Coalesced CAP: An Improved Technique for Frequency Assignment in Cellular Networks

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***Abstract*—This paper presents an elegant technique for solving the channel assignment problem (CAP) for second generation (2G) cellular mobile networks, where channel allocation is made on a quasi-fixed basis and all sessions are connection oriented. It first maps a given CAP P to a modified coalesced CAP P' on a smaller subset of cells of the network, which appreciably reduces the search space. This helps to solve the problem P' by applying approximate algorithms very efficiently, reducing the computing time drastically. This solution to P' is then used to solve the original problem P by using a modified version of the forced assignment with rearrangement (FAR) operation reported by Tcha *et al.* (*IEEE Trans. Veh. Technol.*, vol. 49, p. 390, 2000). The proposed technique has been tested on well-known benchmark problems. It has produced optimal solutions for all cases with an improved computation time. For instance, it needs only around 10 and 20 s (on an unloaded DEC Alpha station 200 4/233) to get an optimal assignment for the two most difficult benchmark problems 2 and 6, respectively, with zero call blocking, in contrast to around 60 and 72 s (on an unloaded Sun Ultra 60 workstation) reported by Ghosh *et al.* Moreover, as a by-product of this approach, there remain, in general, many unused or redundant channels that may be used for accommodating small perturbations in demands dynamically.**

***Index Terms*—Benchmark problems, cellular networks, channel assignment, fixed bandwidth, minimum span.**

I. Introduction

IN RECENT years, the number of mobile users has grown up rapidly, whereas the communication bandwidth for providing service to them has grown very moderately. Hence, the problem of using the radio spectrum efficiently to satisfy the customers' demands has become a critical research issue. This paper considers second generation (2G) cellular network systems, where it is assumed that the demands of the cells are known *a priori*, and the channels are to be allocated to the cells statically to cater sessions that are basically connection oriented. Here, the key factor is the reuse of radio spectrum in cells avoiding channel interference. Neglecting other influencing factors, we assume that channel interference is primarily a function of frequency and distance. A channel can simultaneously be used by multiple base stations if their mutual separation is more than the reuse distance, i.e., the minimum distance at which two signals of the same frequency do not interfere. In a cellular environment, reuse distance is usually expressed in units of number of cells. Based on that, three types of interference are generally taken into consideration: 1) cochannel interference, due to which the same channel is not allowed to be simultaneously assigned to a pair of cells that are not sufficiently far apart, 2) adjacent channel interference, for which adjacent channels are not allowed to be assigned to certain pairs of cells simultaneously, and 3) co-site interference, which implies that any pair of channels assigned to the same cell must be separated by a certain minimum distance in frequency. The task of assigning frequency channels to cells satisfying the frequency separation constraints with a view to avoiding channel interference and using as small bandwidth as possible is known as the channel assignment problem (CAP). In its most general form, CAP is equivalent to the generalized graph-coloring problem, which is a well-known NP-complete problem [2].

Earlier works on approximate algorithms for channel assignment can be broadly classified into two categories. For the first category of CAP, these approximate algorithms first determine an ordered list of all calls and then assign channels deterministically to the calls to minimize the required bandwidth [6], [16], [18], [20]. For the second category of CAP, given the bandwidth of the system, the approximate algorithms formulate a cost function, such as the number of calls blocked by a given channel assignment, and then tries to minimize this cost function [3]–[5], [10], [12], [13], [17], [19], [23]. The advantage of the first category of algorithms is that the derived channel assignment always fulfills all the interference constraints for a given demand, but it may be hard to find an optimal solution in case of large and difficult problems, even with quite powerful optimization tools. On the other hand, for the second category of algorithms, it may be impossible to minimize the cost function to the desired value of zero, in case of hard problems, with the minimum number of channels. In [9], the authors combined both of the above methods and proposed the combined genetic algorithm (CGA) that generates a call list in each iteration and evaluates the quality of the generated call list following the frequency exhaustive assignment (FEA) strategy.

In order to compare the performance of these algorithms for channel assignment, some well-known benchmark instances,

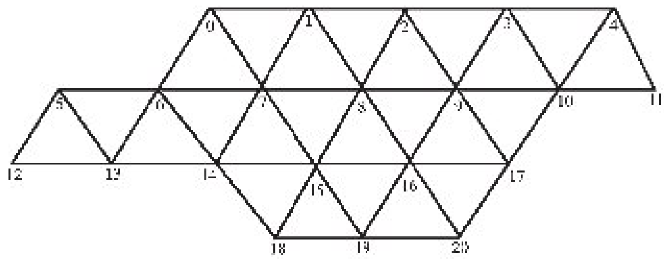

Fig. 1. Benchmark cellular network.

commonly known as Philadelphia benchmarks, are widely used in the literature [6], [7], [9]–[11], [13]–[18], [20]. These benchmarks are defined on a 21-node cellular network shown in Fig. 1. Here, each node represents a cell, and two nodes are connected by an edge, if the corresponding cells share a common boundary. The demands of the cells are represented by any one of the two nonhomogeneous demand vectors D_1 and D_2 shown in Table I. The column i in Table I indicates the channel demand from cell i corresponding to D_1 or D_2. These benchmark instances have been defined on the hexagonal cellular network assuming a two-band buffering restriction, i.e., interference does not extend beyond two cells from the call originating cell. It has been assumed that for avoiding channel interference the calls in the same cell should be separated by at least s_0 channels, and the calls in the cells that are distances of one and two apart should be separated by at least s_1 and s_2 channels, respectively. Table II shows the specifications of these eight problems (problems 1 through 8) in terms of the specific values of s_0, s_1, and s_2 for a two-band buffering system and the corresponding demand vector used for each of them.

Among the eight Philadelphia benchmark instances, it is relatively easier to derive the optimal solutions for all the problems except 2 and 6, because in all those six cases the required number of channels is primarily limited by the co-site interference constraint only. Most difficult is, however, to get the optimal solution for the other two Philadelphia instances—problems 2 and 6 [9], [18]. For example, the assignment algorithm given in [10] required 165 h of computing time for problem 6 on an unloaded HP Apollo 9000/700 workstation but producing only a nonoptimal solution with 268 channels (optimal solution requires only 253 channels). Later, however, the authors in [9] proposed an algorithm that provided optimal solutions for both problems 2 and 6 with a running time of 8 and 10 min, respectively, on the same workstation. Among later works, the frequency exhaustive strategy with rearrangement (FESR) algorithm in [11] and the randomized saturation degree (RSD) heuristic presented in [18] also produce only nonoptimal solutions to benchmark problems 2 and 6. However, combining their RSD heuristic with a local search (LS) algorithm, the authors in [18] were able to find an optimal solution for problem 2 but not for problem 6. Recently, an efficient heuristic algorithm has been proposed in [20], which also produced nonoptimal results for problems 2 and 6 with 463 and 273 channels, respectively. Most recently, given the concept of a critical block of the hexagonal cellular network, the authors in [21] proposed a novel algorithm that provides an optimal assignment for problems 2 and 6 with relatively less computation time than that in [9]. The critical block approach [21] requires only around a few seconds for optimal channel assignment of the other six benchmark instances on an unloaded Sun Ultra 60 workstation. For benchmark problems 2 and 6, however, the approach requires only around 60 and 72 s, respectively, on the same workstation. Hence, so far, the scheme reported in [21] has produced the best results in least time for all the benchmark problems.

In this paper, an elegant technique is presented for solving the second category of CAP, which first maps a given CAP P to a modified problem P' (coalesced CAP) on a small subset of cells of the network, offering a much reduced search space. This helps solving the problem P' by applying approximate algorithms more efficiently. This solution to P' is then used to solve the original problem P. However, based on the solution obtained for P', two possible situations may arise: 1) the solution to P derived from the solution to P' results in zero call blocking, i.e., it is an admissible solution for P or 2) if all requirements for P are not satisfied by the solution to P', resulting in call blocking. An algorithm is then presented that is a modified version of the forced assignment with rearrangement (FAR) operation reported in [11]. Application of this modified FAR (MFAR) operation to well-known benchmarks generates optimal results for all of them. Also, computation time is improved even over that of the critical block approach reported in [21]. Moreover, this approach results, in general, in some unused or redundant channels that may effectively be utilized to solve the perturbation-minimizing frequency assignment problem (PMFAP) [11] dynamically.

The problem is formulated in Section II. Section III describes the construction of coalesced CAP. The technique for solving the original problem is presented in Section IV. Section V shows the simulation results. Finally, concluding remarks are included in Section VI.

II. Problem Formulation

We use here the same model to represent a CAP as described in [1], [6], and [8]. This model is described by the following components:

1) a set X of n distinct cells with labels $0, 1, \ldots, n-1$;
2) a demand vector $W = (w_i)(0 \le i \le n-1)$, where w_i represents the number of channels required for cell i;
3) a frequency separation matrix $C = (c_{ij})$, where c_{ij} represents the minimum frequency separation requirement between a call in cell i and a call in cell j $(0 \le i, j \le n-1)$;
4) a frequency assignment matrix $\Phi = (\phi_{ij})$, where ϕ_{ij} represents the frequency assigned to call j in cell i $(0 \le i \le n-1, 0 \le j \le w_i - 1)$. The assigned frequencies ϕ_{ij}s are assumed to be evenly spaced and can be represented by integers ≥ 0;
5) a set of frequency separation constraints specified by the frequency separation matrix $|\phi_{ik} - \phi_{jl}| \ge c_{ij}$ for all i, j, k, l (except when both $i = j$ and $k = l$).

TABLE I
TWO DIFFERENT DEMAND VECTORS FOR PHILADELPHIA BENCHMARK PROBLEMS

Cell nos	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D_1	8	25	8	8	8	15	18	52	77	28	13	15	31	15	36	57	28	8	10	13	8
D_2	5	5	5	8	12	25	30	25	30	40	40	45	20	30	25	15	15	30	20	20	25

TABLE II
SPECIFICATIONS OF BENCHMARK PROBLEMS

Problems		1	2	3	4	5	6	7	8
Frequency	s_0	5	5	7	7	5	5	7	7
separation	s_1	1	2	1	2	1	2	1	2
constraints	s_2	1	1	1	1	1	1	1	1
Demand vector		D_1	D_1	D_1	D_1	D_2	D_2	D_2	D_2

Based on this model, a CAP P can be characterized by the triplet (X, W, C). A given CAP can be typically represented by means of a graph G, where the k-th call to cell i is represented as a node v_{ik}, and the nodes v_{ik} and v_{jl} are connected by an edge with weight c_{ij} if $c_{ij} > 0$. This graph is referred to the CAP graph in [1]. Then, the channels are assigned to the nodes of the CAP graph in a specific order and a node will be assigned the channel corresponding to the smallest integer that will satisfy the frequency separation constraints with all the previously assigned nodes. It is evident that the ordering of the nodes has a strong impact on the required bandwidth. Suppose there are m nodes in the CAP graph, where m is the total requirement, i.e., $m = \sum_{i=0}^{i=n-1} w_i$. Therefore, the nodes can be ordered in $m!$ ways, and hence, for sufficiently large m, it is impractical to find the best ordering by an exhaustive search. Instead, more time efficient heuristics are necessary to find an optimal or near-optimal solution to the problem.

A frequency assignment Φ for P is said to be admissible if ϕ_{ij}s satisfy component 5 above for all i, j, where $0 \le i \le n-1$ and $0 \le j \le w_i - 1$. The span $S(\Phi)$ of a frequency assignment Φ is the maximum frequency assigned to the system. That is

$$S(\Phi) = \max_{i,j} \phi_{ij}.$$

Thus, the objective of the first category of CAP is to find an admissible frequency assignment with the minimum span $S_0(P)$, where $S_0(P) = \min\{S(\Phi) | \Phi \text{ is admissible for } P\}$. This class of assignment problem is known as the minimum span frequency assignment.

For the second category of CAP, we look for the channel assignment when the bandwidth B of the system is given, which may even be smaller than the required lower bound on bandwidth for the given problem. Depending on B, it may or may not be possible to satisfy all the channel demands of each cell unless B is sufficiently large. Thus, a solution to this variant of CAP may, in general, leave some blocked calls. However, the objective in this case is to minimize call blocking as far as possible. This class of assignment problems is known as the fixed bandwidth channel assignment. Suppose, due to the bandwidth constraint, only w'_i channels are assigned to cell i instead of w_i in an assignment $\Phi = (\phi_{ij})$ for P, where $w'_i < w_i$ for some or all i. Then, the frequency assignment Φ is said to be not admissible, and $b_i = (w_i - w'_i)$ (where $w'_i < w_i$) calls are blocked in the cell i by Φ. We represent the set of blocked calls by means of the vector $BL = (b_i)$. Then, the total blocking BL_{total} of the system is defined as

$$BL_{\text{total}} = \sum_{i=0}^{i=n-1} b_i.$$

Given the bandwidth B of the system, the objective of this fixed bandwidth formulation of CAP is to find Φ for P such that BL_{total} is as low as possible. We find a solution to this problem by using a coalesced CAP as explained below.

III. CONSTRUCTION OF A COALESCED CAP

For a given CAP P, initially the first category of algorithms is applied to find a solution assuming a single demand per cell. This solution is now used to construct the coalesced CAP P'. Here, the algorithm to generate a coalesced CAP P' from the given CAP P follows.

Algorithm Construct_Coalesced_Cap

Step 1: Define the CAP $P^* = (X^*, W^*, C^*)$ from the given CAP $P = (X, W, C)$ such that $X^* = X$, $C^* = C$, but $W^* = (w^*_i) = (1)$, i.e., $w^*_i = 1 \forall i$ $(0 \le i \le n-1)$. Note that P^* is nothing but P with the homogeneous single demand per cell.

Step 2: Find an admissible frequency assignment Φ^* for P^* applying a suitable algorithm of the first category (e.g., the algorithm GA in [22]). Let $a_0, a_1, \ldots, a_{z-1}$ be the z $(z \le n)$ different channels assigned by Φ^*.

Example 1: To demonstrate this step, let us consider a practical assignment problem from Helsinki, Finland [5], [18], [20], to be referred later as problem 9. The example CAP $P = (X, W, C)$ has been formulated on a 25-cell system of nonhexagonal structure whose frequency separation matrix C and demand vector W are shown in Tables III and IV, respectively. The entry corresponding to the ith row and jth column in Table III, i.e., c_{ij}, represents the minimum frequency separation requirement between a call in cell i and a call in cell j $(0 \le i, j \le 24)$. The column i of the row D_3 in Table IV indicates the channel demand w_i from cell i. Next, the problem $P^* = (X^*, W^*, C^*)$ is derived from P (problem 9 defined above), where $X^* = X$, $C^* = C$, but $W^* = (w^*_i) = (1)$. Table V shows an admissible solution Φ^* for P^*, where column i indicates the channel assigned to cell i. Note that Φ^* needs $z = 8$ channels, namely, $a_0 = 0, a_1 = 1, \ldots, a_7 = 7$, respectively.

TABLE III
FREQUENCY SEPARATION MATRIX FOR PROBLEM 9

$$C = \begin{bmatrix}
2&1&1&0&1&0&1&1&1&1&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\
1&2&1&0&1&0&1&1&0&1&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\
1&1&2&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&2&0&0&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&1&1&1\\
1&1&1&0&2&0&0&0&0&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&2&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&0&1&2&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&0&1&1&2&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&1&0\\
1&0&1&1&0&1&1&1&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&1&1\\
1&1&1&1&1&1&1&1&1&2&1&1&1&1&1&1&0&0&0&0&0&1&0&1&0\\
0&0&1&1&1&0&1&1&1&1&2&0&1&1&1&1&0&1&1&1&1&1&1&1&1\\
1&1&1&1&1&0&1&1&1&1&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0\\
1&1&1&1&1&0&1&1&0&1&1&1&2&1&1&1&1&1&1&1&0&0&0&0&0\\
1&1&1&0&1&0&0&0&0&1&1&1&1&2&1&1&1&1&1&1&0&0&0&0&0\\
1&1&0&0&1&0&0&0&0&1&1&0&1&1&2&1&1&1&1&1&1&1&0&0&0\\
0&0&0&0&1&0&0&0&0&1&1&0&1&1&1&2&1&1&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&2&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&3&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&1&1&2&1&1&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&1&0&1&1&2&1&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&1&3&1&1&0&0\\
0&0&0&0&0&0&0&0&0&1&1&0&0&0&1&0&0&0&1&1&1&2&1&1&1\\
0&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&1&1&1&2&1&1\\
0&0&0&1&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&1&1&2&1\\
0&0&0&1&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&1&1&1&2
\end{bmatrix}$$

TABLE IV
DEMAND VECTOR FOR PROBLEM 9

Cell nos:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
D_3	10	11	9	5	9	4	5	7	4	8	8	9	10	7	7	6	4	5	5	7	6	4	5	7	5

TABLE V
HOMOGENEOUS SINGLE-CHANNEL ASSIGNMENT FOR PROBLEM 9

Cell nos:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Φ^*	5	6	3	5	1	2	1	0	6	4	2	2	7	0	3	6	2	5	1	4	0	7	6	1	0

TABLE VI
MATRIX $c'(a_i, a_j)$ FOR PROBLEM 9

$c'(a_i,a_j)$	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_0	2	1	1	1	1	1	1	1
a_1	1	2	1	1	1	1	1	1
a_2	1	1	2	1	1	1	1	1
a_3	1	1	1	2	1	1	1	1
a_4	1	1	1	1	2	1	1	1
a_5	1	1	1	1	1	2	1	1
a_6	1	1	1	1	1	1	2	1
a_7	1	1	1	1	1	1	1	2

TABLE VII
CHANNEL ASSIGNMENT FOR COALESCED CAP P' OF PROBLEM 9

$Nodes \rightarrow$	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7
	21	0	4	1	14	2	3	12
	25	5	7	6	26	9	13	20
	27	11	10	8	29	16	17	22
	40	34	31	15	32	19	23	30
	44	45	47	18	39	24	36	35
	58	48	51	28	55	33	38	37
	71	60	63	43	62	53	42	41
		65	66	57	72	61	50	46
		68	70	67		64	52	49
						69	56	54
							59	
$T_a \rightarrow$	7	9	9	9	8	10	11	10
$T_r \rightarrow$	7	9	9	9	8	10	11	10

Step 3: Construct the coalesced CAP $P' = (X', W', C')$ from Φ^* using the following four substeps.

Step 3.1: Let $Y(i)$ be the set of all cells where channel a_i has been assigned in Φ^*. Since every cell demands only one channel, $Y(i) \cap Y(j) = \text{NULL}$ for $i \neq j$. On the other hand, since Φ^* is admissible, $X^* = \cup_{i=0}^{i=z-1} Y(i)$.

Example 2: For the problem P^* in Example 1, the admissible assignment Φ^* is shown in Table V. Note that channel 0 has been assigned to cells 7, 13, 20, and 24. Hence, $Y(0) = \{7, 13, 20, 24\}$. Similarly, $Y(1) = \{4, 6, 18, 23\}$, $Y(2) = \{5, 10, 11, 16\}$, $Y(3) = \{2, 14\}$, $Y(4) = \{9, 19\}$, $Y(5) = \{0, 3, 17\}$, $Y(6) = \{1, 8, 15, 22\}$, and $Y(7) = \{12, 21\}$.

Step 3.2: Compute the maximum entry in C (the frequency separation matrix of P) for every pair of cells u and v, $u \in Y(i)$, $v \in Y(j)$, and denote it by $c'(a_i, a_j)$. That is

$$c'(a_i, a_j) = c'\left(Y(i), Y(j)\right) = \max_{u \in Y(i), v \in Y(j)} \{c_{uv}\}$$

where $0 \leq i, j \leq z - 1$.

TABLE VIII
COMPLETE CHANNEL ASSIGNMENT FOR PROBLEM 9

$Cells \rightarrow$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
	2	3	1	2	0	4	0	21	3	14	4	4	12	21	1	3	4	2	0	14	21	12	3	0	21
	9	13	6	9	5	7	5	25	13	26	7	7	20	25	6	13	7	9	5	26	25	20	13	5	25
	16	17	8	16	11	10	11	27	17	29	10	10	22	27	8	17	10	16	11	29	27	22	17	11	27
	19	23	15	19	34	31	34	40	23	32	31	31	30	40	15	23	31	19	34	32	40	30	23	34	40
	24	36	18	24	45	47	45	44	36	39	47	47	35	44	18	36	47	24	45	39	44	35	36	45	44
	33	38	28	33	48	51	48	58	38	55	51	51	37	58	28	38	51	33	48	55	58	37	38	48	58
	53	42	43	53	60	63	60	71	42	62	63	63	41	71	43	42	63	53	60	62	71	41	42	60	71
	61	50	57	61	65	66	65		50	72	66	66	46		57	50	66	61	65	72		46	50	65	
	64	52	67	64	68	70	68		52		70	70	49		67	52	70	64	68			49	52	68	
	69	56		69					56				54			56		69				54	56		
		59							59							59							59		
$\mathcal{T}_\alpha \rightarrow$	10	11	9	10	9	9	9	7	11	8	9	9	10	7	9	11	9	10	9	8	7	10	11	9	7
$\mathcal{T}_\beta \rightarrow$	10	11	9	5	9	4	5	7	4	8	8	9	10	7	7	6	4	5	5	7	6	4	5	7	5

Example 3: For the problem in Example 1, from Table III, we get $c'(Y(0), Y(1)) = c'(a_0, a_1) = 1$, i.e., 1 is the maximum among all c_{ij}s, where $i \in Y(0) = \{7, 13, 20, 24\}$ and $j \in Y(1) = \{4, 6, 18, 23\}$. All $c'(a_i, a_j)$s $(0 \leq i, j \leq 7)$ are shown in Table VI.

Step 3.3: Find the maximum weight among all cells in $Y(i)$, and denote it by $M(a_i)$. That is

$$M(a_i) = M(Y(i)) = \max_{j \in Y(i)} \{w_j\}$$

where $0 \leq i \leq z - 1$.

Example 4: For the problem in Example 1, the demands of each cell are given in Table IV. From this table, $M(Y(0)) = M(a_0) = 7$, i.e., 7 is the maximum demand among the cells in $Y(0) = \{7, 13, 20, 24\}$. Similarly, $M(a_1) = 9$, $M(a_2) = 9$, $M(a_3) = 9$, $M(a_4) = 8$, $M(a_5) = 10$, $M(a_6) = 11$, and $M(a_7) = 10$.

Step 3.4: Represent all the cells in $Y(i)$ by a single node $N(Y(i))$ of a weighted graph G', where the weight of a node $N(Y(i))$ is $M(Y(i))$. Connect nodes $N(Y(i))$ and $N(Y(j))$ by an edge with weight $c'(Y(i), Y(j))$ if $c'(Y(i), Y(j)) > 0$, and terminate.

This graph G' is termed as the coalesced CAP graph. The corresponding coalesced CAP $P' = (X', W', C')$ is represented by the following components:

1) a set $X' = (N(Y(i)))$ $(0 \leq i \leq z - 1)$ of z distinct nodes, where node $N(Y(i))$ represents the set $Y(i)$ in Φ^* of P^*;
2) a demand vector $W' = (M(Y(i)))$, where $M(Y(i))$ represents the weight of the node $N(Y(i))$ in G', $0 \leq i \leq z - 1$;
3) a frequency separation matrix $C' = (c'(Y(i), Y(j)))$, where $c'(Y(i), Y(j))$ represents the weight of the edge between nodes $N(Y(i))$ and $N(Y(j))$ in G', $0 \leq i, j \leq z - 1$;
4) a frequency assignment matrix $\Phi' = (\phi'_{ij})$, where ϕ'_{ij} represents the frequency assigned to call j in the node $N(Y(i))$ $(0 \leq i \leq z - 1, 0 \leq j \leq M(Y(i)) - 1)$;
5) a set of frequency separation constraints specified by the frequency separation matrix $|\phi'_{ik} - \phi'_{jl}| \geq c'(Y(i), Y(j))$ for all i, j, k, l (except when both $i = j$ and $k = l$). ■

Once P' is constructed as above, the objective is now to find an assignment Φ' for this P' with a given bandwidth B.

Example 5: The coalesced CAP $P' = (X', W', C')$ for the problem $P = (X, W, C)$ in Example 1 is given by $X' = (N(a_i))$, $0 \leq i \leq 7$, $W' = (7, 9, 9, 9, 8, 10, 11, 10)$, and C' is as given in Table VI.

Lemma 1: The given CAP P and the coalesced CAP P' are equivalent if $z = n$. For $z < n$, the total search space of P' is always less than that of P.

Proof: Clearly, if $z = n$, problems P and P' have the same number of nodes having the same weights, and $C = C'$. Hence, there is no reduction in search space after transferring P to P'.

As explained in Section II, the number of nodes and their demands in a CAP graph actually determine the total search space for a given CAP. Let the sum of demands on all cells for problems P and P' be $\mathcal{T}$ and $\mathcal{T}'$, respectively. Hence, the CAP graphs for P and P' will have $\mathcal{T}$ and $\mathcal{T}'$ nodes, respectively. If $z < n$, $\mathcal{T}'$ must be less than $\mathcal{T}$ because the sum of the demands of all the cells in $Y(i)$ has been replaced by the maximum of those to contribute to $\mathcal{T}'$. Hence, we have the proof. ■

IV. PROPOSED TECHNIQUE FOR SOLVING THE ORIGINAL CAP FROM THE COALESCED CAP

Transforming the original CAP $P = (X, W, C)$ to the coalesced CAP $P' = (X', W', C')$, we apply a suitable algorithm

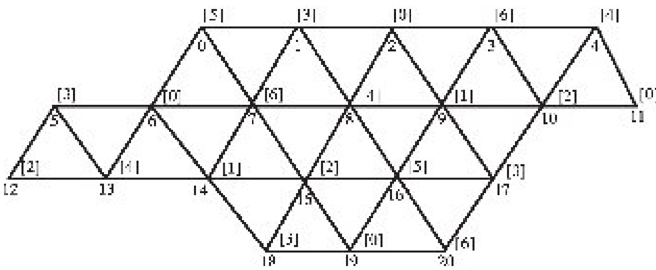

Fig. 2. Single-channel assignment of the benchmark problem 5.

TABLE IX
FREQUENCY SEPARATION MATRIX FOR P^* OF PROBLEM 5

$c'(\alpha_i, \alpha_j)$	α_0	α_1	α_2	α_3	α_4	α_5	α_6
α_0	7	1	1	1	1	1	1
α_1	1	7	1	1	1	1	1
α_2	1	1	7	1	1	1	1
α_3	1	1	1	7	1	1	1
α_4	1	1	1	1	7	1	1
α_5	1	1	1	1	1	7	1
α_6	1	1	1	1	1	1	7

of the second category of CAP to find a frequency assignment Φ' of P' with a view to minimizing the number of blocked calls BL_{total}. The assignment Φ' may or may not be admissible, depending on the available bandwidth B. Hence, to derive the required assignments for P, we consider the following two cases.

Case 1: Assignment Φ' is Admissible: In this case, an admissible frequency assignment for P can be derived by using Φ' by means of the following theorem.

Theorem 1: Given the problem $P = (X, W, C)$ and the bandwidth B, if the frequency assignments Φ' for P' are admissible, an admissible frequency assignment for P can be derived from Φ'.

Proof: To get an assignment of P from Φ', all the cells in $Y(i)$ $(0 \leq i \leq z-1)$ are assigned the same set of channels assigned to $N(Y(i))$ in Φ'. This assignment must satisfy the interference constraints because in P', $c'(Y(i), Y(j))$ is the maximum among all c_{ij}s in C, where $i \in Y(i)$ and $j \in Y(j)$. This assignment must also satisfy the demand vector $W = (w_i)$, since in P', $M(Y(i))$ is the maximum among all w_is in W, where $i \in Y(i)$. ■

When it is admissible, Φ' not only satisfies all the requirements of P but also provides some redundant channels. If cell i has been assigned w'_i channels while the requirement was w_i and $w'_i > w_i$, then $r_i = (w'_i - w_i)$ number of channels remains unused or redundant in cell i. This set of redundant channels is represented by the vector $R = (r_i)$. The total number of redundant channels R_{total} of the system is

$$R_{\text{total}} = \sum_{i=0}^{i=n-1} r_i.$$

Example 6: For the problem in Example 1, the derived problem P' has been completely described in Example 5 above. One solution to P' has been obtained by the algorithm in [9] (a GA-based algorithm for the second category of CAP), as shown in Table VII, where entries in row T_a indicate the total number of channels assigned to each node, and those in row T_r indicate the total number of channels actually required for each node (in all subsequent tables, T_a and T_r will indicate the same meaning as mentioned here). In Table VII, the rows T_a and T_r are identical with the demand vector W' of P'. In other words, the frequency assignment of Table VII is admissible. The complete assignment following Theorem 1 has been shown

TABLE X
DERIVED CHANNEL ASSIGNMENT FOR P' OF PROBLEM 5

Nodes →	a_0	a_1	a_2	a_3	a_4	a_5	a_6
	0	1	2	3	4	159	96
	5	6	7	8	9	164	103
	10	11	12	13	14	169	108
	15	16	17	18	19	174	113
	20	21	22	23	24	184	118
	25	26	27	28	29	189	123
	30	31	32	33	34	194	128
	35	36	37	38	39	199	133
	40	41	42	43	44	209	138
	45	46	47	48	49	214	143
	50	51	52	53	54	219	148
	55	56	57	58	59		153
	60	61	62	63	64		158
	65	66	67	68	69		163
	70	71	72	73	74		168
	75	76	77	78	79		173
	80	81	82	83	84		178
	85	86	87	88	89		183
	90	91	92	93	94		188
	95	101	97	98	99		193
	100	106	102	154	104		198
	105	111	107	177	109		203
	110	116	112	182	114		208
	115	121	117	187	119		213
	120	126	122	192	124		218
	125	131	127	197	129		
	130	136	132	202	134		
	135	141	137	207	139		
	140	146	142	212	144		
	145	151	147	217	149		
	150	156	152				
	155	161	157				
	160	166	162				
	165	171	167				
	170	176	172				
	175	181	179				
	180	186	201				
	185	191	206				
	190	196	211				
	195	204	216				
	200						
	205						
	210						
	215						
	220						
T_a →	45	40	40	30	30	11	25
T_r →	45	40	40	30	30	15	25

TABLE XI
DERIVED CHANNEL ASSIGNMENT FOR PROBLEM 5

Cells →	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	159	3	0	98	4	3	0	96	4	1	2	0	2	4	1	2	159	3	3	0	96
	164	8	5	103	9	8	5	103	9	6	7	5	7	9	6	7	164	8	8	5	103
	169	13	10	108	14	13	10	108	14	11	12	10	12	14	11	12	169	13	13	10	108
	174	18	15	113	19	18	15	113	19	16	17	15	17	19	16	17	174	18	18	15	113
	184	23	20	118	24	23	20	118	24	21	22	20	22	24	21	22	184	23	23	20	118
	189	28	25	123	29	28	25	123	29	26	27	25	27	29	26	27	189	28	28	25	123
	194	33	30	128	34	33	30	128	34	31	32	30	32	34	31	32	194	33	33	30	128
	199	38	35	133	39	38	35	133	39	36	37	35	37	39	36	37	199	38	38	35	133
	209	43	40	138	44	43	40	138	44	41	42	40	42	44	41	42	209	43	43	40	138
	214	48	45	143	49	48	45	143	49	46	47	45	47	49	46	47	214	48	48	45	143
	219	53	50	148	54	53	50	148	54	51	52	50	52	54	51	52	219	53	53	50	148
		58	55	153	59	58	55	153	59	56	57	55	57	59	56	57		58	58	55	153
		63	60	158	64	63	60	158	64	61	62	60	62	64	61	62		63	63	60	158
		68	65	163	69	68	65	163	69	66	67	65	67	69	66	67		68	68	65	163
		73	70	168	74	73	70	168	74	71	72	70	72	74	71	72		73	73	70	168
		78	75	173	79	78	75	173	79	76	77	75	77	79	76	77		78	78	75	173
		83	80	178	84	83	80	178	84	81	82	80	82	84	81	82		83	83	80	178
		88	85	183	89	88	85	183	89	86	87	85	87	89	86	87		88	88	85	183
		93	90	188	94	93	90	188	94	91	92	90	92	94	91	92		93	93	90	188
		98	95	193	99	98	95	193	99	101	97	95	97	99	101	97		98	98	95	193
		154	100	198	104	154	100	198	104	106	102	100	102	104	106	102		154	154	100	198
		177	105	203	109	177	105	203	109	111	107	105	107	109	111	107		177	177	105	203
		182	110	208	114	182	110	208	114	116	112	110	112	114	116	112		182	182	110	208
		187	115	213	119	187	115	213	119	121	117	115	117	119	121	117		187	187	115	213
		192	120	218	124	192	120	218	124	126	122	120	122	124	126	122		192	192	120	218
		197	125		129	197	125		129	131	127	125	127	129	131	127		197	197	125	
		202	130		134	202	130		134	136	132	130	132	134	136	132		202	202	130	
		207	135		139	207	135		139	141	137	135	137	139	141	137		207	207	135	
		212	140		144	212	140		144	146	142	140	142	144	146	142		212	212	140	
		217	145		149	217	145		149	151	147	145	147	149	151	147		217	217	145	
			150				150			156	152	150	152		156	152				150	
			155				155			161	157	155	157		161	157				155	
			160				160			166	162	160	162		166	162				160	
			165				165			171	167	165	167		171	167				165	
			170				170			176	172	170	172		176	172				170	
			175				175			181	177	175	177		181	177				175	
			180				180			186	201	180	201		186	201				180	
			185				185			191	206	185	206		191	206				185	
			190				190			196	211	190	211		196	211				190	
			195				195			201	216	195	216		201	216				195	
			200				200					200								200	
			205				205					205								205	
			210				210					210								210	
			215				215					215								215	
			220				220					220								220	
$T_\alpha \rightarrow$	11	30	45	25	30	30	45	25	30	40	10	45	40	30	10	10	11	30	30	15	25
$T_\gamma \rightarrow$	5	5	5	8	12	25	30	25	30	40	40	45	20	30	25	15	15	30	20	20	25

in Table VIII. Note that the same set of channels assigned to node $N(Y(0))$ has also been assigned to all the cells in $Y(0) = \{7, 13, 20, 24\}$. It is easy to verify that this assignment is also admissible for P. In addition, this assignment keeps five redundant channels in cell 3. Similarly, the redundant channels for other cells can be computed, and we get $R = (0, 0, 0, 5, 0, 5, 4, 0, 7, 0, 1, 0, 0, 0, 2, 5, 5, 5, 4, 1, 1, 6, 6, 2, 2)$ and $R_{\text{total}} = 61$.

Case 2: Assignment Φ' is not Admissible: In this case, the given bandwidth B is not enough to satisfy all the requirements for P'. Let us assume that Φ' satisfies the demand vector $W'' = (w''_i)$ instead of W', where $w''_i < w'_i$ for some or all i. From Φ', if we assign all the cells in $Y(i)$ $(0 \le i \le z-1)$ the same set of channels assigned to $N(Y(i))$, there will be, in general, some blocked calls in some cells, as well as some redundant channels in some other cells. We denote the blocked calls and redundant channels produced by this assignment as $BL = (b_i)$ and $R = (r_j)$, respectively, where $b_i = w_i - w''_i$, if $w''_i < w_i$ and 0 otherwise, and $r_j = (w''_j - w_j)$, if $w''_j > w_j$ and 0 otherwise. We then try to assign the blocked calls in BL by appropriately using these redundant channels in R and other available free channels by an approach similar to the FAR operation in [11]. For the sake of completeness, we briefly describe below the essential features of the FAR operation reported in [11].

Essence of FAR operation [11]: Let b_i be an unassigned requirement and Q denote the set of already assigned frequencies. Suppose for b_i there is no frequency available to be assigned without any conflict to the already assigned frequencies of Q. Then, FAR attempts to assign a frequency in L (where L is the given list of available frequencies) to satisfy the requirement b_i with minimum change or perturbation on the present assignment Q. The essence of FAR is to identify a minimal subset $S(b_i)$ of Q, where each requirement can be simultaneously reassigned with an alternative feasible frequency so that b_i can be assigned a frequency without conflict to the present assignment of Q. Let $B(b_i, f_i)$ denote the subset

TABLE XII
COMPLETE CHANNEL ASSIGNMENT FOR PROBLEM 5

Cells →	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	159	3	*0	98	4	3	0	98	4	1	2	0	2	4	1	2	0	3	3	*0	98
	164	8	*5	103	9	8	5	103	9	6	7	5	7	9	6	7	5	8	8	*5	103
	169	13	*10	108	14	13	10	108	14	11	12	10	12	14	11	12	10	13	13	*10	108
	174	18	*15	113	19	18	15	113	19	16	17	15	17	19	16	17	15	18	18	*15	113
	184	23	20	118	24	23	20	118	24	21	22	20	22	24	21	22	159	23	23	20	118
	189	28	25	123	29	28	25	123	29	26	27	25	27	29	26	27	164	28	28	25	123
	194	33	30	128	34	33	30	128	34	31	32	30	32	34	31	32	169	33	33	30	128
	199	38	35	133	39	38	35	133	39	36	37	35	37	39	36	37	174	38	38	35	133
	209	43	40	138	44	43	40	138	44	41	42	40	42	44	41	42	184	43	43	40	138
	214	48	45	143	49	48	45	143	49	46	47	45	47	49	46	47	189	48	48	45	143
	219	53	50	148	54	53	50	148	54	51	52	50	52	54	51	52	194	53	53	50	148
		58	55	153	59	58	55	153	59	56	57	55	57	59	56	57	199	58	58	55	153
		63	60	158	64	63	60	158	64	61	62	60	62	64	61	62	209	63	63	60	158
		68	65	163	69	68	65	163	69	66	67	65	67	69	66	67	214	68	68	65	163
		73	70	168	74	73	70	168	74	71	72	70	72	74	71	72	219	73	73	70	168
		78	75	173	79	78	75	173	79	76	77	75	77	79	76	77		78	78	75	173
		83	80	178	84	83	80	178	84	81	82	80	82	84	81	82		83	83	80	178
		88	85	183	89	88	85	183	89	86	87	85	87	89	86	87		88	88	85	183
		93	90	188	94	93	90	188	94	91	92	90	92	94	91	92		93	93	90	188
		98	95	193	99	98	95	193	99	101	97	95	97	99	101	97		98	98	95	193
		154	100	198	104	154	100	198	104	106	102	100	102	104	106	102		154	154	100	198
		177	105	203	109	177	105	203	109	111	107	105	107	109	111	107		177	177	105	203
		182	110	208	114	182	110	208	114	116	112	110	112	114	116	112		182	182	110	208
		187	115	213	119	187	115	213	119	121	117	115	117	119	121	117		187	187	115	213
		192	120	218	124	192	120	218	124	126	122	120	122	124	126	122		192	192	120	218
		197	125		129	197	125		129	131	127	125	127	129	131	127		197	197	125	
		202	130		134	202	130		134	136	132	130	132	134	136	132		202	202	130	
		207	135		139	207	135		139	141	137	135	137	139	141	137		207	207	135	
		212	140		144	212	140		144	146	142	140	142	144	146	142		212	212	140	
		217	145		149	217	145		149	151	147	145	147	149	151	147		217	217	145	
			150				150			156	152	150	152		156	152				150	
			155				155			161	157	155	157		161	157				155	
			160				160			166	162	160	162		166	162				160	
			165				165			171	167	165	167		171	167				165	
			170				170			176	172	170	172		176	172				170	
			175				175			181	177	175	177		181	177				175	
			180				180			186	201	180	201		186	201				180	
			185				185			191	206	185	206		191	206				185	
			190				190			196	211	190	211		196	211				190	
			195				195			204	216	195	216		204	216				195	
			200				200					200								200	
			205				205					205								205	
			210				210					210								210	
			215				215					215								215	
			220				220					220								220	
T_a →	11	30	41	35	30	30	45	35	30	40	40	45	40	30	40	40	15	30	30	41	25
T_r →	5	5	5	8	13	35	30	35	30	40	40	45	30	30	25	15	15	30	20	20	25

of requirements in Q, which are conflicting, if we assign frequency f_i to requirement b_i. In other words, f_i becomes a feasible frequency for b_i if the frequency assignments for $B(b_i, f_i)$ are undone. To identify one $S(b_i)$, we examine a sequence of f_is such that each time a $B(b_i, f_i)$ is generated, we undo the corresponding portion of frequency assignment in Q and try to assign an alternative feasible frequency to each requirement of $B(b_i, f_i)$ by the unforced assignment (UA) operation. The UA operation finds the lowest frequency in L feasible to the present assignments in Q. If the frequency assignment of $B(b_i, f_i)$ is successfully made, $B(b_i, f_i)$ becomes $S(b_i)$ itself. In case such a frequency reassignment cannot be made for some $b_j \in B(b_i, f_i)$, one proceeds to identify $B(b_j, f_j)$ and attempts to assign an alternative feasible frequency to each $b_k \in B(b_j, f_j)$. Such $B(b_j, f_j)$s are blockers at the second depth level. Generalizing this, the FAR operation is encoded to render the so-called vth breadth-level and wth depth-level $(Bv{-}Dw)$ procedures. In $(Bv{-}Dw)$, we consider blockers only within the cardinality of v (i.e., $|B(t, f_i)| \leq v$) and limit the number of successive downward search to w. The complexity of FAR operation actually prohibits the direct implementation of this general $(Bv - Dw)$ procedure.

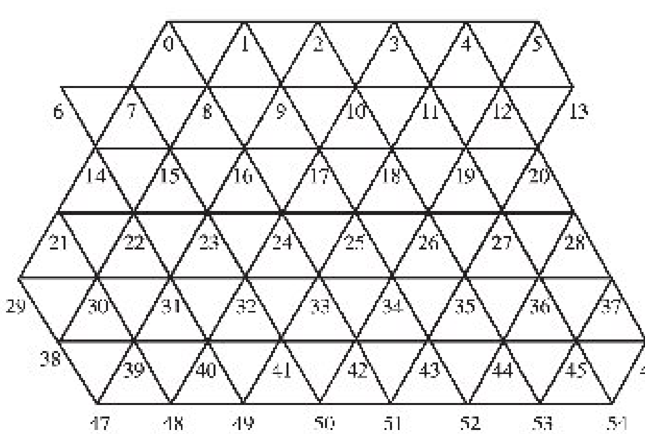

Fig. 3. Cellular graph corresponding to the cellular network of 55-node benchmark.

TABLE XIII
TWO DIFFERENT DEMAND VECTORS FOR 55-NODE BENCHMARK PROBLEMS

Cell nos.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
D_4	5	5	5	8	12	25	30	25	30	40	40	45	20	30	25	15	15	30	20	20	25	8	5	5	5	5	5	5
D_5	10	11	9	5	9	4	5	7	4	8	8	9	10	7	7	6	4	5	5	7	6	4	5	7	5	10	11	9

Cell nos.	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
D_4	8	12	25	30	25	30	40	40	45	20	30	25	15	15	30	20	20	25	8	5	5	5	25	8	5	5	5
D_5	5	9	4	5	7	4	8	8	9	10	7	7	6	4	5	5	7	6	4	5	7	5	6	4	5	7	5

In our proposed algorithm, we have implemented $(B1-D1)$, $(B2-D1)$, and $(B1-D2)$ incorporating the concept of redundant channels as described above. We call this operation as the MFAR operation. This modification actually lies in the notion of a free channel to be assigned to an unassigned requirement, say, t in BL. We consider a channel to be free and suitable to be assigned to t even if it conflicts with the requirements of some other cells containing some redundant channels. However, when we choose such a channel for assigning it to t, we may need to undo some of the assignments in neighboring cells and adjust the assignments in other cells as well to keep the degree of perturbation (number of changes in the existing assignments) as low as possible, following the techniques similar to FAR operation [11].

Here, a formal description of the algorithm (Derive-Assignment (P, P')) to derive the solution to $P = (X, W, C)$ from the solution to P' follows.

Algorithm Derive-Assignment (P, P')

Step 1: Assign all the cells in $Y(i)$ $(0 \le i \le z-1)$ the same set of channels assigned to $N(Y(i))$ in Φ' of P'.

Remark 1: By similar arguments as in the proof of Theorem 1, this assignment satisfies the interference constraints as specified by C.

Step 2: Apply MFAR operation to minimize the blocked calls in $BL = (b_i)$ using the redundant channels in $R = (r_j)$ appropriately. ■

Example 7: To illustrate this step, we consider the Philadelphia benchmark problem 5 (Table II). We first construct P' for this problem as follows. A solution Φ^* of P^* is shown in Fig. 2, where the label $[\alpha]$ associated with a node indicates that a frequency α is assigned to that node. Only seven channels $(0, 1, \cdots, 6)$ have been assigned repeatedly in the assignment of Fig. 2, where $Y(0) = \{2, 6, 11, 19\}$, $Y(1) = \{9, 14\}$, $Y(2) = \{10, 12, 15\}$, $Y(3) = \{1, 5, 17, 18\}$, $Y(4) = \{4, 8, 13\}$, $Y(5) = \{0, 16\}$, and $Y(6) = \{3, 7, 20\}$. Corresponding frequency separations $c'(a_i, a_j)$ $(0 \le i, j \le 6)$ are shown in Table IX. We find $M(a_0) = 45$, $M(a_1) = 40$, $M(a_2) = 40$, $M(a_3) = 30$, $M(a_4) = 30$, $M(a_5) = 15$, and $M(a_6) = 25$. Therefore, $P' = (X', W', C')$ is given by $X' = (N(Y(i)))$, $0 \le i \le 6$, $W' = (45, 40, 40, 30, 30, 15, 25)$, and C' is as given in Table IX.

Now, to get Φ', we partition P' into several subnetworks with homogeneous weights, following the critical block approach reported in [21]. However, during the multiple weight

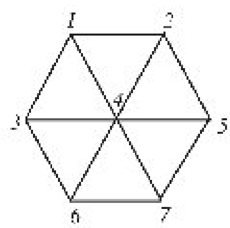

Fig. 4. Distance-2 clique of the hexagonal cellular network.

assignment, we consider the assignments with channels not exceeding the given bandwidth B only. One solution to P' as obtained by this approach has been shown in Table X. It follows that all the requirements for P' as given by $W' = (45, 40, 40, 30, 30, 15, 25)$ are not satisfied; this assignment results in four blocked calls in node $N(Y(5))$. In other words, the derived assignment is not admissible.

We now apply step 1 of the algorithm Derive-Assignment (P, P') to get the assignment for P from the solution to P' in Table X. The derived assignment for P has been shown in Table XI. Note that the same set of channels assigned to node $N(Y(0))$ has also been assigned to all the cells in $Y(0) = \{2, 6, 11, 19\}$. This assignment leaves four blocked calls $(15 - 11 = 4)$ in cell 16 and, at the same time, also produces many redundant channels in some other cells (e.g., cell 2 has 40 redundant channels).

We now apply step 2 of the algorithm Derive-Assignment (P, P') to get the assignment as shown in Table XII. Note that in the assignment of Table XI, there were four blocked calls in cell 16. However, MFAR operation finds that channels 0, 5, 10, and 15 can be assigned to cell 16 if the assignments of channels 0, 5, 10, and 15 from cells 2 and 19 are undone. Note that there are more than four redundant channels in both cells 2 and 19. The assignments of channels 0, 5, 10, and 15 in cell 16 have been underlined, and those of cells 2 and 19 have been marked by an asterisk $(*)$.

V. SIMULATION RESULTS

We have simulated the proposed coalesced CAP approach on all Philadelphia benchmark problems as well as on problem 9 defined above. Other than these benchmarks, we have also considered two other benchmarks defined on a 55-node cellular network [20] shown in Fig. 3. These two benchmarks have also been defined on a two-band buffering system where s_0, s_1, and s_2 are given as 7, 1, and 1, respectively. The demand vectors of these two problems (termed as Problems 10 and 11) are given by D_4 and D_5, respectively, as shown in Table XIII.

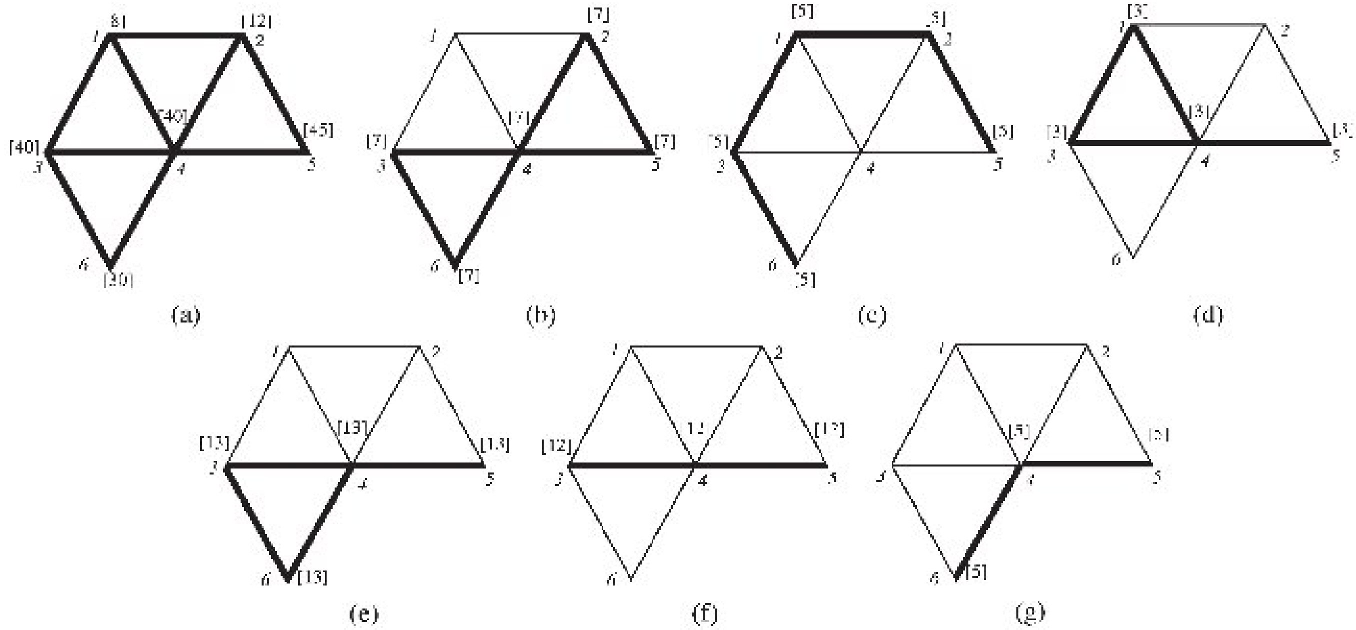

Fig. 5. Critical block and its homogeneous partitions.

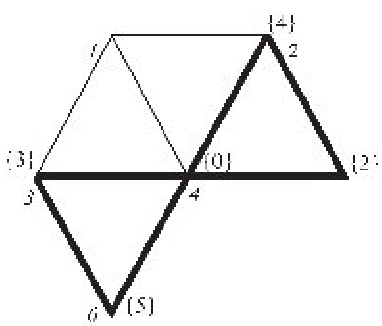

Fig. 6. Optimal partition assignment for partition P_1 in Fig. 5(b).

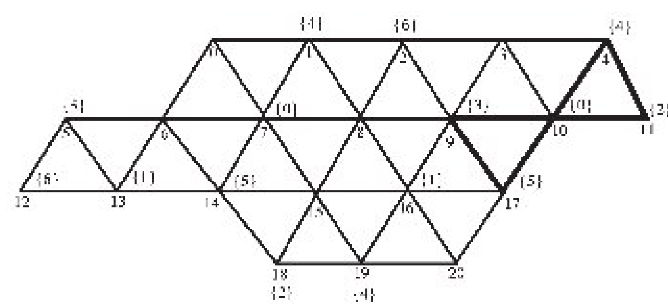

Fig. 7. Extension of partition P_1 of the critical block of problem 6.

The computation time needed to solve a given CAP following the proposed coalesced CAP technique is determined by the following three phases.

Phase 1: finding an admissible frequency assignment for P^* in step 2 of the Construct_coalesced_Cap algorithm by applying a suitable algorithm for the first category of CAP;

Phase 2: finding a frequency assignment Φ' of P' as derived in step 3 of the Construct_coalesced_Cap algorithm by applying a suitable algorithm for the second category of CAP;

Phase 3: the MFAR operation, if all requirements of P are not satisfied by Phase 2 above, i.e., the derived Φ' is not admissible.

It is to be noted that the computation time will depend on the performance of the algorithms chosen for phases 1 and 2 above.

For all the benchmark problems, except Philadelphia problems 2 and 6, we obtain assignments with zero call blocking if the given bandwidth B is equal to the respective lower bound of the problems. Also, significant improvement in computation time has been achieved as compared to earlier works, even to the critical block approach in [21]. Improvement is more significant for all such problems for which solutions to P' generate solutions to P, e.g., problems 1, 3, 4, 7, 9, and 10. For these problems, we need not execute the MFAR operation, and as a result, the computation time is of the order of a second on an unloaded DEC Alpha station 200 4/233. However, for other problems, e.g., problems 2, 5, 6, 8, and 11, all requirements for P are not satisfied by the solution to P', and we need to apply the MFAR operation to satisfy the remaining requirements. Out of these, for problems 5, 8, and 11, the solutions to P' generate a small number of call blocking in some cells and at the same time produce a large number of redundant channels in some other cells. As a result, the MFAR algorithm can easily accommodate these blocked calls by using redundant channels appropriately. For these problems, the computation time is around 3–4 s on the same workstation.

Most difficult is, however, to get the solution with zero call blocking for problems 2 and 6 by this approach when the given bandwidth B is equal to the respective lower bounds. For these benchmark problems, the solutions to P' generate a few blocked calls in some cells, whereas they produce only few redundant channels in some other cells. As a result, MFAR

TABLE XIV
COMPARISONS OF REQUIRED BANDWIDTH

Problems	1	2	3	4	5	6	7	8	9	10	11
Lower Bounds	381	427	533	533	221	253	309	309	73	309	71
Proposed approach	381	427	533	533	221	253	309	309	73	309	71
(2003)[28]	381	427	533	533	221	253	309	309	–	–	–
(2001)[26]	381	463	533	533	221	273	309	309	73	309	79
(2001)[24]	381	427	533	533	221	254	309	309	73	–	–
(2000)[17]	381	433	533	533	–	260	–	309	–	–	–
(1998)[15]	381	427	533	533	221	253	309	309			
(1998)[16]	–	–	–	–	221	268	–	309	–	–	–
(1997)[19]	381		533	533	221		309	309			
(1997)[20]	381	436	533	533	–	268	–	309	–	–	–
(1997)[32]	381	433	533	533	221	263	309	309	73		
(1996)[21]	381	–	533	533	–	–	–	–	–	–	–
(1994)[22]	381	464	533	536	–	293	–	310	–	–	–
(1992)[23]	381	–	533	533	221	–	309	309	73	–	–
(1991)[5]	–	–	–	–	–	–	–	–	73	–	–
(1989)[8]	381	447	533	533		270		310			

fails to accommodate all these blocked calls. In the next section, we present a modification in our algorithm to reduce this problem.

A. Modification in Coalesced CAP Construction

Definition 1: Suppose $G = (V, E)$ is a cellular graph. A subgraph $G' = (V', E')$ of the graph $G = (V, E)$ is defined to be a distance-2 clique if every pair of nodes in G' is connected in G by a path of length at most 2 [1].

Example 8: Fig. 4 shows a distance-2 clique of a hexagonal cellular structure.

Definition 2: Given a cellular graph G with a demand vector W, and the set of all possible distance-2 cliques $\{G_j\}$, each with minimum bandwidth requirement B_j, the critical block CB_2 is that distance-2 clique, whose minimum bandwidth requirement is the maximum of all B_js.

While using the MFAR operation in the earlier section, we make an important observation that MFAR fails to accommodate mostly the blocked calls at the cells of a critical block. It appears that the assignment of the critical block is so tight that it becomes difficult to find alternative frequencies to be assigned to blocked calls existing in the critical block. This observation motivated us to a modification in the construction of coalesced CAP before we apply the MFAR operation as discussed below. We will see that with this modification Philadelphia problems 2 and 6 would finally require around 10 and 20 s, respectively, on the same platform to produce zero call blocking.

Algorithm Derive-Assignment (P, P')

Step 1: Following the approach in [21], find a critical block of P along with its homogeneous demand partitions and then assign the critical block. Let $P_1, P_2, \ldots, P_k$ be the k partitions with homogeneous weights $\alpha_1, \alpha_2, \ldots, \alpha_k$, respectively.

Example 9: For problem 6, the critical block is the distance-2 clique centered around node 10, i.e., consisting of nodes $\{3, 4, 9, 10, 11, 17\}$, which is isomorphic to G_2, as shown in Fig. 5(a). Fig. 5(b)–(g) shows the homogeneous partitions $P_1, P_2, \ldots, P_6$ (obtained through an integer programming formulation) with weights 7, 5, 3, 13, 12, and 5, respectively. In Fig. 5, the label $[\alpha]$ associated with a node indicates the demand of that node. After the partitioning of demands into homogeneous weights, the assignment of the critical block is obtained following an optimal ordering of partitions (see [21] for details). As an example, Fig. 6 shows an optimal partition assignment for the partition P_1 of Fig. 5(b).

Step 2: Define coalesced CAP $P' = (X', W', C)$, where X' is the subset of X containing the cells of the critical block, and W' is the actual demand of the cells of the critical block. (Here, the coalesced CAP is defined on the cells of the critical block.)

Step 3: Extend the assignment of each partition P_i $(i = 0, 1, \ldots, k)$ to consider the assignment of the remaining network. Combine all these assignments in the optimal ordering of the partitions $P_1, P_2, \ldots, P_k$. Compute the total blocking $BL_{\text{total}} = (b_i)$ in this combined assignment.

Remark 2: The assignment of each partition can be extended in many different ways. As a result, their combined assignment will generate different call blocking BL_{total} for different assignments, but the most important thing is that there never be blocked calls in the cells of X'. Our objective is to search heuristically to find such a combined assignment with minimal call blocking.

Step 4: Repeat step 3 for all possible assignments to obtain an assignment Φ_m with the minimal value of BL_{total}.

Step 5: Apply MFAR operation for reallocating the channels in Φ_m to minimize the blocked calls resulting from step 4 above. ■

Example 10: Consider the partition P_1 of the critical block of problem 6 and its optimal partition assignment shown in Figs. 5(b) and 6, respectively. One possible extension of the assignment of P_1 to consider the whole network has been shown in Fig. 7. This assignment may lead to call blocking in some cell but not in the critical block.

TABLE XV
DERIVED CHANNEL ASSIGNMENT FOR PROBLEM 2

Cells →	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	149	2	145	149	2	0	128	125	0	124	146	0	3	350	1	3	264	1	147	195	145
	155	7	151	155	7	5	133	130	5	129	152	5	8	355	6	8	271	6	153	201	151
	161	12	157	161	12	10	138	135	10	134	158	10	13	360	11	13	278	11	159	207	157
	167	17	163	167	17	15	143	140	15	139	164	15	18	365	16	18	285	16	165	213	163
	173	22	169	173	22	20	352	146	20	193	170	20	23	370	21	23	292	21	171	219	169
	179	27	175	179	27	25	357	152	25	199	176	25	28	375	26	28	299	26	177	225	175
	185	32	181	185	32	30	362	158	30	205	182	30	33	380	31	33	306	31	183	231	181
	191	37	187	191	37	35	367	164	35	211	188	35	38	385	36	38	313	36	189	237	187
	197	42		197	42	40	372	170	40	217	349	40	43	390	41	43	320	41	126	243	
	203	47		203	47	45	377	176	45	223	354	45	48	395	46	48	327	46	131	249	
	209	52		209	52	50	382	182	50	229	359	50	53	400	51	53	334	51		255	
	215	57		215	57	55	387	188	55	235	364	55	58	405	56	58	341	56		261	
	221	62		221	62	60	392	194	60	241	369	60	63	410	61	63	348	61		128	
	227	67		227	67	65	397	200	65	247	374	65	68	415	66	68	353	66			
	233	72		233	72	70	402	206	70	253	379	70	73	420	71	73	358	71			
	239	77		239	77	75	407	212	75	259	384	75	78	425	76	78	363	76			
	245	82		245	82	80	412	218	80	266	389	80	83		81	83	368	81			
	251	87		251	87	85	417	224	85	273	394	85	88		86	88	373	86			
	257	92		257	92	90		230	90	280	399	90	93		91	93	378	91			
	263	97		263	97	95		236	95	287	404	95	98		96	98	383	96			
		102			102	100		242	100	294	409	100	103		101	103	388	101			
		107			107	105		248	105	301	414	105	108		106	108	393	106			
		112			112	110		254	110	308	419	110	113		111	113	398	111			
		117			117	115		260	115	315	424	115	118		116	118	403	116			
		122			122	120		265	120	322		120	123		121	123	408	121			
						148		272	127	329		127	144		270	144	413	268			
						154		279	132	336		132	150		277	150	418	275			
						160		286	137	343		137	156		284	156	423	282			
						166		293	142			142	162		291	162		289			
						172		300	148			148	168		298	168		296			
						178		307	154			154	174		305	174		303			
						184		314	160			160	180		312	180		310			
						190		321	166			166	186		319	186		317			
						196		328	172			172	192		326	192		324			
						202		335	178			178	198		333	198		331			
						208		342	184			184	204		340	204		338			
						214		349	190			190	210		347	210		345			
						220		354	196			196	216			216					
						226		359	202			202	222			222					
						232		364	208			208	228			228					
						238		369	214			214	234			234					
						244		374	220			220	240			240					
						250		379	226			226	246			246					
						256		384	232			232	252			252					
						262		389	238			238	258			258					
						269		394	244			244	267			267					
						276		399	250			250	274			274					
						283		404	256			256	281			281					
						290		409	262			262	288			288					
						297		414	269			269	295			295					
						304		419	276			276	302			302					
						311		424	283			283	309			309					
						318			290			290	316			316					
						325			297			297	323			323					
						332			304			304	330			330					
						339			311			311	337			337					
						346			318			318	344			344					
									325			325									
									332			332									
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									391			391									
									396			396									
									401			401									
									406			406									
									411			411									
									416			416									
									421			421									
									426			426									
T_a →	20	25	8	20	25	57	18	52	77	28	21	77	57	16	37	57	28	37	10	18	8
T_b →	8	25	8	8	8	15	18	52	77	28	19	15	31	15	30	57	28	8	10	18	8

TABLE XVI
DERIVED CHANNEL ASSIGNMENT FOR PROBLEM 6

Cells →	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	51	4	228	48	4	1	3	49	1	3	0	2	4	7	0	76	8	5	9	4	2
	56	11	233	53	11	45	10	54	6	10	7	9	11	14	5	82	15	12	16	11	49
	61	18	238	58	18	53	17	59	13	17	14	16	18	21	12	88	22	19	23	18	54
	66	25	243	63	25	58	24	64	20	24	21	23	25	28	19	98	29	26	30	25	59
	71	32	248	68	32	63	31	69	27	31	28	30	32	35	26	104	36	33	37	32	64
		39		73	39	68	38	74	34	38	35	37	39	42	33	110	43	40	44	39	69
		46		79	46	73	45	80	41	45	42	44	46	50	40	116	183	47	79	46	74
				85	51	79	77	86	78	50	77	49	101	55	47	122	188	52	85	51	80
					56	85	83	92	84	55	83	54	149	60	52	128	193	57	91	56	86
					61	91	89	100	90	60	89	59	155	65	57	134	198	62	101	61	94
					66	97	95	106	96	65	95	64	161	70	62	140	203	67	107	66	100
					71	107	103	112	102	70	101	69	176	75	67	146	208	73	113	71	106
						113	109	118	108	75	107	74	181	81	72	152	213	91	119	171	112
						119	115	124	114	81	113	80	196	87	167	158	218	97	169	176	118
						125	121	130	120	87	119	86	201	93	172	164	223	103	184	181	124
						131	127	136	126	93	125	92	206	99	177			109	189	196	130
						137	133	142	132	99	131	98	211	105	182			115	194	201	136
						143	139	148	138	105	137	104	231	111	187			121	199	206	142
						173	145	154	144	111	143	110	236	117	192			127	204	211	148
						178	151	160	150	117	149	116	241	123	197			133	209	216	154
						183	157	227	156	123	155	122	246	129	202			139	214	221	160
						188	163	234	162	129	161	128	251	135	207			145	219	228	166
						193	175	239	168	135	167	134		141	212			151	224	235	174
						198	180	244	173	141	172	140		147	217			157		240	179
						203	195	249	178	147	177	146		153	222			163		245	186
						208	200		230	153	182	152		159	229			229		250	191
						213	205		236	159	187	158		165				234			226
						218	210		241	165	192	164		170				239			231
						223	232		246	170	197	169		185				244			
						228	237		251	175	202	174		190				249			
							242			180	207	179		215							
							247			185	212	184		220							
							252			190	217	189		225							
										195	222	194									
										200	227	199									
										205	232	204									
										210	237	209									
										215	242	214									
										220	247	219									
										225	252	224									
												230									
												235									
												240									
												245									
												250									
T_d →	5	7	5	8	12	30	33	25	30	40	40	45	22	33	26	15	15	30	23	26	28
T_r →	5	5	5	8	12	25	30	25	30	40	40	45	20	30	25	15	15	30	20	20	25

The required bandwidths from earlier works along with that from our proposed approach have been shown in Table XIV for the purpose of comparison. The row *Lower Bound* in Table XIV corresponds to the lower bound for each of the problems as reported in [21]. The derived channel assignments for the two most difficult problems, i.e., problems 2 and 6, are shown in Tables XV and XVI, respectively.

VI. CONCLUSION

We have presented an elegant technique for solving CAP, which is applicable even to a cellular network of nonhexagonal structure. The assignment is done for a given bandwidth B with a view to minimizing call blocking. The proposed technique is able to achieve the optimal solution for all the well-known benchmark problems when the given bandwidth B is equal to the lower bound of the corresponding problem. The required computation time has also been improved even over the critical block approach in [21]. Moreover, it can be further studied how unused or redundant channels provided by this approach can be utilized for accommodating small changes in demands dynamically.

ACKNOWLEDGMENT

The authors would like to thank the anonymous referees for their help in improving the presentation of this paper.

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