Point and Interval Estimation for the Lifetime Distribution of a k-Unit Parallel System Based on Progressively Type-II Censored Data

Biswabrata Pradhan

Abstract: This work considers point and interval estimation for the lifetime distribution of a k-unit parallel system assuming exponential distribution for the components lifetime. The parameter of the distribution is estimated by the maximum likelihood method based on samples which are progressively Type-II censored. The expression for Fisher's information is derived. Approximate confidence intervals are constructed based on the asymptotic distribution of the maximum likelihood estimator and log-transformed maximum likelihood estimator. The confidence levels of the approximate confidence intervals are examined via simulation. They turn out to be quite satisfactory. An approximate β-expectation tolerance interval for the life distribution is also discussed.

Keywords: Progressively Type-II censoring, maximum likelihood estimator (MLE), coverage probability, β-expectation tolerance interval

1 Introduction

It is common practice in a life testing experiment to terminate the experiment before all the units have failed. The observations obtained in such situation are called censored samples. Censoring is done to save time and cost associated with testing. Common type of censoring schemes are Type-I censoring and Type-II censoring. In Type-I censoring, experiment continues up to a pre-specified time T. Failures after the time T are not observed. In Type-II censoring, the experiment continues up to the pre-specified number of failures r, $r \leq n$, where n is the number of items put on life test. Several authors have discussed these censoring schemes which are also called single stage censoring.

In many practical situations censoring is done in several stages. For example, in many industrial experiments, units may have to be removed from the experiment prior to failure due to lack resources or because of cost constraints. Removal of the units at the points other than the terminal points are not allowed in conventional Type-I and Type-II censoring. A more general censoring scheme called progressive censoring is useful where censoring is carried out at different time points rather than at the terminal points. In progressive Type-I censoring scheme, m censoring times T_1, T_2,..., T_m are fixed. At time

T_i remove R_i of the remaining units randomly: the experiments terminates at time T_m with R_m units still surviving.

This paper considers the progressive Type-II censoring scheme. Consider a life testing experiment in which n identical units are put on life test. The number m and the numbers R_i, $i = 1, ..., m-1$ are fixed prior to the test. At the first failure, R_1 units are randomly removed from the remanining $n-1$ units. At the second failure, R_2 units are randomly removed from the remaining $n-2-R_1$ units. The test continues until the mth failure, when all remaining R_m units are removed, where $R_m = n - m - \sum_{i=1}^{m-1} R_i$. In this general censoring scheme, if $R_1 = R_2 = \ldots = R_m = 0$, then $n = m$, which corresponds to the complete sample. If $R_1 = R_2 = \ldots = R_{m-1} = 0$, then $R_m = n - m$ which correspond to the conventional Type-II censoring scheme.

Several authors have discussed progressive Type-II censoring schemes for different life distributions. Some of the earlier work on progressive censoring was conducted by Cohen [7] and Mann [10, 11]. Viveros et al. [14] studied interval estimation of parameters of lifetime distributions from progressively censored data. Balakrishnan et. al. [4] discussed point and interval estimation for the Gaussian distribution based on progressively Type-II censored samples. In a recent study Balakrishnan et al. [5] discussed inference for the extreme value distribution under progressive Type-II censoring. Kundu et al. [9] discussed Type-II progressively Hybrid censored data in the exponential case. Details about progressive censoring may also be obtained from Balakrishnan & Aggarwala [3]. A recent account of progressive censoring schemes can be obtained in the excellent review article by Balakrishnan [6]. But no effort has been made to draw inference on the life distribution of a system based on progressively Type-II censored data.

In this work, we consider point and interval estimation for the lifetime distribution of a k-unit parallel system based on progressively Type-II censored data. We derive the distribution for a k-unit parallel system based on the distribution of the individual components. The inference for the parameter of the distribution is made based on the lifetimes of the system. Sometime it may not be possible to estimate the parameters of lifetime distribution of individual components of a system because of non-observability of the lifetimes of components. If we have lifetimes of the system, we can estimate the parameters of the lifetime distribution of individual components by this method. This work is motivated by the work of Kumbhar and Shirke [8]. They studied a tolerance interval for the lifetime distribution of k-unit parallel system based on complete lifetime data.

This article focuses on the lifetime distribution of a k-unit parallel system with exponentially distributed lifetimes of the components. Inferences on the life distribution of k-unit parallel system is drawn based on progressively Type-II censored data. The parameter of the distribution is estimated by the maximum likelihood method, which is discussed in Section 2, where Fisher's information is used to calculate the standard error of the maximum likelihood estimate. Section 3 provides the coverage probabilities for pivotal quantities based on asymptotic normality. β-expectation tolerance interval for the life distribution of k-unit parallel system is discussed in Section 4. A simulation study has been carried out to investigate the performance of the maximum likelihood estimator and examine the coverage probabilities in Section 5. A simulation study has also been carried out in Section 5 for the β-expectation tolerance interval to compare the simulated and approximate expected coverage. In Section 6 the results obtained are discussed.

2 Estimation of the Model Parameter

Consider a k-unit parallel system with independent and identically distributed components. Let X_i be the lifetime of the ith component with $X_i \sim Exp(\lambda)$. Let X be the lifetime of the system, then $X = \max\{X_1, X_2, \ldots, X_k\}$.

The distribution function of X is given by:

$$F(x;\lambda) = (1 - e^{-\lambda x})^k \tag{1}$$

with $\lambda > 0$ and density function:

$$f(x;\lambda) = k(1 - e^{-x\lambda})^{k-1}\lambda e^{-\lambda x} \quad \text{for } x \geq 0 \tag{2}$$

Suppose n identical k-unit parallel units are put on life test. Let $x_{(i)}$ be the order-statistic from a progressively Type-II censored sample of size n with $(R_1, R_2, \ldots, R_m)$ being the progressive censoring scheme.

The likelihood function based on a progressively Type-II censored sample can be written as:

$$\begin{aligned} L(\lambda) &= C\prod_{i=1}^{m} f(x_{(i)};\lambda)[1 - F(x_{(i)};\lambda)]^{R_i} \\ &= C\prod_{i=1}^{m} k\lambda e^{-\lambda x_{(i)}}(1 - e^{-\lambda x_{(i)}})^{k-1}[1 - (1 - e^{-\lambda x_{(i)}})^k]^{R_i} \end{aligned} \tag{3}$$

where $C = n(n-1-R_1)(n-2-R_2-R_1)\ldots(n-m+1-R_1-\ldots-R_{m-1})$.

Equating the derivative of $\ln L(\lambda)$ with 0, the likelihood equation for the estimate $\hat{\lambda}$ is obtained:

$$\frac{m}{\lambda} - \sum_{i=1}^{m} x_{(i)} + (k-1)\sum_{i=1}^{m}\frac{x_{(i)}e^{-\lambda x_{(i)}}}{1-e^{-\lambda x_{(i)}}} - k\sum_{i=1}^{m} R_i\frac{x_{(i)}e^{-\lambda x_{(i)}}(1-e^{-\lambda x_{(i)}})^{k-1}}{1-(1-e^{-\lambda x_{(i)}})^k} = 0 \tag{4}$$

Equation (4) must be solved iteratively, because even in the complete sample case, i.e., when $R_1 = R_2 = \ldots = R_m = 0$, we do not get explicit solutions. We apply the Newton-Raphson method for obtaining the maximum likelihood estimate $\hat{\lambda}$. For obtaining the confidence intervals, Fisher's information is derived, which is defined by $I(\lambda) = -\frac{d^2 \ln L}{d\lambda^2}$.

The second derivative of $\ln L$ is obtained as follows:

$$\begin{aligned}\frac{d^2 \ln L(\lambda)}{d\lambda^2} &= -\frac{m}{\lambda^2} - (k-1)\sum_{i=1}^{m} \frac{x_{(i)}^2 e^{-\lambda x_{(i)}}}{y_{(i)}^2} \\ &+ k\sum_{i=1}^{m} R_i x_{(i)}^2 e^{-\lambda x_{(i)}} y_{(i)}^{k-2} \left(\frac{k y_{(i)} - y_{(i)}^k - (k-1)}{(1-y_{(i)}^k)^2} \right)\end{aligned} \tag{5}$$

with $y_{(i)} = 1 - e^{-\lambda x_{(i)}}$.

Guessing an initial value for the Newton-Raphson method is a difficult task. The least square estimate of λ can be used as a initial value in Newton-Raphson method. Ng [13] discussed estimation of model parameters of modified Weibull distribution based on progressively Type-II censored data where the empirical distribution function is computed as (see Meeker & Escober [12]):

$$\hat{F}(x_{(i)}) = 1 - \prod_{j=1}^{i}(1-\hat{p}_j) \tag{6}$$

with

$$\hat{p}_j = \frac{1}{n - \sum_{k=2}^{j} R_{k-1} - j + 1} \quad \text{for } j = 1, 2, ..., m \tag{7}$$

The estimate of the parameters can be obtained by least squares fit of simple linear regression:

$$y_i = \beta x_{(i)} \tag{8}$$

with

$$\begin{aligned}&\beta = -\lambda \\ &y_i = \ln\left(1 - (\hat{F}^{1/k}(x_{(i-1)}) + \hat{F}^{1/k}(x_{(i)}))/2\right) \quad \text{for } i = 1, 2, ..., m \\ &\hat{F}(x_{(0)}) = 0\end{aligned} \tag{9}$$

The least squares estimate of λ is given by:

$$\tilde{\lambda} = \frac{\sum_{i=1}^{m} x_{(i)} y_i}{\sum_{i=1}^{m} x_{(i)}^2} \tag{10}$$

In our simulation study, we use $\tilde{\lambda}$ as the initial value for the Newton-Raphson method to obtain the maximum likelihood estimate of the parameters.

3 Confidence Intervals

Here we discuss the construction of an asymptotic confidence interval for λ by two methods namely first based on maximum likelihood estimation and secondly on log-transformed maximum likelihood estimation.

Let $\hat{\lambda}_n$ be an unbiased estimator of λ, then the asymptotic distribution of the quantity

$$P_0 = \frac{\hat{\lambda}_n - \lambda}{\sqrt{\hat{\sigma}^2(\hat{\lambda})}} \tag{11}$$

is standard normal, where $\hat{\sigma}^2(\hat{\lambda}_n)$ is the estimated variance of $\hat{\lambda}_n$ and is given by $\hat{\sigma}^2(\hat{\lambda}_n) = I^{-1}(\hat{\lambda}_n)$. Therefore, we propose the following $100(1-\alpha)\%$ confidence interval for λ:

$$\left[\hat{\lambda}_n^{(l)}, \hat{\lambda}_n^{(u)}\right] = \left[\hat{\lambda}_n - \tau_{\frac{\alpha}{2}}\sqrt{\hat{\sigma}^2(\hat{\lambda}_n)}\,,\ \hat{\lambda}_n + \tau_{\frac{\alpha}{2}}\sqrt{\hat{\sigma}^2(\hat{\lambda}_n)}\right] \tag{12}$$

Meeker and Escober [12]) reported that the asymptotic confidence interval for λ based on $\ln(\hat{\lambda}_n)$ is superior to the one based on $\hat{\lambda}_n$. The approximate $100(1-\alpha)\%$ confidence interval for $\ln(\lambda)$ is

$$\left[\ln(\hat{\lambda}_n) - \tau_{\frac{\alpha}{2}}\sqrt{\hat{\sigma}^2(\ln(\hat{\lambda_n}))}\,,\ \ln(\hat{\lambda}_n) + \tau_{\frac{\alpha}{2}}\sqrt{\hat{\sigma}^2(\ln(\hat{\lambda_n}))}\right] \tag{13}$$

where $\hat{\sigma}^2(\ln(\hat{\lambda_n}))$ is the estimated variance of $\ln(\hat{\lambda_n})$ which is approximately obtained by $\hat{\sigma}^2(\ln(\hat{\lambda_n})) \approx \frac{\hat{\sigma}^2(\hat{\lambda_n})}{\hat{\lambda_n}^2}$

Hence, an approximate $100(1-\alpha)\%$ confidence interval for λ is given by:

$$\left[\hat{\lambda_n}^{(l)*}, \hat{\lambda_n}^{(u)*}\right] = \left[\hat{\lambda_n}\, e^{\left(-\frac{\tau_{\alpha/2}\sqrt{\hat{\sigma}^2(\hat{\lambda_n})}}{\hat{\lambda_n}}\right)}, \hat{\lambda_n}\, e^{\left(\frac{\tau_{\alpha/2}\sqrt{\hat{\sigma}^2(\hat{\lambda_n})}}{\hat{\lambda_n}}\right)}\right] \tag{14}$$

4 Tolerance Intervals

Kumbhar et al. [8] derived the expression for a β-expectation tolerance interval for the lifetime distribution of a k-unit parallel system. They investigated the performance of the tolerance interval based on complete data. But in practice, a complete data set may not be always available. Generally only censored data are available. Therefore, we study the performance of the tolerance interval based on progressively Type-II censored data. Let $\ell_\beta(\lambda)$ be the lower quantile of order β of the distribution function $F(x;\lambda)$.

Then, we have:

$$\ell_\beta(\lambda) = -\frac{1}{\lambda}\ln(1-\beta^{1/k}) \tag{15}$$

Thus, an upper β-expectation tolerance interval for $F(x;\lambda)$ is obtained by:

$$I_\beta = (0, \ell_\beta(\lambda)) \tag{16}$$

The maximum likelihood estimate of $\ell_\beta(\lambda)$ is given by:

$$\ell_\beta(\hat{\lambda}_n) = -\frac{1}{\hat{\lambda}_n}\ln(1-\beta^{1/k}) \tag{17}$$

yielding the approximate β-expectation tolerance interval:

$$\hat{I}_\beta = (0, \ell_\beta(\hat{\lambda}_n)) \tag{18}$$

The expectation of $\hat{I}_\beta$ can be obtained approximately using the approach suggested by Atwood [1] (see also Kumbhar et al. [8]) and given below:

$$E\left[F(\ell_\beta(\hat{\lambda}_n);\lambda)\right] \approx \beta - 0.5F_{02}\sigma^2(\hat{\lambda_n}) + \frac{F_{01}\sigma^2(\hat{\lambda_n})F_{11}}{F_{10}} \tag{19}$$

where $F_{10} = \frac{\partial F}{\partial x}$, $F_{01} = \frac{\partial F}{\partial \lambda}$, $F_{11} = \frac{\partial^2 F}{\partial x \partial \lambda}$, $F_{02} = \frac{\partial^2 F}{\partial^2 \lambda}$ with

$$\begin{aligned} F_{10} &= k\lambda e^{-\lambda x}(1-e^{-x\lambda})^{k-1} \\ F_{01} &= kxe^{-\lambda x}(1-e^{-x\lambda})^{k-1} \\ F_{11} &= ke^{-\lambda x}(1-e^{-\lambda x})^{k-2}[(1-\lambda x)(1-e^{-\lambda x}) + (k-1)\lambda xe^{-\lambda x}] \\ F_{02} &= kx^2e^{-\lambda x}(1-e^{-\lambda x})^{k-2}[(k-1)e^{-\lambda x} - (1-e^{-\lambda x})] \end{aligned} \tag{20}$$

The derivatives of F are all evaluated at $x = \ell_\beta(\lambda)$ and $\hat{\lambda_n}$.

With (20) we obtain:

$$E\left[F(\ell_\beta(\hat{\lambda}_n);\lambda)\right] \approx \beta + \frac{\sigma^2(\hat{\lambda}_n)}{\lambda^2}ktg(1-g)^{k-2}[(0.5kt-1)g - 0.5t + 1] \tag{21}$$

where t = λx = $-\ln(1-\beta^{1/k})$ and g = e^{-t}.

The expression (21) gives the approximate expectation of the β-expectation tolerance interval given by (18). Note that the actual expression for $\sigma^2(\hat{\lambda})$ is not explicitly available and numerical integration is necessary to compute it. Instead of the actual value of $\sigma^2(\hat{\lambda})$ we use an estimate of it in (21). In the following section the approximate expectations given by (21) are examined by means of a simulation study.

5 Simulation Study

A simulation study is conducted to asses the performance of the maximum likelihood estimator with respect to bias and MSE for different progressively Type-II censoring scheme. The confidence levels of the approximate confidence intervals based on MLE and log-transformed MLE are computed and compared through simulation study. The expectations of the β-expectation tolerance intervals are also examined via simulation study. Progressively Type-II censored samples are generated from the distribution (1) of a k-unit parallel system using the algorithm presented in Balakrishnan et al. [2]. The steps of the algorithm for generating progressively Type-II censored sample are given below:

1. Generate an independent random sample $(W_1, W_2, \ldots, W_m)$ from the uniform distribution $U[0, 1]$.
2. Set $E_i = 1/(i + R_m + R_{m-1} + \cdots + R_{m-i+1})$ for $i = 1, \ldots, m$.
3. Set $V_i = W_i^{E_i}$ for $i = 1, 2, \ldots, m$.
4. Set $U_i = 1 - V_m V_{m-1} ... V_{m-i+1}$ for $i = 1, 2, \ldots, m$
   Then $(U_1, U_2, \ldots, U_m)$ is the required progressively Type-II censored sample from $U[0, 1]$.
5. For given values of the parameters λ, set

$$x_{(i)} = -\frac{1}{\lambda} \ln(1 - U_i^{1/k}), \text{for } i = 1, \ldots, m. \tag{22}$$

Then the resulting set $(x_{(1)}, x_{(2)}, \ldots, x_{(m)})$ is the required progressively Type-II censored sample from the distribution of a k-unit parallel system with exponential distribution as the component life distribution.

The simulation study was performed for 16 different progressively Type-II censored schemes. These schemes are specified in Table 1 below. Moreover, the following situations were considered in the study:

- The simulation was carried out for $k = 3$ and $k = 6$ with $\lambda = 0.8$. Thus, a 3-unit parallel system and a 6-unit parallel system are considered.
- The maximum likelihood estimate of the parameter λ was obtained by the Newton-Raphson method using equation (4) and (5).
- The confidence levels for the asymptotic confidence intervals given in (12) and (14) were calculated and compared with the simulation results.
- The expectations of the approximate β-expectation tolerance intervals were calculated, using (21) by replacing λ and $\sigma^2(\hat{\lambda}_n)$ by their respective estimates, and were compared with the corresponding simulation results.

- For each particular progressive censoring scheme, 1000 sets of observation were generated. The average bias, the MSE and the confidence levels for the corresponding approximate confidence intervals of λ are displayed in the Tables 2(a) and 2(b) for $k = 3$ and $k = 6$, respectively.
- In Tables 3(a) and 3(b) for $k = 3$ and $k = 6$, respectively, the simulated mean coverage and the estimated expectation of the tolerance interval are given.

Table 1: The 16 considered censoring schemes.

n	m	$(R_1, R_2, ..., R_m)$	Scheme
15	5	$R_1 = 10$, $R_i = 0$ for $i \neq 1$	[1]
		$R_5 = 10$, $R_i = 0$ for $i \neq 5$	[2]
		$R_i = 2$ for all i	[3]
15	10	$R_1 = 5$, $R_i = 0$ for $i \neq 1$	[4]
		$R_{10} = 5$ and $R_i = 0$ for $i \neq 10$	[5]
		$R_1 = 3, R_2 = 2$, $R_i = 0$ for $i \neq 1, 2$	[6]
30	15	$R_1 = 15$, $R_i = 0$ for $i \neq 1$	[7]
		$R_{15} = 15$ and $R_i = 0$ for $i \neq 15$	[8]
		$R_1 = R_2 = R_3 = 5$, $R_i = 0$ for $i \neq 1, 2, 3$	[9]
30	20	$R_1 = 10$, $R_i = 0$ for $i \neq 1$	[10]
		$R_{20} = 10$ and $R_i = 0$ for $i \neq 20$	[11]
		$R_2 = R_3 = 5$, $R_i = 0$ for $i \neq 2, 3$	[12]
50	20	$R_1 = 30$, $R_i = 0$ for $i \neq 1$	[13]
		$R_{20} = 30$ and $R_i = 0$ for $i \neq 20$	[14]
50	35	$R_1 = 15$, $R_i = 0$ for $i \neq 1$	[15]
		$R_{35} = 15$ and $R_i = 0$ for $i \neq 35$	[16]

As expected, the bias and MSE decrease with sample size (n) and the same trend follows when effective sample size (m) increases for a particular sample size (n). The bias and MSE decreases significantly as k increases, as expected. That is, if more components are added to the system the precision of the estimates improves significantly. As far as different censoring schemes are concerned, the bias is relatively smaller in progressively Type-II censoring scheme for small sample size than the bias in conventional Type-II censoring schemes. Whereas for large sample size, the bias is less in conventional Type-II censoring scheme. It can be seen that MSE is less for conventional Type-II censoring scheme in all cases. The confidence levels for the asymptotic confidence intervals by both the methods are quite satisfactory. There is no significant difference between the confidence levels in the MLE case and the log-transformed MLE case. The expectation of the β-expectation tolerance intervals are quite satisfactory. It performs very well for large sample, as expected. The simulated mean coverage and estimated expectation match very well for large sample size. Hence, one can construct the tolerance interval proposed by Kumbhar et. al. [8] for the lifetime distribution of k-unit parallel system based on progressively Type-II censored data.

Though any optimum censoring scheme cannot be suggested based on the simulation study, we can conclude that given the option it is better to use conventional Type-II censoring scheme as far as MSE is concerned. However, one can use progressively Type-II censoring schemes for small sample sizes.

Table 2(a): Average Bias, MSE and confidence levels for k =3 and λ= 0.8

n	m	scheme	Bias	MSE	Confidence level (MLE)		Confidence level (log MLE)	
					90%	95%	90%	95%
15	5	[1]	.0447	.0624	.893	.941	.894	.938
		[2]	.0497	**.0408**	.925	.97	.908	.953
		[3]	**.0382**	.0445	.906	.954	.905	.948
15	10	[4]	**.0239**	.0289	.916	.95	.913	.962
		[5]	.0345	**.0266**	.897	.948	.883	.935
		[6]	.0251	.0279	.908	.956	.901	.944
30	15	[7]	.0163	.0180	.898	.846	.893	.949
		[8]	**.0113**	**.0133**	.897	.938	.894	.939
		[9]	.0156	.0179	.895	.951	.895	.950
30	20	[10]	.0120	.0144	.892	.943	.889	.944
		[11]	**.0092**	**.0112**	.891	.931	.91	.949
		[12]	.0103	.0115	.92	.962	.913	.961
50	20	[13]	.0110	.0127	.915	.961	.911	.955
		[14]	**.0078**	**.0095**	.897	.956	.901	.946
50	35	[15]	.0077	.0074	.911	.951	.904	.951
		[16]	**.0058**	.0060	.904	.955	.904	.951

Table 2(b): Average Bias, MSE and confidence level for k =6 and λ= 0.8.

n	m	scheme	Bias	MSE	Confidence level (MLE)		Confidence level (log MLE)	
					90%	95%	90%	95%
15	5	[1]	**.0175**	.0301	.889	.950	.893	.931
		[2]	.0203	**.0186**	.90	.961	.907	.958
		[3]	.0263	.0229	.89	.95	.887	.941
15	10	[4]	.0160	.0164	.887	.942	.878	.941
		[5]	.0089	**.0121**	.909	.942	.897	.954
		[6]	**.0069**	.0151	.905	.943	.897	.944
30	15	[7]	.0120	.0094	.921	.957	.918	.961
		[8]	.0102	**.0070**	.905	.949	.902	.948
		[9]	**.0075**	.0088	.918	.960	.910	.956
30	20	[10]	.0105	.0080	.896	.946	.895	.944
		[11]	**.0048**	**.0059**	.908	.953	.907	.957
		[12]	.0069	.0071	.910	.949	.906	.952
50	20	[13]	.0078	.0072	.909	.944	.904	.945
		[14]	**.0063**	**.0049**	.901	.953	.902	.948
50	35	[15]	.0061	.0042	.925	.961	.924	.961
		[16]	**.0046**	**.0036**	.904	.952	.897	.956

Table 3(a): Simulated mean and estimated expectation of the approximate β-expectation tolerance interval for k =3 and λ= 0.8.

n	m	Scheme	Simulated Mean			Estimated Expectation		
			90%	95%	99%	90%	95%	99%
15	5	[1]	.871	.925	.977	.884	.933	.981
		[2]	.872	.928	.980	.889	.939	.984
		[3]	.876	.930	.980	.888	.934	.983
15	10	[4]	.884	.937	.984	.892	.942	.986
		[5]	.880	.935	.983	.893	.943	.986
		[6]	.884	.937	.984	.892	.942	.986
30	15	[7]	.892	.943	.986	.895	.945	.987
		[8]	.892	.943	.987	.896	.946	.988
		[9]	.893	.944	.988	.895	.946	.988
30	20	[10]	.889	.942	.986	.896	.950	.988
		[11]	.897	.946	.988	.897	.947	.988
		[12]	.892	.944	.987	.896	.946	.988
50	20	[13]	.890	.942	.989	.896	.946	.988
		[14]	.894	.945	.988	.897	.947	.988
50	35	[15]	.896	.946	.988	.897	.947	.988
		[16]	.896	.947	.988	.898	.948	.989

Table 3(b): Simulated mean and estimated expectation of the approximate β-expectation tolerance interval for k =3 and λ= 0.8.

n	m	Scheme	Simulated Mean			Estimate Expectation		
			90%	95%	99%	90%	95%	99%
15	5	[1]	.879	.933	.982	.886	.937	.984
		[2]	.882	.936	.984	.890	.941	.986
		[3]	.878	.933	.983	.890	.940	.986
15	10	[4]	.885	.938	.985	.892	.943	.987
		[5]	891	.943	.987	.894	.944	.987
		[6]	.889	.941	.986	.892	.943	.987
30	15	[7]	.890	.943	.987	.895	.945	.988
		[8]	.892	.943	.987	.896	.947	.988
		[9]	.893	.944	.988	.895	.946	.988
30	20	[10]	.893	.945	.988	.896	.956	.988
		[11]	.892	.944	.988	.897	.947	.989
		[12]	.893	.945	.988	.896	.946	.988
50	20	[13]	.893	.945	.988	.896	.947	.988
		[14]	.894	.946	.988	.898	.948	.990
50	35	[15]	.895	.946	.989	.897	.947	.988
		[16]	.899	.949	.989	.898	.948	.989

6 Discussion

In this work we derive and examine point and interval estimation for the lifetime distribution of k-unit parallel system of identical components based on progressively Type-II censored data. The parameter of the lifetime distribution is estimated by maximum likelihood method. Confidence levels for asymptotic confidence intervals are examined via simulation. A β-expectation tolerance interval for the lifetime distribution is also discussed.

Sometimes we need to estimate the lifetime distribution of individual components of a system based on observations of the lifetime of the system as the individual components can not be put on life test. Assessing the reliability of individual component is an important aspect in reliability study. In that case our method of inference would be very much helpful. In this work we have considered that the lifetime of the individual components cannot be observed but we observe only the lifetime of the system. We first derive the lifetime distribution of the system based on the lifetime distribution of the components. Then we estimate the parameters of the lifetime distribution of the components based on the overall lifetimes of the system. In this work we have assumed the exponential distribution as the component life distribution. Inference for the life distribution of k-unit parallel system with Weibull distribution as the component lifetime distribution is the future topic for study.

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Biswabrata Pradhan
SQC & OR Unit, Indian Statistical Institute
203, B. T. Road
Kolkata -700 108
India