Accelerated life testing in the presence of dependent competing causes of failure

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Abstract

In accelerated life testing, the products are tested at high stress conditions and the results are used to draw inferences about the product lifetime at the normal stress condition. The products can fail due to one of the several possible causes of failure which need not be independent. In this paper, we consider the accelerated life testing in the presence of dependent competing risks. We propose suitable models using the subsurvival functions for the data arising from such situations and carry out the estimation of the parameters involved in the models. Finally, the method is applied to a real data.

Key words: Dependent competing risks, accelerated testing, maximum likelihood estimation, stress levels, subsurvival function

1 Introduction

Engineers and management spend time and money in assessing new designs, identifying causes of failure and trying to eliminate them so that the product produced is acceptable to the consumer. Due to the longer lifetimes of the products, accelerated life testing is used to determine the reliability of the products. In accelerated life testing, the products are tested at high stress conditions and the results are used to draw inferences about the product at the normal stress condition. We refer to Nelson (1990) for the examples of accelerated life testing.

A few examples where the system can fail due to more than one cause are: (1) fatigue specimens of a certain sintered superalloy can fail from a surface defect or an interior one, (2) in ball bearing assemblies, a ball or the race can fail, (3) in motors, the Turn, Phase or Ground insulation can fail, (4) a cylindrical fatigue specimen can fail in the cylindrical portion , in the fillet (or radius), or in the grip, (5) a semiconductor device can fail at a junction or at a lead and (6) cars can fail due to electrical failure or mechanical failure. Hence, studying accelerated lifetesting data in the presence of competing risks is essential.

The simplest accelerated model is described by the modification in the failure time due to the application of stress or a treatment or an exposure. Let T_0 be the failure time under the normal condition. The failure time T_s under stress s is defined as $T_0/\theta(s)$ for some fixed scaling factor. Under this model, the survival function of T_s is $S(t \mid s) = P(T_0 > \theta(s)t) = S_0(\theta(s)t)$ and the hazard rate of T_s is $h(t \mid s) = \theta(s)h_0(\theta(s)t)$, where $h_0(t)$ is the hazard rate of T_0. This can also be described by a regression model in terms of the logarithm of the failure times as $logT_s = logT_0 - log\theta(s)$.

Denote $X_s = logT_s$, $X_0 = logT_0$ and $W_s = log\theta(s)$. The accelerated model is $X_s = X_0 - W_s$ where X_0 and W_s are independent. This model is similar to the random clipping model for competing risks described in Cooke *et al.* (1993) and it is logical that the failures occur sooner after the application of the stress. Parametric models under the accelerated failure time assumptions are obtained by specifying the underlying distribution $S_0(.)$ and the form of dependence on the stress s through $\theta(s)$. Some important special cases of the underlying distribution include

$$\begin{aligned} S_0(t) &= \exp(-kt^{\alpha}) \ (Weibull) \\ S_0(t) &= \frac{1}{1+kt^{\alpha}} \ (log-logistic) \end{aligned} \tag{1}$$

and the lognormal where $\log T_s$ has normal distribution. The commonly used models for $\theta(s)$ are discussed in Nelson (1990) and Bagdonavicius and Nikulin (2002). Some of them are listed below

$$\begin{aligned} \theta(s) &= \exp(-\beta_0 - \beta_1 s) \text{ (Log-linear)} \\ \theta(s) &= \alpha_1 s^{\beta_1} \text{ (Power rule model)} \\ \theta(s) &= \exp(\beta_0 - \beta_1/s) \text{ (Arrhenius model)} \\ \theta(s) &= \alpha_1 (\frac{s}{1-s})^{-\beta_1}, 0 < s < 1 \text{ (Meeker-Luvalle model)} \end{aligned} \tag{2}$$

An accelerated failure time model is generally proposed as an alternative to Cox's proportional hazards model. In practice, which one of the two is valid will depend on the operating mechanisms. The only distribution that satisfies both the accelerated failure time and proportional hazards condition is the Weibull distributions with underlying hazard rate of the form $h(t) = \alpha k t^{\alpha-1}$.

There has been little done in modelling accelerated failure time in the presence of competing causes of failure for the obvious reasons of the difficulty in modelling the joint effect of stress on both the failure time and the cause of failure. Consider a latent failure model where k competing risks are acting simultaneously with hypothetical failure times $X_1, X_2, \ldots, X_k$, which are assumed to be independent. The data consist of the minimum of these hypothetical times $T = \min(X_1, X_2, \ldots, X_k)$, and an indicator function δ giving the cause of the failure, ($\delta = j$ if $T = X_j$). Trivially, the reliability of the system will be $S(t) = P[T > t] = \prod_{i=1}^{k} P[X_i > t]$. Let $v_1 < v_2 < \ldots < v_m$ be m stress levels at which the accelerated life test is conducted. At stress v_s, let X_{sj} denote the failure time under the jth risk, $j = 1, 2, \ldots, k$ and $s = 1, 2, \ldots, m$. Under the above latent failure time model, Nelson (1990) gave the maximum likelihood estimators of the parameters at normal stress for each risk. He also worked out the maximum likelihood estimator of the product life that would result if certain failure modes were eliminated by product design. He also discussed graphical procedures for fitting a parametric distribution at each stress level.

Klein and Basu(1981, 1982a, 1982b) studied accelerated life times when the latent failure times have exponential distribution or Weibull with equal and unequal shape parameters in the presence of censoring. They assumed that the component lifetimes are independent and each lifetime is exponentially distributed with survival function

$$P[X_{sj} > x] = \exp(-\lambda_{sj} x),\ x > 0, \lambda_{sj} > 0, j = 1, 2, \ldots, k, s = 1, 2, \ldots, m.$$

They further assumed that the stress and component life distributions are related by the power rule model, $\lambda_{sj} = \exp(A_j) v_s^{B_j}$, where A_i, B_j are constants to be estimated from the accelerated life test.

For an item put to test at stress v_s we observe $Y_s = \min(X_{s1}, X_{s2}, \ldots, X_{sk})$ and the corresponding indicator function which identifies the cause of failure.

Under Type I censoring , n_s items are put to test at stress level v_s, and testing is continued until fixed time τ_s. The $\tau_s' s$ may differ from one stress level to another to allow for increased testing time at low stress levels.

Let r_s be the number of items failing before τ_s. Let $Y_{s1}, Y_{s2}, \ldots, Y_{sr_s}$, denote the corresponding failure times. That is, $Y_{s\ell} = \min(X_{s1\ell}, \ldots, X_{sk\ell})$ where $X_{sj\ell}$ is the lifetime of the jth component of the ℓth system which failed prior to time τ_s at stress v_s. Let m_{sj} denote the number of items which failed doe to failure of jth component. Let $r_s = \sum_{j=1}^{k} m_{sj}$ The total time on test is given by $T_s = \sum_{\ell=1}^{r_s} Y_{s\ell} + (n_s - r_s)\tau_s$.

The likelihood equations are given by

$$\begin{aligned} \hat{A}_j &= \log(\sum_{i=1}^{m} m_{ij}) - \log(\sum_{i=1}^{m} T_i v_i^{B_j}) \\ (\sum_{i=1}^{m} m_{ij} \log v_i)(\sum_{i=1}^{m} T_i v_i^{B_j}) &= (\sum_{i=1}^{m} m_{ij})(\sum_{i=1}^{m} T_i v_i^{B_j} \log v_i) \end{aligned}$$

These estimators are then used to estimate the parameters at used or normal conditions. They also consider the case for Type II censoring.

Starting from Bernoulli (1760), there has been debate about how the competing risks data are to be modelled. Because of the inherent identifiability problems associated with the latent failure times, in the last few years researchers have tried to analyse the data in terms of the subsurvival functions or cause-specific hazards (Kalbfleisch and Prentice, 1980, Deshpande, 1990 and Aras and Deshpande, 1992). For a recent review of the competing risks literature see Crowder (2001). In the situations where the failure time T and cause of failure δ are independent, it is straight forward to model the survival function $S(t) = P(T > t)$ using proper parametric distribution and a multinomial distribution for δ. This then provides the model for the subsurvival function $S_j(t) = P(T > t, \delta = j) = P(\delta = j)S(t)$. In fact, inferences can be drawn separately for T and δ. The situations where the assumptions of independence of T and δ is not justifiable, it is essential to model the subsurvival functions directly (see Deshpande, 1990, Aras and Deshpande, 1992 and Dewan *et al.*, 2004). There has been attempt in using the proportional hazards model for the cause-specific hazards. It might be just simpler to propose models for the cause-specific hazards and for the survival function of T which can then provide the subsurvival function. This approach has been suggested by Crowder (2001). In such cases, the subsurvival functions need not be in a nice analytic form. Other approaches are to specify the conditional distribution of T given δ or the conditional distribution of δ given T.

To the best of our knowledge accelerated life testing in the presence of competing risks has been studied only under the latent failure time models. In the present paper, we attempt to combine the competing risks models and accelerated failure time models to propose some likely models for the accelerated life test in the presence of dependent competing risks. We also discuss estimation of the parameters involved in the proposed models.

One way to define models for competing risks is to assume that the cause-specific hazards with parameters which depend on stress as well as risk. Depending on the situation in hand, one can build up the appropriate model possibly using the guide lines given in section 2. In section 3, we discuss the estimation of the parameters. In section 4, we reanalyse a part of the data given in Klein and Basu (1981). We conclude in the last section.

2 Models

In modelling the competing causes in the accelerated life tests, there are two distinct situations: one when the causes of failure remain the same at all the stresses, the other when the causes of failure are affected by the stress in the sense that the probability of failure due to a specific cause depends on the stress. Let (T_s, δ_s) denote the failure time and the cause of failure at stress v_s, $s = 1, 2, \ldots, m$ and (T_0, δ_0) denote these variables at the normal stress condition denoted by v_0. Without loss of generality, we assume that $v_0 < v_1 < \ldots < v_m$.

2.1 Stress does not change the causes of failure

In case when the stress does not change the cause of failure, we assume that δ_s, cause of failure at the stress v_s does not depend on v_s, that is δ_s and δ_0 are identical in distribution. This implies that $P(\delta_s = \delta_0) = 1$ and also that $P(\delta_s = j) = P(\delta_0 = j) = p_j$ for all v_s and $\sum_{j=1}^{k} p_j = 1$.

A natural extension of the simplest accelerated model to the competing risks set-up is to allow the shift function to vary with the cause as well as stress. The subsurvival function at stress v_s for failure mode $j = 1, 2, ..., k$, and $t > 0$ can be defined as

$$\begin{aligned} S_j(t \mid v_s) &= P(T_s > t, \delta_s = j) \\ &= P(T_0 > \theta(v_s)t, \delta_0 = j) = S_{0j}(\theta(v_s)t), \end{aligned} \tag{3}$$

where $S_{0j}(t) = P(T_0 > t, \delta_0 = j)$ is the subsurvival function corresponding to the cause j at the normal stress. Let the overall survival function at stress v_s be denoted by $S(t \mid v_s)$ and at the normal stress by $S_0(t)$. Then $S(t \mid v_s) = \sum_{j=1}^{k} S_{0j}(\theta(v_s)t) = S_0(\theta(v_s)t)$. Let $f_j(t \mid v_s)$ and

$h_j(t \mid v_s)$ denote the subdensity function and the cause-specific hazard rate for cause j and $f(t \mid v_s)$ and $h(t \mid v_s)$ denote the overall density function and the hazard rate at the stress level v_s.

Further, when the failure time and the cause of failure are also independent at all stresses, $S_{0j}(t) = p_j S_0(t)$ and $S_j(t \mid v_s) = p_j S_0(\theta(v_s)t)$. Hence, the standard underlying distributions used in accelerated life test can be employed for the survival function of the failure time. The cause of failure can be studied separately.

In case when the failure time and cause are not independent, the model proposed by Dewan *et al.* (2004) to specify the subsurvival function at the normal stress when there are only two causes of failure can be used

$$S(t \mid v_s) = S_0(\theta(v_s)t) \quad (4)$$

$$S_1(t \mid v_s) = [pS_0(\theta(v_s)t)]^\alpha \quad (5)$$

$$S_2(t \mid v_s) = S_0(\theta(v_s)t) - [pS_0(\theta(v_s)t)]^\alpha. \quad (6)$$

When the shift function varies with the cause also, the overall survival function does not have a simple form as given above. Due to the lack of the models proposed in terms of the subsurvival functions, it is difficult to say more here.

2.2 Stress changes the causes of failure

Here, we assume that $P(\delta_s = j) = p_j(v_s)$ so that the probability of failure due to cause j depends on stress. Note that $\sum_{j=1}^{k} p_j(v_s) = 1$ at any stress v_s. It is also implicit here that the stress does not introduce new causes of failure but it might eliminate some cause. If some cause has the probability zero at all the stresses except at the normal stress then it will never be identified. So, we assume that $p_j(v_s)$ is nonzero at least at some stress for all j. One way and may be the best way to model in such situations is to specify the conditional survival functions of the type

$$S(t \mid v_s, j) = P(T_s > t \mid v_s, \delta_s = j) \quad (7)$$

If T_s and δ_s are not independent then the equation (7) is defined as

$$S(t \mid v_s, j) = P(T_0 > \theta(v_s, j)t) = S_0(\theta(v_s, j)t), \quad (8)$$

which is the survival function at normal stress with shift $\theta(v_s, j)$. Note that the shifting parameter is a function of stress as well as the cause of failure. The unconditional subsurvival function

$$S_j(t \mid v_s) = p_j(v_s) S_0(\theta(v_s, j)t) \quad (9)$$

and the overall survival function is the mixture of such k components.

If T_s and δ_s are independent then

$$S(t \mid v_s, j) = P(T_s > t \mid v_s) = P(T_0 > \theta(v_s)t) = S_0(\theta(v_s)t), \quad (10)$$

and the unconditional subsurvival function is $S_j(t \mid v_s) = p_j(s)S_0(\theta(v_s)t)$. Further, if δ_s does not depend on v_s then $S_j(t \mid v_s) = p_j S_0(\theta(v_s)t)$ which is the same as that in the earlier subsection.

A general form of the shift function which can be used here (see equation 2.3, page 2077, Klein and Basu, 1981) is

$$\theta(v_s, j) = \exp\{\sum_{l=0}^{k_j} \beta_{jl}\theta_{jl}(v_s)\}, \; j = 1, 2, \ldots, k, s = 1, 2, \ldots, m, \tag{11}$$

where $\theta_{j0}(v_s) = 1$ and $\theta_{j1}(v_s), \ldots, \theta_{jk_j}(v_s)$ are k_j non-decreasing functions of v_s. The power model with $\theta(v_s, j) = \beta_{j0} v_s^{\beta_{j1}}$, the Arrhenius model with $\theta(v_s, j) = \exp\{\beta_{j0} - \beta_{j1}/v_s\}$, and the Eyring model with $\theta(v_s, j) = v_s^{\beta_{j2}} \exp\{\beta_{j0} - \beta_{j1}/v_s\}$ are special cases of the above shift function.

The probabilities $p_j(v_s)$ can be modelled using any standard distribution like logistic distribution so that $p_j(v_s) = e^{\mu+\gamma v_s}/(1 + e^{\mu+\gamma v_s})$. Depending on whether γ is positive or negative, the effect of stress on causes can be studied. The survival function $S_0(t)$ can assume any of the standard functional form used in the accelerated life tests.

In general, it is easy to see that when the effect of stress is multiplicative independence of the failure time and failure cause at the normal stress level implies independence at higher stresses and vice-versa.

3 Estimation

Estimation procedure will depend on the distributional assumption regarding the failure time at the normal stress. Below, we give an expression for likelihood function which can be used in various situations.

Let n independent and identical copies of systems be put to test and out of these n_s systems are put at stress level v_s, $s = 1, ..., m$. Let (T_{si}, δ_{si}), $i = 1, 2, ..., n_s$ be the data obtained from n_s i.i.d. copies put on test at stress v_s. Note that $n = \sum_{s=1}^{m} n_s$. Let Θ denote the parameters involved in the accelerated model and $(\underline{t}, \underline{\delta})$ denote the data.

The likelihood function in terms of the subdensity functions is given by

$$L_1(\underline{t}, \underline{\delta}; \Theta) = \prod_{s=1}^{m} \prod_{i=1}^{n_s} \prod_{j=1}^{k} f_j(t_{si} \mid v_s)^{I(\delta_{si}=j)} \tag{12}$$

and the likelihood function in terms of the cause-specific hazards is given by

$$L_2(\underline{t}, \underline{\delta}; \Theta) = \prod_{s=1}^{m} \prod_{i=1}^{n_s} \prod_{j=1}^{k} h_j(t_{si} \mid v_s)^{I(\delta_{si}=j)} exp(-\int_0^{t_{si}} \sum_{j=1}^{k} h_j(u \mid v_s))du. \tag{13}$$

In case an independent right censoring is imposed, the likelihood contribution from the censored observation at stress v_s is $S(t \mid v_s) = exp(-\int_0^t h(u \mid v_s)du)$ and the above likelihood function can be used directly since the censoring intensity does not include the parameters of interest.

3.1 Stress does not change the causes of failure

In case if the failure time and the cause of failure are independent then the likelihood function $L_1(\underline{t}, \underline{\delta}; \Theta)$ simplifies to

$$
\begin{aligned}
L(\underline{t}, \underline{\delta}; \Theta) &= \prod_{s=1}^{m} \prod_{i=1}^{n_s} \prod_{j=1}^{k} p_j^{I(\delta_{si}=j)} f(t_{si} \mid v_s) \\
&= \prod_{s=1}^{m} \prod_{i=1}^{n_s} \prod_{j=1}^{k} p_j^{I(\delta_{si}=j)} h(t_{si} \mid v_s) exp(-\int_0^{t_{si}} h(u \mid v_s)) du. \qquad (14)
\end{aligned}
$$

Under this model, the standard theory of estimating the survival function in the accelerated life testing applies (Nelson, 1990).

Consider the case when failure time and the cause of failure are dependent and there are two risks operating in the population. Let the accelerated model be specified by equations (4)-(6) with $S_0(t) = exp(-\lambda t)$. The likelihood function is given as

$$
\begin{aligned}
L_1(\underline{t}, \underline{\delta}; \Theta) &= \prod_{s=1}^{m} \prod_{i=1}^{n_s} (p^{\alpha} \alpha \lambda \theta(v_s) exp(-\alpha \lambda t_{si} \theta(v_s)))^{I(\delta_{si}=1)} \\
&\quad (\lambda \theta(v_s) exp(-\lambda t_{si} \theta(v_s)) - (p^{\alpha} \alpha \lambda \theta(v_s) exp(-\alpha \lambda t_{si} \theta(v_s)))^{I(\delta_{si}=2)}.
\end{aligned}
$$

The shift function $\theta(v_s)$ can be appropriately specified according to the equation (2). Numerical method may be required to obtain the maximum likelihood estimates of the parameters.

3.2 Stress changes the causes of failure

Let us consider the model specified by (9) and the shift function given by Arrhenius model $\theta(v_s, j) = \exp\{\beta_{j0} - \beta_{j1}/v_s\}$, for $j = 1, 2, \ldots, k$ and $s = 1, 2, \ldots, m$. Here,

$$
\begin{aligned}
S_j(t|v_s) &= p_j(v_s) \exp(-\sum_{l=1}^{k} \exp\{\beta_{l0} - \beta_{l1}/v_s\} t^{\alpha}) \qquad (15) \\
f_j(t|s) &= p_j(v_s) (\sum_{l=1}^{k} \exp\{\beta_{l0} - \beta_{l1}/v_s\}) \alpha t^{\alpha-1} \exp(-\sum_{l=1}^{k} \exp\{\beta_{l0} - \beta_{l1}/v_s\} t^{\alpha}).
\end{aligned}
$$

Then, the likelihood function (13) is given by

$$
L_1(\underline{t}, \underline{\delta}; \Theta) = \prod_{s=1}^{m} \prod_{i=1}^{n_s} \prod_{j=1}^{k} [f_j(t_{si} \mid s)]^{I(\delta_{si}=j)}.
$$

The likelihood factorises into two parts - function of $p_j(v_s)$ and function of $(\alpha, \beta_{0j}, \beta_{1j})$ for $j = 1, 2, \ldots, k$ and $s = 1, 2, \ldots, m$. Differentiating the log-likelihood function with respect to β_{0j}, β_{1j}, and α, following likelihood equations are obtained

$$
\beta_{0j} = \log \left[\frac{\sum_{s=1}^{m} n_{sj}}{\sum_{s=1}^{m} \sum_{i=1}^{n_s} I(\delta_{si} = j) t_{si}^{\alpha} \exp\{-\beta_{1j}/v_s\}} \right],
$$

$$\sum_{s=1}^{m} \frac{n_{sj}}{v_s} = \sum_{s=1}^{m} \frac{1}{v_s} \sum_{i=1}^{n_s} I(\delta_{si} = j) t_{si}^{\alpha} \exp\{\beta_{0j} - \beta_{1j}/v_s\},$$

$$\alpha = n \left[\sum_{s=1}^{m} \sum_{i=1}^{n_s} t_{si}^{\alpha} \log t_{si} (\sum_{l=1}^{k} \exp\{\beta_{0l} - \beta_{1l}/v_s\} - \sum_{s=1}^{m} \sum_{i=1}^{n_s} \log t_{si} \right]^{-1},$$

where $n_{sj} = \sum_{i=1}^{n_s} I(\delta_{si} = j)$.

Solving the above equations, the maximum likelihood estimates of the parameters are obtained. The probabilities $p_j(v_s)$ can be modelled appropriately as a function of stress and the estimators can be obtained using the second part of the likelihood, so as the probabilities at the normal stress levels can be inferred.

As a special case, take

$$p_1(v_s) = P(\delta = 1 \mid v_s) = \frac{exp(\mu + \eta v_s)}{1 + exp(\mu + \eta v_s)}.$$

The maximum likelihood estimators of μ and η are obtained by maximising the first part of the likelihood in the usual way. The log-likelihood is

$$\begin{aligned} L(\underline{\delta}; p_1, p_2, p_3) &= \sum_{s=1}^{m} n_{s1} log(p_1(v_s)) + n_{s2} log(1 - p_1(v_s)) \\ &= n_1 \mu + \alpha \sum_{s=1}^{m} n_{s1} v_s - \sum_{s=1}^{m} n_s log(1 + exp(\mu + \eta v_s)). \end{aligned}$$

Now, it is straight forward to obtain the maximum likelihood estimators of μ and η. An estimate of the probability of failures due to risk 1 at normal stress is obtained by allowing stress $v_s = v_0$.

All the standard procedures for testing parameters and for finding confidence intervals are valid.

4 Illustration

We reanalyse the data given in Klein and Basu (1981) by considering the model (15) with $\alpha = 1$. Here, we consider three stress levels $v_0 = (190 + 273.16)/1000$, $v_1 = (220 + 273.16)/1000$, and $v_2 = (240 + 273.16)/1000$. There are three possible types of failures corresponding to the distinct parts of insulation system namely turn (risk 1), phase (risk 2) and ground (risk 3). We also assume that a priori the number of failures due to each cause is known. Here, the stress level v_0 is the normal stress. Equal number of units ($n_s = 20$) are tested at each stress level. Table 1 gives the proportion of failures due to each cause at various stress levels. Table 2 gives the maximum likelihood estimates of the parameters $(\beta_{01}, \beta_{02}, \beta_{03}, \beta_{11}, \beta_{12}, \beta_{13})$. Their respective variances are $(1.28, 0.19, 2.13, 0.31, 0.47, 0.49)$ and the covariances between β_{0j} and β_{1j} are $(0.63, 0.96, 1.21)$, respectively for $j = 1, 2, 3$. Note that the covariance between the extomators of β_{0j} and $\beta_{1j'}$ is zero for $j \neq j'$.

Figures 1a, 1b and 1c show the comparison of the empirical estimates of the subsurvival functions corresponding to turn failure at stress levels v_1, v_2 and v_0, respectively. Similarly, Figures 2a-2c and 3a-3c show the comparison of the empirical estimates of the subsurvival functions corresponding to phase and ground failures at stress levels v_1, v_2 and v_0, respectively. The comparison of overall survival function at various stress levels is shown in Figures 4a-4c. It is clear from figures that the fitted distributions match the corresponding empirical functions in almost all the cases. The estimates of the parameters using stress levels v_1 and v_2 are used to estimate the subsurvival functions at the normal stress. It can be seen from Figures 1c, 2c, 3c and 4c that the estimated subsurvival functions are close to the empirical functions obtained using the data from the normal stress.

5 Discussion

The present paper proposes the models to analyse the accelerated failure data in the presence of dependent competing risks. To the best of our knowledge this approach has been described in detail for the first time here. We have also suggested procedures for estimating the subsurvival functions under normal stress in the presence of accelerated testing when the risks are dependent. In the illustration, the proportions of failures due to various causes are assumed to be known. In general, when the units are tested at several stress levels, unlike just two stress levels in our iluustration, it is possible to model the $p_j(s)$ for $j = 1, 2, \dots, k$ and use it to infer the proportions at the normal stress level.

The proposed methods can also be used for any form of subdensity function, stress function, and adjusted to include step-stress models and also situations where stress is time dependent.

In practical situations, it is likely that the stress affects some competing causes and increase in stess levels decreases the subsurvival function corresponding to a specific cause. We are currently working on nonparametric estimation of subsurvival functions under such ordered restriction.

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Table 1: Proportion of failures due to the three causes at various stress levels

Stress	n_s	Turn	Phase	Ground
v_0	20	0.4	0.25	0.35
v_1	20	0.55	0.25	0.20
v_2	20	0.5	0.35	0.15

Table 2: Maximum likelihood estimates of β_0 and β_1 parameters for the three causes

Cause	β_0	β_1
Turn	3.087	4.696
Phase	3.262	4.911
Ground	-1.302	2.522

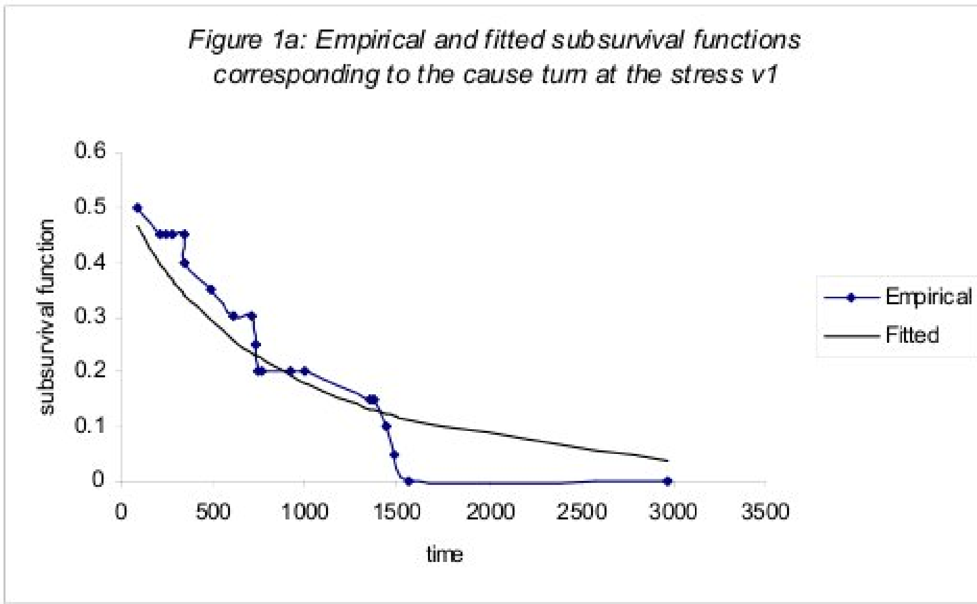
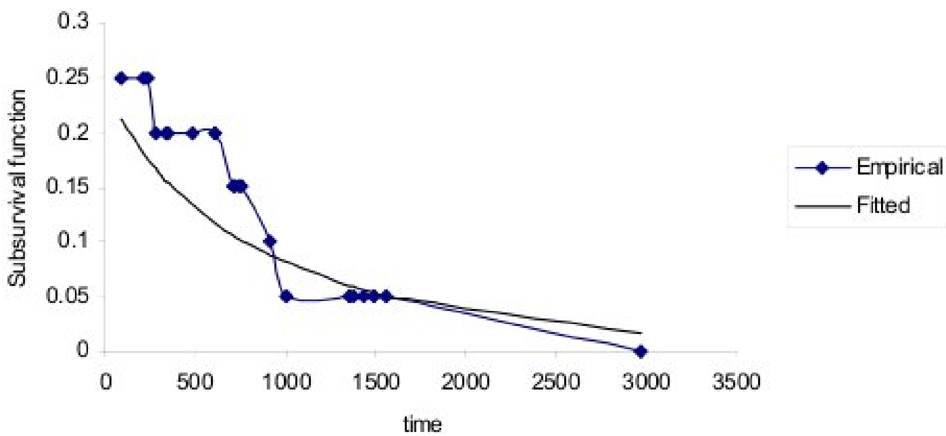


Figure 1a: Empirical and fitted subsurvival functions corresponding to the cause turn at the stress v1

Figure 2a: Empirical and fitted subsurvival functions corresponding to the cause phase at the stress v1



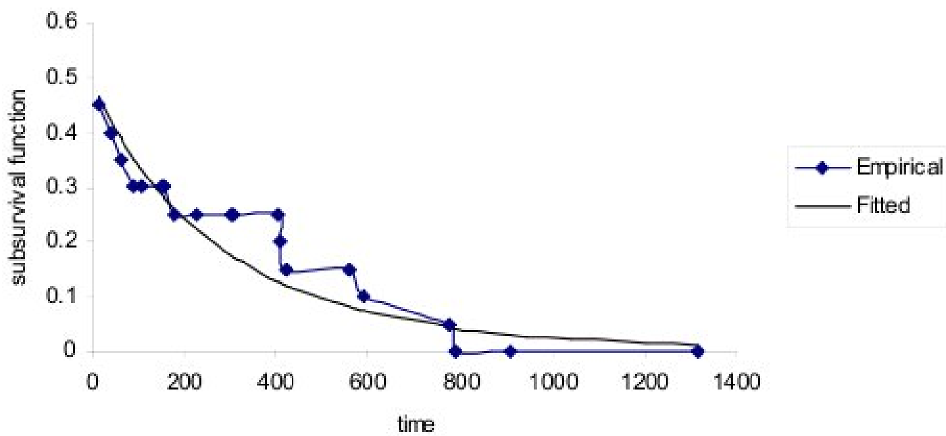
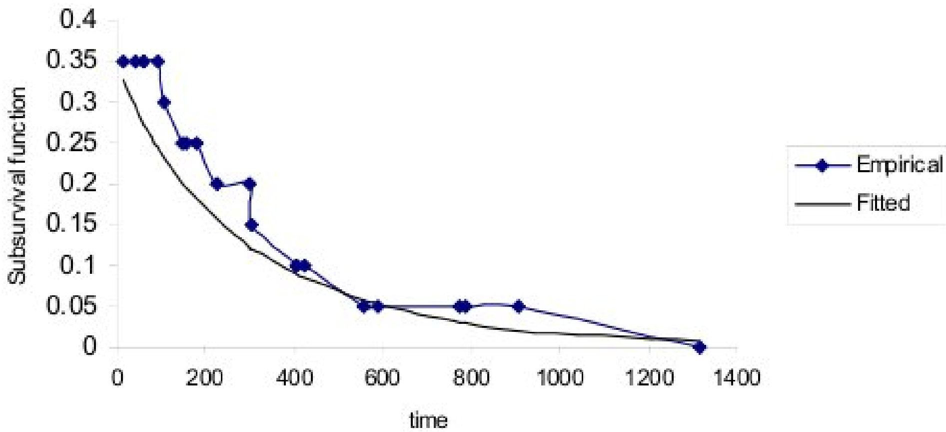


Figure 1b: Empirical and fitted subsurvival functions corresponding to the cause turn at the stress v2

Figure 2b: Empirical and fitted subsurvival functions corresponding to the cause phase at the stress v2



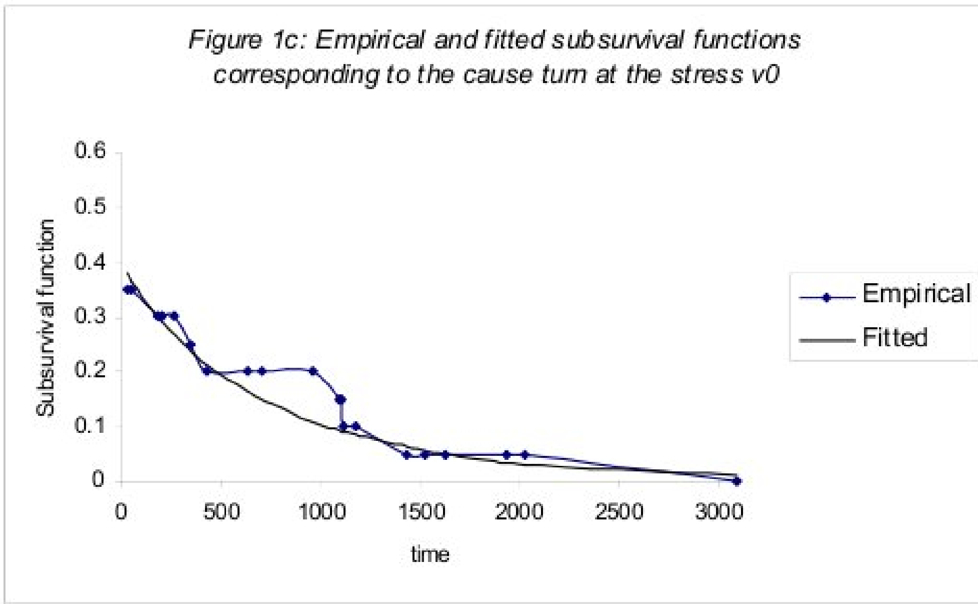


Figure 1c: Empirical and fitted subsurvival functions corresponding to the cause turn at the stress v0
Subsurvival function
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500
2000
2500
3000
time
Empirical
Fitted

Figure 2c: Empirical and fitted subsurvival functions corresponding to the cause phase at the stress v0

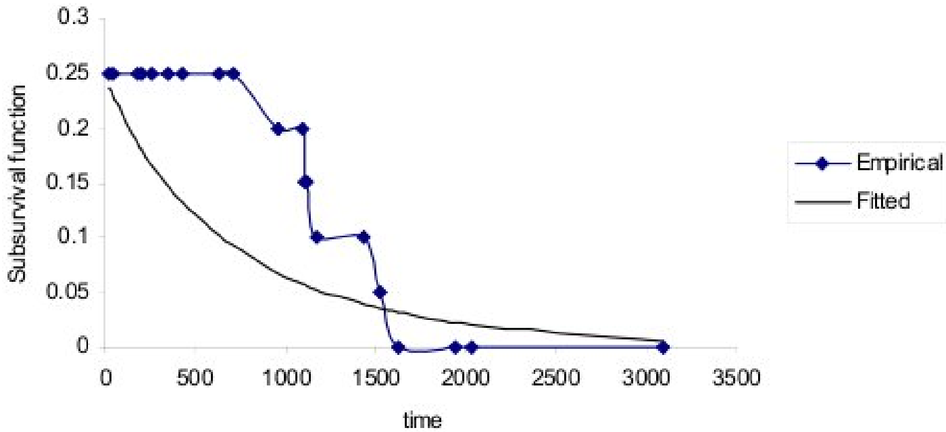
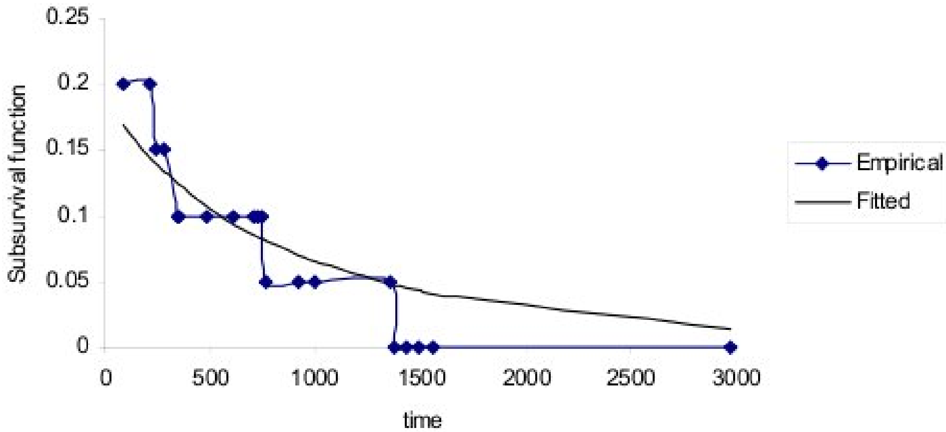


Figure 3a: Empirical and fitted subsurvival functions corresponding to the cause ground at the stress v1



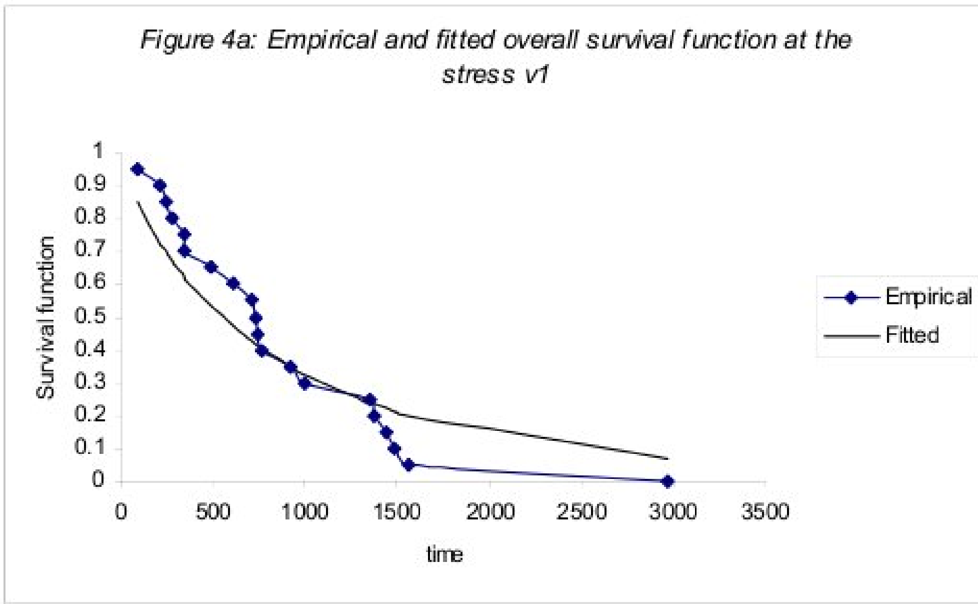


Figure 4a: Empirical and fitted overall survival function at the stress v1
Survival function
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500
2000
2500
3000
3500
time
Empirical
Fitted

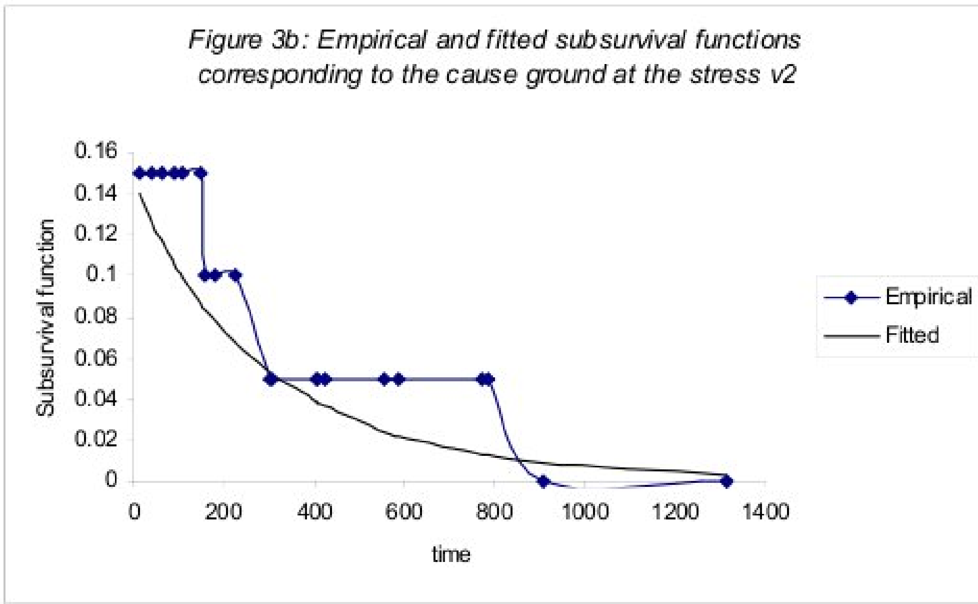


Figure 3b: Empirical and fitted subsurvival functions corresponding to the cause ground at the stress v2
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Subsurvival function
0
200
400
600
800
1000
1200
1400
time
Empirical
Fitted

Figure 4b: Empirical and fitted overall survival function at the stress v2

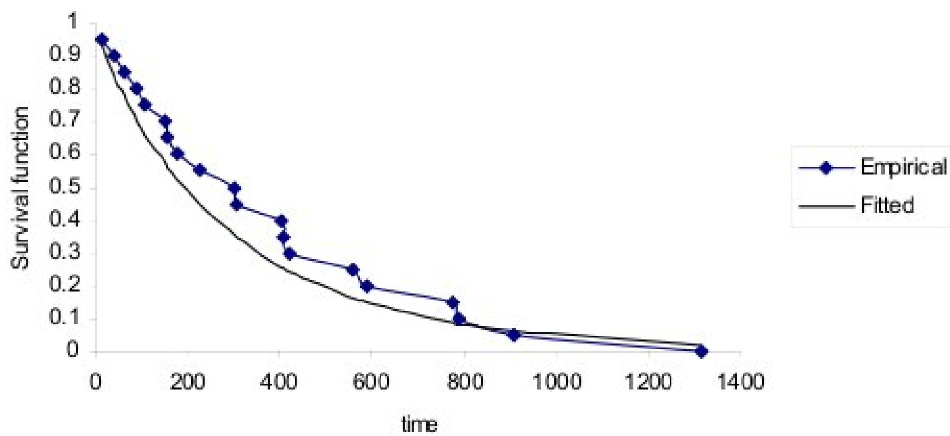


Figure 3c: Empirical and fitted subsurvival functions corresponding to the cause ground at the stress v0

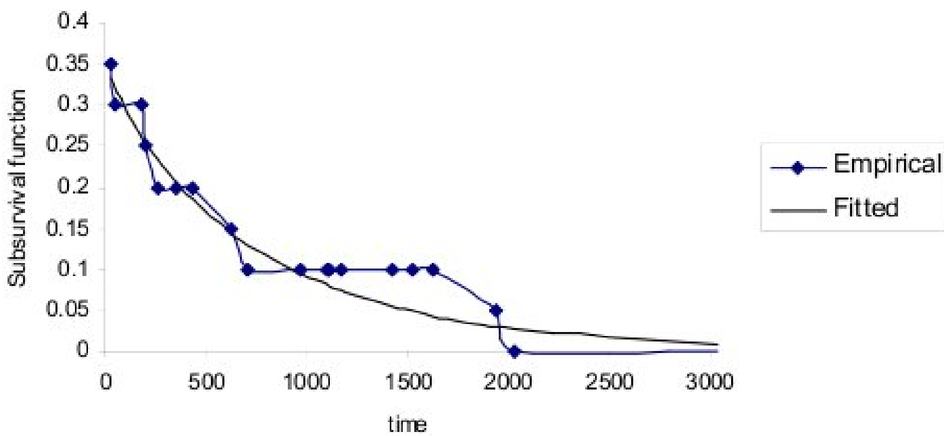


Figure 4c: Empirical and fitted overall survival function at the stress v0