

THE ONE-TENTH PER CENT LEVEL OF THE RATIO OF VARIANCES

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Fisher defines
$$z = \log_e (s_1/s_2),$$

where s_1 and s_2 are the estimated standard deviations of two samples drawn from the same normal population. For a test of significance, therefore, one has to calculate the Napierian logarithm of the ratio of the two standard deviations (or what is the same thing, the difference between the Napierian logarithms of the two standard deviations). Finding the Napierian logarithms however takes time, and to avoid this, Mahalanobis published in 1932, 5 per cent and 1 per cent tables of s_1^2/s_2^2 (and also of s_1/s_2) and similar tables have also been given in G. W. Snedecor's book on the Analysis of Variance and Covariance*. Now that C. G. Colcord and L. S. Deming are publishing a one-tenth per cent table of z , it will be convenient to have a table of one-tenth per cent values of (s_1^2/s_2^2) and (s_1/s_2) .

The present Table has been derived directly from Colcord and Deming's* "One-tenth per cent Level of z " in the following way:—

$$z = \frac{1}{2} \log_e (s_1^2/s_2^2) = \frac{1}{2} \log_e (x) = 1.1513 \log_{10} (x)$$

Thus
$$x = \text{Antilog}_{10} (z/1.1513) = \text{Antilog}_{10} 0.8686 (z)$$

The following example will illustrate the use of the Table. In a randomised block experiment the following analysis of variance was obtained.

		Mean Square	Ratio of Variance
Block	... 5	23.146	—
Treatment	... 4	19.708	—
Error	... 20	2.525	7.23

* Since the table has been constructed from the corresponding table by Colcord and Deming, given correct to 4 places of decimals, the present table is correct within one unit in the fifth figure; but this will not in any way affect the usual test of significance of two observed variances.

$$\begin{aligned}
 z &= \frac{1}{2} \log_e (19.708/2.725) \\
 &= 1.1513 (\log_{10} 19.708 - \log_{10} 2.725) \\
 &= 1.1513 \times 0.8593 \\
 &= 0.9893
 \end{aligned}$$

To test the significance, we have the following table:—

$$n_1 = 4 \quad \text{and} \quad n_2 = 20$$

	Values of z	Values of x
5.0 per cent	0.5265 (Fisher)	2.866 (Mahalanobis)
1.0 per cent	.7443 (..)	4.431 (..)
0.1 per cent	.9798 (Colcord & Deming)	7.102 (Present Tables)

Since the observed value of z ($=0.9893$) exceeds not only the 1 per cent but also the 0.1 per cent value of z given in Colcord and Deming's Table, the treatment is significant on the 0.1 per cent level.

The same result is reached more easily by calculating the ratio of variances and using the present Table. We have, in the present example,

$$x = \text{the ratio of variances} = \frac{19.708}{2.725} = 7.23$$

But 7.23 exceeds the 0.1 per cent value of the ratio of variances (7.102) given in the table, so that we reach the same conclusion almost immediately.

REFERENCES

1. Mahalanobis P. C. (1932):—Auxiliary Tables for Fisher's x -test in Analysis of Variance (*Ind. Jour. Ag. Sc.* Vol. II (6)).
2. Snedecor, G. W. (1934):—Calculation and Interpretation of Analysis of Variance and Covariance (*Collegiate Press, Ames, Iowa.*)

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ONE-TENTH PER CENT. LEVEL OF THE RATIO OF VARIANCES

ONE-TENTH PERCENT VALUES OF THE RATIO OF VARIANCES (σ_1^2/σ_2^2)

Values of n_1	Values of n_2									
	1	2	3	4	5	6	7	8	9	10
1	406401	409976	413637	417387	421228	425160	429185	433305	437521	441834
2	608	609	610	611	612	613	614	615	616	617
3	16746	1485	14111	13708	13438	13181	12936	12702	12478	12264
4	74176	61238	56181	53430	51706	50058	48488	46988	45558	44198
5	81059	68612	63201	59057	55748	52855	50268	47988	45914	44048
6	85509	70969	63702	57907	52909	48429	44369	40629	37209	34099
7	29218	18403	12628	8728	5688	3408	1908	1068	568	288
8	23416	14303	9228	5628	3408	1908	1068	568	288	148
9	21625	13143	8228	5228	3028	1708	968	528	268	138
10	21028	12906	8228	5228	3028	1708	968	528	268	138
11	19687	12613	7660	4860	2920	1660	920	500	260	130
12	18641	12372	7403	4603	2763	1503	863	483	253	120
13	17843	12130	7146	4346	2606	1346	826	466	246	118
14	17286	11888	6889	4089	2449	1189	800	450	240	116
15	16866	11646	6632	3832	2292	1032	764	434	234	114
16	16480	11404	6375	3575	2135	875	728	418	228	112
17	16120	11162	6118	3318	1978	818	702	402	222	110
18	15780	10920	5861	3061	1821	761	686	386	216	108
19	15460	10678	5604	2804	1664	724	670	370	210	106
20	15160	10436	5347	2547	1507	687	654	354	204	104
21	14880	10194	5090	2290	1350	650	638	338	198	102
22	14620	9952	4833	2033	1193	613	622	322	192	100
23	14380	9710	4576	1776	1036	578	606	306	186	98
24	14160	9468	4319	1519	879	543	590	290	180	96
25	13960	9226	4062	1262	722	508	574	274	174	94
26	13780	8984	3805	1005	565	473	558	258	168	92
27	13610	8742	3548	748	422	438	542	242	162	90
28	13460	8500	3291	491	271	303	526	226	156	88
29	13320	8258	3034	234	114	168	510	210	150	86
30	13200	8016	2777	117	17	23	494	194	144	84
31	13080	7774	2520	110	10	16	478	178	138	82
32	12980	7532	2263	84	4	10	462	162	132	80
33	12880	7290	2006	58	0	4	446	146	126	78
34	12790	7048	1749	32	0	0	430	130	120	76
35	12710	6806	1492	6	0	0	414	114	114	74
36	12640	6564	1235	0	0	0	398	98	108	72
37	12580	6322	978	0	0	0	382	82	102	70
38	12530	6080	721	0	0	0	366	66	96	68
39	12480	5838	464	0	0	0	350	50	90	66
40	12440	5596	207	0	0	0	334	34	84	64
41	12400	5354	0	0	0	0	318	18	78	62
42	12360	5112	0	0	0	0	302	2	72	60
43	12320	4870	0	0	0	0	286	0	66	58
44	12280	4628	0	0	0	0	270	0	60	56
45	12240	4386	0	0	0	0	254	0	54	54
46	12200	4144	0	0	0	0	238	0	48	52
47	12160	3902	0	0	0	0	222	0	42	50
48	12120	3660	0	0	0	0	206	0	36	48
49	12080	3418	0	0	0	0	190	0	30	46
50	12040	3176	0	0	0	0	174	0	24	44
51	12000	2934	0	0	0	0	158	0	18	42
52	11960	2692	0	0	0	0	142	0	12	40
53	11920	2450	0	0	0	0	126	0	6	38
54	11880	2208	0	0	0	0	110	0	0	36
55	11840	1966	0	0	0	0	94	0	0	34
56	11800	1724	0	0	0	0	78	0	0	32
57	11760	1482	0	0	0	0	62	0	0	30
58	11720	1240	0	0	0	0	46	0	0	28
59	11680	998	0	0	0	0	30	0	0	26
60	11640	756	0	0	0	0	14	0	0	24
61	11600	514	0	0	0	0	0	0	0	22
62	11560	272	0	0	0	0	0	0	0	20
63	11520	30	0	0	0	0	0	0	0	18
64	11480	0	0	0	0	0	0	0	0	16
65	11440	0	0	0	0	0	0	0	0	14
66	11400	0	0	0	0	0	0	0	0	12
67	11360	0	0	0	0	0	0	0	0	10
68	11320	0	0	0	0	0	0	0	0	8
69	11280	0	0	0	0	0	0	0	0	6
70	11240	0	0	0	0	0	0	0	0	4
71	11200	0	0	0	0	0	0	0	0	2
72	11160	0	0	0	0	0	0	0	0	0
73	11120	0	0	0	0	0	0	0	0	0
74	11080	0	0	0	0	0	0	0	0	0
75	11040	0	0	0	0	0	0	0	0	0
76	11000	0	0	0	0	0	0	0	0	0
77	10960	0	0	0	0	0	0	0	0	0
78	10920	0	0	0	0	0	0	0	0	0
79	10880	0	0	0	0	0	0	0	0	0
80	10840	0	0	0	0	0	0	0	0	0
81	10800	0	0	0	0	0	0	0	0	0
82	10760	0	0	0	0	0	0	0	0	0
83	10720	0	0	0	0	0	0	0	0	0
84	10680	0	0	0	0	0	0	0	0	0
85	10640	0	0	0	0	0	0	0	0	0
86	10600	0	0	0	0	0	0	0	0	0
87	10560	0	0	0	0	0	0	0	0	0
88	10520	0	0	0	0	0	0	0	0	0
89	10480	0	0	0	0	0	0	0	0	0
90	10440	0	0	0	0	0	0	0	0	0
91	10400	0	0	0	0	0	0	0	0	0
92	10360	0	0	0	0	0	0	0	0	0
93	10320	0	0	0	0	0	0	0	0	0
94	10280	0	0	0	0	0	0	0	0	0
95	10240	0	0	0	0	0	0	0	0	0
96	10200	0	0	0	0	0	0	0	0	0
97	10160	0	0	0	0	0	0	0	0	0
98	10120	0	0	0	0	0	0	0	0	0
99	10080	0	0	0	0	0	0	0	0	0
100	10040	0	0	0	0	0	0	0	0	0

ONE-TENTH PERCENT VALUES OF THE RATIO OF VARIANCES (s_1^2/s_2^2)—Contd.

		Values of n_1											
		12	15	20	24	30	40	60	120	240			
1	610487										635487		
2	12930										12930		
3	12930	12735	12641	12548	12454	12366	12274	12180	12081	11984	12340		
4	47407	46758	46098	45438	44778	44118	43458	42798	42138	41478	41828		
5	20416	20000	19586	19172	18758	18344	17930	17516	17102	16688	17044		
6	17080	17480	17110	16780	16450	16120	15790	15460	15130	14800	15166		
7	18708	19324	18934	18544	18154	17764	17374	16984	16594	16204	16570		
8	11194	10812	10430	10048	9666	9284	8902	8520	8138	7756	8122		
9	9570	9290	8998	8706	8414	8122	7830	7538	7246	6954	7320		
10	8445	8129	7802	7475	7148	6821	6494	6167	5840	5513	5879		
11	7625	7301	6968	6635	6302	5969	5636	5303	4970	4637	5003		
12	7005	6700	6395	6090	5785	5480	5175	4870	4565	4260	4626		
13	6519	6231	5934	5637	5340	5043	4746	4449	4152	3855	4221		
14	6130	5848	5557	5265	4974	4682	4391	4099	3808	3517	3883		
15	5812	5536	5248	4961	4673	4385	4097	3809	3521	3233	3600		
16	5548	5274	4992	4706	4420	4134	3848	3562	3276	2990	3357		
17	5284	5014	4737	4450	4163	3876	3590	3303	3016	2730	3100		
18	5152	4887	4607	4319	4031	3743	3455	3167	2880	2592	2960		
19	4967	4704	4427	4140	3852	3564	3276	2988	2700	2412	2780		
20	4823	4562	4280	4000	3720	3440	3160	2880	2600	2320	2690		
21	4697	4437	4157	3876	3596	3316	3036	2756	2476	2196	2566		
22	4563	4303	4023	3742	3462	3182	2902	2622	2342	2062	2432		
23	4482	4222	3942	3661	3381	3101	2821	2541	2261	1981	2351		
24	4393	4133	3853	3572	3292	3012	2732	2452	2172	1892	2262		
25	4311	4051	3771	3490	3210	2930	2650	2370	2090	1810	2180		
26	4238	3978	3698	3417	3137	2857	2577	2297	2017	1737	2107		
27	4170	3910	3630	3349	3069	2789	2509	2229	1949	1669	2039		
28	4100	3840	3560	3279	2999	2719	2439	2159	1879	1599	1969		
29	4033	3773	3493	3212	2932	2652	2372	2092	1812	1532	1902		
30	4000	3740	3460	3179	2899	2619	2339	2059	1779	1499	1869		
40	3652	3400	3150	2870	2590	2310	2030	1750	1470	1190	1560		
60	3815	3570	3320	3040	2760	2480	2200	1920	1640	1360	1730		
100	4016	3760	3510	3230	2950	2670	2390	2110	1830	1550	1920		
∞	2712	2518	2306	2132	1960	1788	1616	1444	1272	1100	1468		