Chaos in eco-epidemiological problem of the Salton Sea and its possible control

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Abstract

The Salton Sea, which is located in the southeast desert of California, came into the limelight due to deaths of fish and fish catching birds on a massive scale. Recently, Chattopadhyay and Bairagi [J. Chattopadhyay, N. Bairagi, Pelicans at risk in Salton Sea – an eco-epidemiological model, Ecol. Model. 136 (2001) 103–112] proposed and analysed an eco-epidemiological model on Salton Sea. In the present paper, we modified their model by taking into account the bilinear mass action incidence rate and performed extensive numerical simulations. Our studies show that the system exhibits chaotic dynamics when some key parameters attain their critical values. We have tried to explain the unusual deaths of fish and fish eating birds in the Salton Sea using the simulation results. We have also suggested some possible measures to avoid chaos in such natural systems.

Keywords: Susceptible fish; Infected fish; Pelican; Limit cycle; Chaos

1. Introduction

Most studies of chaos in model ecological systems have focused their attention on single population dynamics rather than the dynamical behavior of interacting populations of the system. May [2] found that one of the simplest equations used to model single population growth, popularly known as linear logistic equation or logistic (quadratic) map, exhibited chaotic dynamics when the maximal per capita rate of increase exceeded some threshold value. Soon after this pioneering discovery, attempts were made to observe chaos in two dimensional prey–predator model [3]. The existence of chaos in Lotka–Volterra one predator, two-prey model was first observed by Gilpin [4]. Later on, Schaffer and Kot [5] and Schnabl et al. [6] did extensive numerical studies of Gilpin's model. Many attempts have also been made to determine whether chaos occurs in natural

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populations [7–10]. Klebanoff and Hasting [11] showed that chaos is the expected outcome of the dynamics of a class of one-predator, two-prey models. Upadhyay et al. [10] showed that dynamical complexity is possible without structural complexity. McCann and Yodzis [12] observed that chaotic dynamics which exist in biologically plausible regions in the parameter space are not very common and productive environment is a pre-requisite for a system to support a chaotic dynamical behavior. Suarez [13] proposed certain techniques to achieve control over a chaotic dynamical system in an ecological context.

Epidemiological processes like disease may cause vital changes in the dynamics of an ecosystem [14–19]. Birth rate [20], contact rate [21], and age structure [22] may be considered as possible parameters which influence the onset of chaotic behavior in an epidemiological model. Chattopadhyay and Bairagi [1] modeled and studied an eco-epidemiological problem of Salton Sea. The sea is sustained mostly by inflow of agricultural waste water from the Coachella and Imperial Valleys. The lack of out-flowing streams has resulted in a gradual build up of salts and nutrients in the Salton Sea, causing massive algal blooms in the sea. These algae die as quickly as they grow. When it dies, dissolved oxygen from the sea water is wasted. This usually happens during the late summer when there is little dissolved oxygen in the water and becomes a suitable medium for botulism bacteria to grow and produce toxin (for details see, Gonzalez et al. [23] and Horvitz [24]). Millions of Tilapia fish die-off every year due to this vibrio infection. When the fish struggle in their death, they tend to rise to the sea surface and become vulnerable as well as attractive to fish eating birds, like Pelican. This interaction between birds like Pelican and the sick Tilapia fish is a cause of disease in the birds due to the toxins. Also vibrio infection is passed from one fish to another susceptible fish. This is a possible reason for high mortality rates of fish in the Salton Sea and the birds that feed on them. On August 12, 1999, almost 8 millions of fish died. Also, over 14000 water birds, mostly white Pelican, died of avian botulism after eating the infected Tilapia fish during the summer of 1996 (Ocean update, November 1996). In August 1997, a high mortality rate in birds was found to be caused by avian botulism. The infected Tilapia fish was suspected to be the source of botulism toxin for the birds. Similar events also happened in 1992 and 1994. Recently, Chattopadhyay and Bairagi [1] proposed and analysed a three dimensional eco-epidemiological model of Salton Sea consisting of infected Tilapia fish and their predator, the Pelican population. They observed that if the search rate level of the predator is low, the system around the positive interior equilibrium is stable. But, instability sets in with the increase of search rate level of the predator. They reached at the conclusion that for preventing the epidemic in the Sea, considerable harvesting of Tilapia fish population is needed. Sarkar et al. [25], Bairagi and Chattopadhyay [26] and Chattopadhyay et al. [27] also modified the model of Chattopadhyay and Bairagi [1] from different biological point of view. The main outcome of the above studies was almost the same – harvesting of Tilapia fish population is needed for ecological sustainability of the Sea.

However, further investigations are required to answer the following questions. What are the reasons behind the massive deaths of fish and fish eating birds? How does increased value of the nutrient influence the overall system? It is known that Tilapia fish has stunning reproduction rate [28]. Is this higher reproduction rate responsible for the instability of the system? How do enhanced values of different key parameters jointly affect the system? Did the system witness a chaotic dynamics that caused massive fish and bird mortality events? To give some answers for the above questions, it is essential to study the dynamical behavior of the system over wider ranges of parameter values. Further, to predict the real life situation from a mathematical model, it is also necessary to test the model under different initial conditions.

The most widely used method to control chaos is the OGY method that was proposed by Ott et al. [29]. It works by slightly adjusting system parameters according to a rule, which has to be extracted from information about the dynamics of the system. They showed how one could use the sensitive dependence of initial conditions to direct and control the trajectories of the chaotic systems. By making use of the attributes of chaos, one can stabilize the otherwise unstable periodic orbits by adjusting system parameters. Because of the delicate and complicated geometry of chaotic attractors, very small adjustments in the control parameters are sufficient to achieve control [29]. This method has been successfully used to a variety of ecological and evolutionary modeling [30] and experimental systems in physics [31] and chemistry [32].

In this paper, we have done extensive numerical studies of the model system to determine the regions in parameter spaces, which support different dynamical behavior of the system. The main objectives of this paper are the following:

(i) to investigate the dynamical behavior of the model to explore the possibility of chaotic behavior, if any, of the model system (2.4) for different ranges of system parameters through numerical simulation,
(ii) to give an insight about the control of chaos, if possible.

Section 2 deals with the details of the mathematical model. In Section 3, we present the numerical simulation studies. Discussion on the results is presented in Section 4.

2. The basic ecological assumptions and the mathematical model

We assume that there are two populations:

(i) The prey population, Tilapia fish, whose population density is denoted by N, the number of Tilapia fish per unit designated area.
(ii) The predator population, Pelican birds, whose population density is denoted by P, the number of birds per unit designated area.

The following assumptions are made for formulating the basic differential equations:

Assumption 1. In the absence of bacterial infection, the fish population grows according to a logistic law with carrying capacity $K(K \in R_+)$, with an intrinsic birth rate constant $r(r \in R_+)$ such that

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right). \tag{2.1}$$

Assumption 2. In the presence of bacterial infection, the total fish population N is divided into two classes, namely, susceptible fish population, denoted by S, and infected fish population, denoted by I. Therefore, at any time t, the total density of prey (i.e., fish) population is

$$N(t) = S(t) + I(t). \tag{2.2}$$

Assumption 3. Only the susceptible fish population, S, is capable of reproducing with the logistic law (Eq. (2.1)) and the infected fish population, I, dies before having the capacity of reproduction. However, the infected fish, I, still contributes with S to population growth towards the carrying capacity.

Assumption 4. Liu et al. [33] concluded that the bilinear mass action incidence rate due to saturation or multiple exposures before infection could lead to a non-linear incidence rate as $\lambda S^p I^q$ with p and q near 1. Therefore, in our studies, we also assume the non-linear incidence rate as $\lambda S^p I^q$, without a periodic forcing term, which have much wider range of dynamical behaviors in comparison to bilinear incidence rate λSI. Here, $\lambda \in R_+$ is the force of infection or rate of transmission. We also assume that the disease is spread among the prey population only and the disease is not genetically inherited. The infected populations do not recover or become immune. Therefore, the evolution equation for the susceptible fish population, S, according to Eq. (2.1) and Assumptions 3 and 4 can be written as

$$\frac{dS}{dt} = rS\left(1 - \frac{S+I}{K}\right) - \lambda SI. \tag{2.3}$$

Assumption 5. It is assumed that Pelicans cannot distinguish the infected and healthy fish. They consume the fish that are readily available. Since the prey population is infected by a disease, infected preys are weakened and become easier to predate, while susceptible (healthy) preys easily escape predation. Considering this fact, it is assumed that the Pelicans mostly consume the infected fish only. The natural death rate of infected prey (not due to predation) is denoted by $\mu(\mu \in R_+)$. The natural death rate of predator population is denoted by $e_1(e_1 \in R_+)$ and the rate of death due to predation of infected prey is denoted by $e_2(e_2 \in R_+)$. From the above assumptions, we can write down the following differential equations:

$$\begin{aligned}
\frac{dS}{dt} &= rS\left(1-\frac{S+I}{K}\right)-\lambda IS,\\
\frac{dI}{dt} &= \lambda IS-\frac{mIP}{(I+a)}-\mu I,\\
\frac{dP}{dt} &= \frac{\theta IP}{(I+a)}-dP
\end{aligned} \tag{2.4}$$

with $S(0)=S_0>0$, $I(0)=I_0>0$ and $R(0)=R_0>0$. Here $d=e_1+e_2(d\in R_+)$ is the total death of predator population, m is the search rate, θ is the conversion factor and a is the half saturation coefficient. To account for the stability of aquatic ecosystem of multi-species fisheries, Holling type II or Holling type III functional response is more appropriate than the Lotka–Volterra predational form [34]. We shall study the system governed by Eq. (2.4) under the initial conditions $S_0>0$, $I_0>0$, $R_0>0$ in the positive octant, $\{(S,I,P){:}S>0, I>0, P>0\}$.

The model system described by Eq. (2.4) is bounded, dissipative and persistent [1]. The system possesses four equilibrium points:
$E_0(0,0,0), E_1(K,0,0)$, $E_2(\frac{\mu}{\lambda}, r\frac{\lambda K-\mu}{\lambda(r+\lambda K)},0)$, and a unique positive equilibrium point

$$E^*(S^*,I^*,P^*)\quad \text{where } S^*=K-\frac{ad(r+\lambda K)}{r(\theta-d)},\ I^*=\frac{ad}{\theta-d},\ P^*=\frac{1}{m}(a+I^*)(\lambda S^*-\mu).$$

Linear stability analysis can be done by computing the eigen values of the Jacobian matrix about each equilibrium point. It was shown that E_0 is an unstable (saddle) point, which repels in S direction and attracts in I–P coordinate plane. E_1 is also an unstable (saddle) point in the direction orthogonal to S–P coordinate plane, if $\lambda K-\mu>0$. If $\lambda K-\mu<0$, then E_1 is a locally asymptotically stable point. E_1 is also an unstable point in the direction orthogonal to S–I coordinate plane. Further, it is found that the system (2.4) is globally asymptotically stable, around the positive equilibrium point E^*, if the conversion factor (θ) has an upper threshold value given by $\theta<m-\lambda Ka$. Chattopadhyay et al. [1] carried out the stability analysis of the model system (2.4) and found the criteria for the persistent and global asymptotic stability. The loss of stability of E^* happens due to a supercritical Hopf bifurcation. It is interesting to note that if the level of search rate of the predator is low, then the system (around the positive interior equilibrium) is stable, but loss of stability sets in with the increase of search rate level of the predator. The increase in search rate implies the increase of the functional response of the predator and hence predator will be infected rapidly as they are preying on the infected Tilapia fish [1].

3. Numerical simulation and results

We investigate the global dynamical behavior of the system described by Eq. (2.4) by numerical integration of the model system. The objective is to explore the possibility of chaotic behavior. Extensive numerical simulations of the model system (2.4) are carried out for various parameter values and different sets of initial conditions. The parameter values are chosen on the basis of biological principles and correspond to quantitative measures of attributes of the susceptible Tilapia fish and infected Tilapia-Pelican bird population. We have used the ODE workbench software from AIP (American Institute of Physics) for simulation. The model was numerically integrated to get the time series corresponding to the variables of the model systems. Since every nonlinear system has a finite amount of transients, the data points representing transient behavior were discarded. Phase portraits were drawn using this data to obtain the geometry of the attractors. The geometrical object (phase portrait) with zero phase volume and represented by an isolated point in the phase plane is called a stable focus. The presence of a stable focus in the phase space of a dynamical system suggests that the system trajectories would tend to an equilibrium state as $t\to\infty$. In the case of limit cycle solutions, the equilibrium state is not a point, but oscillates persistently between top and bottom limits. The system's trajectory evolves strictly on a closed path, in the phase space. On the other hand, if the trajectory meanders in a bounded phase space of finite volume and there is only one asymptotic attractor, then the corresponding attractor is called a chaotic attractor. As we change the parameter values, the dynamics of the model system changes and it may finally settle on a regular (stable focus or stable limit cycle) state or a chaotic attractor.

The following typical computational procedure is used to distinguish between the chaotic and regular behavior:

(i) First we fixed the initial conditions, for example $S(0) = I(0) = P(0) = 10$ and plotted the time series. Then, we changed one of the initial conditions as $S(0) = 10.01$. If the new time series intersects with the earlier time series, then it confirms that the dynamics of the system is chaotic and if it does not intersect or completely overlaps with the previous time series or the system trajectories would tend to an equilibrium state as $t \to \infty$, then it confirms that the dynamics of the system is regular.
(ii) We draw the phase diagram in the 2D and 3D planes. If the dynamics does not truncate and goes to newer and newer places, but it never goes beyond the region specified by the vector field then we say that the system's dynamics is chaotic and if the dynamics is represented by a fixed point or closed loop in the phase space, then the system's dynamics is regular.

The simulation experiments were done to determine the regions in the parameter spaces, which support different dynamical behaviors for the above system. The values of the parameters were chosen such that the system is biologically feasible. The ranges of values of the parameters are chosen on the basis of the values reported in Jorgensen [35]. Varying one of the critical parameters in its range while keeping all the other constant (i.e., on the limit cycle), the changes in the dynamical behavior of the system are studied, thereby fixing the regimes in which the system may exhibit chaotic dynamics. Simulations were done for large sets of values. We have varied the parameter values in large ranges with small step sizes, for example, 0.25 for the parameters r, d, a; 0.1 for μ, 0.01 for λ and 25 for K. The limit cycles of different periods are clubbed to define a stable limit cycle. The computed results are given in Table 1. From these results, we conclude that the system supports stable equilibrium point and stable limit cycle attractors in reasonably large parameter regimes, whereas chaotic behavior is exhibited in narrow parameter regimes. The parameter r, which is the intrinsic birth rate of the susceptible Tilapia fish population; K, the carrying capacity of the environment and a, the half saturation coefficient are mainly responsible for the chaotic behavior. Chaotic dynamics has been observed for the values of a (the half saturation coefficient) in the ranges $[2.25, 2.80]$, $[3.1, 3.5]$, $[3.6, 4.7]$ and at a discrete point $a = 6.0$. No chaos is found with respect to the parameters λ, μ, d and θ for the given ranges of parameters values. The system supports stable focus and stable limit cycle behavior in the cases of these parameters. For all values of parameter m (the search rate), the system exhibits only limit cycle attractor. A typical strange chaotic attractor is observed for $r = 22$ per day, $K = 400$ tones, $\lambda = 0.06$ per day, $\mu = 3.4$ per day, $m = 15.5$ per day, $a = 15$ tones, $d = 8.3$ per day, $\theta = 10$ per day (see Fig. 1).

The calculation of basin boundaries of coexisting attractors is useful for our discussion. For a dynamical behavior to be of any practical value it is essential that it should exist in a wide parameter range and the corresponding natural measure in 2D parameter scan should be nonzero. In addition, it must fulfill the requirement that it should possess a phase space of initial condition whose natural measure (area or volume) is nonzero. When these two conditions are met by a dynamical system for a particular dynamical behavior, then the same is understood to be a robust one and is considered to have some significance. In this case, stability demands that the basin boundary of the coexisting attractors should be smooth (non-fractal). For systems which possess smooth basin boundaries [36] with one of the attractors existing in narrow regimes, it is difficult to discuss its stability. The basin boundary calculations are performed using the basin and attractor structure (BAS) routine developed by *Maryland Chaos group* (see also [10]). As the model systems are approximate representations of reality, it is necessary to study the nature of basin boundaries. The nature of these boundaries gives us an insight into how a system would behave in fluctuating environments. Keeping this in mind, we have computed the basin boundaries for typical chaotic attractors (Fig. 2) of the system (2.4) for the above set of parameter values. But a change in the value of the parameter r, which is the intrinsic birth rate of the susceptible prey population, from 22 to 9 brings the system to a limit cycle attractor (Fig. 3). Fig. 2 gives the XY view ($-300 \leqslant X = S \leqslant 550$, $-300 \leqslant Y = I \leqslant 550$) of the chaotic attractor in system (2.4). The interesting feature we observe from this figure is the existence of a repeller in the domain of chaotic attractor. The repeller repels the system's trajectories to infinity. Fig. 2 shows that the chaotic attractor is the dominant one. There are no visible competing attractors. Therefore, we can expect that the model dynamics at the suitable parameter values will evolve and continue to evolve on a strange chaotic attractor even in the case when it is challenged by exog-

Table 1
Simulation experiments of model system (2.4) with the fixed parameter values $r = 9$, $K = 400$, $\lambda = 0.06$, $\mu = 3.4$, $m = 15.5$, $a = 15$, $d = 8.3$, $\theta = 10$

Parameters kept constant	Parameter varied	Ranges in which parameter was varied	Dynamical behavior
K, λ, μ, d, a	r	5.00–7.25	Stable focus
	$5 \leqslant r \leqslant 25$	7.30–21.10	Stable limit cycle
		21.15–22.20	Chaos
		22.25–23.35	Stable limit cycle
		23.39–23.86	Chaos
		23.87–23.97	Stable limit cycle
		23.98–25.00	Chaos
r, λ, μ, d, a	K	75–340	Stable focus
	$75 \leqslant K \leqslant 1000$	350–690	Stable limit cycle
		700–770	Chaos
		775–780	Stable limit cycle
		785–850	Chaos
		875	Stable limit cycle
		880–890	Chaos
		900–925	Stable limit cycle
		930–950	Chaos
		960–1000	Stable limit cycle
K, r, μ, d, a	λ	0.01–0.035	Stable focus
		0.04–0.08	Stable limit cycle
	$0.01 \leqslant \lambda \leqslant 5.0$	0.085–5.00	Stable focus
K, λ, r, d, a	μ	0.1–4.65	Stable limit cycle
	$0.1 \leqslant \mu \leqslant 10.0$	4.7–10.0	Stable focus
K, λ, μ, r, a	d	0.5–8.45	Stable limit cycle
	$0.5 \leqslant d \leqslant 25.0$	8.5–25.0	Stable focus
K, λ, μ, d, r	a	1.0–2.5	Stable Limit cycle
	$1.0 \leqslant a \leqslant 20.0$	2.55–2.80	Chaos
		2.85–3.0	Stable limit cycle
		3.1–3.5	Chaos
		3.55	Stable limit cycle
		3.6–4.7	Chaos
		4.75–5.95	Stable limit cycle
		6.0	Chaos
		6.1–16.95	Stable limit cycle
		17.00–20.00	Stable focus

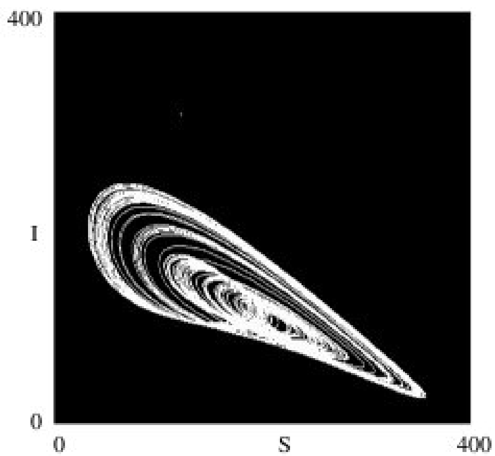

Fig. 1. Typical chaotic attractor for the model system (2.4) obtained for the parameter values $r = 22$, $K = 400$, $\lambda = 0.06$, $\mu = 3.4$, $m = 15.5$, $a = 15$, $d = 8.3$, $\theta = 10.0$.

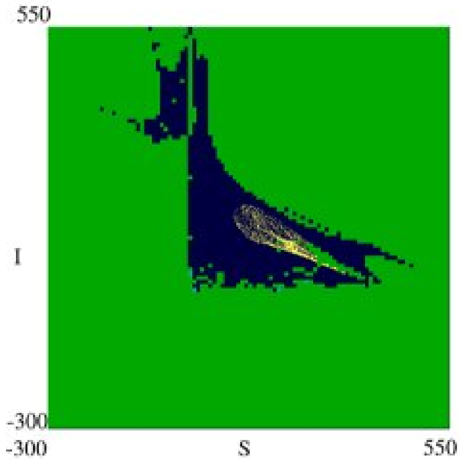

Fig. 2. XY view of the basin boundary structure for the chaotic attractor in model system (2.4) for $r = 22$, $K = 400$, $\lambda = 0.06$, $\mu = 3.4$, $m = 15.5$, $a = 15$, $d = 8.3$, $\theta = 10.0$.

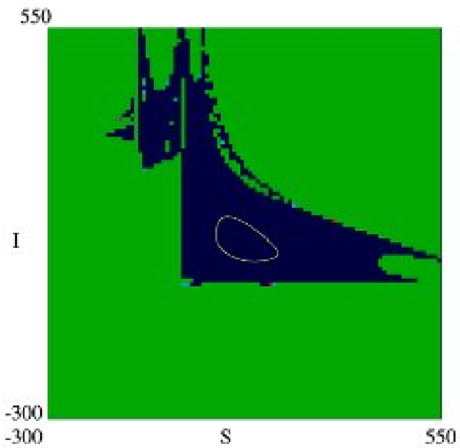

Fig. 3. XY view of the basin boundary structure for the limit cycle structure in model system (2.4) for $r = 9.0$. Rest of the parameter values are the same as those used for generating Fig. 2.

enous chance fluctuations (environmental stochasticity). Fig. 3 shows the XY view ($-300 \leqslant X = S \leqslant 550$, $-300 \leqslant Y = I \leqslant 550$) of the basin boundary structure for the limit cycle attractor in the model system (2.4). The upper boundary of the basin looks like saw tooth and lower boundary is smooth. The basin structure suggests that in the event of disturbances, the system behavior would be of mixed nature. In certain situations, the dynamical behavior of the entire system may depend only on the limit cycle attractor. In other situations, it may often be either unpredictable or meaningless depending on the strength of disturbances. In this case, the dynamics would continue to oscillate between two extreme asymptotic states after transients (initial meandering of the system trajectories before the dynamics is locked on to the attractor) have died out.

We have performed two-dimensional (2D) parameter scan to identify the parameter regimes in which chaos exists. The basis of 2D scans is the belief that the changes in physical/ecological conditions may bring corresponding changes in at most two parameters at a time. The changes in the nature of dynamics in monitored. The parameters used for 2D scans are r, K and λ which represent intrinsic birth rate of susceptible fish population, environmental carrying capacity and the force of infection, respectively. The computed results are given in graphical form in Figs. 4 and 5. From Fig. 4, we find that for lower values of r and K, we observe stable focus and for higher values we observe limit cycles, which intermixed with chaotic attractor. Chaos

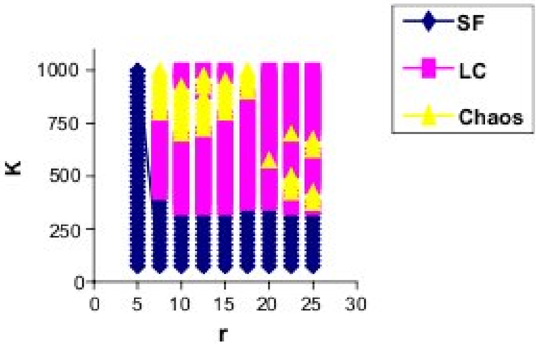

Fig. 4. 2D scan diagram between (r, K) parameter space with $\lambda = 0.06$, $\mu = 3.4$, $m = 15.5$, $a = 15$, $d = 8.3$, $\theta = 10$. Here SF = stable focus, LC = limit cycle.

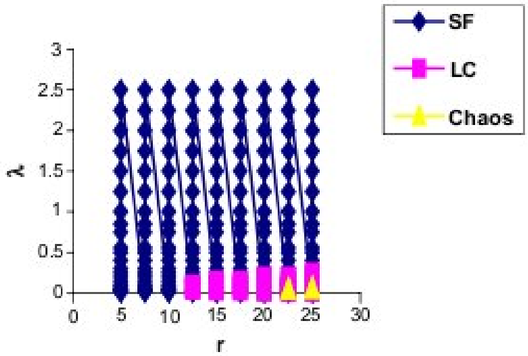

Fig. 5. 2D scan diagram between (r, λ) parameter space with $K = 400$, $\mu = 3.4$, $m = 15.5$, $a = 15$ $d = 8.3$, $\theta = 10$. Here SF = stable focus, LC = limit cycle.

was mainly observed at lower values of r and higher values of K. But in the (r, λ) parameter space, stable focus is dominant and we observe chaos only at few discrete points (Fig. 5).

4. Discussion

In this paper, we have studied the existence of chaotic behavior in the model system described by Eq. (2.4) for different ranges of system parameters. We have also tried to suggest some viable measures to control the chaotic dynamics, if any. The simulation results suggest that the system may show different interesting dynamical behavior starting from stable focus/stable limit cycle to chaos.

From the simulation results, it is observed that the parameters r, K and a are the key parameters responsible for the chaotic behavior of the system, whereas no chaos was found due to the parameters λ, μ, d, m and θ. The system supports stable limit cycles or stable focus in reasonably large parameter regimes, whereas chaotic behavior is exhibited, intermixed with stable limit cycles, in narrow parameter regimes.

It is observed that the possibility of existence of chaos is high when r, the intrinsic birth rate of the susceptible fish population, increases. When r increases (the number of fish population increases), the number of infected fish population also increases and unpredictable dynamics is observed. Similar events also occur when K, the carrying capacity increases (Fig. 4). The situation is different in case of a, the half saturation constant. In this case, we observe from the table that the system exhibits chaotic behavior for lower values of a (up to 6.0) but for higher values, the system supports stable limit cycles or stable focus. We note that a quantifies the extent to which environment provides protection to the prey (the infected fish) and can be thought of as a mea-

The Salton Sea receives a huge amount of nutrients from its surrounding farmlands. Thus, the Sea is becoming more and more eutrophic over the years and this high eutrophication causes high productivity of the fish. These two events, as a whole, increase the values of K and r. The 2D scan results (Figs. 4 and 5) shows that when the values of K and r are high, keeping other parameter values unaltered, the system exhibits chaotic dynamics. McCann and Yodzis [12] also pointed out similar observations in a different context. In the late summer, when water level is low, nutrient concentration of sea water increases and consequently the value of carrying capacity K also increases. High values of K enhance the productivity of the fish population and thus increase the value of r. The cumulative effect of increased values of r and K is to force the system to settle on the chaotic regime. Perhaps, Salton Sea witnessed such chaotic dynamics when 8 million fish died in a single day on 12th August 1999 (see, http://ens.lycos.com/ens/aug99/1999L-08-12-06.html). To control the chaotic behavior of the system, the value of the carrying capacity, K, should be kept low and this can be done by reducing the nutrients from the sea water. Also, the value of r (the intrinsic birth rate of the fish population) should not be very high. It is to be noted that for narrow ranges of r in $[21.15, 22.20]$, $[23.39, 23.86]$ and $[23.98, 25.00]$ the system exhibits chaotic behavior. Proper harvesting of prey species, at this stage, can reduce the value of r and the system will return to a stable state from the chaotic state. Thus, we may conclude that when the intrinsic birth rate of the prey population increases, the chaotic behavior of the system can be avoided by implementing a proper harvesting policy. It is known that the Tilapia fish has a stunning reproduction rate. This higher reproduction rate increases the value of r and in turn increases the number of infected fish. To preserve the endangered Pelican population, the fish with higher reproduction rate can be partially replaced, if possible, from the Salton Sea with another type of fish, which has lower reproduction rate. This may help to reduce the possibility of chaos caused by an increased value of r.

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