Contemporary Issues and Ideas in Social Sciences
December 2006

On the Relationship between the Number of Firms and the Endogenous Growth Rate for a Model with Public Infrastructure

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Abstract: The paper considers a model of endogenous growth where public infrastructure acts as the engine of growth. The service of infrastructure in production is assumed to be a pure public good. The analytical implication of this assumption is that even with identical firms in any given sector of production, the numbers of firms in the different sectors appear as explicit parameters in the model. This allows for the possibility of comparative statics exercises with respect to the number of firms. The paper explains the manner in which changes in the number of firms can improve or worsen the balanced rate of growth for the system both under autarky and free trade.

JEL Classification: F43, H23, H41, H54, O11

Key Words: Endogenous growth, Infrastructure, Autarky, Free Trade, Number of Firms, Public goods, Tax.

1 Introduction

In a recent paper, Dasgupta and Shimomura (2005) studied the possibility of sustained endogenous growth in a developing economy characterized by the existence of surplus labour and scarce private and public capital. The services of public capital were assumed to be a pure public good that the private sector, a profit maximizer, would normally be averse to supplying. Consequently, the government was assumed to finance infrastructure accumulation on a non-profit basis. It raised revenues from the private sector in the form of lump sum taxes and spent these on purchasing private capital and labour services from the market. Thus, the government contracted out public infrastructure accumulation to the private sector and the size of the contract was limited by the extent of the government's tax generated budget.

The economy in question was assumed to be small compared to world trade and Dasgupta and Shimomura (DS) studied the existence and stability of a balanced growth path for such an economy under autarky as well as free trade in the presence as well as absence of free foreign direct investment. The general spirit of the conclusions arrived at in that paper was that, under mild restrictions on world relative to autarkic prices, free trade was a growth wise superior policy choice than autarky. The analytical structure of the model was typically macro, comprising of three producers, a dynastic household and an altruistic government. Of the three producers, two belonged to the profit driven private sector, whereas the third producer was the infrastructure accumulating government.

Contrary to typical macro models, however, the fact that all the firms employed a pure public good, viz. the infrastructure services, implied, as will be shown below, that none of the macro firms could be assumed to represent aggregates of small profit maximizing firms. The existence of the pure public input implied that each macro firm was in fact a single firm treating market prices parametrically by assumption. This somewhat unsatisfactory feature can, however, be removed by specifying exogenously the number of firms in the private sector and treating the infrastructure accumulating government as the only macro firm in the model. Introducing this change on the other hand brings along with it new parameters, the number of firms in each sector and this in turn introduces scope for comparative statics exercises with respect to changes in the number of firms. The purpose of this paper is to study the nature of such comparative statics and shed new light on the manner in which changes in the number of firms can affect the rate of growth of the system.

In particular, we shall demonstrate that the growth rate of the economy can go up under autarky with a rise in the number of firms, i.e., even without an opening up of the economy. The reason for this positive link between the number of firms and the growth rate lies in the fact that a larger number of firms generates more revenue for the government for infrastructure growth. Equilibrium values of government revenues, the different commodity prices as well as the real rate of interest can respond to increasing numbers in a manner that is beneficial to growth. An obvious conclusion that follows is that integrating with the world economy is not the only policy choice before developing economies for improving their growth performance, unless of course such integration raises the number of producers.

This intuition is supported by the extension of our results to open economies, where the growth rate need not depend, as per the DS criterion, on the relationship between international and domestic prices alone. Even if world prices moved in the appropriate direction relative to domestic prices a fall in the number of firms could generate forces in the opposite direction.

The next section sets up the model and derives the preliminary result on the analytical importance of the number of firms. Section 3 proves results for the closed economy model. Section 4 extends the results to a small open economy. Section 5 concludes the paper.

2 A Model with a Single Firm for Each Sector

As in DS, the model economy has three sectors of production, denoted Y, Z and G. Sector Y produces a pure consumption good (denoted Y), the Z-sector produces a (Solow (1956) type) consumption-cum-investment good Z. The output of sector G, written $\dot{G}$, is identically the same as investment in public infrastructure. Commodities Y and Z are produced under competitive conditions and investment in G is under government control. In this sense, the model below represents a Mixed Economy. All outputs are produced with the help of the services of private and public capital as well as semi-skilled labour, denoted by K, G and L respectively.

There is a surplus of semi-skilled workers available in unlimited numbers at a subsistence wage rate $\bar{w}$, *á la* Lewis (1954). The government views accumulation of industrial capital as an important vehicle for employment generation. This is captured by assuming labour to be complementary with private capital. Thus, we assume that $L_i/K_i = \lambda = constant,\ i = y,\ z,\ g$ where y, z and g index the Y, Z and G-sectors respectively and L_i and K_i denote aggregate amount of L and K used in sector i. (The coefficient λ can be assumed to vary across sectors at the cost of extra algebra, but without the benefit of additional insight.)

Commodity Z acts as the *numéraire*. The price of commodity Y is p, the rate of interest r and the services of G are supplied free of user charge. Commodities Y and Z are produced under competitive conditions, the rate of interest r being equated to the private marginal product of capital services. The government finances its purchases of private capital by imposing lump-sum taxes T_y and T_z on sectors Y and Z, which could vary across time points as in Barro (1990) and Barro & Sala-i-Martin (2004). The entire tax revenue is spent on purchasing K and L-services at the market rate of interest. Thus, the government does not directly organize the production of infrastructure stocks. Instead, as is often the case for both developed as well as developing economies, it floats tenders to contract out production of infrastructure stocks to private capitalists. The demand for capital is restricted by the government's budget constraint.

Labour and capital being complementary, the single notation K may be employed to denote the joint input of the two factor services. Given this convention, technologies in the two sectors are represented by neoclassical production functions satisfying the Inada conditions. In particular, we write

$$Y = Gf_y(k_y)$$

$$
\begin{aligned}
&= GA_y k_y^{\alpha},\ 1 > \alpha > 0; &(1)\\
Z &= Gf_z(k_z) \\
&= A_z k_z^{\beta},\ 1 > \beta > 0; &(2)\\
\dot{G} &= Gf_z(k_z) \\
&= GA_g k_g^{\gamma},\ 1 > \gamma > 0, &(3)
\end{aligned}
$$

where Y, Z and $\dot{G}$ are the aggregate productions in the three sectors and $k_i = K_i/G$ is the ratio of K and G in sector i.

As with the joint input K, let us use r *wlog* to denote the term $r + \bar{w}\ \lambda$. Given the meaning assigned to r, the necessary *foc*'s for profit maximization are

$$
\begin{aligned}
r &= pf_y'(k_y), &(4)\\
&= f_z'(k_z). &(5)
\end{aligned}
$$

The standard practice in macroeconomics is to interpret the production functions (1) - (3) as representing the aggregate production activities of several identical firms. However, in the presence of the pure public good, this interpretation breaks down.

Proposition 1 *In either of the private sectors, there must be a single firm producing each good.*

Proof: Assume to the contrary that there are $n_y > 1$ identical firms producing commodity Y. Suppose, moreover, that each firm employs κ_y units of capital and G units of the public input. Then, profit maximization implies

$$r = pf_y'(\kappa_y/G) \tag{6}$$

Moreover, by definition, $k_y = n_y\kappa_y/G$. Using, (6) and (4) it follows that $n_y = 1$. Similarly, $n_z = 1$. ■

Comment 1 Note that the above proposition is dependent on the assumption that G is a pure public good. If it were a private good, then each firm would be employing G/n_y units of G and Proposition 1 would not hold. This result shows that the DS model assumed that there is exactly one firm each in sectors Y and Z. Given the perfectly competitive market structure, this amounted to the further assumption that each firm treats prices of commodities parametrically. In the present note, the assumption of a single firm characterizing the Y and Z sectors is replaced by the assumption that there are multiple, but finitely many, identical firms producing these commodities. Of course, the assumption of parametric treatment of prices must be retained even in the changed scenario, as pointed out by Hildenbrand (1974).

3 A Model with Multiple Firms in the Private Sector: Autarky

We assume now that there are n_y identical firms in the Y sector and n_z identical firms in the Z sector. Further, let k_y and k_z represent the K/G ratio employed by each firm in the two sectors. Similarly, k_g is the K/G ratio employed by the government sector.

Profits for each firm in the Y and Z-sectors are given by

$$\left.\begin{array}{rcl} \Pi_y^i &=& pY^i - (r + \bar{w}\lambda)K_y^i - T_y \\ \Pi_z^j &=& Z^j - (r + \bar{w}\lambda)K_z^j - T_z \end{array}\right\} \qquad (7)$$

where the superscripts $i = 1, 2, \cdots n_y$ and $j = 1, 2, \cdots n_z$ denote variables specific to each firm in a sector. Also, T_y and T_z are assumed to be uniform across firms *wlog*. At each point of time, the government fixes T_y and T_z at levels consistent with competitive shares. In other words,

$$\begin{aligned} \frac{T_y}{pY^i} &= 1 - \alpha \\ \frac{T_z}{Z^j} &= 1 - \beta. \end{aligned}$$

This amounts to assuming that the government taxes away all super-normal profits of firms. The assumption is justified by the fact that the government charges the firm for the part of the inputs supplied by itself. Charging a lesser amount would lead to a smaller budget for infrastructure accumulation. Given that infrastructure is a primary constraint on growth, the government wishes to siphon off as large a sum as possible for infrastructure accumulation. Quite apart from this, the assumption is made partly to simplify the algebra. Allowing firms to retain a part of the super-normal profits would imply a payment to private capital that exceeds its marginal product and this in turn would need changes in the assumed market structure.

Since T_y and T_z are lump sum taxes, profit maximization ensures that irrespective of the quantum of K_y and K_z, the entire existing supply of the free input G will be used up by the two private sectors. The G-sector, though, is not a profit maximizer and is *assumed* to employ G to capacity.

The values of marginal products of G in sectors Y and Z are

$$
\begin{aligned}
q_y &= (1-\alpha)pf_y(k_y), \\
q_z &= (1-\beta)f_z(k_z).
\end{aligned}
$$

Noting that q_y and q_z are merely effective prices underlying the lump-sum taxes T_y and T_z, it follows from Euler's Theorem that

$$
\begin{aligned}
T_y &= q_yG \\
T_z &= q_zG.
\end{aligned}
$$

Full employment of capital services implies that at each point of time t,

$$k(t) = n_yk_y(t) + n_zk_z(t) + k_g(t), \tag{8}$$

where $k = K/G$ and K is the aggregate private capital in the economy. The entire tax revenue is spent on purchasing K at the market rate of interest. Thus, capital may not earn its marginal product in the G-sector. The government's budget constraint is written

$$\left.\begin{array}{rcl} rk_g & = & n_yT_y/G + n_zT_z/G \\ & = & n_yq_y + n_zq_z \\ & = & n_y(1-\alpha)pf_y(k_y) + n_z(1-\beta)f_z(k_z) \end{array}\right\} . \tag{9}$$

Dividing (9) by r and substituting from (4), (5), we have

$$k_g = \frac{1-\alpha}{\alpha}\, n_yk_y + \frac{1-\beta}{\beta}\, n_zk_z \; . \tag{10}$$

Also, adding $n_yk_y + n_zk_z$ to both sides of (10),

$$k = n_y\frac{k_y}{\alpha} + n_z\frac{k_z}{\beta} \; . \tag{11}$$

The savings rate at each t is chosen optimally by a dynastic household. The latter is endowed with an instantaneous felicity function

$$u(Y_c(t), Z_c(t)) = ln[v(Y_c(t), Z_c(t))],$$

where $Y_c(t)$ and $Z_c(t)$ are the consumptions of Y and Z by the household at time t and v is an increasing, linearly homogeneous, strictly quasi-concave function in Y_c and Z_c. The household's demand for the two commodities at any t is found by maximizing u subject to

$$\left.\begin{array}{rcl} E(t) + \dot{K}(t) & = & r(t)K(t) \\ p(t)Y_c(t) + Z_c(t) & = & E(t) \end{array}\right\} , \tag{12}$$

where $E(t)$ represents the consumption budget at t. In particular, assuming v to have the Cobb-Douglas form

$$v(Y_c, Z_c) = Y_c^{\delta} Z_c^{1-\delta},\ 0 < \delta < 1,$$

the demand function for $Y_c(t)$ is

$$Y_c(t) = \frac{\delta E(t)}{p(t)}. \tag{13}$$

Under autarky, Y market clearance at each instant requires $Y_c = Y = n_y G f_y(k_y)$, dropping t for simplicity.[4]

The model is completed by noting that the household maximizes

$$U = \int_0^{\infty} ln[Y_c^{\delta} Z_c^{1-\delta}]\ e^{-\rho t}\ dt,$$

subject to (12) at each t. The solution to the problem yields

$$\dot{E} = E(r - \rho). \tag{14}$$

3.1 Relationship between the Number of Firms and the Balanced Growth Rate

Using (13) and market clearance, we have

[4] This implies, by Walras Law, that the Z market clears also.

$$x = \frac{n_y p f_y(k_y)}{\delta}, \tag{15}$$

where $x = E/G$. Further, (14) and (3) reduce to

$$\dot{x} = x(r - \rho - f_g(k_g)). \tag{16}$$

Also, (12) and (3) yield

$$\dot{k} = k\left\{r - \frac{x}{k} - f_g(k_g)\right\}. \tag{17}$$

Under balanced growth, $\dot{x} = \dot{k} = 0$. Using this fact in (16) and (17), we obtain $\rho = x/k$, or, $x = \rho k$. In other words, under balanced growth, optimal expenditure is a multiple of the total wealth of the household, a natural result to expect under the logarithmic specification of the utility function. Now, substituting from (15),

$$\delta\rho k = n_y p f_y(k_y). \tag{18}$$

The last equation must hold if markets are to clear under balanced growth. The LHS of the equation stands for the value of aggregate demand for Y per unit of G and the RHS stands for the value of its aggregate supply per unit of G. Substituting from (4) and (5), we may write $p = f_z'/f_y'$. Using this along with (1) and (11), equation (18) reduces to

$$\frac{n_y k_y f_z'}{\alpha} = \delta\rho\left(\frac{n_y k_y}{\alpha} + \frac{n_z k_z}{\beta}\right). \tag{19}$$

Once again, the LHS is the value of supply of Y and the RHS is the value of demand for Y. We are now in a position to prove the following

Proposition 2 *Suppose a strictly positive balanced growth path exists for each specification of the number of firms under autarky. Then a ceteris paribus rise in the number of firms in the investment goods sector increases the balanced rate of growth of the economy. On the other hand, a ceteris paribus rise in the number of firms in the pure consumption goods sector cannot cause a fall in the the balanced rate of growth.*

Proof: By transfering terms from the RHS to the LHS, we may rewrite the equilibrium condition (19) as

$$\frac{n_y k_y}{\alpha}(f_z' - \delta\rho) - \delta\rho\frac{n_z k_z}{\beta} = 0. \tag{20}$$

The LHS of (20) stands for the value of aggregate excess supply relative to G in the Y market. Thus, (20) says that the value of excess supply of Y must equal zero in equilibrium.

Now assume a *ceteris paribus* rise in n_z. If the result is false then the rise in n_z leads to a fall in the rate of growth or leaves it invariant. In this case, we know from (5) that k_z does not fall. Further, (3) implies that k_g must not rise. Consider now equation (10). Since n_z rises, k_z does not fall and k_g does not rise, it follows that k_y must fall. Intuitively speaking, if the government's choice of k_g does not rise despite a rise in the number of firms in the Z sector and hence a rise in the tax revenue at initial unchanged prices, then equilibrium variables must change so as to negate the initial rise in tax revenue in the Z sector by causing a fall in the revenue raised from the Y sector through a fall in k_y.

It is easy to see that under the assumed behaviour of n_y, k_y and k_z, the expression in (20) turns negative. In other words, a fall in the rate of balanced growth in response to a rise in n_z leads to an excess demand in the Y market so that equilibrium is violated. Hence, a rise in the number of firms in the Z sector must lead to a rise in the balanced rate of growth in the economy.

Consider next a *ceteris paribus* rise in n_y. Suppose that the rate of growth falls. Then k_z must rise and k_g must fall. Consequently, (10) implies that $n_y k_y$ must fall. Following the argument for the case where n_z rises, it is straightforward to see that (20) is violated. Thus, the rate of growth of the economy cannot fall if n_y increases. ■

COROLLARY 1 *A simultaneous rise in the number of firms in both the consumption and investment goods sectors must raise the equilibrium balanced rate of growth of the economy under autarky.*

Proof: Obvious.

Comment 2 Proposition 2 assumes the existence of a balanced growth path. It is a relatively straightforward exercise to verify the existence of a unique balanced growth path by repeating the proof of Proposition 1 in DS, once the equations are modified, as above, to recognize the multiplicity of firms.

4 A Model with Multiple Firms in the Private Sector: Free Trade

DS argued that a movement from autarky to free trade raises the balanced rate of growth of the economy if the free trade value of $p = p^f$ happens to be higher than the autarky equilibrium value of $p = p^a$. This result too must change with the introduction of a multiplicity of firms. In particular, we may prove

Proposition 3 *Suppose that the economy opens up and that the numbers of firms in the pure consumption good sector and the investment good sector do not fall. Then the balanced rate of growth rises if the free trade price of the pure consumption good exceeds its price under autarky.*

Proof: Suppose n_y and n_z do not fall and $p^f > p^a$. If the proposition is false, then $r^f \leq r^a$, where r^f and r^a are the equilibrium rates of interest under free trade and autarky. Hence (4) implies that $k_y^f > k_y^a$, whereas (5) implies that $k_z^f \geq k_z^a$. This means that the RHS of (10) must rise. On the other hand, under balanced growth

$$f_g(k_g) = r - \rho. \tag{21}$$

Since $r^f \leq r^a$ implies $k_g^f \leq k_g^a$, equation (10) is violated. Hence, under the assumed conditions, the free trade rate of growth must be higher than that under autarky. ■

Comment 3 DS argued that it was natural to assume that p would rise with free trade, since the developing economy was likely to be characterized by a comparative advantage in producing commodity Y. Proposition 3 implies that the rate of growth can be improved if trade leads to a rise in the number of firms even if p falls. In other words, the rate of growth could increase independent of comparative advantage considerations. It also suggests that an opening of the economy can worsen the rate of growth if it is accompanied by a reduction in the number of firms.

Comment 4 As with the case of autarky, we do not supply a proof of the existence of a unique balanced growth rate under free trade since it is an immediate extension of the the arguments of Proposition 3 in DS.

5 Conclusion

Externalities play an important role in a large part of endogeonous growth theory. In the present paper, the externalities appear on account of the existence of a pure public input, viz. infrastructure. However, the paper has shown that the introduction of such a public input requires the explicit specification of the number of firms characterizing each sector of production. Once these numbers are specified, it is natural to investigate the comparative statics properties of changes in the number of firms, in particular their implications for the rate of growth of the system. The paper derives results in that direction. However, it also opens up the possibility of studying the growth rate of the system under varying market stuctures. The present model looks into perfectly competitive market structures alone.

References

Barro, R. (1990) Government Spending in a Simple Model of Endogenous Growth, Journal of Political Economy, 98, S 103 - S 125.

Barro, R., and X. Sala-i-Martin (2004), Economic Growth, 2nd Edition, MIT Press, Cambridge.

Dasgupta, D. and Koji Shimomura (2005) Public Infrastructure, Employment and Sustainable Growth in a Small Open Economy With and Without Foreign Direct Investment, ERU Discussion Paper Series, No. ERU/2005-20, Indian Statistical Institute, October, 2005. To appear in Journal of International Trade and Economic Development, 2006, Volume 15, No. 3, Special Issue.

Hildenbrand, W. (1974) Core and Equilibria of a Large Economy, Princeton University Press, Princeton.

Lewis, W. A. (1954) Economic Development with Unlimited Supplies of Labour, The Manchester School, 28, 139 - 91.