Collective Punishments: Incentives and Examinations in Organisations

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Abstract

The paper investigates the impact of examinations on incentives and decision-making in bureaucracies and similar organisations. When one amongst a group of bureaucrats can be appointed to give policy advise whose outcome affects all parties, with advisory ability increasing in personal effort, a free-riding problem is generated if preferences are aligned, leading to an ex ante inefficiency. Free-riding may be mitigated by an examination with a pass-mark, i.e., a minimum ability requirement as a necessary criterion for advisory appointment. By collectively punishing all experts when maximal ability is low, it raises private incentive to enhance ability, and improves decision quality.

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1 Introduction

Bureaucracies, as the primary operational and advisory arm of governments, play a central role in modern economies as providers of public goods and services. Bureaucrats implement tasks, and acquire and report information and provide a variety of other advisory services which aid in the formulation of rules, policies and laws. One aspect of these organisations which has received significant attention in the recent past is the absence or peripherality of standard incentive devices such as piece-rates or efficiency wages, in spite of the potential presence of free-riding, moral hazard and other agency problems. A variety of explanatory issues have been discussed, including the possibilities that individuals who choose to join bureaucracies may be motivated by the output of such organisations or the desire for public service or that explicit incentives may dampen intrinsic motivation.[1]

This paper studies a simple free-riding problem in bureaucracy and attempts to examine how agency problems can be ameliorated in spite of the infeasibility of standard incentive instruments. Consider a Ministry consisting of a minister and a group of bureaucrats. The minister has to choose a policy. He could do so on the basis of available prior information, or could appoint one of the bureaucrats as his advisor (or secretary). Depending on her ability, the advisor may get better information, which would lead to superior policy choice. Assume the payoffs of all parties are dependent on the quality of policy choice and that the choice affects the payoffs of all parties in the same way, i.e., there are no divergences of preferences. If bureaucrats have different abilities, then it is optimal to appoint as advisor the bureaucrat with the highest ability. But if building ability is privately costly, a free-riding problem can emerge. All bureaucrats prefer that the one among them with highest ability be appointed as advisor. However, since the value of advisory services is expressed through the quality of policy choice, which affects all parties equivalently, each bureaucrat has an incentive to underinvest in ability. This reduces expertise levels within the organisation, leading to inferior policy choice and lower payoffs for everyone.

How can the Ministry attempt to control this problem? One way is to institute a

[1] The literature on these topics is by now quite large; recent interesting contributions include Francois (2000), Murdock (2002), Bénabou and Tirole (2003), Besley and Ghatak (2005) and Akerlof and Kranton (2005). These essays, along with Dixit (2002), provide an extensive survey.

minimal or least acceptable ability, which can be thought of as an *examination* with a *pass-mark*, as a necessary criterion for advisory appointment. In such a setting, a bureaucrat whose ability is less than the pass-mark cannot be appointed as advisor. This introduces an inefficiency: given the profile of abilities, if no bureaucrat achieves the minimal level, an advisor is not appointed, while appointing any bureaucrat with some positive ability can improve expected policy quality. At the same time, an examination can act as an incentive device: by collectively punishing all bureaucrats when the maximal ability level is low, it increases each bureaucrat's incentive to augment her own. In turn, this raises the overall level of expertise, and hence the quality of policy choice. This trade-off determines whether imposing an examination can be beneficial.

We build a simple formal model, along the lines indicated above, in Section 2. Sections 3 and 4 describe the equilibrium and the free-riding problem. Examinations are introduced in Section 5, where we show that committing to them can have useful incentive implications. Although the example above was couched in terms of advisor selection by a minister from amongst a group of bureaucrats, the spirit of the discussion extends to wider organisational and collective contexts such as community participation and institution building. It may also help understand other aspects of bureaucracy such as how bureaucrats themselves are to be selected from a wider population. If interested candidates have some commonality of interests with the organisation, in terms of, say, social policy or public service goals, and bureaucrats' actions influence organisational outcomes, a similar free-riding problem can manifest itself, and lead to low quality of potential candidates. In such situations, civil service examinations may play important incentive roles. The argument of the paper may also contribute to understanding why examinations exist and indeed are ubiquitous in bureaucratic organisations around the world, both as devices for selection into the services, as well as instruments which aid in promotion, transfer and task allocation processes.

The article argues that the absence of such control devices can lead to inadequate policy quality in collective contexts. Consider as an example Sweden's recent attempts to modify the entire business regulatory structure with a view to augmenting simplicity and efficiency. NNR[2], one of the stakeholders in the process, has argued that a signifi-

[2] Näringslivets Regelnämnd NNR (Arbetar för Färre och Enklare Regler). In English: Board of Swedish Industry and Commerce for Better Regulation

cant proportion of proposals, stemming from a variety of government offices, regulatory agencies, stakeholder groups etc., ultimately selected for consultative discussions were of inadequate quality (see NNR (2005)). In its view, the "key reason" is insufficient quality control (over which proposals are promoted to the consultation stage) and sanctions (for inferior quality). Our analysis suggests that a clearer demarcation of how quality can be measured together with denial of entry to the consultation stage for poorer quality proposals may have beneficial impacts. Of course, the use of such an examination style selection protocol will presumably yield better results if articulated explicitly rather than implicitly. As an illustration, consider the instance of Brown Field Municipal Airport, managed by the City of San Diego in California, U.S.A. Despite city officials insisting that it is a "priority" and a "valuable asset" for the city and the airport community, Brown Field appears in the eyes of many as a symbol of opportunities lost (see San Diego Union-Tribune (2006)). A variety of stakeholders have cited examples of proposals, over several years from a diversity of sources, with beneficial impact on airfield activity, upkeep and maintenance, and revenue augmentation, which have been rejected, for reasons which remain less than fully clear. The representative of one such group complains that proposals got rejected, in spite of apparently meeting preliminary selection guidelines, because "no proposal was good enough" (see San Diego Union-Tribune (2005)). While numerous allegations, including incompetence and corruption, have been levelled, especially in the context of a wider malaise confronting Brown Field, this paper argues that rejecting a proposal which meets initial criteria because it is of inadequate quality is not necessarily without benefit. Naturally, a situation where participants remain unsure about the use and implementation of such a selection protocol is unhelpful.

The essay assumes that a bureaucrat derives utility only from policy direction, but not directly from advising the policy-maker. Our results also hold when there is a direct benefit from being the advisor, as long as such benefit is relatively small. When the direct benefit from being advisor is very large relative to the benefit stemming from appropriate policy choice, collective punishments generated through positive pass-marks are no longer necessary to provide incentives. The assumption that there are no asymmetries between bureaucrats is also unnecessary. In Section 6, we consider two extensions of the basic model described above. Firstly, we show that if the bureaucrats differ in terms of their cost of building ability, our basic results may still hold. Secondly, we allow bureaucrats to have preference divergences. Suppose in such situations getting appointed as advisor carries the additional benefit of direct influence over policy choice. Preference conflicts between bureaucrats can then lead to competition for advisory appointment, and increase each bureaucrat's incentive to build ability. In such environments, we show that the benefit of instituting an examination decreases with the degree of preference divergence. Thus, examinations and preference divergences can be substitutes for each other, as each may reduce the extent of the free-riding problem.[3]

The article also contributes to the emergent literature studying how agency problems can be mitigated through organisational design in environments with limited scope for standard incentive instruments. This literature has focussed on such issues as the selection of advisory agents, the restrictions on agents' choices and organisational decision-making protocols which can lead to overall incentive benefits, the allocation of control amongst various groups, etc. (see Aghion and Tirole (1997), Dewatripont and Tirole (1999), Dessein (2002), Li (2001), Aghion, Dewatripont and Rey (2002, 2004), Li and Suen (2004), Szalay (2005), Dur and Swank (2005), Banerjee (2006a, b) etc.). A recurrent theme is that such decisions are significantly influenced by the structure of preference congruence over policies or organisational choices. In reality, agent preferences may not be known exactly or there may not be much scope in certain organisational settings for their instrumental usage because of supply constraints, anti-discrimination statutes, etc. Further, if we view organisations as collections of parties with relatively allied interests, there may not be much variation in preferences across different agents. Finally, factors such as ability are also known to be important in determining selection and allocation. Existing studies assume that all agents within a given class have the same ability, which is exogenously determined. Yet in actuality organisations expend significant thought on how to acquire agents with more suitable ability, or how to encourage ability-building among potential or actual members. The essay shows that the commonly observed institution of examinations can help improve incentives for augmenting ability, and thereby enhance the quality of decision-making.

The theoretical literature on educational standards (Costrell (1994) and Betts (1998))

[3] Banerjee (2006b) also points out that preference conflicts between members of an organisation can help reduce free-riding. The problem considered there is quite different: abilities are exogenous, while conflicts are chosen endogenously, and change the way members compete and cooperate with each other. The paper does not allow for examinations or other such preliminary selection devices.

has analysed the impact of examinations on student effort, and the resulting ramifications for lifetime expected earnings. These are models with autonomous students who respond only to financial incentives and impose no externalities on each other, and so cannot explore the impact of examinations in enhancing the quality of collective decision-making.[4] Empirical support for the idea that examinations can improve incentives can be found in Bishop (1997), Betts and Grogger (2003) and Figlio and Lucas (2004).

The incentive effects of examinations in collective contexts have also received attention in the literature on the history and evolution of examination and assessment systems in different regions of the world. The earliest known examples of selection protocols using written examinations, instituted in Imperial China, arose in the context of recruitment to the bureaucracy, and were influenced by concerns over the dominance of narrowly constituted groups and the resulting decline in the quality of functionaries (for a broad narrative, see, for example, Chaffee (1985)). Later modifications and extensions of various examination systems in China as well as in the other "Mandarinates" of Korea and Vietnam, were also motivated in part by such factors (see, for example, Burns and Bowornwathana (2001) and Woodside (2006)). It is well-known that modern civil service examinations, as well as examinations in educational institutions, have evolved from forms that emerged in Europe in the 17^{th} and 18^{th} centuries (see, for example, Tilly (1975) and Makdisi (1981)). In turn, these were substantially influenced by the Chinese experience, with Jesuit schools forming the principal mediating link (see, for example, Scaglione (1987) and Wilbrink (1997)).

2 The basic model

A *Decision-maker* ($\mathcal{D}$) has to choose a policy on behalf of a group of agents, whom we call *Experts* ($\mathcal{E}$). There are $T \geq 2$ experts, and three possible policies, 0, 1 and 2. $\mathcal{D}$ may make the policy choice on his own, or he may appoint one expert as his *Advisor* to help him make the decision.[5]

[4] Sobel (2001) also considers the incentive effects of minimum entry requirements to a club with autonomous candidates in a dynamic setting. His main interest is in studying how such standards can decline with time.

[5] In principle, the maximum number of advisors could exceed 1. The basic intuition does not depend on what this maximum is. We assume it is 1 for simplicity.

If policy 0 is chosen, each expert receives gross payoff 0. It is known that exactly one of the other two policies will yield gross payoff 1 to any expert, while the other will yield gross payoff $-\beta$, $\beta > 0$. It is not known for sure which of policies 1 and 2 yields positive payoff. Assume β is sufficiently large so that in the absence of any further information, $\mathcal{D}$ will choose policy 0.[6]

$\mathcal{D}$ has to decide between policies at date 5. If no advisor has been appointed, then he receives no further information regarding payoffs from policies 1 and 2, and hence chooses policy 0, and so each expert receives gross payoff 0.[7] On the other hand, if $\mathcal{E}_i$ has been appointed advisor, with *ability* $\alpha_i \in [0, 1]$, then at date 4 she may discover which amongst policies 1 and 2 yields positive gross payoff. She does so with probability α_i. With the complementary probability, she receives no further information. Assume information received by the advisor is public.[8] Thus, if an advisor is appointed, and she receives information, each expert's gross payoff is 1, while if an appointed advisor receives no further information, each expert receives gross payoff 0.[9]

At date 3, $\mathcal{D}$ has to decide whether to appoint an advisor or not, and, if he appoints one, which expert to appoint. At this time, the abilities of all experts are known.[10] The ability of any expert $\mathcal{E}_j$ is $\alpha_j \in [0, 1]\ \forall j \in \{1, ..., T\}$.

Hence, any expert's gross expected payoff at date 3 is 0 if no advisor is appointed, while it is α_i if $\mathcal{D}$ appoints $\mathcal{E}_i$ as his advisor. It is efficient at date 3 for $\mathcal{D}$ to appoint the

[6] The decision-maker in this model is largely a metaphor for a collective decision-making procedure for the community of agents. Since all agents have common preferences over policies, assuming that an exogenous decision-maker (with identical preferences) is responsible for policy choice presents no singular difficulties. We study heterogeneous preferences later, at which time it will be more useful to think in terms of a separate agent who has decision-making power.

[7] Allowing $\mathcal{D}$ to receive additional information does not change our qualitative results, as long as his informational capacity is less than that of the experts.

[8] Since all parties have aligned preferences, whether or not signals received by the advisor are public makes no difference. If there were preference misalignments between the decision-maker and an advisor, an issue we do not consider in this paper, such an assumption would not be so innocuous: see, for example, Dessein (2002), Li and Suen (2004), Dur and Swank (2005), Szalay (2005) and Banerjee (2006a).

[9] The assumption that an expert, regardless of ability, can only receive information if she is appointed as advisor, is important. It reflects the idea that the provision of advisory services requires access to some information generating processes, which are not open access or indivisible.

[10] Qualitative results remain unchanged if, given unknown abilities, information regarding abilities were symmetric across parties.

highest-ability expert as his advisor, and he will clearly do so in the absence of any prior commitments, except perhaps in the trivial case where all experts have 0 ability.

Experts' abilities are not exogenous however. At date 1, each expert has to decide how much effort to exert towards building ability. Experts make this decision simultaneously. Abilities are realised publicly at date 2. If $\mathcal{E}_i$ exerts 0 effort, she bears no private cost, and her ability is 0. If she exerts high effort, her private cost is $c > 0$, and her ability is the realisation of a random variable distributed over [0, 1] according to the continuously differentiable, strictly increasing distribution function F, whose density is denoted as f.[11,12] Thus experts are symmetric at date 1.[13] $\mathcal{D}$ observes the experts' abilities and makes her appointment decision at date 3.[14]

We now analyse this model to determine the outcome of the experts' effort choice game. The timeline is given below in Figure 1.

[Figure 1 about here]

3 Incentives

We study the experts' private investment game in this section. We use backward induction to determine an expert's payoff at date 1 as a function of her own effort choice, and the choices of the other experts. We then examine the configuration of equilibrium effort choices.

Suppose the ability at date 3 of any $\mathcal{E}_i$ is α_i, and let $\alpha_j > \max_{i \neq j} \alpha_i$, $j, i \in \{1,..., T\}$. Then $\mathcal{D}$ appoints $\mathcal{E}_j$ as his advisor, and each expert obtains gross expected payoff α_j. Let X_k^m denote the $m^{th}-$highest order statistic of a sample of size k drawn from [0, 1] according

[11] All qualitative results extend immediately if instead the probability of realising 0 ability is always a positive fraction, diminishing in the level of effort.

[12] Binary effort choice is assumed for simplicity. The intuition does not seem to depend on this specification.

[13] We show later that our results can extend to situations where the experts differ in cost of high effort.

[14] Implicitly, we assume that experts are not responsive to monetary transfers, so piece-rates or other standard incentive devices are ineffective. Our results hold otherwise as well, provided the weight given to such payments is relatively small.

to distribution function F, $k \geq 1$, $m \in \{1,...,k\}$. Let F_k^m and f_k^m respectively denote the distribution and density functions of X_k^m, and let $E_{F_k^m}$ be its expectation.[15]

Let $E_{F_0^1} = 0$, and for $t \in \{1,...,T\}$, define

$$b_t = E_{F_t^1} - E_{F_{t-1}^1}, \text{ so } b_1 = E_{F_1^1} - E_{F_0^1} = E_{F_1^1} \tag{1}$$

b_t is therefore the increment in the expected value of the highest order statistic when the sample size is increased from $t-1$ to t, $t \in \{1,...,T\}$. We have

Lemma 1: (i) $b_t > 0$, $t \in \{1,...,T\}$. (ii) $b_t > b_{t+1}$, $t \in \{1,...,T-1\}$.

Proof. Follows from the proof of Lemma 3 below. ■

Consider the game at date 1. Suppose in equilibrium t experts choose high effort, and the remaining $T-t$ experts choose low effort, $t \in \{0,...,T\}$. We call such a pure strategy equilibrium a $t-$equilibrium.

For a $T-$equilibrium, i.e., a symmetric pure strategy equilibrium with all experts choosing high effort, to exist, we need

$$\int_0^1 x dF_T^1(x) - c \geq \int_0^1 x dF_{T-1}^1(x)$$

Similarly, a $0-$equilibrium, i.e., a symmetric pure strategy equilibrium with no expert choosing high effort, exists if and only if

$$\int_0^1 x dF_1^1(x) - c \leq E_{F_0^1} = 0$$

Now let $t \in \{1,...,T-1\}$. A $t-$equilibrium exists when

$$\int_0^1 x dF_t^1(x) - c \geq \int_0^1 x dF_{t-1}^1(x) : \text{ Incentive constraint for a high-effort expert}$$

$$\int_0^1 x dF_t^1(x) \geq \int_0^1 x dF_{t+1}^1(x) - c : \text{ Incentive constraint for a low-effort expert}$$

Since the experts are symmetric ex ante, we shall restrict attention to symmetric equilibria.[16,17] The following result describes equilibrium.

[15] See Tan (1992) for a model of information acquisition which also uses results from order statistics, although in a very different context.

[16] For more discussion of the focus on symmetric equilibrium, see Section 5 below.

[17] Throughout the paper, the terms ex ante and ex post are used with respect to the timing of investment.

Proposition 1: A unique symmetric equilibrium exists for all c. It is in mixed strategies if $c \in [b_T, b_1]$, while all experts choose high effort if and only if $c \leq b_T$.

Proof. Follows from the proof of Proposition 3 below. ■

The structure of symmetric equilibrium is then very simple. If private cost is sufficiently low ($c \leq b_T = E_{F_T^1} - E_{F_{T-1}^1}$), all experts choose high effort, while if private cost is very high ($c \geq b_1 = E_{F_1^1}$), all experts choose low effort. For intermediate cost, experts randomise between high and low effort, with the equilibrium probability of choosing high effort decreasing in the cost. Asymmetric equilibria also exist with a $t-$equilibrium, $t \in \{1,..., T-1\}$, existing if and only if $c \in [b_{t+1}, b_t]$.

We now show that experts may have a tendency to free-ride on other experts' efforts, and so all experts could be better off by committing to some effort level ex ante.

4 Free-riding

When the payoffs from policies 1 and 2 are known in our environment, all experts wish to adopt the policy which yields positive payoff, while if the payoffs are not known for certain, they all wish to guarantee themselves a payoff of 0 by choosing policy 0. Thus, all experts benefit if greater information about the payoffs from policies 1 and 2 is available. Given the set-up of the problem, such information is a pure public good. Since the degree of information availability is dependent on experts' effort choices, and each expert alone bears her private cost for effort, a standard free-riding problem is generated. Each expert underinvests in effort, and overall this free-riding leads to inadequate information gathering, here leading to a first order stochastic dominated distribution for maximum ability.

To understand this, first consider the date 1 problem of choosing investment probabilities $(\sigma_i)_{i=1}^T$ for the experts to maximise the sum of ex ante net expected payoffs. Suppose the solution is symmetric, i.e., $\sigma_1 = \sigma_2 ... = \sigma_T = \sigma$.[18] Now suppose all experts could commit to some $\sigma \in [0, 1]$ prior to making investment choices at date 1. They would then commit to choosing high effort with probability $\widetilde{\sigma}$, where $\widetilde{\sigma}$ maximises

[18] We show below in Appendix B that any solution to this problem is necessarily symmetric.

$$R_0(\sigma) - \sigma c \tag{2}$$

where

$$R_0(\sigma) = \sum\nolimits_{t=0}^{T} \binom{T}{t} \sigma^t (1-\sigma)^{T-t} E_{F_t^1} \tag{3}$$

Let

$$S_0(\sigma) = \sum\nolimits_{t=0}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} b_{t+1} \tag{4}$$

$S_0(\sigma)$ is thus the increment in expected gross payoff to an expert from taking high effort, relative to taking low effort, when all other experts choose high effort with probability σ. We find, using (3) and (4),

$$R_0'(\sigma) = TS_0(\sigma) > 0; R_0''(\sigma) = -T(T-1)\sum\nolimits_{t=0}^{T-2} \binom{T-2}{t} \sigma^t (1-\sigma)^{T-t-2}(b_{t+1} - b_{t+2}) < 0$$

Thus $R_0(\sigma)$ is strictly concave. Using (2) and (3), we have

Claim 2: $\widetilde{\sigma} = 1$ if and only if $c \leq Tb_T$, while $\widetilde{\sigma} = 0$ if and only if $c \geq Tb_1$. For $c \in (Tb_T, Tb_1)$, $\widetilde{\sigma} \in (0, 1)$.

Proof. Clearly, $\widetilde{\sigma} = 0$ if and only if $c \geq R_0'(0)$, while $\widetilde{\sigma} = 1$ if and only if $c \leq R_0'(1)$. For $c \in (Tb_T, Tb_1)$, $\widetilde{\sigma} \in (0, 1)$ is the unique solution to

$$S_0(\sigma) = \frac{c}{T}$$

The proof is complete as $R_0'(0) = Tb_1$, and $R_0'(1) = Tb_T$. ■

To see formally when free-riding manifests itself, let $\widetilde{\Pi}(c; \sigma)$ be an expert's net expected payoff in symmetric equilibrium at date 1, prior to making private investment choice, when all experts choose high effort with probability σ, and let $\widetilde{\Pi}'(c; \sigma)$ be the derivative of $\widetilde{\Pi}(c; \sigma)$ with respect to σ.

Then, using Proposition 1 and (4),

$$\widetilde{\Pi}(c; \sigma) = \begin{array}{l} E_{F_T^1} - c, \text{ if } c \leq b_T \\ \sum_{t=0}^{T} \binom{T}{t} \sigma^t (1-\sigma)^{T-t} E_{F_t^1} - \sigma c, \text{ if } c \in [b_T, b_1] \\ 0, \text{ if } c \geq b_1 \end{array} \tag{5}$$

where σ in (5) is the unique solution to the indifference condition for symmetric mixed strategy equilibrium:

$$S_0(\sigma) = c \tag{6}$$

We have

Proposition 2: Let $c \in (b_T,\, b_1]$, and suppose in symmetric equilibrium all experts choose high effort with probability σ^*, with σ^* solving (6). Then date 1 expected payoff of any expert is higher if all experts instead choose high effort with probability $\sigma^* + \epsilon$, where $\epsilon > 0$, and ϵ sufficiently small.

Proof. Given $c \in (b_T,\, b_1]$, we know from Proposition 1 that experts choose high effort with probability σ^* in symmetric equilibrium, where, using (1), (4) and (6)

$$\sum\nolimits_{t=0}^{T-1} \binom{T-1}{t} \sigma^{*t}(1-\sigma^*)^{T-t-1}(E_{F_{t+1}^1} - E_{F_t^1}) = c$$

An expert's ex ante net expected payoff is, using (5)

$$\widetilde{\Pi}(c; \sigma = \sigma^*) = \sum\nolimits_{t=0}^{T} \binom{T}{t} \sigma^{*t}(1-\sigma^*)^{T-t} E_{F_t^1} - \sigma^* c$$

It suffices to show therefore that $\widetilde{\Pi}'(c;\, \sigma = \sigma^*) > 0$. We find, since $E_{F_0^1} = 0$,

$$\begin{aligned}
\frac{\partial \widetilde{\Pi}}{\partial \sigma} &= \sum\nolimits_{t=1}^{T-1} \binom{T}{t} \{t\sigma^{t-1}(1-\sigma)^{T-t} - (T-t)\sigma^t(1-\sigma)^{T-t-1}\} E_{F_t^1} + T\sigma^{T-1} E_{F_T^1} - c \\
&= T(1-\sigma)^{T-1} E_{F_1^1} + T \sum\nolimits_{t=1}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} b_{t+1} - c \\
&= T S_0(\sigma) - c, \text{ using (4)}
\end{aligned}$$

and so, using (6)

$$\widetilde{\Pi}'(c; \sigma = \sigma^*) = (T-1)c > 0$$

■

Thus, free-riding exists in equilibrium whenever $c \in (b_T,\, b_1]$. When $c \leq b_T$, all experts choose high effort in equilibrium, and so no free-riding manifests itself. When $c > b_1 = E_{F_1^1}$, all experts choose low effort in equilibrium. Free-riding may be present in this situation as well since each expert is better off if all experts choose high effort with some positive probability whenever $c \in (b_1, Tb_1)$. Further, for c greater than b_T, yet sufficiently close to b_T, it is optimal for all experts to choose high effort always, though each expert chooses high effort with probability less than 1 in symmetric equilibrium, because of free-riding.

5 Examinations

We now analyse whether $\mathcal{D}$ can mitigate free-riding and increase all experts' ex ante net expected payoff by setting a simple examination as a selection rule, prior to experts choosing effort levels. Through the examination, $\mathcal{D}$ sets a pass-mark or a minimum ability level $\mu \in (0, 1]$, and commits to selecting the highest ability expert if and only if her ability exceeds the pass-mark.[19]

An examination with $\mu > 0$ is inefficient ex post. At date 3, given the profile of abilities, it is efficient to appoint as advisor the expert with the highest ability, irrespective of whether her ability exceeds μ or not. At the same time, setting an examination can improve ex ante effort incentives for any expert by jointly penalising all experts when realised expertise levels are very low. An examination can hence increase the expected maximum level of expertise at date 3.[20]

To proceed, recall the definitions of X_k^m, F_k^m, f_k^m and $E_{F_k^m}$, and define

$$b_1(\mu) = \int_\mu^1 x dF_1^1(x), \text{ and, for } t \in \{2, ...T\}, \, b_t(\mu) = \int_\mu^1 x dF_t^1(x) - \int_\mu^1 x dF_{t-1}^1(x) \quad (7)$$

[19] Commitment is critical as any potential incentive benefit from an examination can arise only if experts believe, prior to effort choices, that $\mathcal{D}$ will abide by the pass-mark rule. Such commitment may be generated through the organisation's concern for reputation, though a full dynamic analysis is beyond the scope of the paper.

[20] The idea that it may be sensible to commit to instruments yielding ex post inefficiencies because of overall ex ante gains is hardly new, and has been used extensively in many studies, including several recent papers analysing incentives in organisations such as Aghion and Tirole (1997), Li (2001), Szalay (2005), Banerjee (2006a, b) etc. Although the specific issues and some underlying processes studied in these essays are different from those analysed in the current article, the basic mechanics of the trade-off are related. The closest is Li (2001), who analyses a similar incentive problem, but uses a different information aggregation process.

We have, using (1) and (7)

Lemma 3: (a) For $t \in \{1,...,T\}$, (i) $b_t(0) = b_t$, (ii) $b_t(1) = 0$, (iii) if $\mu < 1$, $b_t(\mu) > 0$. (b) (i) $b_1'(0) = 0$; $b_1'(\mu) < 0$ if $\mu \in (0, 1]$, (ii) for $t \in \{2,...,T\}$, $b_t'(\mu) = 0$ if $\mu \in \{0, F^{-1}(\frac{t-1}{t})\}$; $b_t'(\mu) > 0$ if $\mu \in (0, F^{-1}(\frac{t-1}{t}))$; $b_t'(\mu) < 0$ if $\mu \in (F^{-1}(\frac{t-1}{t}), 1]$. (c) For $t \in \{1,...,T-1\}$, and $\mu < 1$, $b_t(\mu) > b_{t+1}(\mu)$.

Proof. See Appendix A. ■

$b_t(\mu)$ is the increment in the conditional expected value of the highest order statistic, conditional on the random variable being at least μ, when the sample size is increased from $t-1$ to t. Part (c) of the Lemma says that $b_t(\mu)$ is decreasing in t, while part b(i) says that $b_1(\mu)$ is decreasing in μ. Part b(ii) observes that for $t \geq 2$, $b_t(\mu)$ is non-monotone in μ: it is increasing in μ for μ small, and decreasing otherwise. Given μ, and considering the game at date 1, suppose in equilibrium t experts choose high effort, and the remaining $T-t$ experts choose low effort, $t \in \{0,...,T\}$. As before, we call such a pure strategy equilibrium a $t-$equilibrium.

For a $T-$equilibrium to exist, we need

$$\int_{\mu}^{1} x dF_T^1(x) - c \geq \int_{\mu}^{1} x dF_{T-1}^1(x) \tag{8}$$

Similarly, a $0-$equilibrium exists if and only if

$$\int_{\mu}^{1} x dF_1^1(x) - c \leq 0 \tag{9}$$

Now let $t \in \{1,...,T-1\}$. A $t-$equilibrium exists when

$$\int_{\mu}^{1} x dF_t^1(x) - c \geq \int_{\mu}^{1} x dF_{t-1}^1(x) \tag{10}$$

$$\int_{\mu}^{1} x dF_t^1(x) \geq \int_{\mu}^{1} x dF_{t+1}^1(x) - c \tag{11}$$

The following result describes symmetric equilibrium.

Proposition 3: If $\mu = 1$, all experts choose low effort. If $\mu < 1$, a unique symmetric equilibrium exists for all c. It is in mixed strategies if $c \in [b_T(\mu), b_1(\mu)]$, while all experts choose high effort if and only if $c \leq b_T(\mu)$.

Proof. See Appendix A. ■

In symmetric equilibrium, if private cost is sufficiently low ($c \leq b_T(\mu)$), all experts choose high effort, while if private cost is very high ($c \geq b_1(\mu)$), all experts choose low effort. For intermediate cost, experts randomise between high and low effort, with the equilibrium probability of choosing high effort decreasing in the cost.

We now show that committing to an examination can improve all experts' ex ante net expected payoffs. Suppose for now $\mathcal{D}$ wants all experts to take high effort, and wishes to set an examination to induce such behaviour. Clearly, he sets $\mu = 0$ if $c \leq b_T(0)$, since experts choose high effort in equilibrium in the absence of any examination. So let $c > b_T(0)$. Define

$$\widetilde{\mu}_T = F^{-1}(\frac{T-1}{T}) \in (0,1) \tag{12}$$

We have

Lemma 4: (a) $b_T(\widetilde{\mu}_T) \in (b_T(0), b_1(0))$. (b) For every $c \in (b_T(0), b_T(\widetilde{\mu}_T)]$, there exists $\widehat{\mu}_T(c) \in (0, \widetilde{\mu}_T]$, with $\widehat{\mu}_T(c)$ monotone increasing in c, such that for every $\mu \in [\widehat{\mu}_T(c), \widetilde{\mu}_T]$, the unique equilibrium involves all experts choosing high effort at date 1.

Proof. See Appendix A. ■

For $c \in (b_T(0), b_T(\widetilde{\mu}_T)]$, let $\widehat{\mu}_T(c)$ be the lowest μ in [0, 1] such that $b_T(\mu) = c$. The lemma shows that for every $c \in (b_T(0), b_T(\widetilde{\mu}_T)]$, $\mathcal{D}$ can always set an examination such that all experts will choose high effort in equilibrium. It is clear that he will set $\mu = \widehat{\mu}_T(c)$ in such a case, as a higher μ cannot increase effort, yet raises ex post inefficiency.

We also see that if $c > b_T(\widetilde{\mu}_T)$, $\mathcal{D}$ can never induce all experts to always choose high effort in equilibrium. Let $c \in (b_T(0), b_T(\widetilde{\mu}_T)]$, and suppose $\mathcal{D}$ has committed to an examination, with $\mu = \widehat{\mu}_T(c)$, at the beginning of date 1. Further let $c - b_T(0) = \delta$. We now show that such an examination can make all experts better off.

Proposition 4: Suppose when $c \in (b_T(0), b_T(\widetilde{\mu}_T))$, $\mathcal{D}$ commits to an examination $\mu = \widehat{\mu}_T(c)$. If $\delta = c - b_T(0)$ is sufficiently small, all experts receive higher net expected payoff than when no examination is imposed.

Proof. See Appendix A. ■

Thus, if $c > b_T(0)$, with $c - b_T(0)$ sufficiently small, $\mathcal{D}$ can always set an examination such that all experts choose high effort in equilibrium, and are better off compared to when there was no examination. Whenever $\delta > 0$, each expert suffers a loss compared to when $\delta = 0$, irrespective of whether an examination is set or not. When δ is small, and the extent of the free-riding problem is not acute, collective punishment works well in the sense that a small μ is adequate to induce all experts to choose high effort. The payoff loss from the examination regime, stemming from the ex post inefficiency, is then less than the payoff loss from ex ante free-riding when no examination is set.

In general, if $c \leq b_T(0)$, setting a positive pass-mark has no benefit. The same is true if $c \geq b_1(0)$. To see that, observe by Lemma 3, $b_1(\mu) > b_2(\mu) > .. > b_T(\mu)$, for all $\mu \in [0, 1)$, and $b_1(\mu)$ is non-increasing in μ. Then, whether or not an examination is set, all experts choose low effort.

Therefore, an examination can only bring benefits if $c \in (b_T(\mu), b_1(\mu))$. Then, if μ is sufficiently small, we know from Lemma 3 and Proposition 3 that experts are indifferent between high and low effort, and choose high effort with probability $\sigma^*(\mu)$, where $\sigma^*(\mu)$ is the unique solution to

$$\sum\nolimits_{t=0}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} b_{t+1}(\mu) = c$$

Ex ante net expected payoff is then

$$\widetilde{\Pi}(c; \mu) = \sum\nolimits_{t=0}^{T} \binom{T}{t} \sigma^*(\mu)^t [1-\sigma^*(\mu)]^{T-t} \int_{\mu}^{1} x dF_t^1(x) - \sigma^*(\mu) c$$

And

$$\frac{d\widetilde{\Pi}}{d\mu} = \frac{c\mu f(\mu)[-(1-\sigma^*)^{T-1} + \sum_{t=1}^{T-1} \binom{T-1}{t} \sigma^{*t} (1-\sigma^*)^{T-t-1} F(\mu)^{t-1} \{\frac{t}{t+1} - F(\mu)\}]}{\sum_{t=0}^{T-2} \binom{T-2}{t} \sigma^{*t} (1-\sigma^*)^{T-t-2} [b_{t+1}(\mu) - b_{t+2}(\mu)]}$$

Then, for $\mu > 0$, yet sufficiently small, it is easy to show that the above expression is negative for c sufficiently close to $b_1(\mu)$. This is because, if $b_1(\mu) - c$ is small, experts choose high effort with a very low probability in the absence of an examination. Imposing a pass-mark in this situation depresses ex post payoff, and because of the high cost of effort, makes an increase in ex ante effort unattractive. If c is sufficiently close to $b_T(\mu)$, however, $\frac{d\widetilde{\Pi}}{d\mu} > 0$, and so an examination can make experts better off.

We have proceeded thus far assuming that experts always choose to play symmetric equilibrium strategies if the pass-mark is set to 0. This seems natural given the ex ante symmetry among experts. We further justify that focus below. We first show that examinations can still yield benefits if experts instead play asymmetric equilibrium strategies.

To see that, suppose $\mu = 0$ and $c = b_T(0) + \delta$, where δ is small and positive. Using Lemma 3, (10) and (11), we know that multiple pure strategy equilibria exist in this case, and that in any such equilibrium, $T - 1$ experts choose high effort and 1 expert chooses low effort.

Let aggregate ex ante net expected payoff of all experts, given any such equilibrium, be denoted as $\Pi^A(\delta)$. Using (10) and (11), we have

$$\Pi^A(\delta) = T \int_0^1 x dF^1_{T-1}(x) - (T-1)[b_T(0) + \delta]$$

Now suppose an examination is set with $\mu = \widehat{\mu}_T(c)$, where $\widehat{\mu}_T(c)$ is defined above. We know then from Lemma 4 that there is a unique equilibrium with all experts taking high effort. Let the aggregate ex ante net expected payoff in equilibrium to all experts be denoted as $\Pi^{AS}(\delta; \widehat{\mu}_T)$. Using the proof of Proposition 4, we see that

$$\Pi^{AS}(\delta; \widehat{\mu}_T) = T \int_{\widehat{\mu}_T}^1 x dF^1_{T-1}(x)$$

Since $\lim_{\delta \to 0} \widehat{\mu}_T = 0$, we see that

$$\lim_{\delta \to 0} [\Pi^{AS}(\delta; \widehat{\mu}_T) - \Pi^A(\delta)] = \lim_{\delta \to 0} [(T-1)\{b_T(0) + \delta\} - T \int_0^{\widehat{\mu}_T} x dF^1_{T-1}(x)] = (T-1)b_T(0) > 0$$

Hence we see that examinations can be advantageous even if experts do not necessarily always play symmetric equilibrium strategies.

Finally, we show that aggregate ex ante net expected payoff of all experts is strictly higher when symmetric equilibrium strategies are played than when asymmetric pure equilibrium strategies are played, for sufficiently small δ. If experts play pure strategies, aggregate payoff is given by $\Pi^A(\delta)$ above. If, on the other hand, they play symmetric equilibrium strategies, payoff is, using (4) through (6)

$$\Pi^S(\delta) = T[\sum_{t=0}^{T} \binom{T}{t} \sigma^t (1-\sigma)^{T-t} E_{F^1_t} - \sigma\{b_T(0) + \delta\}]$$

where σ is given by

$$\sum_{t=0}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} b_{t+1} = b_T(0) + \delta$$

We know that $\Pi^A(\delta)$ and $\Pi^S(\delta)$ are continuous in δ at $\delta = 0$, and $\lim_{\delta \to 0} \sigma = 1$. We find

$$\lim_{\delta \to 0} [\Pi^S(\delta) - \Pi^A(\delta)] = (T-1) b_T(0) > 0$$

Thus, aggregate ex ante net expected payoff of all experts is strictly higher when symmetric equilibrium strategies are played than when asymmetric pure equilibrium strategies are played, for sufficiently small δ.

6 Non-identical experts

So far, we have assumed that the experts are identical in all respects. In such a context, committing to an examination could benefit them by mitigating the ex ante free-riding problem. In this section, we briefly study whether our results hold when the experts are not identical. We study two sources of difference. First, the experts could have different costs of high effort, and second, they could have different preferences over policy choice. For simplicity, we shall assume throughout that $T = 2$. All qualitative results hold when the number of experts exceeds 2.

6.1 Asymmetric costs

Suppose first the two experts face different costs of high effort, but are otherwise identical ex ante, with the cost borne by $\mathcal{E}_i$ equal to c^i. Suppose without loss of generality $c^2 > c^1 > 0$. Given an examination with pass-mark $\mu \in [0, 1]$, using (7) and the same line of argument as in Proposition 3, it is easy to show that both experts take high effort if and only if $c^1 < c^2 \leq b_2(\mu)$, while neither takes high effort if and only if $c^2 > c^1 \geq b_1(\mu)$. When $c^2 \geq b_2(\mu)$ and $c^1 \leq b_1(\mu)$, if in addition $c^2 \leq b_1(\mu)$ and $c^1 \geq b_2(\mu)$, there are multiple equilibria. Two of these equilibria are in pure strategies; in each, one expert takes high effort, while the other takes low effort. There is also a mixed strategy equilibrium. If σ_i is the probability of $\mathcal{E}_i$ taking high effort in mixed strategy equilibrium, we have

$$\sigma_i = \frac{b_1(\mu) - c^j}{b_1(\mu) - b_2(\mu)}, i, j \in \{1, 2\}, i \neq j \tag{13}$$

If parameters satisfy none of these restrictions, i.e., if either $c^2 \in [b_2(\mu),\ b_1(\mu)]$ and $c^1 \leq b_2(\mu)$, or $c^2 \geq b_1(\mu)$ and $c^1 \leq b_1(\mu)$, there is a unique equilibrium in which $\mathcal{E}_1$ (the low-cost expert) takes high effort, while $\mathcal{E}_2$ (the high-cost expert) takes low effort.

Now let

$$c^1 = b_2(0) + \lambda_1\epsilon, c^2 = b_2(0) + \lambda_2\epsilon; \epsilon > 0, \lambda_2 > \lambda_1 > 0$$

For sufficiently small ϵ and μ, we know, by appropriately extending Lemma 3 and Proposition 3, a unique mixed strategy equilibrium exists in the date 1 effort choice game. Suppose experts select this equilibrium. Then $\mathcal{E}_i$'s net expected ex ante payoff is

$$\overline{\Pi}_i(\epsilon; \mu) = \sigma_j(\epsilon; \mu) b_1(\mu), i, j \in \{1, 2\}, i \neq j \tag{14}$$

where $\sigma_j(\epsilon; \mu)$ is given by (13). Then, using (7), (13), (14) and Lemma 3, we have, for ϵ and μ sufficiently small,

$$\frac{d\overline{\Pi}_i(\epsilon; \mu)}{d\mu} \cong \frac{\mu f(\mu) b_2(\mu)}{b_1(\mu) - b_2(\mu)} > 0$$

So an examination can improve experts' ex ante net expected payoff in this case as well.

6.2 Divergent preferences

We now analyse preference divergence among experts, when they have the same cost of high effort. We augment the model slightly. Suppose there are four possible policies, 0, 1, 2 and 3. As before, $\mathcal{D}$ chooses the policy. If policy 0 is chosen, $\mathcal{D}$, as well as the two experts receive 0 payoff. It is known that exactly one (say policy i) of the other three policies will yield payoff $-\beta$, $\beta > 0$, to all three players. Any of the remaining two (say j and k) will yield $\mathcal{D}$ some positive payoff (say $p > 0$). Of these two, exactly one (say j) will yield $\mathcal{E}_1$ payoff 1, and $\mathcal{E}_2$ payoff z, with $z \in [0,\ 1]$. The other (k) will yield $\mathcal{E}_1$ payoff z, and $\mathcal{E}_2$ payoff 1. The identities of these three policies, i.e., which of policies 1, 2 and 3 is i, and which are j and k, are not known for sure. Assume that β is sufficiently large so that if no additional information is obtained, policy 0 will be chosen.

The above is a simple extension of the model used till now. With $z = 1$, it reduces to the earlier model, but with $z < 1$, the two experts have divergent preferences over policy. The current model allows preference divergence across experts, but continues to assume there are no conflicts over policy choice between the decision-maker and any of the experts.

Suppose $\mathcal{E}_i$ is appointed advisor at date 3 with ability α_i. She receives no additional information with probability $1 - \alpha_i$. With the complementary probability, assume she observes the identities of i, j and k. In such a case, we assume, since $\mathcal{D}$ and the advisor have no preference conflict at this stage, that $\mathcal{D}$ chooses the advisor's preferred policy.

It is clear that at date 3, if $\mathcal{D}$ appoints an advisor, he will choose the expert with greater ability. Suppose no examination has been instituted. Then, at date 1, if $\mathcal{E}_i$ anticipates that $\mathcal{E}_j$ will choose high effort with probability σ_j, her payoffs from choosing high and low effort levels are respectively

$$(1 - \sigma_j)E_{F_1^1} + \frac{\sigma_j}{2}(1 + z)E_{F_2^1} - c \text{ (high effort)}; \ \sigma_j z E_{F_1^1} \text{ (low effort)}$$

When $z < 1$, the experts' ex post policy preferences are divergent. Since, when an advisor receives additional information, $\mathcal{D}$ chooses the advisor's favoured policy, each expert now prefers that she herself becomes the advisor. Hence, in the presence of preference divergence, there is competition between the experts for obtaining the advisory appointment. This competition mitigates the free-riding problem, and hence has a tendency to increase ex ante effort. Free-riding remains a concern however, as long as $z > 0$, because of the positive externality stemming from an advisor's effort choice. Indeed, it is easy to show, using the same logic as in Proposition 2, that each expert retains a tendency to free-ride whenever $z > 0$. Let, using (1) and (7)

$$b_2(0; z) = b_1 - \frac{(1 + z)}{2}(b_1 - b_2)$$

We see that $b_2(0; z)$ is decreasing in z, and $b_2(0; 0) < b_1$. Then, using arguments along the lines of Proposition 3, it is easy to show that a unique symmetric equilibrium always exists in the date 1 effort choice game. Both experts take high effort in equilibrium if and only if $c \leq b_2(0; z)$, while both take low effort if and only if $c \geq b_1(0)$. When $c \in [b_2(0; z),\ b_1(0)]$, a mixed strategy equilibrium exists with each expert taking high effort with probability

$$\sigma = \frac{2(b_1 - c)}{(1+z)(b_1 - b_2)} \tag{15}$$

Suppose $c > b_2(0; z)$, with $c - b_2(0; z) = \delta$, and δ small. Can $\mathcal{D}$ then induce the experts to take high effort with an examination, and can that improve the experts' net expected ex ante payoff, when $z < 1$? If no examination is set, then an expert's ex ante net expected payoff is $\sigma z b_1$, which approaches, as $\delta \to 0$, $z b_1$, using (15). Further, using (15) again

$$\lim_{\delta \to 0} \frac{d(\sigma z b_1)}{d\delta} = -\frac{2 z b_1}{(1+z)(b_1 - b_2)} \tag{16}$$

Now suppose $\mathcal{D}$ wishes to set an examination with pass mark μ. Let

$$\widetilde{\mu}_2(z) = F^{-1}(\frac{z}{1+z}) \in (0, 1)$$

It is straightforward to show, using the arguments of Lemma 4 and Proposition 4, that if δ is small, there exists $\widehat{\mu}_2(c; z) \in (0, \widetilde{\mu}_2(z)]$, such that for every $\mu \in [\widehat{\mu}_2(c; z), \widetilde{\mu}_2(z)]$, the unique equilibrium involves both experts choosing high effort at date 1. Suppose $\mathcal{D}$ sets $\mu = \widehat{\mu}_2(c; z)$. An expert's net expected ex ante payoff is

$$z \int_{\widehat{\mu}_2(c;z)}^{1} x dF_1^1(x) \text{ and } \lim_{\delta \to 0} z \int_{\widehat{\mu}_2(c;z)}^{1} x dF_1^1(x) = z b_1$$

Also,

$$\lim_{\delta \to 0} \frac{z d \int_{\widehat{\mu}_2(c;z)}^{1} x dF_1^1(x)}{d\delta} = -1 \tag{17}$$

Comparing (16) and (17), we find that an examination can make the experts better off when δ is small if

$$-\frac{2 z b_1}{(1+z)(b_1 - b_2)} < -1 \Leftrightarrow z > \frac{b_1 - b_2}{b_1 + b_2}$$

Hence if z is very small, such an examination cannot make the experts better off. This is simply because, when z is small, there is substantial competition between the experts for the advisory position. Since the level of free-riding is then low, the ability of an examination to increase ex ante effort is limited, and so it may be better not to impose one at all. Another way of seeing this is by looking at the expression determining $\widehat{\mu}_2(c; z)$.

Using Propositions 3 and 4, we find that $\widehat{\mu}_2(c; z)$ is the lowest $\mu \in (0, 1)$ satisfying the relationship

$$\frac{(1+z)}{2}\int_{\mu}^{1} x dF_2^1(x) - c = z\int_{\mu}^{1} x dF_1^1(x)$$

Then

$$\frac{d\widehat{\mu}_2(c; z)}{dz} = \frac{2\int_{\widehat{\mu}_2(c;z)}^{1} x[1 - F(x)]f(x)dx}{\widehat{\mu}_2(c; z)f(\widehat{\mu}_2(c; z))[z - (1+z)F(\widehat{\mu}_2(c; z))]} > 0$$

In other words, given δ small, the greater the degree of conflict or divergence between experts, the lower is the pass mark needed to induce both experts to take high effort. Thus, conflict induces competition between the experts, and so can reduce free-riding ex ante.

7 Conclusion

The ability profile of bureaucrats is an important factor driving organisational performance. Abilities can influence policy choices, which in turn govern outcomes, and payoffs of members of the bureaucracy. When individual ability is endogenously determined through private effort, a free-riding problem may be generated in such settings if agents share similar preferences over policy choice. With limited preference divergences, an increase in an individual's ability has a public benefit, and each member therefore has a private incentive to underinvest in building ability. This lowers overall ability, worsens the quality of policy-making, and hence organisational outcomes. An examination with a pass-mark, i.e., a minimum ability requirement for an agent to participate in decision-making, can mitigate this free-riding problem, and benefit the organisation. Such an examination is ex post inefficient, as bureaucrats with positive ability may be excluded from decision-making. However, since it imposes a joint penalty on all members when ability levels are low, it can increase ex ante effort, and thereby improve the quality of policy-making. Overall, we show that if the extent of free-riding in the absence of an examination is not too severe, an examination can benefit the organisation.

This is always true when the members have perfectly aligned preferences. With a limited number of decision-making positions, preference divergences across agents can induce competition between them for access to such positions if advisory appointment gives direct influence over policy choice. By reducing free-riding, this competition can yield benefits. An examination may still be useful in such an environment. However, since competition induced by preference divergence acts as a substitute for examination-based incentives, the benefits of examinations are limited in the presence of such preference divergences. As discussed earlier, these benefits are also limited if an expert derives utility not only from policy direction, but also directly from being the advisor. If preference divergences or utility from occupying a particular position are more important in higher tiers of an organisation, then examinations may more commonly prevail in lower organisational tiers. Relating the structure of examinations to the hierarchical structure of organisations may therefore be an issue of future research interest.

Examinations are common in many settings, with selection and promotion of agents often dependent on examination results. It may be interesting to study the incentive effects of different kinds of examinations, or examination-based selection rules, in other environments such as when candidate performances are endogenously linked through joint effort, or when information on performance is not symmetric. Such issues are left for future research.

8 Appendix

A: Proofs

Proof of Lemma 3. (a i) and (a ii) are obvious. To prove (a iii), we first show that $b_t > 0$. We recall, from standard results on order statistics, that

$$\text{If } t \geq 1, F_t^1 = F^t, \text{ so } f_t^1 = tfF^{t-1}, \text{ and, if } t \geq 2,\ F_{t-1}^1 - F_t^1 = F^{t-1}(1-F)$$

F_t^1 therefore first order stochastic dominates F_{t-1}^1, and so using (1), $b_t > 0$, for all $t \in \{1,...,T\}$.

(a iii) then follows from (a i), (a ii) and (b). To prove (b), we use the results on order statistics described earlier, and obtain

$$b_1'(\mu) = -\mu f(\mu), \text{ so } b_1'(0) = 0 \text{ and } b_1'(\mu) < 0 \text{ if } \mu \in (0, 1]$$

proving (b i). For $t \in \{2,..., T\}$, we find

$$b_t'(\mu) = t\mu f(\mu)F(\mu)^{t-2}[\frac{t-1}{t} - F(\mu)]$$

So we see that $b_t'(\mu) = 0$ if $\mu \in \{0,\ F^{-1}(\frac{t-1}{t})\}$, $b_t'(\mu) > 0$ if $\mu \in (0,\ F^{-1}(\frac{t-1}{t}))$, and $b_t'(\mu) < 0$ if $\mu \in (F^{-1}(\frac{t-1}{t}),\ 1]$, completing the proof of (b). To prove (c), we recall that

$$\begin{aligned} \text{If } t &\geq 2, F_t^2 = tF^{t-1}[1 - (\frac{t-1}{t})F], \text{ so } f_t^2 = t(t-1)fF^{t-2}(1-F), \\ \text{and, if } t &\geq 3,\ F_{t-1}^2 - F_t^2 = (t-1)F^{t-2}(1-F)^2 \end{aligned}$$

Let $t \in \{1,..., T-1\}$, and $\mu < 1$. Then

$$b_t(\mu) > b_{t+1}(\mu)$$

$$\Leftrightarrow \int_\mu^1 xF(x)^{t-1}[(t+1)F(x) - t]f(x)dx < \int_\mu^1 xF(x)^{t-2}[tF(x) - (t-1)]f(x)dx$$

$$\Leftrightarrow \int_\mu^1 xF(x)^{t-2}[\{1 - F(x)^2\} - t\{1 - F(x)\}^2]f(x)dx > 0$$

$$\Leftrightarrow \int_\mu^1 xF(x)^{t-2}[1 - F(x)][(t+1)F(x) - (t-1)]f(x)dx > 0$$

$$\Leftrightarrow t(t+1)\int_\mu^1 xF(x)^{t-1}[1 - F(x)]f(x)dx > (t-1)t\int_\mu^1 xF(x)^{t-2}[1 - F(x)]f(x)dx$$

$$\Leftrightarrow \int_\mu^1 xdF_{t+1}^2(x) > \int_\mu^1 xdF_t^2(x)$$

But, we see after integrating by parts,

$$\int_\mu^1 xdF^2_{t+1}(x) - \int_\mu^1 xdF^2_t(x) > 0$$

$$\Leftrightarrow \mu[F^2_t(\mu) - F^2_{t+1}(\mu)] - \int_\mu^1 [F^2_{t+1}(x) - F^2_t(x)]dx > 0$$

which is true, since F^2_{t+1} first order stochastic dominates F^2_t, by the results on order statistics above. The proof is thus complete. Setting $\mu = 0$, we see that parts (a) and (c) of this lemma imply Lemma 1. ■

Proof of Proposition 3. It is clear that no expert wishes to invest any effort if $\mu = 1$. So let $\mu < 1$. $b_t(\mu) > 0$, for $t \in \{1,..., T\}$, and $b_1(\mu) > b_2(\mu) > .. > b_T(\mu)$, by Lemma 3. From (7) and (8), we know that all experts choose high effort in equilibrium if and only if $c \leq b_T(\mu)$. Further, from (7) and (9), we know that all experts choose low effort in equilibrium if and only if $c \geq b_1(\mu)$. So whenever a pure strategy symmetric equilibrium exists, it is the unique pure strategy symmetric equilibrium. Asymmetric pure strategy equilibria also exist. From (7), (10) and (11), a $t-$equilibrium, $t \in \{1,..., T-1\}$, exists if and only if $c \in [b_{t+1}(\mu),\ b_t(\mu)]$. To complete the proof, we now study mixed strategy equilibrium, and show that a unique symmetric equilibrium exists in mixed strategies when $c \in [b_T(\mu),\ b_1(\mu)]$.

Let $\sigma \in [0,\ 1]$. Consider the expression

$$S(\sigma; \mu) = \sum\nolimits_{t=0}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} b_{t+1}(\mu)$$

Then, using Lemma 3

$$\frac{dS(\sigma; \mu)}{d\sigma} = -(T-1) \sum\nolimits_{t=0}^{T-2} \binom{T-2}{t} \sigma^t (1-\sigma)^{T-t-2} [b_{t+1}(\mu) - b_{t+2}(\mu)] < 0$$

For an expert to be indifferent between choosing high and low effort, given all other experts are choosing high effort with probability σ, we need

$$\sum\nolimits_{t=1}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} \int_\mu^1 xdF^1_t(x) =$$

$$\sum\nolimits_{t=0}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} \int_\mu^1 xdF^1_{t+1}(x) - c$$

$$\text{or, } S(\sigma;\mu) = c$$

We see that $S(\sigma;\mu)$ is strictly decreasing in σ, $\sigma = 0$ when $c = b_1(\mu)$, and $\sigma = 1$ when $c = b_T(\mu)$. Thus a unique symmetric equilibrium exists in mixed strategies when $c \in [b_T(\mu), b_1(\mu)]$. Setting $\mu = 0$, we see that Proposition 1 is also proved. ■

Proof of Lemma 4. (a) From (7), (12) and Lemma 3, we know that $b_T(\mu)$ has a unique maximum on [0, 1], and the maximiser is $\widetilde{\mu}_T$. Since $b_T(\mu)$ is strictly increasing in μ for $\mu \in (0, \widetilde{\mu}_T)$, $b_T(\widetilde{\mu}_T) > b_T(0)$. Furthermore, $b_1(\mu)$ is strictly decreasing in μ for $\mu > 0$, and $b_1(\mu) > b_T(\mu)$ for $\mu < 1$, so $b_T(\widetilde{\mu}_T) < b_1(0)$.

(b) We see from Lemma 3 and part (a) above that for every $c \in [b_T(0), b_T(\widetilde{\mu}_T))$, the relation

$$b_T(\mu) = c$$

has two solutions on [0, 1], with the lesser of the two solutions lying in $[0, \widetilde{\mu}_T)$. Let this solution be denoted $\widehat{\mu}_T(c)$. We see that $\widehat{\mu}_T(c)$ is monotone increasing in c, approaches 0 as c approaches $b_T(0)$, and approaches $\widetilde{\mu}_T$ as c approaches $b_T(\widetilde{\mu}_T)$. Further, for every $\mu \in (\widehat{\mu}_T(c), \widetilde{\mu}_T]$,

$$b_T(\mu) > c$$

and so all experts choose high effort in equilibrium, by Proposition 3.

Further, if $c = b_T(\widetilde{\mu}_T)$, then $\widehat{\mu}_T(c) = \widetilde{\mu}_T$, and all experts choose high effort in equilibrium if and only if $\mu = \widetilde{\mu}_T$. ■

Proof of Proposition 4. Given $\delta > 0$, suppose $\mu = 0$. Using (1) and (4) through (6), Proposition 1, and Lemma 4, we know any expert's ex ante net expected payoff is

$$\widehat{\Pi}(\delta;\sigma) = \sum\nolimits_{t=0}^{T} \binom{T}{t} \sigma^t (1-\sigma)^{T-t} E_{F_t^1} - \sigma[\delta + b_T(0)]$$

where σ is the unique solution to

$$\sum\nolimits_{t=0}^{T-1} \binom{T-1}{t} \sigma^t (1-\sigma)^{T-t-1} b_{t+1} = \delta + b_T(0)$$

Then

$$\frac{d\widehat{\Pi}(\delta;\sigma)}{d\delta} = (T-1)[\delta + b_T(0)]\frac{\partial\sigma}{\partial\delta} - \sigma$$

and

$$\frac{\partial\sigma}{\partial\delta} = -\frac{1}{(T-1)\sum_{t=0}^{T-2}\binom{T-2}{t}\sigma^t(1-\sigma)^{T-t-2}(b_{t+1}-b_{t+2})} < 0$$

Now, $\sigma \to 1$ as $\delta \to 0$, and so

$$\lim_{\delta\to 0}\frac{d\widehat{\Pi}(\delta;\sigma)}{d\delta} = -[1 + \frac{b_T(0)}{b_{T-1}(0) - b_T(0)}] < -1$$

Also, using (1)

$$\lim_{\delta\to 0}\widehat{\Pi}(\delta;\sigma) = E_{F_T^1} - b_T(0) = E_{F_{T-1}^1}$$

With $\mu = \widehat{\mu}_T(c)$, an expert's ex ante net expected payoff is, using (7) and (8), and the definition of $\widehat{\mu}_T(c)$

$$\widehat{\Pi}(\delta;1,\widehat{\mu}_T) = \int_{\widehat{\mu}_T}^1 x dF_T^1(x) - [\delta + b_T(0)] = \int_{\widehat{\mu}_T}^1 x dF_{T-1}^1(x)$$

Therefore

$$\lim_{\delta\to 0}\widehat{\Pi}(\delta;1,\widehat{\mu}_T) = E_{F_{T-1}^1}, \text{ as } \lim_{\delta\to 0}\widehat{\mu}_T(c) = 0$$

Using the definitions of $\widehat{\Pi}(\delta;1,\widehat{\mu}_T)$ found above and $\widehat{\mu}_T$ from Lemma 4, we see that

$$\frac{\partial\widehat{\mu}_T(c)}{\partial\delta} = \frac{1}{T\widehat{\mu}_T f(\widehat{\mu}_T)F(\widehat{\mu}_T)^{T-2}[\frac{T-1}{T} - F(\widehat{\mu}_T)]}$$

and

$$\frac{\partial\widehat{\Pi}(\delta;1,\widehat{\mu}_T)}{\partial\widehat{\mu}_T} = -(T-1)\widehat{\mu}_T f(\widehat{\mu}_T)F(\widehat{\mu}_T)^{T-2}$$

So

$$\lim_{\delta\to 0}\frac{d\widehat{\Pi}(\delta;1,\widehat{\mu}_T)}{d\delta} = -\lim_{\delta\to 0}(\frac{T-1}{T})[\frac{T-1}{T} - F(\widehat{\mu}_T)]^{-1} = -1$$

Summarising the results, we find

$$\lim_{\delta\to 0}\widehat{\Pi}(\delta;1,\widehat{\mu}_T) = E_{F^1_{T-1}} = \lim_{\delta\to 0}\widehat{\Pi}(\delta;\sigma)$$

and

$$\lim_{\delta\to 0}\frac{d\widehat{\Pi}(\delta;1,\widehat{\mu}_T)}{d\delta} = -1 > \lim_{\delta\to 0}\frac{d\widehat{\Pi}(\delta;\sigma)}{d\delta} = -[1+\frac{b_T(0)}{b_{T-1}(0)-b_T(0)}]$$

Thus, if c is sufficiently close to $b_T(0)$, then imposing an examination with pass-mark $\widehat{\mu}_T(c)$ increases any expert's ex ante net expected payoff compared to the situation when no examination is set. ■

B: Symmetric solution in the planner's problem

Let P^t_T, $t \in \{0, ..., T\}$, denote the probability that t out of T experts take high effort, when any $\mathcal{E}_i$ invests with probability $\sigma_i \in [0, 1]$. Now consider the date 1 problem of choosing investment probabilities $(\sigma_i)^T_{i=1}$ for the experts such that the sum of ex ante net expected payoffs is maximised. Since $E_{F^1_0} = 0$, the problem can be written as

$$I : \max_{\sigma_1,...\sigma_T} T\sum\nolimits_{i=1}^{T} P^t_T E_{F^1_t} - c\sum\nolimits_{i=1}^{T}\sigma_i$$

Suppose the solution is $(\widehat{\sigma}_i)^T_{i=1}$. Pick any two experts, say 1 and 2. Suppose $\widehat{\sigma}_1 + \widehat{\sigma}_2 = \widehat{\tau}$, and let $\widehat{P}^t_T$ denote the probability that t out of T experts take high effort, given that $\mathcal{E}_1$ and $\mathcal{E}_2$ take high effort with some probabilities σ_1 and σ_2 respectively, while any other expert $\mathcal{E}_i$ takes high effort with probability $\widehat{\sigma}_i$. The restricted problems below are then equivalent and both have the same solution: $\sigma_1 = \widehat{\sigma}_1$, $\sigma_2 = \widehat{\sigma}_2$.

$$\begin{aligned} II &: \max_{\sigma_1,\sigma_2} T\sum\nolimits_{i=1}^{T}\widehat{P}^t_T E_{F^1_t} - c(\sigma_1+\sigma_2+\sum\nolimits_{i=3}^{T}\widehat{\sigma}_i) \\ s.t. &: \sigma_1+\sigma_2 = \widehat{\tau} \end{aligned}$$

$$\begin{aligned} III &: \max_{\sigma_1,\sigma_2} \sum\nolimits_{i=1}^{T}\widehat{P}^t_T E_{F^1_t} \\ s.t. &: \sigma_1+\sigma_2 = \widehat{\tau} \end{aligned}$$

We now show that $\widehat{\sigma}_1 = \widehat{\sigma}_2 = \frac{\widehat{\tau}}{2}$. It suffices to show, from Problem III, that $\widehat{P}_T^t$ is maximised when $\widehat{\sigma}_1 = \widehat{\sigma}_2$, for all $t \in \{1, ..., T\}$. Let $\widehat{P}^k$ denote the probability that k out of the remaining experts (3 through T), $k \in \{0, ..., T-2\}$, take high effort.

First suppose $T \geq 4$, $t \notin \{1, T-1, T\}$. Consider the following problem:

$$\begin{aligned} IV &: \max_{\sigma_1, \sigma_2} \widehat{P}_T^t \\ s.t. &: \sigma_1 + \sigma_2 = \widehat{\tau} \end{aligned}$$

The problem can be rewritten as

$$\max_{\sigma_1} (1-\sigma_1)(1-\widehat{\tau}+\sigma_1)\widehat{P}^t + \{\sigma_1(1-\widehat{\tau}+\sigma_1) + (1-\sigma_1)(\widehat{\tau}-\sigma_1)\}\widehat{P}^{t-1} + \sigma_1(\widehat{\tau}-\sigma_1)\widehat{P}^{t-2}$$

The first-order condition is

$$(\widehat{\tau} - 2\sigma_1)(\widehat{P}^t - 2\widehat{P}^{t-1} + \widehat{P}^{t-2}) = 0$$

Thus, $\widehat{\sigma}_1 = \widehat{\sigma}_2 = \frac{\widehat{\tau}}{2}$. A similar method can be used to derive the same conclusion when $T \geq 3$, $t \in \{1, T-1, T\}$,and also when $T = 2$, $t \in \{1, 2\}$. Since experts 1 and 2 were chosen arbitrarily, we conclude that $\widehat{\sigma}_i = \widehat{\sigma}_j \ \forall \ i, j$.

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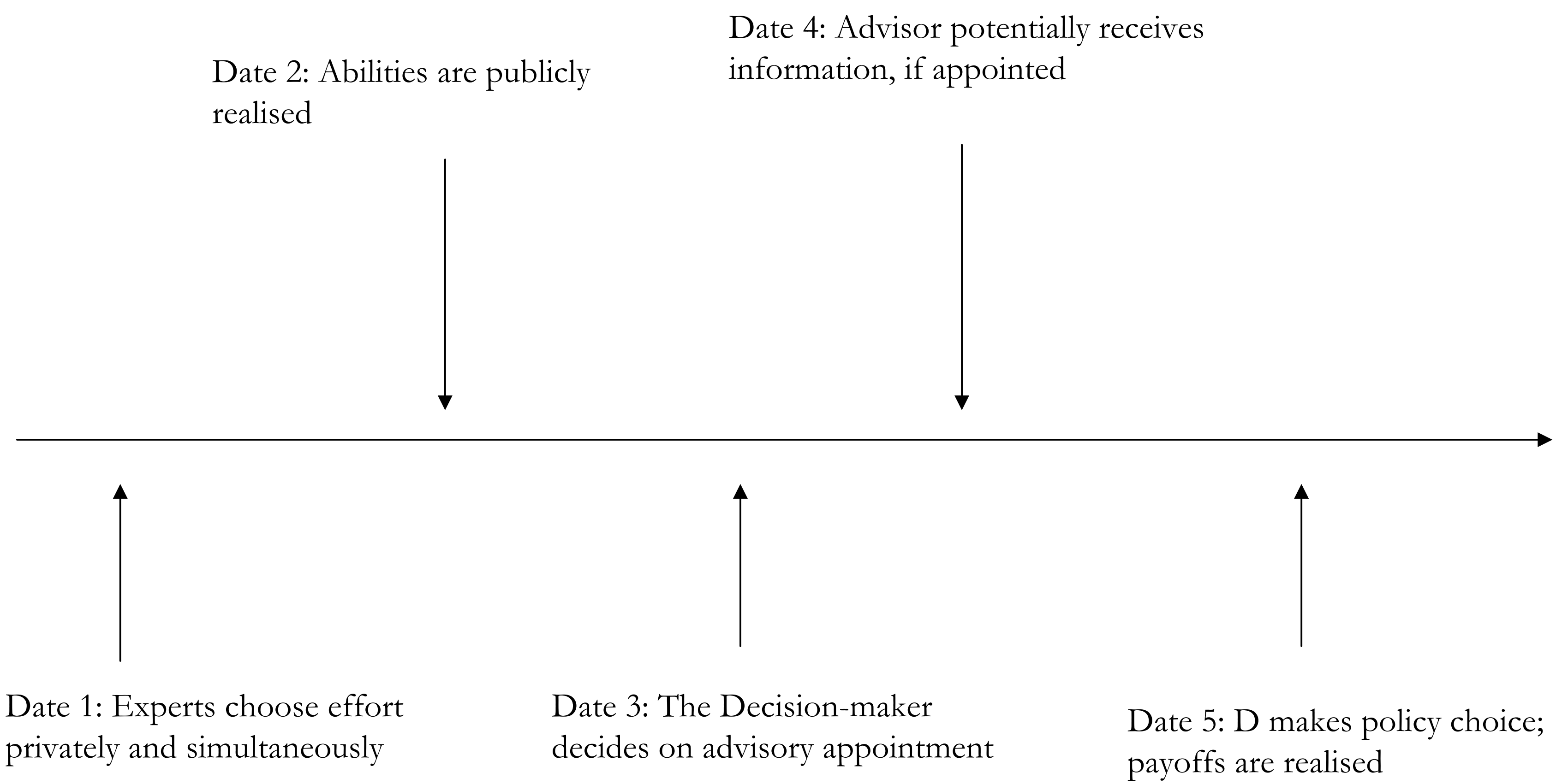
Date 2: Abilities are publicly realised
Date 4: Advisor potentially receives information, if appointed
Date 1: Experts choose effort privately and simultaneously
Date 3: The Decision-maker decides on advisory appointment
Date 5: D makes policy choice; payoffs are realised

Figure 1