A stochastic manpower planning model under varying class sizes

Asis Kumar Chattopadhyay · Arindam Gupta

Abstract Solution related to different types of manpower planning problems arising in different industries and organizations are very much helpful for proper planning and implementation of different objectives. Previously those type of problems are mostly solved under the deterministic set up. Gradually several scientists have developed different types of stochastic models appropriate for solving such types of problems. The present study is an attempt to develop a stochastic manpower planning model under the set up where the classes are of varying sizes and promotion occurs only on the basis of seniority.

Keywords Manpower planning · Stochastic · Promotion · Probability distribution

1 Introduction

One of the important aspects of manpower planning relates to the study of social systems that are subjected to continuous transformations. These changes may come within the system itself as a need of the hour, or may well be dictated by the circumstances prevailing outside the system. In order to have a better understanding of the turmoil within the system, whatever may be the cause of it, one has to observe the system through its constituents. Study of manpower corresponding to any such system or an organization in particular, therefore necessarily boils down to observing its members moving along various grades or set of classes.

Taking up the case of an organization, the size of these grades is fixed by the budget or amount of work to be done at each level. Recruitment and promotion can only occur

The work of second author was supported by a research fellowship from Council of Scientific and Industrial Research (Sanction No. 9/28(611)/2003-EMR-I), India.

A.K. Chattopadhyay (✉)
Calcutta University, 35, B.C. Road, Kolkata 700019, India
e-mail: chattopadhyayasis@hotmail.com

A. Gupta
Department of Statistics, Calcutta University, 35, B.C. Road, Kolkata 700019, India
e-mail: arindamdeep@yahoo.com

when vacancies arise through leaving the system or expansion of the same. These vacancies thereafter generate a sequence of internal movements within the organization similar to the changes mentioned above. McClean (1978, 1980) and Abodunde and McClean (1980) has considered several manpower planning situations and developed models based on Markov and Semi-Markov processes. The present work aims at explaining these movements under a probabilistic set up by restricting ourselves within the framework of an organization characterizing such social systems.

In order to study the changes an organization is going through in terms of the recruitment and promotion to fill up the available vacancies, one of the available approaches is replacement theory. Sasieni et al. (1959) discussed many of such problems under the deterministic set up. Originally such problems have arisen in connection to the renewal of human population through births and deaths. In recent years the main application of this type of problems found its way in the context of industrial replacement and reliability theory.

Mukherjee and Chattopadhyay (1989) developed stochastic solution to the problems related to recruitment and promotion of staff members in case of an airline. Recently Chattopadhyay and Khan (2004) have extensively studied the nature of job changes of staff members of a university on the basis of stochastic modelling. Khan and Chattopadhyay (2003) have also derived corresponding prediction distribution on the basis of job offers received by the employees. These types of works are very useful to investigate the manpower planning condition prevailing in different organizations. The present study attempts to develop stochastic solution to the classical staffing problem under a more general type.

2 Staffing problem

The primary objective of a staffing problem is to study the dynamics of an organization through the movements of its employees over different categories. Problems related to the recruitment and promotion within an organization involving its members lead themselves to analysis similar to that used in industrial replacement. The key random variable in studying the recruitment and promotion pattern prevailing in an organization is the time interval an entity remains active in a particular grade. The age distribution corresponding to these grades while recruiting and promoting an individual is relevant to the organization for its manpower planning.

2.1 Parameters of the study

$r =$ Number of graded job categories in an organization.

$k(t) =$ Total size of the organization at a particular time point t.

$k_i(t) =$ Number of workers to be recruited at the ith category as decided by the management previously corresponding to the time point t.

$x_h =$ Initial recruitment age (fixed for all employees).

$x_w =$ Retirement age (fixed for all employees).

$\lambda_0'(t) =$ The number of persons recruited each year at age x_h at a particular time point t and at the first category.

$\lambda_x(t) =$ The number of persons of age x in service at a particular time point t.

$p_x(t) =$ The proportion of the population at age x that continues to be in service at age $(x+1)$ (survival rate from age x to age $(x+1)$) at a particular time point t.

$Z_x(t) =$ The number of persons at age x who survive up to age $(x+1)$ at a particular time point t.

$x_{i-1}(t) =$ The minimum possible age at which $(i-1)$th promotion takes place at a particular time point t.

$p_{0x}(t) =$ Probability of survival of a person at age x in the system at a particular time point t.

Here $p_x(t)$ can be interpreted as the conditional probability that a person in service at age x will be in service at age $(x+1)$ at a particular time point t while $p_{0x}(t)$ can be interpreted as the unconditional probability that a person will survive in the system up to age x at a particular time point t.

2.2 Characteristics of the organization

Depending on the circumstances related to the organization that we are interested in, different assumption with respect to the parameters of the system are made. The various grades constituting the system are categorized on the basis of remuneration offered and promotion occur on the basis of seniority from a category which is just below category where promotion would take place ignoring any efficiency aspect since such a set up is very common in different organizations. The total size of such a system ($k(t) = k_i(t)$) is time dependent. The initial age of recruitment (x_b) and the age of retirement (x_w) are fixed. All employees will be recruited and retired at the same age. The number of staff members at age x ($\lambda_x(t)$) will be determined by voluntary and involuntary wastage. Involuntary wastage include losses due to death, ill-health and retirement. Voluntary wastage arises out of withdrawals by individuals on their own. It is also assumed that there is no demotion in the system. The survival rates from age x to age $(x+1)$ ($p_x(t)$) are expected to depend on both age and grade. Under the present set up we have considered them to be depended only on age.

Given the $\lambda_x(t)$ values and the staff strength requirement at the different grades,one can easily find out exact ages at which promotions to different categories should take place. Solution of the above problem under stochastic set up have been developed by Mukherjee and Chattopadhyay (1989). A more general form of the above problem is the situation where the age distribution of the staff members at different categories are known at a particular time point and the number of workers to be increased or decreased in the next few years can also be estimated. The present work attempts to develop a stochastic solution of such a problem. The problem and its classical solution have been illustrated in terms of the following example by Sasieni et al. (1959).

An airline requires 200 assistant flight attendants, 300 flight attendants and 50 supervisors. Assistant flight attendants are recruited at age 21 and, if still in service, retire at age 60. It is estimated that for the next year staff requirements will increase by 10% . If persons are to be recruited age 21, at what ages will promotions take place? The survival rates and age distribution are given in Tables 1 and 2 (Sasieni et al. 1959).

2.3 Solution under deterministic set up:

We multiply the number of staff at each age by the survival rate p_x thus obtaining the age distribution one year later.

For example, the number of supervisors in one year's time will be:

$5 \times 0.947 = 4.735$, Aged 43,
$4 \times 0.942 = 3.768$, Aged 44,
$5 \times 0.936 = 4.68$, Aged 45,
$3 \times 0.930 = 2.79$, Aged 46,
$3 \times 0.923 = 2.769$, Aged 47,

Table 1 Survival rates at different ages

x	p_x	x	p_x
21	0.600	41	0.952
22	0.800	42	0.947
23	0.800	43	0.942
24	0.800	44	0.936
25	0.800	45	0.930
26	0.850	46	0.923
27	0.875	47	0.915
28	0.900	48	0.906
29	0.925	49	0.896
30	0.950	50	0.885
31	0.958	51	0.873
32	0.962	52	0.860
33	0.965	53	0.846
34	0.967	54	0.831
35	0.968	55	0.815
36	0.968	56	0.798
37	0.968	57	0.780
38	0.965	58	0.761
39	0.961	59	0.741
40	0.957		

and so on. By proceeding in this way we finally get the total number (rounded) as 43.

Thus, in a year's time there will be 43 remaining supervisors. Since total of 55 ($50 + 10\%$ of 50) will be required, 12 persons have to be promoted from the flight attendant group. Assuming that promotion is based solely on age, the most senior 12 flight attendants will be promoted. In a year's time, there will be

$$6 \times 0.952 = 5.712 \text{ flight attendant aged } 42$$

$$8 \times 0.957 = 7.656 \text{ flight attendant aged } 41.$$

Thus, these two age groups together (13.368) will be more than suffice to fill the vacancies for supervisors. One flight attendant aged 41 will not be promoted. In a similar fashion, we can compute the number of remaining flight attendants and thus obtain the number of assistants to be promoted. Finally, we obtain the number of recruits required by subtraction.

3 Probabilistic formulation

Since human behavior depends on several factors it is natural to consider the event of "withdrawal" as a random event. As a result, the survival rates $p_x(t)$ and the number of persons at different ages $\lambda_x(t)$ should be treated as random. Empirical findings have also established beyond doubt that the completed length of service (CLS) is unpredictable and should be treated as random variable. The numbers $\lambda_x(t)$ observed in any organization should be treated as random samples from the corresponding CLS distribution depending upon the $p_x(t)$ values.

Table 2 Staff age distribution at a particular time point

Assistant flight attendants		Flight attendants		Supervisors	
Age	Number	Age	Number	Age	Number
21	90	26	40	42	5
22	50	27	35	43	4
23	30	28	35	44	5
24	20	29	30	45	3
25	10	30	28	46	3
Total	200	31	26	47	3
		32	20	48	6
		33	18	49	2
		34	16	50	0
		35	12	51	0
		36	10	52	4
		37	8	53	3
		38	0	54	5
		39	8	55	0
		40	8	56	3
		41	6	57	2
		Total	300	58	0
				59	2
				Total	50

In the above context of unpredictable numbers remaining in service at different ages, promotions should be given at ages which themselves may have to be random in order to meet the staff-strength requirements which may also be random. All this implies that the life table functions as regarded deterministic previously should be looked upon as random variables, amenable to established statistical analysis.

Let $Z_x(t)$ be the numbers of persons at age x who survive up to age $x+1$, then $Z_x(t) = [\lambda_x(t)p_x(t)]$ =rounded value of the product $\lambda_x(t)p_x(t)$.

Hence the actual number of persons in category i is given by

$$\sum_{x=x_{i-1}(t)}^{x_i(t)-1} Z_x(t), \tag{1}$$

where $x_{i-1}(t)$ is the minimum age at which $(i-1)$th promotion takes place and the range of x values corresponds to only those persons who have got their promotions at age $x_{i-1}(t)$. The above quantity is a random sum of random variables.

Since the staff strength in category i should not exceed $k_i(t)$ we have the condition

$$\sum_{x=x_{i-1}(t)}^{x_i(t)-1} Z_x(t) < k_i(t). \tag{2}$$

Here $x_i(t)$ determines the minimum age at which ith promotion takes place. In case the number of persons to be promoted is larger than the requirement, the management may promote on the basis of date of joining or similar other criteria. Although the persons join their services at same age and at some time point, these ages correspond to age at last birth day. Under the above situation, management may go to more micro level by considering year, month, and day of birth. Of course this solution is quite ad hoc and crude but it is difficult to find a better solution. Under the probabilistic set up, the above condition only may be satisfied with a high probability. In order to get a solution, we have to make some assumptions regarding the underlying population distributions of the different random variables involved in the model. We may assume under general set up

$$p_x(t) \sim Beta(a_x, b_x).$$

Since the data set is available only for a particular time point we have to assume that the distributions of the different random variables involved are time homogeneous. Otherwise the related parameters can not be estimated from data.

Under this assumption

$$p_x(t) \sim Beta(a, b)$$

as p_x takes values between 0 and 1 and $p_x(t)$ values gradually increase up to a value of x and then start decreasing.

Given λ_0',

$$\lambda_x(t) \sim Binomial(\lambda_0', p_{0x})$$

(according to Chiang 1968).

Here it is also quite reasonable to assume that

$$k_i(t) \sim Poisson(m_i)$$

since for large number of categories the probability that a particular person belongs to the ith category is very small.

The problem is to find $x_i(t)$ in such a way that the probability of the random event given in (1) is sufficiently large.

The ages at which successive promotions take place cannot be exactly predicted. Thus $x_1(t)$ (i.e. the age at which the first promotion takes place) can be treated as a random variable distributed over (x_h, x_w) with a reasonable value of the average. Without losing much significance, we may reasonably assume (Mukherjee and Chattopadhyay 1989) $x_1(t)$ to have a geometric distribution with probability mass function

$$f(x_1(t)) = pq^{x_1(t)-1}, \quad x_1(t) = 1, 2, \ldots, \infty.$$

In general, $x_i(t)$ (i.e. the age at which the ith promotion takes place) may be similarly assumed to have a geometric distribution over $(x_{i-1}(t), \infty)$. The average age at promotion may of course vary from one stage to another.

Our problem now involves four sets of random variables viz $\lambda_x(t)$, $p_x(t)$,$x_i(t)$ and $k_i(t)$. For the first promotion, we are required to find the value of $x_1(t)$, which will make the probability of the event stated in (2) sufficiently large.

4 Determination of age at promotion

Considering $x_i(t)$ as random here, it is difficult to derive the distribution of the above sum generally, since $Z_x(t)$s have varying parameters.

$$
\begin{aligned}
&P\left[\sum_{x=x_{i-1}(t)}^{x_i(t)-1} Z_x(t) \le k_i(t)\right]\\
&=\sum_{\mathrm{k}_i=1}^{\infty} P\left[\sum_{x_{i-1}(t)}^{x_i(t)-1} Z_x(t) \le \mathrm{k}_i\right] P[k_i(t)=\mathrm{k}_i]\\
&=\sum_{\mathrm{k}_i=1}^{\infty}\sum_{j=1}^{\infty} P\left[\sum_{x=0}^{j-1} Z_x(t) \le \mathrm{k}_i | x_i(t)-x_{i-1}(t)=j\right] f(j) P[k_i(t)=\mathrm{k}_i], \qquad (3)
\end{aligned}
$$

where, $f(j)=P[x_i(t)-x_{i-1}(t)=j]$.

Here, $x_{i-1}(t)$ is known, since in the first promotion case it is x_h, and in all other cases it may be assumed known from the previous step.

Using the relation, $f(j)=pq^{j-1}$, $j=1,2,\ldots,\infty$, (3) becomes

$$
\sum_{\mathrm{k}_i=1}^{\infty}\sum_{j=1}^{\infty} P\left[\sum_{x=0}^{j-1} Z_x(t) \le \mathrm{k}_i\right] pq^{j-1} P[k_i(t)=\mathrm{k}_i]. \qquad (4)
$$

It is easy to show that this series is convergent.

Under the conditional set up one can find the value of j from the following relation:

$$
P\left[\sum_{x=0}^{j-1} Z_x(t) \le \mathrm{k}_i\right] \ge (1-\alpha), \qquad (5)
$$

where α may be chosen sufficiently small so that the system requirement is satisfied.

5 A simulation study

On the basis of the example of Sasieni et al. (1959) we have developed the following solution under probabilistic set up using simulation study.

Using the data of Table 1, it has been found that *Beta*(14.36, 1.0933) fits the data on p_x well under the assumption of time homogeneity. By estimating the p_{0x} values from Table 2, λ_x values have been generated from Binomial distribution using computer simulation (after adjusting for the total size of the system). Hence it is easy to generate the Z_x values. Figure 1 shows the nature of the $E(Z_x)$ values.

Since here it is difficult to estimate the Poisson parameters m_i's corresponding to the different job categories, the k_i values are assumed to be given. But it is easy to extend the analysis to the case where k_i's are random variables when we have sufficient number of observations at each age corresponding to different organizations.

On the basis of the simulated values of Z_x (at a particular time point), we have estimated the age at first promotion from the probability statement given in (5) by taking α as 0.04. Under this probabilistic set up it is more realistic to specify an age interval rather than a particular age to identify the most probable age at promotion. From our simulation study we

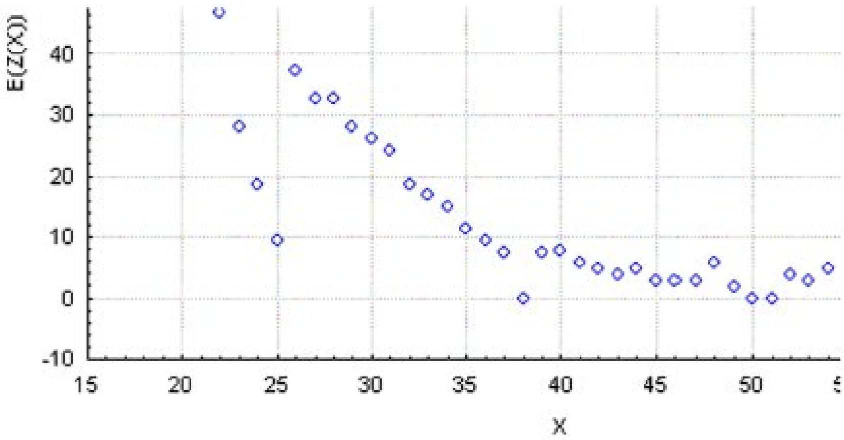

Fig. 1 Curve of $E(Z(X))$ values

Table 3 Age intervals at promotions under the stochastic set up

Promotion	Age interval	Probability
1	26–27	0.96
2	50–57	0.96

found that most probable age interval is 26–27 for the 1st promotion. Similarly by starting at age 26 the most probable age interval at second promotion has been found to be 50–57. The findings are presented in tabular form in Table 3.

Here, the age interval for 2nd promotion is large. But it is possible to obtain small age interval for 2nd promotion depending on the data set.

6 Concluding remarks

The present work is an attempt to establish the necessity for using the probabilistic set up to find the solutions for different types of scheduling problems. In fact although the underlying distributions may vary over the different situations, the overall solution technique should remain more or less the same. Since, in the large companies data base for the several past years are available it is very easy to find out the underlying distribution of different random variables involved. So, instead of using the data at the current time point and calculating the deterministic solution, it is much more worthwhile to derive a stochastic solution. This type of analysis is also helpful to predict the future situation.

Acknowledgement The authors are grateful to the referees for their valuable suggestions and helpful comments at different stages.

References

Abodunde, T. T., & McClean, S. I. (1980). Production planning for a manpower system with a constant level of recruitment. *Applied Statistics*, *29*, 43–49.

Chattopadhyay, A. K., & Khan, S. (2004). A general occupational mobility model-a salary based measure. *Hacetappe Journal of Mathematics and Statistics*, 33.

Chiang, C. L. (1968). *Introduction to stochastic processes in biostatistics*. New York: Wiley.

Khan, S., & Chattopadhyay, A. K. (2003). Predictive analysis of occupational mobility based on number of job offers. *Journal of Applied Statistical Science*, *12*(1), 11–22.

McClean, S. I. (1978). Continuous-time stochastic models of a multigrade population. *Journal of Applied Probability*, *15*, 26–37.

McClean, S. I. (1980). A semi-Markov model for a multigrade population with Poisson recruitment. *Journal of Applied Probability*, 846–852.

Mukherjee, S. P., & Chattopadhyay, A. K. (1989). A stochastic analysis of a staffing problem. *Journal of the Operational Research Society*, *40*(5), 489–494.

Sasieni, M., Yaspan, A., & Friedman, L. (1959). *Operations research: methods and problems*. New York: Wiley.