BALANCED CONFOUNDED ARRANGEMENTS FOR THE 5^n TYPE OF EXPERIMENT

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Introduction

In a previous paper[3] I discussed a method of getting confounded arrangements for the general symmetrical type of experiment, that is to say, of n factors at p levels each. The method was made up of two systems of interchanges from a p-sided hyper-Græco-Latin Square; and could be used to obtain balanced sets of replications, partially confounding all the degrees of freedom of the high order interactions affected, provided $(p-1)$ also was a prime or a power of a prime. This method was illustrated by discussing in detail confounded arrangements for the 3^n and 4^n types of experiment.

Yates[4] had given designs for the 3^n type of experiment before me but did not indicate the method of getting them. The 4^n type was dismissed by him as one reducible to the 2^n type, which had been exhaustively given by Barnard[1]; mine was the first attempt to deal with the 4^n type without breaking it up into a 2^{2n} type. The method indicated by me being general for p^n when p and $(p-1)$ are both primes or powers of primes, I have discussed in this paper a hitherto unsolved case, namely, the confounding of a 5^n type of experiment.

The Two Systems of Interchanges for 5 × 5 Square

Fisher[2] has found by complete enumeration that there are six sets of 5 × 5 hyper-Græco-Latin squares. One only of these sets has been given by him in *The Design of Experiments.* In Table I is reproduced this hyper-Græco-Latin Square.

Table I. A 5 × 5 Hyper-Græco-Latin Square

$A_1\ \alpha_1$	$B_2\ \beta_2$	$C_3\ \gamma_3$	$D_4\ \delta_4$	$E_5\ \epsilon_5$
$B_3\ \delta_5$	$C_4\ \epsilon_1$	$D_5\ \alpha_2$	$E_1\ \beta_3$	$A_2\ \gamma_4$
$C_5\ \beta_4$	$D_1\ \gamma_5$	$E_2\ \delta_1$	$A_3\ \epsilon_2$	$B_4\ \alpha_3$
$D_2\ \epsilon_3$	$E_3\ \alpha_4$	$A_4\ \beta_5$	$B_5\ \gamma_1$	$C_1\ \delta_2$
$E_4\ \gamma_2$	$A_5\ \delta_3$	$B_1\ \epsilon_4$	$C_2\ \alpha_5$	$D_3\ \beta_1$

The four orthogonal squares are represented respectively by the Latin letters A, B, C, D, E, the Greek letters α, β, γ, δ, ϵ, the suffixes 1, 2, 3, 4, 5 attached to the Latin letters and to the Greek letters. That the four squares are mutually orthogonal, besides being orthogonal to rows and columns can be verified from the fact that (*i*) each Latin letter occurs once and only once with each Greek letter, each Latin suffix and each Greek suffix (*ii*) each Greek letter occurs once and only once with each Latin suffix and each Greek suffix and finally (*iii*) each Latin suffix occurs once and only once with each Greek suffix.

TABLE 2. THE SIXTEEN STANDARD 5×5 SQUARES.

1					2					3					4				
1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1
2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2
3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3
4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4
5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5
5					6					7					8				
1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1
4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4
2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2
5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5
3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3
9					10					11					12				
3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3
5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5
2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2
4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4
1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1
13					14					15					16				
5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5
4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4
3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3
2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2
1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1

From Table (1) it is clear that the Latin letters form a square in the standard position

1	2	3	4	5
2	3	4	5	1
3	4	5	1	2
4	5	1	2	3
5	1	2	3	4

The Greek letters, Latin suffixes and Greek suffixes can be brought to the same standard position by *three* interchanges of the columns (rows) such that the positions (1, 2, 3, 4, 5) of the columns (rows) are (1, 4, 2, 5, 3) ; (3, 5, 2, 4, 1) and (5, 4, 3, 2, 1) respectively. These constitute the first system of interchanges associated with a 5 × 5 square. From the fact that the rows (columns) of the standard square are got by cyclical interchange of the previous row (column), the second system of interchanges associated with a 5 × 5 square is cyclical. In fact for every odd value of p, the second system of interchanges associated with a $p \times p$ square is cyclical.

THREE FACTORS AT FIVE LEVELS EACH

Let A, B, C be the three factors at five levels each $(a_1, a_2, a_3, a_4, a_5)$ $(b_1, b_2, b_3, b_4, b_5)$ and $(c_1, c_2, c_3, c_4, c_5)$. There are 64 d. f. belonging to the second order interaction ABC In sub-blocks of 25 plots, one replication will have 5 sub-blocks thus confounding 4 d. f. of ABC ; 16 replications are thus needed for balanced partial confounding.

The 5 × 5 square in the standard position can generate 15 more squares by performing the first system of interchanges (5 × 5) on the rows and columns. These are given in Table 2. Keep the levels of C arranged according to these sixteen squares, making the columns and rows represent the (a)-levels and (b)-levels respectively. These 16 (a, b, c)-squares represent a key sub-block from each of the 16 balanced replications. The remaining 4 sub-blocks of each replication are obtained by performing the second system of interchanges (5 × 5) on the rows (columns) of each key sub-block. The 16 sets of 4 d. f. of ABC obtained by this method are denoted by $s_1, s_2, \ldots\ldots\ldots s_{16}$ and Table (3) gives the levels of the three factors occurring in each sub-block of each of the 16 replications. A and B are taken as the first two factors and C as the third factor.

FOUR FACTORS AT FIVE LEVELS EACH

Let A, B, C, D be the four factors. There are 64 d. f. belonging to each of the second order interactions ABC, ABD, ACD and BCD and 256 d. f. belonging to the third order interaction ABCD. In sub-blocks of 25 plots, 24 d. f. get confounded among the 25 sub-blocks of a replication. These 24 d. f. can be separated into 6 orthogonal sets of 4 d. f. each. Four of these sets belong one each to ABC, ABD, ACD and BCD and the remaining two sets to ABCD. For balanced partial confounding of all the second and third order interactions 64 replications will be found to be necessary. In this section besides getting the 64 replications for balancing the second and third order interaction it will be shown that they fall into four sets of 16 replications, each set confounding with balance all the second order interactions. It will also be seen that for balancing the third order interaction alone 32 replications will suffice.

TABLE 3. 5^3 DESIGNS IN SUB-BLOCKS OF 25 PLOTS CONFOUNDING SECOND ORDER INTERACTIONS.

Levels of first and second factors			s_1					s_2					s_3					s_4					s_5					s_6					s_7					s_8				
			Level of third factor																																							
1	1	1	1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1	1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1
1	2	2	2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2	4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4
1	3	3	3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3	2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2
1	4	4	4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4	5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5
1	5	5	5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5	3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3
2	1	6	2	3	4	5	1	4	2	5	3	1	5	2	4	1	3	4	3	2	1	5	2	3	4	5	1	4	2	5	3	1	5	2	4	1	3	4	3	2	1	5
2	2	7	3	4	5	1	2	5	3	1	4	2	1	3	5	2	4	5	4	3	2	1	5	1	2	3	4	2	5	3	1	4	3	5	2	4	1	2	1	5	4	3
2	3	8	4	5	1	2	3	1	4	2	5	3	2	4	1	3	5	1	5	4	3	2	3	4	5	1	2	5	3	1	4	2	1	3	5	2	4	5	4	3	2	1
2	4	9	5	1	2	3	4	2	5	3	1	4	3	5	2	4	1	2	1	5	4	3	1	2	3	4	5	3	1	4	2	5	4	1	3	5	2	3	2	1	5	4
2	5	10	1	2	3	4	5	3	1	4	2	5	4	1	3	5	2	3	2	1	5	4	4	5	1	2	3	1	4	2	5	3	2	4	1	3	5	1	5	4	3	2
3	1	11	3	4	5	1	2	2	5	3	1	4	2	4	1	3	5	3	2	1	5	4	3	4	5	1	2	2	5	3	1	4	2	4	1	3	5	3	2	1	5	4
3	2	12	4	5	1	2	3	3	1	4	2	5	3	5	2	4	1	4	3	2	1	5	1	2	3	4	5	5	3	1	4	2	5	2	4	1	3	1	5	4	3	2
3	3	13	5	1	2	3	4	4	2	5	3	1	4	1	3	5	2	5	4	3	2	1	4	5	1	2	3	3	1	4	2	5	3	5	2	4	1	4	3	2	1	5
3	4	14	1	2	3	4	5	5	3	1	4	2	5	2	4	1	3	1	5	4	3	2	2	3	4	5	1	1	4	2	5	3	1	3	5	2	4	2	1	5	4	3
3	5	15	2	3	4	5	1	1	4	2	5	3	1	3	5	2	4	2	1	5	4	3	5	1	2	3	4	4	2	5	3	1	4	1	3	5	2	5	4	3	2	1
4	1	16	4	5	1	2	3	5	3	1	4	2	4	1	3	5	2	2	1	5	4	3	4	5	1	2	3	5	3	1	4	2	4	1	3	5	2	2	1	5	4	3
4	2	17	5	1	2	3	4	1	4	2	5	3	5	2	4	1	3	3	2	1	5	4	2	3	4	5	1	3	1	4	2	5	2	4	1	3	5	5	4	3	2	1
4	3	18	1	2	3	4	5	2	5	3	1	4	1	3	5	2	4	4	3	2	1	5	5	1	2	3	4	1	4	2	5	3	5	2	4	1	3	3	2	1	5	4
4	4	19	2	3	4	5	1	3	1	4	2	5	2	4	1	3	5	5	4	3	2	1	3	4	5	1	2	4	2	5	3	1	3	5	2	4	1	1	5	4	3	2
4	5	20	3	4	5	1	2	4	2	5	3	1	3	5	2	4	1	1	5	4	3	2	1	2	3	4	5	2	5	3	1	4	1	3	5	2	4	4	3	2	1	5
5	1	21	5	1	2	3	4	3	1	4	2	5	1	3	5	2	4	1	5	4	3	2	5	1	2	3	4	3	1	4	2	5	1	3	5	2	4	1	5	4	3	2
5	2	22	1	2	3	4	5	4	2	5	3	1	2	4	1	3	5	2	1	5	4	3	3	4	5	1	2	1	4	2	5	3	4	1	3	5	2	4	3	2	1	5
5	3	23	2	3	4	5	1	5	3	1	4	2	3	5	2	4	1	3	2	1	5	4	1	2	3	4	5	4	2	5	3	1	2	4	1	3	5	2	1	5	4	3
5	4	24	3	4	5	1	2	1	4	2	5	3	4	1	3	5	2	4	3	2	1	5	4	5	1	2	3	2	5	3	1	4	5	2	4	1	3	5	4	3	2	1
5	5	25	4	5	1	2	3	2	5	3	1	4	5	2	4	1	3	5	4	3	2	1	2	3	4	5	1	5	3	1	4	2	3	5	2	4	1	3	2	1	5	4

TABLE 3. (*Contd.*) 5^3 DESIGNS IN SUB-BLOCKS OF 25 PLOTS CONFOUNDING SECOND ORDER INTERACTIONS.

Levels of first and second factors			s_9					s_{10}					s_{11}					s_{12}					s_{13}					s_{14}					s_{15}					s_{16}				
			Level of third factor																																							
1	1	1	3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3	5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5
1	2	2	5	1	2	3	4	5	3	1	4	2	2	4	1	3	5	4	3	2	1	5	4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4
1	3	3	2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2	3	4	5	1	2	3	1	4	2	5	5	2	4	1	3	2	1	5	4	3
1	4	4	4	5	1	2	3	4	2	5	3	1	1	3	5	2	4	3	2	1	5	4	2	3	4	5	1	2	5	3	1	4	4	1	3	5	2	1	5	4	3	2
1	5	5	1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1	1	2	3	4	5	1	4	2	5	3	3	5	2	4	1	5	4	3	2	1
2	1	6	4	5	1	2	3	1	4	2	5	3	2	4	1	3	5	1	5	4	3	2	1	2	3	4	5	3	1	4	2	5	4	1	3	5	2	3	2	1	5	4
2	2	7	1	2	3	4	5	3	1	4	2	5	4	1	3	5	2	3	2	1	5	4	5	1	2	3	4	2	5	3	1	4	3	5	2	4	1	2	1	5	4	3
2	3	8	3	4	5	1	2	5	3	1	4	2	1	3	5	2	4	5	4	3	2	1	4	5	1	2	3	1	4	2	5	3	2	4	1	3	5	1	5	4	3	2
2	4	9	5	1	2	3	4	2	5	3	1	4	3	5	2	4	1	2	1	5	4	3	3	4	5	1	2	5	3	1	4	2	1	3	5	2	4	5	4	3	2	1
2	5	10	2	3	4	5	1	4	2	5	3	1	5	2	4	1	3	4	3	2	1	5	2	3	4	5	1	4	2	5	3	1	5	2	4	1	3	4	3	2	1	5
3	1	11	5	1	2	3	4	4	2	5	3	1	4	1	3	5	2	5	4	3	2	1	2	3	4	5	1	1	4	2	5	3	1	3	5	2	4	2	1	5	4	3
3	2	12	2	3	4	5	1	1	4	2	5	3	1	3	5	2	4	2	1	5	4	3	1	2	3	4	5	5	3	1	4	2	5	2	4	1	3	1	5	4	3	2
3	3	13	4	5	1	2	3	3	1	4	2	5	3	5	2	4	1	4	3	2	1	5	5	1	2	3	4	4	2	5	3	1	4	1	3	5	2	5	4	3	2	1
3	4	14	1	2	3	4	5	5	3	1	4	2	5	2	4	1	3	1	5	4	3	2	4	5	1	2	3	3	1	4	2	5	3	5	2	4	1	4	3	2	1	5
3	5	15	3	4	5	1	2	2	5	3	1	4	2	4	1	3	5	3	2	1	5	4	3	4	5	1	2	2	5	3	1	4	2	4	1	3	5	3	2	1	5	4
4	1	16	1	2	3	4	5	2	5	3	1	4	1	3	5	2	4	4	3	2	1	5	3	4	5	1	2	4	2	5	3	1	3	5	2	4	1	1	5	4	3	2
4	2	17	3	4	5	1	2	4	2	5	3	1	3	5	2	4	1	1	5	4	3	2	2	3	4	5	1	3	1	4	2	5	2	4	1	3	5	5	4	3	2	1
4	3	18	5	1	2	3	4	1	4	2	5	3	5	2	4	1	3	3	2	1	5	4	1	2	3	4	5	2	5	3	1	4	1	3	5	2	4	4	3	2	1	5
4	4	19	2	3	4	5	1	3	1	4	2	5	2	4	1	3	5	5	4	3	2	1	5	1	2	3	4	1	4	2	5	3	5	2	4	1	3	3	2	1	5	4
4	5	20	4	5	1	2	3	5	3	1	4	2	4	1	3	5	2	2	1	5	4	3	4	5	1	2	3	5	3	1	4	2	4	1	3	5	2	2	1	5	4	3
5	1	21	2	3	4	5	1	5	3	1	4	2	3	5	2	4	1	3	2	1	5	4	4	5	1	2	3	2	5	3	1	4	5	2	4	1	3	5	4	3	2	1
5	2	22	4	5	1	2	3	2	5	3	1	4	5	2	4	1	3	5	4	3	2	1	3	4	5	1	2	1	4	2	5	3	4	1	3	5	2	4	3	2	1	5
5	3	23	1	2	3	4	5	4	2	5	3	1	2	4	1	3	5	2	1	5	4	3	2	3	4	5	1	5	3	1	4	2	3	5	2	4	1	3	2	1	5	4
5	4	24	3	4	5	1	2	1	4	2	5	3	4	1	3	5	2	4	3	2	1	5	1	2	3	4	5	4	2	5	3	1	2	4	1	3	5	2	1	5	4	3
5	5	25	5	1	2	3	4	3	1	4	2	5	1	3	5	2	4	1	5	4	3	2	5	1	2	3	4	3	1	4	2	5	1	3	5	2	4	1	5	4	3	2

Take a 5×5 square with the columns and rows representing the (*a*)-and (*b*)-levels. The (*c*) and (*d*)-levels should appear in the cells of this square in the form of two orthogonalised Latin squares. The 16 squares of Table 2 fall into four groups: G_1, G_2, G_3, G_4 so that squares in one group are orthogonal only to the squares of the other three groups. G_1 consists of squares 1, 6, 11, 16 ; G_2 of squares 2, 8, 9, 15 ; G_3 of squares 3, 5, 12, 14 and G_4 of squares 4, 7, 10, 13. Arrange the letters G_1, G_2, G_3, G_4 in the form of a 4×4 Latin square in the standard position :—

$$\begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \\ G_2 & G_1 & G_4 & G_3 \\ G_3 & G_4 & G_1 & G_2 \\ G_4 & G_3 & G_2 & G_1 \end{array}$$

Allow the (*c*)-levels to be arranged according to the squares of the groups G_1, G_2, G_3, G_4 of the first column. For (*d*)-levels choose the squares in the group of the second column belonging to the same row as the group to which the square selected for the *c* levels belong. This process will yield 64 (*a*, *b*, *c*, *d*)-squares representing one sub-block from each of the 64 replications that we need for balancing the second and third order interactions. The method of generation will be clear from Table (4).

By choosing the squares for (*d*)-levels from the third and fourth columns of the *G*-square we have two other ways of getting sets of 64 replications each achieving complete balance

From each of the 64 squares, 24 squares more can be generated by performing the second system of interchanges (5×5) on the (*c*)-rows (columns) and the (*d*)-rows (columns). Thus we get all the 25 sub-blocks of the 64 replications.

TABLE 4. SQUARES COMBINED FOR *c* AND *d* LEVELS TO GET THE 64 BALANCED REPLICATIONS.

	I		II		III		IV	
	G_1(c)	G_2(d)	G_2(c)	G_1(d)	G_3(c)	G_4(d)	G_4(c)	G_3(d)
1	1	2	2	1	3	4	4	3
2	1	8	2	6	3	7	4	5
3	1	9	2	11	3	10	4	12
4	1	15	2	16	3	13	4	14
5	6	2	8	1	5	4	7	3
6	6	8	8	6	5	7	7	5
7	6	9	8	11	5	10	7	12
8	6	15	8	16	5	13	7	14
9	11	2	9	1	12	4	10	3
10	11	8	9	6	12	7	10	5
11	11	9	9	11	12	10	10	12
12	11	15	9	16	12	13	10	14
13	16	2	15	1	14	4	13	3
14	16	8	15	6	14	7	13	5
15	16	9	15	11	14	10	13	12
16	16	15	15	16	14	13	13	14

TABLE 5. A 5^4 DESIGN IN SUB-BLOCKS OF 25 PLOTS CONFOUNDING SECOND AND THIRD ORDER INTERACTIONS.

Levels of first and Second factors			Levels of third and fourth factors																								
1	1	1	1	7	13	19	25	9	15	16	22	3	12	18	24	5	6	20	21	2	8	14	23	4	10	11	17
1	2	2	7	13	19	25	1	15	16	22	3	9	18	24	5	6	12	21	2	8	14	20	4	10	11	17	23
1	3	3	13	19	25	1	7	16	22	3	9	15	24	5	6	12	18	2	8	14	20	21	10	11	17	23	4
1	4	4	19	25	1	7	13	22	3	9	15	16	5	6	12	18	24	8	14	20	21	2	11	17	23	4	10
1	5	5	25	1	7	13	19	3	9	15	16	22	6	12	18	24	5	14	20	21	2	8	17	23	4	10	11
2	1	6	9	15	16	22	3	12	18	24	5	6	20	21	2	8	14	23	4	10	11	17	1	7	13	19	25
2	2	7	15	16	22	3	9	18	24	5	6	12	21	2	8	14	20	4	10	11	17	23	7	13	19	25	1
2	3	8	16	22	3	9	15	24	5	6	12	18	2	8	14	20	21	10	11	17	23	4	13	19	25	1	7
2	4	9	22	3	9	15	16	5	6	12	18	24	8	14	20	21	2	11	17	23	4	10	19	25	1	7	13
2	5	10	3	9	15	16	22	6	12	18	24	5	14	20	21	2	8	17	23	4	10	11	25	1	7	13	19
3	1	11	12	18	24	5	6	20	21	2	8	14	23	4	10	11	17	1	7	13	19	25	9	15	16	22	3
3	2	12	18	24	5	6	12	21	2	8	14	20	4	10	11	17	23	7	13	19	25	1	15	16	22	3	9
3	3	13	24	5	6	12	18	2	8	14	20	21	10	11	17	23	4	13	19	25	1	7	16	22	3	9	15
3	4	14	5	6	12	18	24	8	14	20	21	2	11	17	23	4	10	19	25	1	7	13	22	3	9	15	16
3	5	15	6	12	18	24	5	14	20	21	2	8	17	23	4	10	11	25	1	7	13	19	3	9	15	16	22
4	1	16	20	21	2	8	14	23	4	10	11	17	1	7	13	19	25	9	15	16	22	3	12	18	24	5	6
4	2	17	21	2	8	14	20	4	10	11	17	23	7	13	19	25	1	15	16	22	3	9	18	24	5	6	12
4	3	18	2	8	14	20	21	10	11	17	23	4	13	19	25	1	7	16	22	3	9	15	24	5	6	12	18
4	4	19	8	14	20	21	2	11	17	23	4	10	19	25	1	7	13	22	3	9	15	16	5	6	12	18	24
4	5	20	14	20	21	2	8	17	23	4	10	11	25	1	7	13	19	3	9	15	16	22	6	12	18	24	5
5	1	21	23	4	10	11	17	1	7	13	19	25	9	15	16	22	3	12	18	24	5	6	20	21	2	8	14
5	2	22	4	10	11	17	23	7	13	19	25	1	15	16	22	3	9	18	24	5	6	12	21	2	8	14	20
5	3	23	10	11	17	23	4	13	19	25	1	7	16	22	3	9	15	24	5	6	12	18	2	8	14	20	21
5	4	24	11	17	23	4	10	19	25	1	7	13	22	3	9	15	16	5	6	12	18	24	8	14	20	21	2
5	5	25	17	22	4	10	11	25	1	7	13	19	3	9	15	16	22	6	12	18	24	5	14	20	21	2	8

TABLE 6. KEY SUB-BLOCKS OF 64 BALANCED REPLICATIONS OF 5^4 DESIGN IN SUB-BLOCKS OF 25 PLOTS

Levels of a and b			I																II															
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
			Levels of c and d																															
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	2	7	9	8	10	17	19	18	20	12	14	13	15	22	24	23	25	7	9	8	10	17	19	18	20	12	14	13	15	22	24	23	25
1	3	3	13	12	15	14	8	7	10	9	23	22	25	24	18	17	20	19	13	12	15	14	8	7	10	9	23	22	25	24	18	17	20	19
1	4	4	19	20	17	18	24	25	22	23	9	10	7	8	14	15	12	13	19	20	17	18	24	25	22	23	9	10	7	8	14	15	12	13
1	5	5	25	23	24	22	15	13	14	12	20	18	19	17	10	8	9	7	25	23	24	22	15	13	14	12	20	18	19	17	10	8	9	7
2	1	6	9	10	7	8	19	20	17	18	14	15	12	13	24	25	22	23	17	19	18	20	22	24	23	25	7	9	8	10	12	14	13	15
2	2	7	15	13	14	12	10	8	9	7	25	23	24	22	20	18	19	17	23	22	25	24	13	12	15	14	18	17	20	19	8	7	10	9
2	3	8	15	16	16	16	21	21	21	21	6	6	6	6	11	11	11	11	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3
2	4	9	22	24	23	25	12	14	13	15	17	19	18	20	7	9	8	10	10	8	9	7	20	18	19	17	15	13	14	12	25	23	24	22
2	5	10	3	2	5	4	3	2	5	4	3	2	5	4	3	2	5	4	11	11	11	11	6	6	6	6	21	21	21	21	16	16	16	16
3	1	11	12	14	13	15	7	9	8	10	22	24	23	25	17	19	18	20	8	7	10	9	18	17	20	19	13	12	15	14	23	22	25	24
3	2	12	18	17	20	19	23	22	25	24	8	7	10	9	13	12	15	14	14	15	12	13	9	10	7	8	24	25	22	23	19	20	17	18
3	3	13	24	25	22	23	14	15	12	13	19	20	17	18	9	10	7	8	20	18	19	17	25	23	24	22	10	8	9	7	15	13	14	12
3	4	14	5	3	4	2	5	3	4	2	5	3	4	2	5	3	4	2	21	21	21	21	11	11	11	11	16	16	16	16	6	6	6	6
3	5	15	6	6	6	6	16	16	16	16	11	11	11	11	21	21	21	21	2	4	3	5	2	4	3	5	2	4	3	5	2	4	3	5
4	1	16	20	18	19	17	25	23	24	22	10	8	9	7	15	13	14	12	24	25	22	23	14	15	12	13	19	20	17	18	9	10	7	8
4	2	17	21	21	21	21	11	11	11	11	16	16	16	16	6	6	6	6	5	3	4	2	5	3	4	2	5	3	4	2	5	3	4	2
4	3	18	2	4	3	5	2	4	3	5	2	4	3	5	2	4	3	5	6	6	6	6	16	16	16	16	11	11	11	11	21	21	21	21
4	4	19	8	7	10	9	18	17	20	19	13	12	15	14	23	22	25	24	12	14	13	15	7	9	8	10	22	24	23	25	17	19	18	20
4	5	20	14	15	12	13	9	10	7	8	24	25	22	23	19	20	17	18	18	17	20	19	23	22	25	24	8	7	10	9	13	12	15	14
5	1	21	23	22	25	24	13	12	15	14	18	17	20	19	8	7	10	9	15	13	14	12	10	8	9	7	25	23	24	22	20	18	19	17
5	2	22	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	16	16	16	16	21	21	21	21	6	6	6	6	11	11	11	11
5	3	23	10	8	9	7	20	18	19	17	15	13	14	12	25	23	24	22	22	24	23	25	12	14	13	15	17	19	18	20	7	9	8	10
5	4	24	11	11	11	11	6	6	6	6	21	21	21	21	16	16	16	16	3	2	5	4	3	2	5	4	3	2	5	4	3	2	5	4
5	5	25	17	19	18	20	22	24	23	25	7	9	8	10	12	14	13	15	9	10	7	8	19	20	17	18	14	15	12	13	24	25	22	23

Table 6. (*Contd.*) Key Sub-Blocks of 64 Balanced Replications of 5^4 Design in Sub-Blocks of 25 Plots

Levels of *a* and *b*			III																IV															
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a	*b*		Levels of *c* and *d*																															
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	2	7	9	8	10	17	19	18	20	12	14	13	15	22	24	23	25	7	9	8	10	17	19	18	20	12	14	13	15	22	24	23	25
1	3	3	13	12	15	14	8	7	10	9	23	22	25	24	18	17	20	19	13	12	15	14	8	7	10	9	23	22	25	24	18	17	20	19
1	4	4	19	20	17	18	24	25	22	23	9	10	7	8	14	15	12	13	19	20	17	18	24	25	22	23	9	10	7	8	14	15	12	13
1	5	5	25	23	24	22	15	13	14	12	20	18	19	17	10	8	9	7	25	23	24	22	15	13	14	12	20	18	19	17	10	8	9	7
2	1	6	15	13	14	12	10	8	9	7	25	23	24	22	20	18	19	17	23	22	25	24	13	12	15	14	18	17	20	19	8	7	10	9
2	2	7	16	16	16	16	21	21	21	21	6	6	6	6	11	11	11	11	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3
2	3	8	22	24	23	25	12	14	13	15	17	19	18	20	7	9	8	10	10	8	9	7	20	18	19	17	15	13	14	12	25	23	24	22
2	4	9	3	2	5	4	3	2	5	4	3	2	5	4	3	2	5	4	11	11	11	11	6	6	6	6	21	21	21	21	16	16	16	16
2	5	10	9	10	7	8	19	20	17	18	14	15	12	13	24	25	22	23	17	19	18	20	22	24	23	25	7	9	8	10	12	14	13	15
3	1	11	24	25	22	23	14	15	12	13	19	20	17	18	9	10	7	8	20	18	19	17	25	23	24	22	10	8	9	7	15	13	14	12
3	2	12	5	3	4	2	5	3	4	2	5	3	4	2	5	3	4	2	21	21	21	21	11	11	11	11	16	16	16	16	6	6	6	6
3	3	13	6	6	6	6	16	16	16	16	11	11	11	11	21	21	21	21	2	4	3	5	2	4	3	5	2	4	3	5	2	4	3	5
3	4	14	12	14	13	15	7	9	8	10	22	24	23	25	17	19	18	20	8	7	10	9	18	17	20	19	13	12	15	14	23	22	25	24
3	5	15	18	17	20	19	23	22	25	24	8	7	10	9	13	12	15	14	14	15	12	13	9	10	7	8	24	25	22	23	19	20	17	18
4	1	16	8	7	10	9	18	17	20	19	13	12	15	14	23	22	25	24	12	14	13	15	7	9	8	10	22	24	23	25	17	19	18	20
4	2	17	14	15	12	13	9	10	7	8	24	25	22	23	19	20	17	18	18	17	20	19	23	22	25	24	8	7	10	9	13	12	15	14
4	3	18	20	18	19	17	25	23	24	22	10	8	9	7	15	13	14	12	24	25	22	23	14	15	12	13	19	20	17	18	9	10	7	8
4	4	19	21	21	21	21	11	11	11	11	16	16	16	16	6	6	6	6	5	3	4	2	5	3	4	2	5	3	4	2	5	3	4	2
4	5	20	2	4	3	5	2	4	3	5	2	4	3	5	2	4	3	5	6	6	6	6	16	16	16	16	11	11	11	11	21	21	21	21
5	1	21	17	19	18	20	22	24	23	25	7	9	8	10	12	14	13	15	9	10	7	8	19	20	17	18	14	15	12	13	24	25	22	23
5	2	22	23	22	25	24	13	12	15	14	18	17	20	19	8	7	10	9	15	13	14	12	10	8	9	7	25	23	24	22	20	18	19	17
5	3	23	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	16	16	16	16	21	21	21	21	6	6	6	6	11	11	11	11
5	4	24	10	8	9	7	20	18	19	17	15	13	14	12	25	23	24	22	22	24	23	25	12	14	13	15	17	19	18	20	7	9	8	10
5	5	25	11	11	11	11	6	6	6	6	21	21	21	21	16	16	16	16	3	2	5	4	3	2	5	4	3	2	5	4	3	2	5	4

The treatment combinations of the 25 sub-blocks of one of these replications are given in Table (5), for purpose of illustration. The 25 columns represent the 25 sub-blocks. Table (5) is in the form of a 25 × 25 Latin square. It is easy to see that given the first column the remaining columns can be formed with little effort. The key sub-blocks (defined as the sub-block containing treatment combination a_1 b_1 c_1 d_1) of the 64 replications are given in Table (6).

Table (7) gives details about the 6 sets of 4 d. f. confounded in each replication. The 64 sets of 4 d. f. each of ABCD are classified under four heads: X, Y, Z, W these having 16 sets in each. In fact we are introducing X, Y, Z, W as alternative third factors to A and B and indentifying the 16 sets of 4 d. f. of ABX, ABY, ABZ and ABW with the help of Table (3). The levels of X, Y, Z, W are defined in Table (8) where the numbers 1, 2, 3, ...25 stand for the CD combinations as marked in the following square:—

	c_1	c_2	c_3	c_4	c_5
d_1	1	6	11	16	21
d_2	2	7	12	17	22
d_3	3	8	13	18	23
d_4	4	9	14	19	24
d_5	5	10	15	20	25

TABLE 8. LEVELS OF X, Y, Z, W IN THE A B C D INTERACTIONS.

X		Y		Z		W	
x_1	1+ 7+13+19+25	y_1	1+ 8+15+17+24	z_1	5+ 8+11+19+22	w_1	5+ 9+13+17+21
x_2	2+ 8+14+20+21	y_2	2+ 9+11+18+25	z_2	1+ 9+12+20+23	w_2	1+10+14+18+22
x_3	3+ 9+15+16+22	y_3	3+10+12+19+21	z_3	2+10+13+16+24	w_3	2+ 6+15+19+23
x_4	4+10+11+17+23	y_4	4+ 6+13+20+22	z_4	3+ 6+14+17+25	w_4	3+ 7+11+20+24
x_5	5+ 6+12+18+24	y_5	5+ 7+14+16+23	z_5	4+ 7+15+18+21	w_5	4+ 8+12+16+25

When looking up Table (3), A and B were taken as the first and second factors for interactions ABC, ABD, ABX, ABY, ABZ and ABW and C and D were taken as the first and second factors for interactions ACD and BCD.

The 64 replications of Table (7) balance all the second and third order interactions. The 32 replications of I and III or II and IV balance third order interactions but not second order interactions. Since ABCD is less important than the second order interac-

TABLE 7. SETS OF SECOND AND THIRD ORDER INTERACTIONS CONFOUNDED IN THE 64 BALANCED REPLICATIONS OF A 5^4 DESIGN IN SUB-BLOCKS OF 25 PLOTS.

	I								II								III								IV							
	ABC	ABD	ACD	BCD	ABCD				ABC	ABD	ACD	BCD	ABCD				ABC	ABD	ACD	BCD	ABCD				ABC	ABD	ACD	BCD	ABCD			
					X	Y	Z	W					X	Y	Z	W					X	Y	Z	W					X	Y	Z	W
1	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
3	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	
5	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	
11	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]	
14	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
16	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]		[illegible]

tions it is useful to get 16 replications which will balance all the four second order interactions. The 16 replications of any of the groups I, II, III and IV of Table (7) may be arranged in the form of a 4 × 4 square

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

It will be found that the four replications of any row or column confound the same set of ABC or ABD interactions. It is necessary therefore to split the 16 replications under I, II, III and IV in 4 groups of 4 each according to a 4 × 4 Latin square imposed on the above square. It is found that only one of the three orthogonalised Latin squares give a successful reshuffling. The resulting 4 sets I′, II′, III′, and IV′, of 16 replications each balancing within itself all the degrees of freedom of the second order interaction are given in Table (9).

FIVE AND SIX FACTORS AT FIVE LEVELS EACH

Taking first the case of $n=6$, if the factors be A, B, C, D, E and F at 5 levels each a sub-block of 25 plots can be formed by arranging the levels of c, d, e, and f according to the four orthogonalised Latin squares of the 5 × 5 square and superimposing them on a 5 × 5 square whose columns and rows are the levels of a and b. One such square is given in Table (10).

The 625 sub-blocks of a single replication are obtained by performing the second system of interchanges (5 × 5) to the (c)-rows (columns), (d)-rows (columns), (e)-rows (columns) and (f)-rows (columns). Here there are 20 second order interactions each with 64 d. f., 15 third order interactions each with 256 d. f., 6 fourth order interactions each

Table 10. A Key Sub-Block with 25 Plots of a 5^6 Confounded Design.

	a_1	a_2	a_3	a_4	a_5
b_1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
b_5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 9. SETS I′, II′, III′ & IV′ OF 16 REPLICATIONS BALANCING SECOND ORDER INTERACTIONS.

	I′	ABC	ABD	ACD	BCD	ABCD X	ABCD Y	ABCD Z	ABCD W
1	I	s_1	s_3	s_7	s_{12}		s_{15}		s_{13}
6		s_4	s_8	s_6	s_{16}		s_{10}		s_2
11		s_{11}	s_9	s_{12}	s_9		s_7		s_{16}
16		s_{10}	s_{13}	s_{10}	s_3		s_4		s_5
2	II	s_2	s_6	s_{15}	s_7		s_2		s_{13}
5		s_6	s_1	s_3	s_{16}	s_7		s_{11}	
12		s_9	s_{14}	s_6	s_1	s_{10}		s_8	
15		s_{15}	s_{11}	s_2	s_{10}		s_{14}		s_1
3	III	s_8	s_{10}	s_{12}	s_{11}	s_5		s_{15}	
8		s_5	s_{13}	s_{13}	s_4		s_6		s_{11}
9		s_{12}	s_6	s_3	s_{13}		s_9		s_8
14		s_{14}	s_7	s_2	s_4	s_{16}		s_1	
4	IV	s_6	s_{16}	s_9	s_2	s_9		s_1	
7		s_1	s_{13}	s_1	s_2	s_{15}		s_4	
10		s_{10}	s_5	s_{14}	s_{13}	s_9		s_{14}	
13		s_{12}	s_5	s_{11}	s_3	s_3		s_{16}	

	II′	ABC	ABD	ACD	BCD	ABCD X	ABCD Y	ABCD Z	ABCD W
1	II	s_2	s_1	s_{12}	s_{15}			s_7	s_{12}
6		s_4	s_6	s_{15}	s_8			s_{13}	s_9
11		s_9	s_{11}	s_1	s_9			s_1	s_{16}
16		s_{15}	s_{14}	s_7	s_1			s_{10}	s_5
2	I	s_3	s_6	s_9	s_{10}	s_{13}	s_2		
5		s_4	s_3	s_6	s_{10}			s_{12}	s_{13}
12		s_{13}	s_{15}	s_9	s_4			s_5	s_1
15		s_{10}	s_5	s_{14}	s_1	s_7	s_{16}		
3	IV	s_1	s_{15}	s_2	s_6			s_{15}	s_6
8		s_7	s_{11}	s_3	s_8	s_1	s_6		
9		s_{10}	s_5	s_{12}	s_{11}	s_{15}	s_9		
14		s_{13}	s_3	s_{15}	s_{12}			s_2	s_{15}
4	III	s_5	s_{15}	s_{13}	s_2	s_6	s_{11}		
7		s_8	s_{13}	s_{16}	s_{12}	s_{12}	s_4		
10		s_{12}	s_7	s_1	s_3	s_5	s_{15}		
13		s_{11}	s_4	s_4	s_{11}	s_9	s_6		

	III′	ABC	ABD	ACD	BCD	ABCD X	ABCD Y	ABCD Z	ABCD W
1	III	s_9	s_6	s_3	s_{15}			s_6	s_9
6		s_5	s_7	s_1	s_6			s_{12}	s_3
11		s_{13}	s_{10}	s_{12}	s_5			s_1	s_{14}
16		s_{11}	s_{13}	s_{10}	s_2			s_{11}	s_8
2	IV	s_6	s_3	s_{14}	s_{10}	s_{11}	s_3		
5		s_1	s_2	s_9	s_{16}			s_6	s_{16}
12		s_{15}	s_{16}	s_8	s_2			s_6	s_1
15		s_{12}	s_{12}	s_5	S_1	s_6	s_{12}		
3	I	s_1	s_5	s_{12}	s_4			s_{16}	s_7
8		s_4	s_{13}	s_{13}	s_5	s_4	s_5		
9		s_{11}	s_2	s_5	s_{11}	s_{15}	s_{12}		
14		s_{16}	s_6	s_2	s_{15}			s_8	s_{10}
4	II	s_2	s_{15}	s_8	s_1	s_5	s_{10}		
7		s_4	s_{11}	s_2	s_{12}	s_{10}	s_1		
10		s_8	s_4	s_{16}	s_5	s_1	s_{13}		
13		s_{13}	s_1	s_{11}	s_{15}	s_{12}	s_4		

	IV′	ABC	ABD	ACD	BCD	ABCD X	ABCD Y	ABCD Z	ABCD W
1	IV	s_4	s_2	s_{13}	s_{12}		s_{16}		s_9
6		s_7	s_3	s_{12}	s_{11}		s_{13}		s_2
11		s_{10}	s_{12}	s_4	s_5		s_6		s_{15}
16		s_{15}	s_{14}	s_7	s_5		s_1		s_6
2	III	s_3	s_5	s_2	s_7		s_1		s_{15}
5		s_2	s_4	s_8	s_{16}	s_6		s_9	
12		s_{12}	s_{13}	s_9	s_2	s_{11}		s_6	
15		s_{11}	s_{10}	s_{14}	s_{10}		s_{12}		s_5
3	II	s_5	s_{11}	s_2	s_{11}	s_4		s_{14}	
8		s_5	s_{12}	s_8	s_4		s_2		s_{10}
9		s_6	s_1	s_{13}	s_{14}		s_{13}		s_9
14		s_{14}	s_6	s_{13}	s_9	s_{11}		s_8	
4	I	s_5	s_{16}	s_{13}	s_6	s_8		s_6	
7		s_8	s_9	s_{16}	s_2	s_{14}		s_6	
10		s_{11}	s_6	s_1	s_{13}	s_2		s_{16}	
13		s_{16}	s_3	s_6	s_3	s_{12}		s_{12}	

with 1024 d. f. and 1 fifth order interaction with 4096 d. f. I have not investigated how the 624 d. f. confounded in a replication are distributed among these interactions. There is reason to believe that only 4 d. f. of ABCDEF get confounded in a single replication so that balancing of this interaction requires 1024 replications. These can be easily obtained by adopting for the *c*, *d*, *e* and *f* levels the squares of columns 1, 2, 3 and 4 respectively of the G-square such that all the four-squares used in one replication belong to the four groups of the same row. We will get from each row 256 replications. It has yet to be investigated how these 1024 replications can be split into sets which balance (say) only the second order interactions.

The case $n=5$ follows if we drop the factor F.

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