows. □

3.6. *Complexity analysis*

The worst case time complexity of computing $\mathcal{R}_{k+1}$ assumes the fact that no element $s_i \in S_k$ exists with $\rho_i \geq d(s_i, s_{k+1})$. If T_i denotes the time complexity of computing $\mathcal{R}_{k+1}^i$, then the total time complexity of computing $\mathcal{R}_{k+1}$ is $k \times Max_{i=1}^k T_i$. We now calculate the worst case value of T_i.

In Step 2, the computation of *maximal-left-cover* (m) needs $O(k+1-m)$ time. But, Lemma 10 says that, the total time needed for computing the *maximal-left-cover* for all the range assignments $\{\mathcal{R}_{k+1}^i, i = k, k-1, \ldots, 1\}$ is $O(k)$.

While computing $\mathcal{R}_{k+1}^i$ for some i, the worst case situation with respect to the time complexity arises when $m \neq 1$. Here, Steps 2.5 and 2.6 execute in $O(\mu)$ time using the preprocessed matrices $T1$ and $T2$. This may be $O(k)$ in the worst case.

Step 2.7 needs to be repeated for each $s_\alpha \in SS_L^i$ with two feasible ranges. In each case, the computation of *critical-gap* for s_α needs amortized $O(1)$ time using the array LC. Finally, Step 3 needs another $O(k)$ time. Thus, we have the following theorem stating the worst case time complexity of the algorithm.

Theorem 2 *The time complexity of our proposed algorithm for the optimal range assignment of the 1D unbounded range assignment problem is $O(n^3)$ in the worst case. The space complexity is $O(n^2)$.*

Proof. The preprocessing time complexity is $O(n^2)$. The above discussions say that $T_i = O(k)$ in the worst case. Thus, the time required for computing R_{k+1} is $O(k^2)$. The time complexity result follows from the fact that our incremental algorithm inserts n radio-stations on the line one by one in order.

The space complexity result follows from the requirement of space for the matrices $T1$ and $T2$, and the space required for storing $\mathcal{R}_i$ for all $i = 1, 2, \ldots, n-1$ while computing $\mathcal{R}_n$. □

4. Conclusion

The time complexity of the proposed algorithm is $O(n^3)$ which is an improvement by a factor n for the unbounded version of 1D range assignment problem over its existing result [6]. We mentioned Lemma 11 for the further acceleration of the algorithm, but could not use it to improve the time complexity result. We hope, a careful analysis may improve both time and space complexity of the problem.

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