Studies on the formation of large amplitude kinetic Alfvén wave solitons and double layers in plasmas

N. Devi[a)] and R. Gogoi[b)]
Department of Mathematics, Cotton College, Guwahati-781001, Assam, India

G. C. Das
Mathematical Sciences Division, Institute of Advanced Study in Science and Technology, Paschim Boragaon, Guwahati-781035, Assam, India

R. Roychoudhury
Physics and Applied Mathematical Unit, Indian Statistical Institute, Kolkata 700108, India

(Received 27 September 2006; accepted 11 December 2006; published online 29 January 2007)

A two fluid model has been employed to study the oblique propagation of solitary kinetic Alfvén waves. Formation of solitary waves and double layers is observed. Amplitude, width (in the case of solitons), and thickness (in the case of double layers) of the nonlinear structures are studied in some detail. Wider solitary structures are found to exist for oblique propagation nearer to the magnetic field direction. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2432049]

I. INTRODUCTION

Kinetic Alfvén waves (KAW) can be described as a coupling effect between the Alfvén wave and the ion acoustic wave modes in ionospheric and space plasmas.[1–5] Here the transverse wavelength of the shear Alfvén wave becomes close to the ion acoustic gyroradius, $\rho_s=\sqrt{k_B T_e/m_i}/\omega_{ci}$, k_B=Boltzmann constant or the electron inertia length $\lambda_e = c/\omega_{pe}$, and the wave becomes modified to be called the kinetic Alfvén waves (KAW). This mode lies between fast and slow magnetohydrodynamic (MHD) modes, where the perpendicular wavelength becomes comparable to ion gyroradius, thereby acquiring dispersion for an oblique propagation.

These KAWs are dispersive waves and can generate a parallel electric field. Hence, they are responsible more effectively, compared to the Alfvén waves, for plasma heating via Landau damping. In the process, charge separation occurs because of the deviations of the ions from magnetic lines of force while the electrons remain attached to the field line because of their small Larmor radius. Extensive works have been done by many workers so far, where one can find explanations in regards to different scientific satellite observations [found in serial papers presented in Space Sci. Rev. **70**, Nos. 3/4 (1994)].[6,7]

Most of the studies relate to the formation and propagation of solitary kinetic Alfvén waves (SKAW). Employing the two fluid model, Hasegawa and Mima,[8] Yu and Shukla[9] investigated the existence of SKAWs propagating in an oblique direction with respect to the ambient magnetic field in a magnetized plasma with $\alpha \gg 1$ ($\alpha=\beta/2Q$, β=thermal pressure by electron/magnetic pressure by ambient magnetic field, Q=electron mass/ion mass). Works are also done for $\alpha>1$ (neglecting displacement current,[10] using parallel ion inertia and neglecting displacement current[11]). Moreover the work of Wu *et al.*[12] gives evidence of higher frequency enhancement of the spectrum related to density fluctuation for $\alpha \sim 1$ as observed by space satellites.

In these studies the generation of electric fields that are parallel to the local magnetic field direction is found to be self-consistent and accordingly the formation and propagation of kinetic Alfvén solitons (KASs) have been studied through the Sagdeev potential.[13] Here, the question of the potential drop along the field lines being extended over large distances or the confinement to a narrow layer may arise. In the later case, a double layer (DL), a typical electrostatic structure producing net potential difference, may be studied to have a reasonable explanation for the acceleration of charged particles generating the aurora as suggested by Alfvén[14] in 1958. Studies related to double layers are getting impetus because of their existence in any plasma system, from discharge tubes to space plasmas and to the Birkeland currents supplying the Earth's aurora. Due to the potential drop across a DL, acceleration of electrons and positive ions occur in opposite directions (irrespective of the width of DL). Such acceleration of the charged particles may result in beams or jets of charged particles. On acquiring relativistic velocities by accelerated electrons and ions, synchrotron radiation may be produced in the form of radio waves, x rays, and gamma rays. Therefore DLs are used to explain some astrophysical observations. Peratt[15] has written: "Since the double layer acts as a load, there has to be an external source maintaining the potential difference and driving the current. In the laboratory this source is usually an electrical power supply, whereas in space it may be the magnetic energy stored in an extended current system, which responds to a change in current with an inductive voltage." DLs in magnetized plasma are mainly applicable to cosmic plasmas. The thickness of the DLs will depend on the violation of quasineutrality in the plasma. Generally quasineutrality can only be violated over a scale of the Debye length. In ionospheric plasmas, the thickness of a DL is of the order of a few centimeters.

[a)]Electronic mail: nirupama_cotton@rediffmail.com
[b)]Electronic mail: runmoni_gogoi@rediff.com

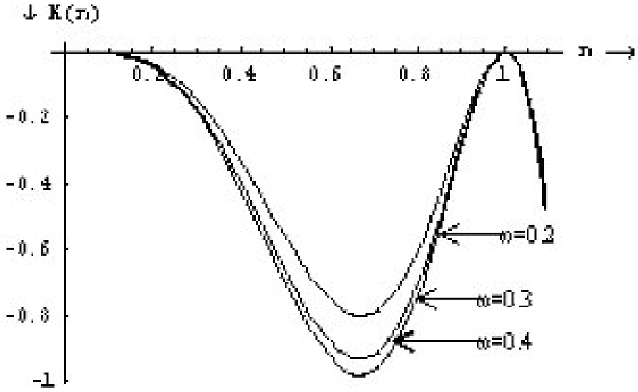

FIG. 1. The Sagdeev potential $K(n)$ vs density n is plotted for different values of $\omega(<1)$ with $k_z=0.1$, $\alpha Q=0.0027$.

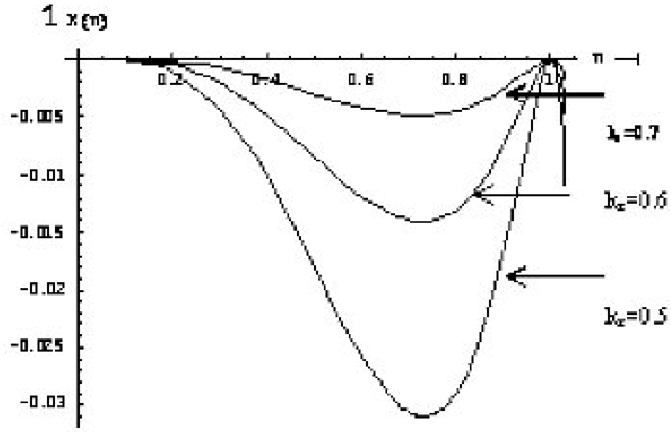

FIG. 2. The Sagdeev potential $K(n)$ vs density n is plotted for different values k_z with $\omega=0.8$ and with $\alpha Q=0.54$.

In most of the works on KAWs double layer solutions were obtained in the small amplitude limit. In this work an exact analytical expression is obtained for the Sagdeev potential. Also exact numerical results were obtained for arbitrary amplitude double layers. Here we have considered a general type of ion motion under the effect of pressure gradient and inertia of electrons moving in the direction of the external magnetic field. We have adopted a fluid plasma model since the rate of Landau damping remains small under inertial effects of electrons.

II. BASIC EQUATIONS

The basic equations are as follows:
For the electrons,

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e v_{ez})=0, \tag{1}$$

$$\frac{\partial v_{ez}}{\partial t}+v_{ez}\frac{\partial v_{ez}}{\partial z}=\alpha\left(-E_z-\frac{1}{n_e}\frac{\partial n_e}{\partial z}\right). \tag{2}$$

For the ions,

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_{ix})+\frac{\partial}{\partial z}(n_i v_{iz})=0, \tag{3}$$

$$\frac{\partial v_{ix}}{\partial t}+v_{ix}\frac{\partial v_{ix}}{\partial x}+v_{iz}\frac{\partial v_{ix}}{\partial z}=\alpha Q E_x+v_{iy}, \tag{4}$$

$$\frac{\partial v_{iy}}{\partial t}+v_{ix}\frac{\partial v_{iy}}{\partial x}+v_{iz}\frac{\partial v_{iy}}{\partial z}=-v_{ix}, \tag{5}$$

$$\frac{\partial v_{iz}}{\partial t}+v_{ix}\frac{\partial v_{iz}}{\partial x}+v_{iz}\frac{\partial v_{iz}}{\partial z}=\alpha Q E_z. \tag{6}$$

From Maxwell's equations

$$-\frac{\partial^3 E_x}{\partial z^2\,\partial x}+\frac{\partial^3 E_z}{\partial x^2\,\partial z}=\frac{1}{\alpha Q}\left[\frac{\partial^2 n_e}{\partial t^2}+\frac{\partial^2}{\partial t\,\partial z}(n_i v_{iz})\right]. \tag{7}$$

Here $Q=m_e/m_i$ (electron to ion mass ratio), $E_x=-\partial\phi/\partial x$, $E_z=-\partial\psi/\partial z$ (two potentials ϕ, ψ are included to justify a low-β plasma model), and $\alpha=\beta/2Q$.

We have normalized densities by the equilibrium plasma density n_0, time by the inverse of the ion cyclotron frequency Ω_{ci}^{-1}, velocities by Alfvén velocity $v_A=cB_0/(4\pi n_0 m_i)^{1/2}$, space by $\rho_s=c/\omega_{pi}$ (the ratio between the velocity of light and the ion plasma frequency), electric fields by $T_e\Omega_{ci}/ev_A$, and magnetic field by B_0.

III. DERIVATION OF THE SAGDEEV POTENTIAL

For a plasma in a uniform ambient magnetic field B_0 along the z direction, the stationary independent variable η is given as $\eta=xk_x+zk_z-\omega t$ with $\omega=V/V_A$, $k_x^2+k_z^2=1$ where ω is the phase velocity of the wave in the unit of the Alfvén velocity V_A. Using this set up for stationary frame, Eqs. (1)–(7) can be reduced to the following:

$$n_e=e^{-(1/k_z)\int E_z d\eta}\left[\exp A\left(1-\frac{1}{n_e^2}\right)\right], \tag{8}$$

$$k_x v_{ix}+k_z v_{iz}=\omega\left(1-\frac{1}{n_i}\right), \tag{9}$$

$$-\frac{\omega}{n_i}\frac{\partial v_{ix}}{\partial\eta}=\alpha Q E_x+v_{iy}, \tag{10}$$

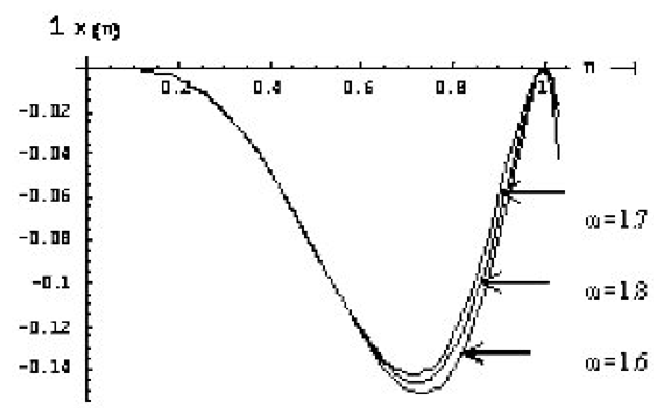

FIG. 3. The Sagdeev potential $K(n)$ vs density n is plotted for different values of $\omega(>1)$ with $k_z=0.3$ and with $\alpha Q=2.0844$.

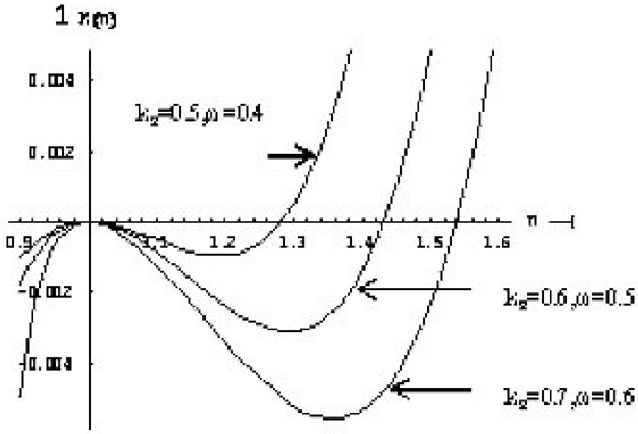

FIG. 4. The Sagdeev potential $K(n)$ vs density n is plotted for different values of k_z and ω with $\alpha Q=0.54$.

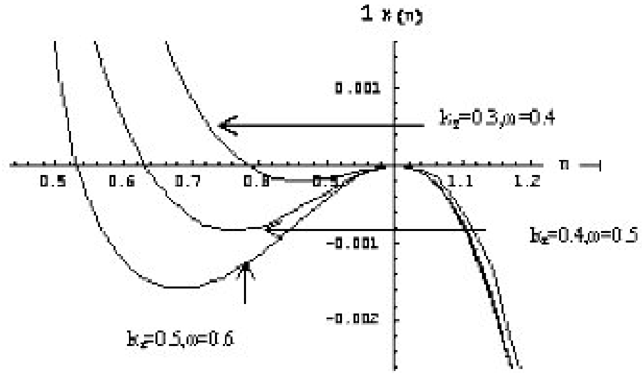

FIG. 5. The Sagdeev potential $K(n)$ vs density n is plotted for different values of k_z and ω with $\alpha Q=2.0844$.

$$\frac{\omega}{n_i}\frac{\partial v_{iy}}{\partial \eta}=v_{ix}, \tag{11}$$

$$-\frac{\omega}{n_i}\frac{\partial v_{iz}}{\partial \eta}=\alpha Q E_z, \tag{12}$$

$$-k_x k_z^2\frac{\partial^3 E_x}{\partial \eta^3}+k_z k_x^2\frac{\partial^3 E_z}{\partial \eta^3}=\frac{1}{\alpha Q}\left[\omega^2\frac{\partial^2 n_e}{\partial \eta^2}-\omega k_z\frac{\partial^2}{\partial \eta^2}(n_i v_{iz})\right] \tag{13}$$

under the boundary conditions $v_{ix}=v_{iz}=v_{ez}=0$ at $n_i=n_e=1$ when $\eta\rightarrow\infty$.

In Eq. (8) the parameter A is given by $A=\omega^2/2\alpha k_z^2$. Using the charge neutrality condition $n_i=n_e=n$ in the above equations we get, after some algebra, the Sagdeev potential equation as

$$\frac{1}{2}\left(\frac{dn}{d\eta}\right)^2+K(n,\alpha,Q,\omega,k_z)=0, \tag{14}$$

where

$$\begin{aligned}K(n,\alpha,Q,\omega,k_z)=&-\frac{n^4\left(1-\frac{k_z^2}{\omega^2}\right)}{k_z^2\left\{\frac{\omega^2}{\alpha Q}-(n^2-2A)\right\}^2}\Bigg[\frac{1}{2}k_z^2n^4-\left\{k_z^2(2A+1)+\frac{\omega^2}{\alpha Q}\right\}n^3+\frac{\omega^2}{\alpha Q}(1-k_z^2)n^2\log n\\&+\left[\frac{\omega^2}{\alpha Q}\left\{(1+A)(1+k_z^2)+\frac{\omega^2}{2\alpha Q}\right\}+\{(1+2A)^2+4A\}\frac{k_z^2}{2}\right]n^2\\&-\left[\frac{\omega^2}{\alpha Q}\left\{\left(2A+\frac{\omega^2}{\alpha Q}\right)+(2A+1)k_z^2\right\}+2Ak_z^2(1+2A)\right]n+\frac{\omega^2}{\alpha Q}\left\{\frac{\omega^2}{2\alpha Q}+A(1+k_z^2)\right\}+2A^2K_z^2\Bigg]\end{aligned} \tag{15}$$

is the Sagdeev potential.

Equation (15) is the main result of this paper.

The boundary condition used in deriving Eq. (14) is given as $dn/d\eta=0$ at $n=1$.

IV. MATHEMATICAL CONDITIONS FOR THE EXISTENCE OF SOLITONS AND DOUBLE LAYER KINETIC ALFVÉN WAVES

Equation (14) can be written as [writing $K(n,\alpha,Q,\omega,k_z)$ as $K(n)$]

$$\frac{d^2n}{d\eta^2}=-\frac{\partial K(n)}{\partial n}. \tag{16}$$

Equation (16) looks like the equation of motion of a particle with coordinate n, moving under a force with potential $K(n)$. This is the reason why $K(n)$ is sometimes referred to as the pseudopotential. For the solitary waves to exist there must be a potential well so that the particle will be reflected back from a point given by $n=N$ ($\neq 1$). The nature of the potential well is depicted in Figs. 1–6, which will be discussed later on.

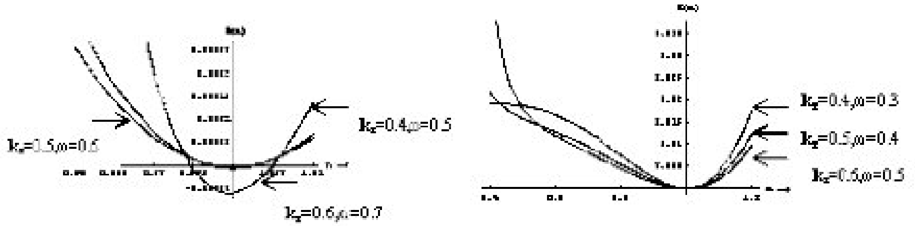

FIG. 6. The Sagdeev potential $K(n)$ vs density n is plotted for different values of k_z and ω with $\alpha Q<1$ and $\alpha Q>1$, respectively.

However we shall discuss here the conditions for the existence of such potential wells.

The conditions are

(i) $K(n)<0$ between $n=1$ and $n=N$ [so that $dn/d\eta$ is real, Eq. (15)], here N gives the amplitude of the solitary wave. N can be both greater than 1 and less than 1. In the former case we have the compressive solitary waves and in the latter case we have the rarefactive solitary waves.

(ii) $K(n)$ must be a maximum at $n=1$ which means

$$\left.\frac{\partial K(n)}{\partial n}\right|_{n=1}=0 \quad \text{and} \quad \left.\frac{\partial^2 K(n)}{\partial n^2}\right|_{n=1}>0.$$

(iii) $K(n)$ should cross the "n" axis from below near $n=N$; and $K(n)>0$ for $n>N$. For the existence of double layers, conditions (i) and (ii) should be valid. But instead of condition (iii) we have the following condition:

(iii)′ $K(n)$ must have a maximum at $n=N$ $(\neq 1)$ which means, $K(n)=0$, $\partial K/\partial n=0$ for $n=N$. In other words $K(n)=0$, has a double root $n=N$. [Note that because of condition (i) $K(n)$ has a double root at $n=1$.]

To get a clear picture we expand $K(n)$ near $n=1$ and $n=N$. Near $n=1$, $K(n)$ is given by

$$K(n)=-\frac{(n-1)^2\left(1-\frac{k_z^2}{\omega^2}\right)}{2k_z^2\left\{\frac{\omega^2}{\alpha Q}-(1-2A)\right\}}\left\{\frac{\omega^2}{\alpha Q}-(1-2A)k_z^2\right\} \quad (17)$$

and near $n=N$, we have

$$K(N)=-\frac{(n-N)(n-1)N^3\left(1-\frac{k_z^2}{\omega^2}\right)}{k_z^2\left\{\frac{\omega^2}{\alpha Q}-(N^2-2A)\right\}}\times\left\{\frac{\omega^2}{\alpha Q}-(N-2A)k_z^2\right\}. \quad (18)$$

Case (i) $N<1$

$n-N>0, \quad n-1<0.$

$$\text{If} \quad \frac{\omega^2}{\alpha Q}<(1-2A)k_z^2<(1-2A) \quad (19)$$

$$\text{and} \quad (N-2A)k_z^2<\frac{\omega^2}{\alpha Q}<(N^2-2A) \quad (20)$$

then $K(n)<0$ between $n=1$ and $n=N$ for $\omega>k_z$.

Case (ii) $N>1$

$n-N<0, \quad n-1>0.$

$$\text{If} \quad (1-2A)k_z^2<\frac{\omega^2}{\alpha Q}<(1-2A) \quad (21)$$

$$\text{and} \quad \frac{\omega^2}{\alpha Q}<(N-2A)k_z^2<(N^2-2A) \quad (22)$$

then $K(n)<0$ for $1<n<N$ for $\omega<k_z$.

Thus we see that both compressive and rarefactive solitons can exist depending on the values of the plasma parameters.

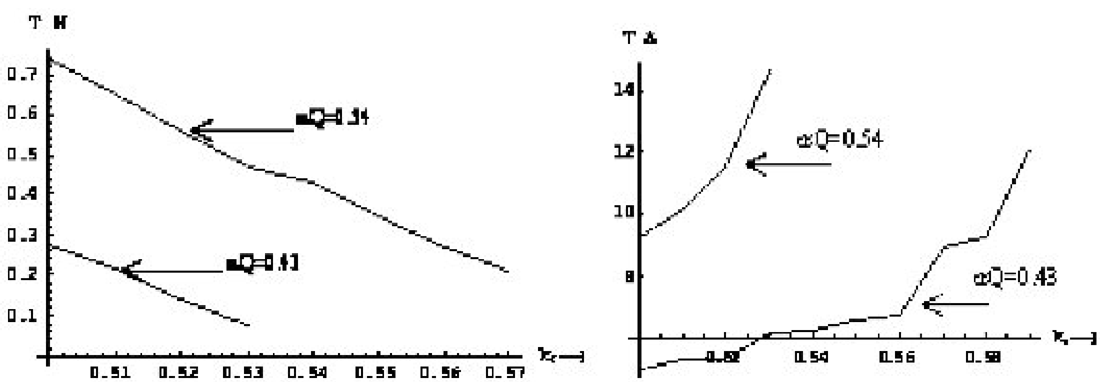

FIG. 7. Amplitude (N) and width (Δ) vs k_z is plotted for different values of $\alpha Q<1$, respectively.

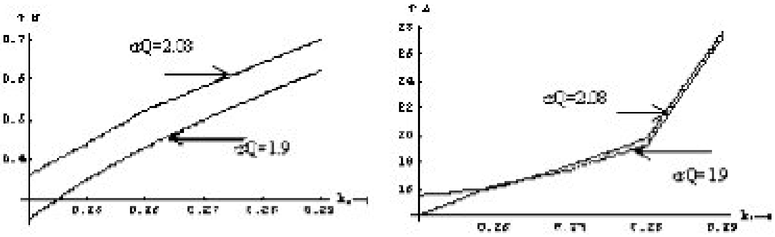

FIG. 8. Amplitude (N) and width (Δ) vs k_z is plotted for different values of $\alpha Q>1$, respectively.

The width of the soliton also can be found from the Sagdeev potential.

If N is the amplitude of the soliton, then the width is $N/\sqrt{d}$ where d is the maximum depth of the Sagdeev potential $K(n)$.

V. RESULTS AND DISCUSSIONS

We first discussed the double layers (DL). Figures 1–3 show the nature of the Sagdeev potential when the conditions for the existence of double layers are satisfied. It is seen from Fig. 1 that for fixed k_z, the depth of the potential increases (so does the depth of the double layer) as ω increases. Figure 2 shows that the case is reversed as far as k_z is concerned and the depth of the DL decreases as k_z increases, provided ω is fixed. The formation of the DL for $k_z=0.3$ and $\alpha Q=2.0844$ is shown in Fig. 3 for different values of ω. Here also the depth increases as ω increases.

Figures 4 and 5 show the nature of the Sagdeev potential when the conditions for the existence of solitary waves, other than double layers, are satisfied. Figure 4 shows the nature of the Sagdeev potential for several sets of values of ω and k_z for $\alpha Q=0.54$. The same is shown in Fig. 5 for a larger value of αQ, $\alpha Q=2.0844$. It is seen that if both k_z and ω increase the depth increases.

It is observed from the numerical analysis of the Sagdeev potential [Eq. (15)] that solitary waves exist when values of k_z and ω are close to each other and (i) $k_z>\omega$, when $\alpha Q<1$ (ii) $k_z<\omega$, when $\alpha Q>1$. Figures 6(a) and 6(b) demonstrate these facts. In Figs. 7(a), 7(b), 8(a), and 8(b), respectively, amplitude (N) and width (Δ) are plotted against k_z for values of $\alpha Q<1$ and $\alpha Q>1$. It appears that both the amplitude and the width of the soliton increases with k_z though the rates of increase are different for these two attributes of solitary waves.

VI. CONCLUSION

A two fluid model is employed to study the oblique propagation of SKAW. An exact analytical expression for the Sagdeev potential is obtained. Existence of solitary waves and double layers are discussed in detail and the Sagdeev potential is evaluated numerically in cases when solitary waves and double layers exist. It is shown how the amplitude and width of the soliton vary with the plasma parameters ω and k_z.

ACKNOWLEDGMENTS

The authors (N.D., R.G., and G.C.D.) are grateful to the head of the Department of Mathematics, Cotton College, Guwahati, Assam, India, for providing the necessary facilities to carry out our research. The authors are also grateful to the anonymous referee for valuable suggestions and constructive comments.

The authors acknowledge the Department of Science and Technology, Government of India for providing a research grant in the form of a project (SP/S2/K-14/2001).

[1]P. Louarn, J. E. Wahlund, T. Chust, H. de Féraudy, A. Roux, B. Hollback, P. O. Dovner, A. I. Erikson, and G. Holmgren, Geophys. Res. Lett. **21**, 1847 (1994).
[2]M. Volwerk, P. Louarn, T. Chust, H. de Féraudy, and A. Roux, J. Geophys. Res. **101**, 13335 (1996).
[3]D. J. Wu, D. Y. Wang, and C. G. Falthammar, Phys. Plasmas **2**, 4476 (1995).
[4]D. J. Wu, and D. Y. Wang, Phys. Plasmas **3**, 4304 (1996).
[5]D. J. Wu and J. K. Chao, Phys. Plasmas **10**, 3787 (2003).
[6]B. Holback, S. E. Jansson, L. Ahlen, G. Lundgren, L. Lyngdol, S. Powell, and A. Meyer, Space Sci. Rev. **70**, 577 (1994).
[7]P. A. Lindqvist, G. T. Marklund, and L. G. Blomberg, Space Sci. Rev. **70**, 594 (1994).
[8]A. Hasegawa and K. Mima, Phys. Rev. Lett. **37**, 690 (1976).
[9]M. Y. Yu and P. K. Shukla, Phys. Fluids **21**, 1457 (1978).
[10]M. K. Kalita and B. C. Kalita, J. Plasma Phys. **35**, 267 (1986).
[11]B. C. Kalita and N. Devi, J. Plasma Phys. **56**, 36 (1996).
[12]D. J. Wu, G. L. Huang, D. Y. Wang, and C. G. Falthamar, Phys. Plasmas **3**, 2879 (1996).
[13]R. Z. Sagdeev, *Review of Plasma Physics* (Consultants Bureau, New York, 1966), Vol. 4, p. 52.
[14]H. Alfvén, Tellus **10**, 104 (1958).
[15]A. Peratt, *Physics of the Plasma Universe* (Springer, New York, 1992), p. 120.