PHASE CHANGES IN NONLINEAR PROCESSES IN INTERACTING FOCK SPACE

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In this paper we study quantum mechanical phase distribution of some nonlinear optical phenomena in a general setting of interacting Fock space. We have investigated the optical phenomena of propagation through a nonlinear medium as in optical fiber and the process of photon absorption from a thermal beam. The input and output phase distribution have been investigated analytically in these two cases.

Keywords: Interacting Fock space; phase operator; phase distribution; nonlinear process.

1. Introduction

To study the fluctuating fields we usually introduce a phase distribution which provides a useful insight into the structure of fluctuations in quantum states. But to define a Hermitian phase operator has a setback starting with the work of Dirac who attempted a definition of a phase operator via a polar decomposition of the annihilation operator. Thereafter, Susskind and Glogower introduced a one-sided unitary decomposition of the phase operator which is used extensively in quantum optics. In the Pegg and Barnett approach a Hermitian phase operator was introduced via a polar decomposition of the annihilation operator in a truncated Hilbert space of dimension $s+1$. Now, given a state in the finite dimensional Hilbert space one first calculates the expectation value in the $s+1-$ dimensional space and then takes the limit as s tends to infinity. However, in this limit the PB phase operator does not converge to a Hermitian phase operator in the full Hilbert space, but the distribution does converge to the SG phase distribution. Thus to describe the quantum-mechanical phase via a phase distribution appears to be computationally advantageous than describing the phase distribution through a phase operator.

However, keeping the ideas of Susskind and Glogower in mind we describe here a phase operator in interacting Fock space and adopt the view point of Agarwal and co-workers to investigate the nonlinear optical phenomena of some states in the interacting Fock space.

The work is organized as follows. In Sec. 2, we discuss in brief the basic concepts of one mode interacting Fock space. In Sec. 3, we describe the phase distribution after introducing the phase operator in interacting Fock space. In Sec. 4, we study the phase distribution of incoherent vector, coherent vector and Kerr vector in the interacting Fock space. In Sec. 5, we consider the evolution of the phase distribution associated with a field as it propagates through nonlinear mediums. Here we discuss two well-known Kerr-like phenomena. In Sec. 6, we observe how the phase distribution changes in the process of photon absorption from a thermal beam and finally in Sec. 7, we give a conclusion.

2. Basic Concepts

Here we discuss some basic concepts which will be utilized throughout the paper.

As a vector space one mode interacting Fock space $\Gamma(\mathbb{C})$ is defined by

$$\Gamma(\mathbb{C}) = \bigoplus_{n=0}^{\infty} \mathbb{C}|n\rangle \,. \tag{1}$$

where $\mathbb{C}|n\rangle$ is called the n-particle subspace. The different n-particle subspaces are orthogonal, that is, the sum in Eq. (1) is orthogonal. The norm of the vector $|n\rangle$ is given by

$$\langle n|n\rangle = \lambda_n \tag{2}$$

where $\{\lambda_n\} \geq 0$ and if for some n we have $\{\lambda_n\} = 0$, then $\{\lambda_m\} = 0$ for all $m \geq n$. The norm introduced in Eq. (2) makes $\Gamma(\mathbb{C})$ a Hilbert space.

An arbitrary vector f in $\Gamma(\mathbb{C})$ is given by

$$f \equiv c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + \cdots + c_n|n\rangle + \cdots \tag{3}$$

for any $n \in \mathbb{N}$ with $\|f\| = (\sum_{n=0}^{\infty} |c_n|^2\lambda_n)^{1/2} < \infty$.

We now consider the following actions on $\Gamma(\mathbb{C})$:

$$\begin{aligned} A^*|n\rangle &= |n+1\rangle \\ A|n+1\rangle &= \frac{\lambda_{n+1}}{\lambda_n}|n\rangle \,. \end{aligned} \tag{4}$$

A^* is called the *creation operator* and its adjoint A is called the *annihilation operator*. To define the annihilation operator we have taken the convention $0/0 = 0$.

We observe that

$$\langle n|n\rangle = \langle A^*(n-1), n\rangle = \langle (n-1), An\rangle = \frac{\lambda_n}{\lambda_{n-1}}\langle n-1, n-1\rangle = \cdots \tag{5}$$

and

$$\||n\rangle\|^2 = \frac{\lambda_n}{\lambda_{n-1}} \cdot \frac{\lambda_{n-1}}{\lambda_{n-2}} \cdots \frac{\lambda_1}{\lambda_0} = \frac{\lambda_n}{\lambda_0} \,. \tag{6}$$

By Eq. (2) we observe from Eq. (6) that $\lambda_0 = 1$.

The commutation relation takes the form

$$[A, A^*] = \frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} \tag{7}$$

where N is the number operator defined by $N|n\rangle = n|n\rangle$.

In a recent paper[2] we have proved that the set $\{\frac{|n\rangle}{\sqrt{\lambda_n}}, n = 0, 1, 2, 3, \ldots\}$ forms a complete orthonormal set and the solution of the following eigenvalue equation

$$A f_\alpha = \alpha f_\alpha \tag{8}$$

is given by

$$f_\alpha = \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n} |n\rangle \tag{9}$$

where $\psi(|\alpha|^2) = \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{\lambda_n}$. We call f_α a **coherent vector** in $\Gamma(\mathbb{C})$.

Now, we observe that

$$AA^* = \frac{\lambda_{N+1}}{\lambda_N}, \qquad A^*A = \frac{\lambda_N}{\lambda_{N-1}}.$$

We further observe that $(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}})$ commutes with both A^*A and AA^*.

3. Phase Distribution

To obtain the phase distribution we obtain first the phase vector f_β which satisfies the eigenvalue equation

$$P f_\beta = \beta f_\beta \tag{10}$$

where P is the phase operator

$$P = \left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} + A^*A\right)^{-1/2} A \tag{11}$$

with $f_\beta = \sum a_n |n\rangle$.

We arrive finally at

$$f_\beta = \sum_{n=0}^{\infty} a_n |n\rangle = a_0 \sum_{n=0}^{\infty} \beta^n \cdot (\lambda_n)^{-1/2} |n\rangle. \tag{12}$$

In Eq. (12) we take $a_0 = 1$ and $\beta = |\beta| e^{i\theta}$.

Then

$$f_\beta = \sum_{n=0}^{\infty} e^{in\theta} (\lambda_n)^{-1/2} |\beta|^n |n\rangle. \tag{13}$$

These vectors are normalizable in the strict sense only for $|\beta| < 1$. In the limit $|\beta| \to 1$, Eq. (13) takes the form

$$f_\theta = \sum_{n=0}^{\infty} e^{in\theta} (\lambda_n)^{-1/2} |n\rangle \tag{14}$$

where $0 \leq \theta \leq 2\pi$ and call f_θ a **phase vector** in $\Gamma(\mathbb{C})$.

The vectors f_θ, though non-normalizable and nonorthogonal, form a complete set and yield the following resolution of identity

$$\frac{1}{2\pi}\int_0^{2\pi} d\theta |f_\theta\rangle\langle f_\theta| = I\,. \tag{15}$$

To prove this we define the operator

$$|f_\theta\rangle\langle f_\theta| : \Gamma(\mathbb{C}) \to \Gamma(\mathbb{C}) \tag{16}$$

by

$$|f_\theta\rangle\langle f_\theta| f = (f_\theta, f) f_\theta \tag{17}$$

with arbitrary $f = \sum a_n |n\rangle$.

Now,

$$(f_\theta, f) = \sum_{n=0}^{\infty} e^{-in\theta}(\lambda_n)^{1/2} a_n$$

and

$$(f_\theta, f) f_\theta = \sum_{m,n=0}^{\infty} e^{i(m-n)\theta}(\lambda_m)^{-1/2}(\lambda_n)^{1/2} a_n |m\rangle\,.$$

Hence

$$\begin{aligned}\frac{1}{2\pi}\int_0^{2\pi} d\theta |f_\theta\rangle\langle f_\theta| f &= \frac{1}{2\pi}\int_0^{2\pi} d\theta \sum_{m,n=0}^{\infty} e^{i(m-n)\theta}(\lambda_m)^{-1/2}(\lambda_n)^{1/2} a_n |m\rangle \\ &= \sum_{m,n=0}^{\infty} \delta_{mn}(\lambda_m)^{-1/2}(\lambda_n)^{1/2} a_n |m\rangle \\ &= \sum_{n=0}^{\infty} a_n |n\rangle \\ &= f\,. \end{aligned} \tag{18}$$

We use the vectors f_θ to associate to a given density operator ρ, a phase distribution as follows:

$$\begin{aligned} P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\ &= \frac{1}{2\pi}\sum_{m,n=o}^{\infty} e^{i(n-m)\theta}\left(\frac{|m\rangle}{\sqrt{\lambda_m}}, \rho\frac{|n\rangle}{\sqrt{\lambda_n}}\right). \end{aligned} \tag{19}$$

The $P(\theta)$ as defined in Eq. (19) is positive, owing to the positivity of ρ, and is normalized

$$\int_0^{2\pi} P(\theta) d\theta = 1\,. \tag{20}$$

For

$$\int_0^{2\pi} P(\theta)d\theta = \int_0^{2\pi} \frac{1}{2\pi} \sum_{m,n=o}^{\infty} e^{i(n-m)\theta} \left(\frac{|m\rangle}{\sqrt{\lambda_m}}, \rho \frac{|n\rangle}{\sqrt{\lambda_n}} \right)$$
$$= \sum_{n=o}^{\infty} \left(\frac{|n\rangle}{\sqrt{\lambda_n}}, \rho \frac{|n\rangle}{\sqrt{\lambda_n}} \right)$$
$$= 1 . \tag{21}$$

The **phase distribution** over the window $0 \leq \theta \leq 2\pi$ for any vector f is then defined by

$$P(\theta) = \frac{1}{2\pi} (f_\theta, |f\rangle\langle f| f_\theta)$$
$$= \frac{1}{2\pi} |(f_\theta, f)|^2 . \tag{22}$$

4. Examples

We now consider some specific vectors in the Hilbert space $\Gamma(\mathbb{C})$ and compute their corresponding phase distributions.

4.1. *Incoherent vectors*

For the incoherent vectors we take the density operator to be

$$\rho = \sum_{n=0}^{\infty} p_n \left| \frac{n}{\sqrt{\lambda_n}} \right\rangle \left\langle \frac{n}{\sqrt{\lambda_n}} \right| \tag{23}$$

with

$$p_n \geq 0 \quad \text{and} \quad \sum_{n=0}^{\infty} p_n = 1 .$$

Now we calculate the phase distribution $P(\theta)$ as

$$P(\theta) = \frac{1}{2\pi}(f_\theta, \rho f_\theta)$$
$$= \frac{1}{2\pi} \sum_{n=0}^{\infty} p_n \left(f_\theta, \left| \frac{n}{\sqrt{\lambda_n}} \right\rangle \left\langle \frac{n}{\sqrt{\lambda_n}} \right| f_\theta \right)$$
$$= \frac{1}{2\pi} \sum_{n=0}^{\infty} p_n \left| \left(f_\theta, \frac{n}{\sqrt{\lambda_n}} \right) \right|^2$$
$$= \frac{1}{2\pi} . \tag{24}$$

4.2. *Coherent vectors*

For the coherent vectors

$$f_\alpha = \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n} |n\rangle \tag{25}$$

we take the density operator to be

$$\rho = |f_\alpha\rangle\langle f_\alpha| \,, \quad \alpha = |\alpha| e^{i\theta_0} \,. \tag{26}$$

and calculate the phase distribution $P(\theta)$ as

$$\begin{aligned} P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\ &= \frac{1}{2\pi}|(f_\alpha, f_\theta)|^2 \\ &= \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta-\theta_0)}|\alpha|^n(\lambda_n)^{-1/2}\psi(|\alpha|^2)^{-1/2}\right|^2 . \end{aligned} \tag{27}$$

For $\lambda_n \sim n!$ we have

$$P(\theta) \sim \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta-\theta_0)}|\alpha|^n(n!)^{-1/2}\psi(|\alpha|^2)^{-1/2}\right|^2 . \tag{28}$$

And with $\psi(|\alpha|^2)^{-1/2} \sim (\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!})^{-1/2} = e^{-|\alpha|^2/2}$ we arrive at

$$P(\theta) \sim \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta-\theta_0)}\frac{|\alpha|^n}{\sqrt{n!}}e^{-|\alpha|^2/2}\right|^2 \tag{29}$$

which is the phase distribution in the bosonic case.

For large $|\alpha|^2$ after using the Gaussian approximation for a Poisson distribution

$$\frac{|\alpha|^{2n}}{n!}e^{-|\alpha|^2} \approx (2\pi|\alpha|^2)^{-1/2} e^{-\frac{(n-|\alpha|)^2}{2|\alpha|^2}} \tag{30}$$

we calculate sum (29) to obtain the following approximate Gaussian form

$$P(\theta) = \frac{1}{2}\left(\frac{|\alpha|^2}{2\pi}\right)^{1/2} e^{-2|\alpha|^2(\theta-\theta_0)^2} . \tag{31}$$

For $\lambda_n \sim [n]!$ we have

$$P(\theta) \sim \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta-\theta_0)}|\alpha|^n([n]!)^{-1/2}\psi(|\alpha|^2)^{-1/2}\right|^2 . \tag{32}$$

And with $\psi(|\alpha|^2)^{-1/2} \sim (\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{[n]!})^{-1/2} = e_q(|\alpha|^2)^{-1/2}$ we arrive at

$$P(\theta) \sim \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta-\theta_0)}\frac{|\alpha|^n}{\sqrt{[n]!}}e_q(|\alpha|^2)^{-1/2}\right|^2 \tag{33}$$

which is the phase distribution in the deformed case.

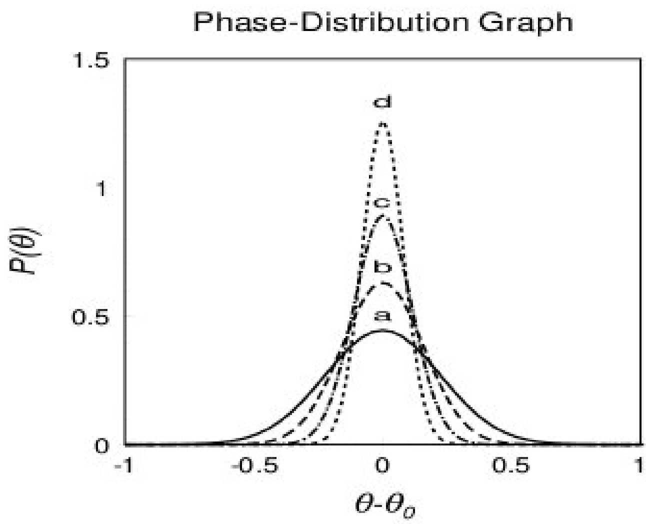

Fig. 1. Plot of $P(\theta)$ as a function of $\theta - \theta_0$ for a coherent state with mean number of photons (a) $|\alpha|^2 = 5$; (b) 10; (c) 20; (d) 40 and $\lambda_n \sim n!$

In Fig. 1 we have plotted $P(\theta)$ when $\lambda_n \sim n!$ and given by Eq. (31) for various values of $|\alpha|^2$. The distribution curve is peaked at $\theta = \theta_0$ and becomes narrower as $|\alpha|^2$ increases.

4.3. *Kerr vectors*

For the Kerr vectors ϕ_α^K,

$$\phi_\alpha^K = \sum_{n=0}^{\infty} q_n |n\rangle \tag{34}$$

where

$$q_n = \psi(|\alpha|^2)^{-1/2} \frac{\alpha^n}{\lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}} - 1)} \tag{35}$$

we take the density operator to be

$$\rho = |\phi_\alpha^K\rangle\langle\phi_\alpha^K|, \quad \alpha = |\alpha| e^{i\theta_0} \tag{36}$$

and calculate the phase distribution $P(\theta)$ as

$$\begin{aligned} P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\ &= \frac{1}{2\pi}|(\phi_\alpha^K, f_\theta)|^2 \\ &= \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta-\theta_0)} |\alpha|^n (\lambda_n)^{-1/2} \psi(|\alpha|^2)^{-1/2} e^{\frac{i}{2}\gamma\frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}}-1)}\right|^2 . \end{aligned} \tag{37}$$

For $\lambda_n \sim n!$ we have the phase distribution of the Kerr vector in the bosonic case

$$P(\theta) \sim \frac{1}{2\pi} \left| \sum_{n=0}^{\infty} e^{in(\theta-\theta_0)} \frac{|\alpha|^n}{\sqrt{n!}} e^{-|\alpha|^2/2} e^{\frac{i}{2}\gamma n(n-1)} \right|^2 . \quad (38)$$

For $\lambda_n \sim [n]!$ we have the phase distribution of the Kerr vector in the deformed case

$$P(\theta) \sim \frac{1}{2\pi} \left| \sum_{n=0}^{\infty} e^{in(\theta-\theta_0)} \frac{|\alpha|^n}{\sqrt{[n]!}} e_q(|\alpha|^2)^{-1/2} e^{\frac{i}{2}\gamma[n]([n]-1)} \right|^2 . \quad (39)$$

5. Propagation Through Nonlinear Mediums

Here, we consider the evolution of the phase distribution associated with a field as it propagates through nonlinear mediums. We shall discuss two well-known Kerr like phenomena that fall in this category.

5.1. *Optical Fiber — type one*

The first dynamic evolution of the density operator for our consideration is given by

$$\rho(t) = e^{-i\gamma A^* A(A^* A-1)t} \rho_0 e^{i\gamma A^* A(A^* A-1)t} \quad (40)$$

where γ is the Kerr constant of the medium. The time evolution of the corresponding phase distribution is given by

$$P(\theta, t) = \frac{1}{2\pi}(f_\theta, \rho(t) f_\theta) . \quad (41)$$

For an initial coherent vector

$$\rho(0) = |f_\alpha\rangle\langle f_\alpha| , \quad \alpha = |\alpha| e^{i\theta_0} . \quad (42)$$

$P(\theta, t)$ is given by

$$\begin{aligned} P(\theta, t) &= \frac{1}{2\pi}(f_\theta, e^{-i\gamma A^* A(A^* A-1)t} |f_\alpha\rangle\langle f_\alpha| e^{i\gamma A^* A(A^* A-1)t} f_\theta) \\ &= \frac{1}{2\pi} |(f_\alpha, e^{i\gamma A^* A(A^* A-1)t} f_\theta)|^2 \\ &= \frac{1}{2\pi} \left| \left(f_\alpha, \sum_{n=0}^{\infty} e^{in\theta} (\lambda_n)^{-1/2} e^{i\gamma t \frac{\lambda_n}{\lambda_{n-1}} (\frac{\lambda_n}{\lambda_{n-1}} - 1)} |n\rangle \right) \right|^2 \\ &= \frac{1}{2\pi} \psi(|\alpha|^2)^{-1} \left| \sum_{n=0}^{\infty} |\alpha|^n e^{in(\theta-\theta_0)} (\lambda_n)^{-1/2} e^{i\gamma t \frac{\lambda_n}{\lambda_{n-1}} (\frac{\lambda_n}{\lambda_{n-1}} - 1)} \right|^2 . \end{aligned} \quad (43)$$

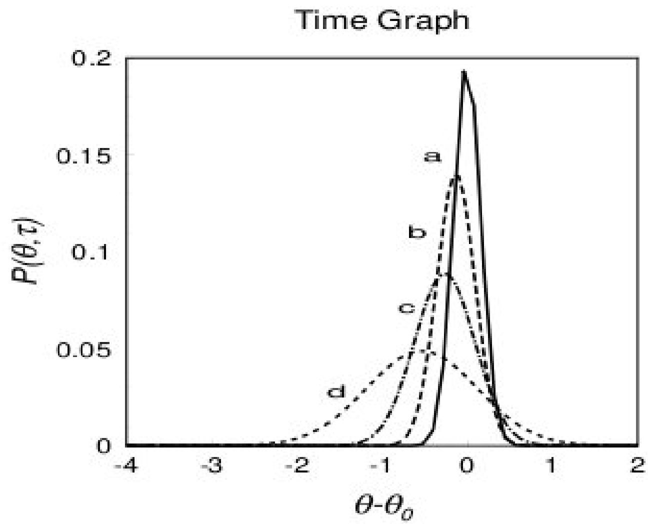

Fig. 2. The phase distribution $P(\theta, \tau)$ with mean number of photons $|\alpha|^2 = 10$ propagating through a nonlinear medium corresponding to (a) $\tau = 0$; (b) $\tau = 0.025$; (c) $\tau = 0.05$; (d) $\tau = 0.1$ and $\lambda_n \sim n!$

For $\lambda_n \sim n!$ and $\tau = \gamma t$, $P(\theta, t)$ can be calculated numerically to be

$$P(\theta,\tau) = \frac{1}{4}\frac{1}{(2\pi\sigma^2)^{1/2}}e^{-1/2} \times \exp\left\{-\frac{\left[-\frac{1}{4|\alpha|^2} + (\theta-\theta_0-\tau)^2 + 4|\alpha|(\theta-\theta_0-\tau)\tau\right]}{2\sigma^2}\right\} \tag{44}$$

with $\sigma^2 = \frac{1+16|\alpha|^4\tau^2}{4|\alpha|^2}$. In Fig. 2, we have shown the distribution and observed that the distribution shifts and broadens as the field propagates through the nonlinear medium. Thus, quantum mechanically, the phase not only shifts but also diffuses.

5.2. *Optical Fiber — type two*

The second dynamic evolution of the density operator for our consideration is given by

$$\rho(t) = e^{-i\gamma(A^*A)^2t}\rho_0 e^{i\gamma(A^*A)^2t} \tag{45}$$

where γ is the Kerr constant of the medium. The time evolution of the corresponding phase distribution is given by

$$P(\theta,t) = \frac{1}{2\pi}(f_\theta, \rho(t)f_\theta)\,. \tag{46}$$

For an initial coherent vector

$$\rho(0) = |f_\alpha\rangle\langle f_\alpha|\,, \quad \alpha = |\alpha|e^{i\theta_0} \tag{47}$$

$P(\theta, t)$ is given by

$$\begin{aligned} P(\theta, t) &= \frac{1}{2\pi}(f_\theta, e^{-i\gamma(A^*A)^2 t}|f_\alpha\rangle\langle f_\alpha|e^{i\gamma(A^*A)^2 t} f_\theta) \\ &= \frac{1}{2\pi}|(f_\alpha, e^{i\gamma(A^*A)^2 t} f_\theta)|^2 \\ &= \frac{1}{2\pi}\left|\left(f_\alpha, \sum_{n=0}^{\infty} e^{in\theta}(\lambda_n)^{-1/2} e^{i\gamma t(\frac{\lambda_n}{\lambda_{n-1}})^2}|n\rangle\right)\right|^2 \\ &= \frac{1}{2\pi}\psi(|\alpha|^2)^{-1}\left|\sum_{n=0}^{\infty}|\alpha|^n e^{in(\theta-\theta_0)}(\lambda_n)^{-1/2} e^{i\gamma t(\frac{\lambda_n}{\lambda_{n-1}})^2}\right|^2 . \end{aligned} \tag{48}$$

6. Process of Photon Absorption from a Thermal Beam

We next consider the phenomenon of photon absorption from a thermal beam. The density operator associated with the process can be written as

$$\rho = cA^{*s}\rho_0 A^s \tag{49}$$

where c is a normalization constant.

If we take the input field as a coherent vector, then the density operators for the input and the absorbed fields are

$$\rho_{\text{in}} = |f_\alpha\rangle\langle f_\alpha|, \quad \alpha = |\alpha|e^{i\theta_0} \tag{50}$$

and

$$\rho_{\text{out}} = cA^{*s}|f_\alpha\rangle\langle f_\alpha|A^s, \quad s > 0 . \tag{51}$$

Having obtained the density operator for the output field, we can now calculate the corresponding phase distribution. For the input field ρ_{in} we have already calculated the phase distribution in Sec. 4. The phase distribution $P_{\text{out}}(\theta)$ for the absorbed field is given by

$$\begin{aligned} P_{\text{out}}(\theta) &= \frac{1}{2\pi}(f_\theta, \rho_{\text{out}} f_\theta) \\ &= \frac{1}{2\pi}(f_\theta, cA^{*s}|f_\alpha\rangle\langle f_\alpha|A^s f_\theta) \\ &= \frac{c}{2\pi}(f_\theta, A^{*s}(f_\alpha, A^s f_\theta) f_\alpha) \\ &= \frac{c}{2\pi}|(f_\alpha, A^s f_\theta)|^2 \\ &= \frac{c}{2\pi}\left|\sum_{n=0}^{\infty}|\alpha|^{n+s} e^{i(n+s)(\theta-\theta_0)}\frac{1}{\sqrt{\lambda_{n+s}}}\psi(|\alpha|^2)^{-1/2}\right|^2 . \end{aligned} \tag{52}$$

7. Conclusion

We have thus shown how the phase distribution associated with the field evolves in various nonlinear processes in the interacting Fock space. We studied phase distribution through Susskind and Glogower type phase operator for incoherent vector, coherent vector and Kerr vector which have no classical analogue and observed various changes there on. We observed how phase distribution evolves when it propagates through Kerr-like mediums and when it undergoes the process of photon absorption from a thermal beam. Experimental observation of phase distribution of the various vectors studied may be quite interesting and may throw some insight into the subject.

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