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State space modelling of the quantum feedback control system in interacting fock space

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In this paper we introduce a formulation of the quantum mechanical feedback system in order to understand the quantum system in interacting Fock space which generalizes the transfer function approach to the quantum mechanical feedback system studied in bosonic mode. A mathematical model of the feedback control of the cavity quantum electrodynamic system in the interacting Fock space with the second cavity in the feedback path is described using a beam splitter.

1. Introduction

The feedback technique has been used for several years by electrical engineers to prevent noise from rendering a system unstable and this technique is now applied in optical systems, for example, to stabilize the phase or intensity of a laser oscillator. Feedback controls are important because, by taking into account the state of the system at each moment, they allow for updating and stabilizing control action. By the feedback method we can improve considerably the performance and robustness of the system, even if the system includes some uncertainty in its environment to which the system is highly structured. In classical engineering, control theory has been applied in space navigation and flight technology. Controlling quantum stochastic evolutions arises naturally in fields like quantum chemistry, quantum information theory and quantum engineering. In modern atomic and molecular physics, preparing atoms and molecules in a predefined state plays an important role. In particular such a problem arises in atom optics and quantum information. At the advent of the progress in quantum electronics, it is now possible to specify the quantum state at our disposal whenever we need it. This means we can control the quantum states for the use of computation and communication.

In this paper we discuss the formulation of the quantum mechanical feedback system in the interacting Fock space in order to introduce the concepts of control theory for the cavity quantum electrodynamics (QED). In recent papers (Yanagisawa and Kimura 2003a,b), the transfer function approach to the quantum feedback system has been developed using interaction of the cavities in the bosonic mode. The feedback control system in the bosonic mode is shown to be a special case of the general model described in the interacting Fock space.

The paper is organized as follows. In § 2, we discuss the basic notions of the interacting Fock space, interaction of the optical cavity and the external field, quantum stochastic process and quantum stochastic differential equation. In § 3, we discuss state space model for the open quantum system. In § 4, we discuss in details quantum mechanical feedback control system. In § 5, we describe state space modelling of the quantum mechanical feedback system and in § 6, we give a conclusion in which the phase margin (PM) and gain margin (GM) of the control system are discussed.

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2. Basic concepts

Here we shall discuss some basic concepts which will be utilized throughout the paper.

2.1 *Interacting Fock space*

As a vector space one mode interacting Fock space $\Gamma(\mathbb{C})$ is defined by

$$\Gamma(\mathbb{C}) = \bigoplus_{n=0}^{\infty} \mathbb{C}|n\rangle. \tag{1}$$

for any $n \in \mathbb{N}$ where $\mathbb{C}|n\rangle$ is called the n-particle subspace. The different n-particle subspaces are orthogonal, that is, the sum in (1) is orthogonal. The norm of the vector $|n\rangle$ is given by

$$\langle n|n\rangle = \lambda_n, \tag{2}$$

where $\{\lambda_n\} \geq 0$ and if for some n we have $\{\lambda_n\} = 0$, then $\{\lambda_m\} = 0$ for all $m \geq n$. The norm introduced in (2) makes $\Gamma(\mathbb{C})$ a Hilbert space.

An arbitrary vector f in $\Gamma(\mathbb{C})$ is given by

$$f \equiv c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + \cdots + c_n|n\rangle + \cdots \tag{3}$$

for any $n \in \mathbb{N}$ with $\|f\| = (\sum_{n=0}^{\infty} |c_n|^2 \lambda_n)^{1/2} < \infty$.

We now consider the following actions on $\Gamma(\mathbb{C})$:

$$\left.\begin{aligned} A^*|n\rangle &= |n+1\rangle \\ A|n+1\rangle &= \frac{\lambda_{n+1}}{\lambda_n}|n\rangle \end{aligned}\right\} \tag{4}$$

A^* is called the creation operator and its adjoint A is called the annihilation operator. To define the annihilation operator we have taken the convention $0/0 = 0$.

We observe that

$$\begin{aligned} \langle n|n\rangle &= \langle A^*(n-1), n\rangle = \langle (n-1), An\rangle \\ &= \frac{\lambda_n}{\lambda_{n-1}} \langle n-1, n-1\rangle = \cdots \end{aligned} \tag{5}$$

and

$$\| |n\rangle \|^2 = \frac{\lambda_n}{\lambda_{n-1}} \cdot \frac{\lambda_{n-1}}{\lambda_{n-2}} \cdots \frac{\lambda_1}{\lambda_0} = \frac{\lambda_n}{\lambda_0}. \tag{6}$$

By (2) we observe from (6) that $\lambda_0 = 1$.

The commutation relation takes the form

$$[A, A^*] = \frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} \tag{7}$$

where N is the number operator defined by $N|n\rangle = n|n\rangle$.

In a recent paper (Accardi and Bozejko 1998) we have proved that the set $\{(|n\rangle/\sqrt{\lambda_n}),\ n = 0, 1, 2, 3, \ldots\}$ forms a complete orthonormal set and the solution of the following eigenvalue equation

$$Af_\alpha = \alpha f_\alpha \tag{8}$$

is given by

$$f_\alpha = \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n} |n\rangle, \tag{9}$$

where $\psi(|\alpha|^2) = \sum_{n=0}^{\infty} (|\alpha|^{2n}/\lambda_n)$. We call f_α a *coherent vector* in $\Gamma(\mathbb{C})$.

Now, we observe that

$$AA^* = \frac{\lambda_{N+1}}{\lambda_N}, \quad A^*A = \frac{\lambda_N}{\lambda_{N-1}}$$

We further observe that $((\lambda_{N+1}/\lambda_N) - (\lambda_N/\lambda_{N-1}))$ commutes with both A^*A and AA^*.

2.2 *Interaction of the cavity and the external field*

We consider the interaction of an interacting single-mode of quantized field confined in a cavity with a noisy external field. Let $\mathcal{H}_A$ and $\mathcal{H}_B$ be Hilbert spaces of the cavity and the external field respectively. The composite system is expressed by the tensor product space $\mathcal{H}_A \otimes \mathcal{H}_B$. The total Hamiltonian is given by

$$H_{total} = H_A \otimes I_B + I_A \otimes H_B + H_{int} \tag{10}$$

where H_A describes the Hamiltonian of the cavity mode. This Hamiltonian may be further decomposed into two parts

$$H_A = H_{cav} + H. \tag{11}$$

Here H is the residual Hamiltonian determined by the optical medium in the cavity, referred to as a free Hamiltonian. H_B is the Hamiltonian of the external field.

The interaction Hamiltonian H_{int} consists of four terms. We drop the energy non-conserving terms corresponding to the rotating wave approximation and obtain the simplified Hamiltonian as

$$H_{int}(t) = i\sqrt{\gamma}\left[a(t)b^+(t) - a^+(t)b(t)\right] \tag{12}$$

with

$$\left[b(t), b^+(t')\right] = \delta(t - t') \tag{13}$$

and γ is a coupling constant. Here a is the annihilation operator of the cavity and b is the annihilation operator of the external field.

The operator $b(t)$ is a driving field at time t and we interpret the parameter t to mean the time at which the initial incoming field will interact with the system and not that $b(t)$ is a time-dependent operator at time t.

2.3 *Quantum stochastic process*

In order to describe quantum stochastic process we define first an operator

$$B_{in}(t, t_0) = \int_{t_0}^{t} b_{in}(s)ds \tag{14}$$

where $b_{in}(t)$ satisfies the commutation relation (13). This $b_{in}(t)$ represents the field immediately before it interacts with the system and we regard it as an input to the system.

Now,

$$\begin{aligned}[B_{in}(t, t_0), B_{in}^{+}(t, t_0)] &= \left[\int_{t_0}^{t} b_{in}(t')dt', \int_{t_0}^{t} b_{in}^{+}(t'')dt''\right] \\ &= \int_{t_0}^{t}\int_{t_0}^{t} [b_{in}(t'), b_{in}^{+}(t'')]dt'dt'' \\ &= \int_{t_0}^{t}\int_{t_0}^{t} \delta(t'-t'')dt'dt'' \\ &= \int_{t_0}^{t}\left(\int_{t_0}^{t} \delta(t'-t'')dt''\right)dt' \\ &= \int_{t_0}^{t} dt' \\ &= t - t_0. \end{aligned} \tag{15}$$

Now, we write down the increments

$$\begin{aligned} dB_{in}(t) = B_{in}(t+dt) - B_{in}(t),\ dB_{in}^{+}(t) \\ = B_{in}^{+}(t+dt) - B_{in}^{+}(t). \end{aligned} \tag{16}$$

Then

$$B_{in}(t+dt, t) = B_{in}(t+dt) - B_{in}(t) = dB_{in}(t) \tag{17}$$

and

$$B_{in}^{+}(t+dt, t) = B_{in}^{+}(t+dt) - B_{in}^{+}(t) = dB_{in}^{+}(t). \tag{18}$$

From (15), (16), (17) and (18) we get

$$\begin{aligned}\left[dB_{in}(t), dB_{in}^{+}(t)\right] &= \left[B_{in}(t+dt, t), B_{in}^{+}(t+dt, t)\right] \\ &= (t+dt) - t = dt. \end{aligned} \tag{19}$$

From (19), we have

$$\left[dB_{in}(t), dB_{in}^{+}(t)\right] = dt. \tag{20}$$

This leads to the natural definition of quantum stochastic process as

$$\begin{aligned} dB_{in}(t)dB_{in}^{+}(t) &= (N'+1)dt \\ dB_{in}^{+}(t)dB_{in}(t) &= N'dt \\ dB_{in}(t)dB_{in}(t) &= Mdt \\ dB_{in}^{+}(t)dB_{in}^{+}(t) &= M^{*}dt. \end{aligned} \tag{21}$$

and all other products higher than the second order in dB_{in} are equal to zero. N' and M are real and complex numbers satisfying

$$N'(N'+1) \geq |M|^2. \tag{22}$$

2.4 *Quantum stochastic differential equation*

Now we shall describe the quantum stochastic differential equation via evolution process. The evolution of an arbitrary operator X is given by

$$X(t) = U^{+}(t)XU(t) \tag{23}$$

in which the unitary operator $U(t)$ is generated by the Hamiltonians (10) and (11). H_{cav} and H_B drive the cavity and the external field respectively. We shall assume here H to be zero. The unitary operator of the system is then given by

$$U(dt) = e^{\sqrt{\gamma}(adB_{in}^{+} - a^{+}dB_{in})}. \tag{24}$$

Also we have

$$U^{+}(dt) = e^{\sqrt{\gamma}(a^{+}dB_{in} - adB_{in}^{+})}. \tag{25}$$

The increment of an arbitrary operator r of the system driven by the stochastic input b_{in} is given by

$$dr(t) = r(t+dt) - r(t) = U^{+}(dt)r(t)U(dt) - r(t). \tag{26}$$

Now

$$
\begin{aligned}
&U^{+}(dt)r(t)U(dt)\\
&= e^{\sqrt{\gamma}(a^{+}dB_{in}-adB_{in}^{+})}r(t)e^{\sqrt{\gamma}(adB_{in}^{+}-a^{+}dB_{in})}\\
&= e^{\sqrt{\gamma}(a^{+}dB_{in}-adB_{in}^{+})}r(t)e^{-\sqrt{\gamma}(a^{+}dB_{in}-adB_{in}^{+})}\\
&= r(t) + \left[\sqrt{\gamma}(a^{+}dB_{in}-adB_{in}^{+}), r(t)\right]\\
&\quad + \frac{1}{2!}\Big[\sqrt{\gamma}(a^{+}dB_{in}-adB_{in}^{+}),\\
&\quad \times\left[\sqrt{\gamma}(a^{+}dB_{in}-adB_{in}^{+}), r(t)\right]\Big] + \cdots - r(t)\\
&= r(t) + \sqrt{\gamma}\left[a^{+}dB_{in}-adB_{in}^{+}, r(t)\right]\\
&\quad + \frac{\gamma}{2}\big[a^{+}dB_{in}-adB_{in}^{+}, a^{+}dB_{in}r\\
&\quad - adB_{in}^{+}r - ra^{+}dB_{in} + radB_{in}^{+}\big]\\
&= r(t) + \sqrt{\gamma}\left[a^{+}dB_{in}-adB_{in}^{+}, r(t)\right]\\
&\quad + \frac{\gamma}{2}\big\{Ma^{+}a^{+}r - (N'+1)a^{+}ar - Ma^{+}ra^{+} + (N'+1)a^{+}ra\\
&\quad - N'aa^{+}r + M^{*}aar + N'ara^{+} - M^{*}ara\\
&\quad - Ma^{+}ra^{+} + (N'+1)a^{+}ra + N'ara^{+} - M^{*}ara\\
&\quad + Mra^{+}a^{+} - (N'+1)ra^{+}a - N'raa^{+} + M^{*}raa\big\}dt\\
&= r(t) + \sqrt{\gamma}\left[a^{+}dB_{in}-adB_{in}^{+}, r(t)\right]\\
&\quad + \frac{\gamma}{2}\{(N'+1)(a^{+}ra - a^{+}ar + a^{+}ra - ra^{+}a)\\
&\quad + N'(ara^{+} + ara^{+} - aa^{+}r - raa^{+})\\
&\quad + M(a^{+}a^{+}r - a^{+}ra^{+} - a^{+}ra^{+} + ra^{+}a^{+})\\
&\quad + M^{*}(aar - ara - ara + raa)\}dt\\
&= r(t) + \sqrt{\gamma}[a^{+}dB_{in}-adB_{in}^{+}, r(t)]\\
&\quad + \frac{\gamma}{2}\big\{(N'+1)(2a^{+}ra - a^{+}ar - ra^{+}a)\\
&\quad + N'(2ara^{+} - aa^{+}r - raa^{+})\\
&\quad + M(a^{+}(a^{+}r - ra^{+}) - (a^{+}r - ra^{+})a^{+})\\
&\quad + M^{*}(a(ar - ra) - (ar - ra)a)\big\}dt.
\end{aligned}
\tag{27}
$$

Hence equation (26) reduces to

$$
\begin{aligned}
dr(t) &= r(t+dt) - r(t)\\
&= U^{+}(dt)r(t)U(dt) - r(t)\\
&= \sqrt{\gamma}\left[a^{+}dB_{in} - adB_{in}^{+}, r(t)\right]\\
&\quad + \frac{\gamma}{2}\big\{(N'+1)(2a^{+}ra - a^{+}ar - ra^{+}a)\\
&\quad + N'(2ara^{+} - aa^{+}r - raa^{+})\\
&\quad + M\left[a^{+},[a^{+},r]\right] + M^{*}[a,[a,r]]\big\}dt.
\end{aligned}
\tag{28}
$$

3. State space model for open-loop quantum system

The first step of analysis and design of quantum control system is the mathematical modelling of the controlled process. In general, given a quantum controlled process, the set of variables that identify the dynamic characteristic of the process should be first defined along with the input–output relation of the system.

3.1 *Dynamics of the cavity*

To describe the dynamics of the operator $a(t)$ in the open quantum system we replace r in (29) by a to get

$$
\begin{aligned}
da &= a(t+dt) - a(t)\\
&= \sqrt{\gamma}\left[a^{+}dB_{in} - adB_{in}^{+}, a\right]\\
&\quad + \frac{\gamma}{2}\big\{(N'+1)(2a^{+}aa - a^{+}aa - aa^{+}a)\\
&\quad + N'(2aaa^{+} - aa^{+}a - aaa^{+})\\
&\quad + M\left[a^{+},[a^{+},a]\right] + M^{*}[a,[a,a]]\big\}dt\\
&= -\sqrt{\gamma}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)b_{in}(t)dt\\
&\quad + \frac{\gamma}{2}\left\{-(N'+1)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)a\right.\\
&\quad \left. + N'a\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right\}dt\\
&= \left\{-\frac{\gamma}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)a\right.\\
&\quad \left. - \sqrt{\gamma}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)b_{in}(t)\right\}dt.
\end{aligned}
\tag{29}
$$

Hence

$$
\begin{aligned}
\dot{a}(t) = &\left\{-\frac{\gamma}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)a(t)\right.\\
&\left. - \sqrt{\gamma}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)b_{in}(t)\right\}.
\end{aligned}
\tag{30}
$$

The state equation represented by equation (30) of the dynamics of the cavity a is a generalization of the well known Langevin equation of the cavity in the boson Fock space. This is due to the fact that the commutator $[A, A^{*}]$ given in (7) of the creation and annihilation operators in the interacting Fock space is not necessarily unity. In the case of boson Fock space the value of the commutator described in equation (7) becomes unity, that is,

$$[A, A^{*}] = 1$$

or,

$$\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} = 1.$$

The dynamics of the cavity given by (30) then reduces to

$$\dot{a}(t) = -\frac{\gamma}{2}a(t) - \sqrt{\gamma}b_{in}(t)$$

which is the usual quantum Langevin equation in the boson Fock space.

3.2 *Input–output relation of the open-loop system*

Due to the interaction of the evolving incoming field with the cavity an outgoing field is produced. To describe this we need to define an operator

$$B_{out}(t, t_0) = \int_{t_0}^{t} b_{out}(s)ds, \tag{31}$$

where

$$b_{out}(t) = U^{+}(dt)b_{in}(t)U(dt). \tag{32}$$

Here b_{out} is the output after the interaction at time t.

We then have

$$\begin{aligned} dB_{out}(t) &= U^{+}(dt)dB_{in}(t)U(dt) \\ &= e^{\sqrt{\gamma}(adB_{in}^{+} - a^{+}dB_{in})^{+}} dB_{in}(t)e^{\sqrt{\gamma}(adB_{in}^{+} - a^{+}dB_{in})} \\ &= dB_{in}(t) + \left[\sqrt{\gamma}(dB_{in}a^{+} - dB_{in}^{+}a), dB_{in}\right] + \cdots \\ &= dB_{in}(t) + \sqrt{\gamma}a\left[dB_{in}, dB_{in}^{+}\right]. \end{aligned} \tag{33}$$

Using equations (20) and (33) we now have

$$b_{out}(t)dt = b_{in}(t)dt + \sqrt{\gamma}adt \tag{34}$$

which implies

$$b_{out}(t) = \sqrt{\gamma}a(t) + b_{in}(t). \tag{35}$$

Taking Laplace transform of the operators we get the transfer function modelling of the input–output system of the cavity as shown in the figure 1.

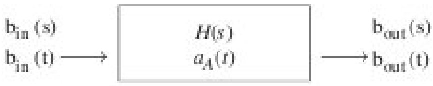

Figure 1. Transfer function of the open-loop system of a single cavity.

3.3 *Transfer function for the open-loop quantum system*

We can now describe the transfer function model of the open-loop quantum system with the help of dynamics and input–output relation of the cavity in the following way.

Equations (30) and (35) can be rewritten as

$$\left.\begin{aligned} \dot{a}(t) &= A'a(t) + B'b_{in}(t) \\ b_{out}(t) &= C'a(t) + D'b_{in}(t), \end{aligned}\right\} \tag{36}$$

where

$$\left.\begin{aligned} A' &= -\frac{\gamma}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ B' &= -\sqrt{\gamma}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ C' &= \sqrt{\gamma} \\ D' &= 1. \end{aligned}\right\} \tag{37}$$

Equations (36) represent the interacted state space model of the cavity.

As in the classical control system the state space model of the quantum control system for an open-loop single cavity is represented as

$$\left[\begin{array}{c|c} A' & B' \\ \hline C' & D' \end{array}\right] = \left[\begin{array}{c|c} -\frac{\gamma}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) & -\sqrt{\gamma}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ \hline \sqrt{\gamma} & 1 \end{array}\right]. \tag{38}$$

In usual bosonic mode (38) reduces to

$$\left[\begin{array}{c|c} A' & B' \\ \hline C' & D' \end{array}\right] = \left[\begin{array}{c|c} -\frac{\gamma}{2} & -\sqrt{\gamma} \\ \hline \sqrt{\gamma} & 1 \end{array}\right] \tag{39}$$

The transfer function relation of the output function $b_{out}(s)$ and the input function $b_{in}(s)$ for bosonic case is given by

$$b_{out}(s) = H(s)b_{in}(s) \tag{40}$$

where

$$\begin{aligned} H(s) &= C'(sI - A')^{-1}B' + D' \\ &= -\gamma\left(sI - \frac{\gamma}{2}\right)^{-1} + 1. \end{aligned} \tag{41}$$

is the open-loop transfer function of the cavity in the bosonic mode.

4. Quantum mechanical feedback system

At the advent of advanced experimental technology in the fields of cavity QED one can now monitor quantum systems continuously with very low noise and can manipulate on the time scales of the system evolution. For this reason it is quite natural to apply feedback to control the individual quantum systems in real time. The control of noise in optical systems has entered the quantum domain, squeezing noise within the constraints of Heisenberg's uncertainty principle which states that no action can be done without introducing inevitable disturbances to quantum system. The noise reducing capabilities of feedback in quantum optical systems have been recently explored by Wiseman and Milburn (1993) by deriving a master equation which describes quantum-limited feedback of a homodyne current to control an optical cavity. Wiseman–Milburn theory was applied to protect and generate non-classical states of light field and to manipulate motional state of atoms or the mirrors of optical cavities.

A cavity is thought of as a unit of a quantum system with a single operator valued state variable, an operator valued input and output on the associated Hilbert space. A quantum mechanical feedback is constructed through the input–output which store the information of the cavity. We consider here two cavities, A and B, that are positioned to interact with each other through the external field as depicted in figure 2. Let a_A and a_B be the annihilation operators for the modes of A and B respectively. These fields are treated as statistically independent. Because of the closed loop of the travelling field, A is influenced by B and vice versa. That is, these two systems are entangled, and this entanglement gives the control resources. The cavities are connected through a beam splitter forming a composite feedback control system.

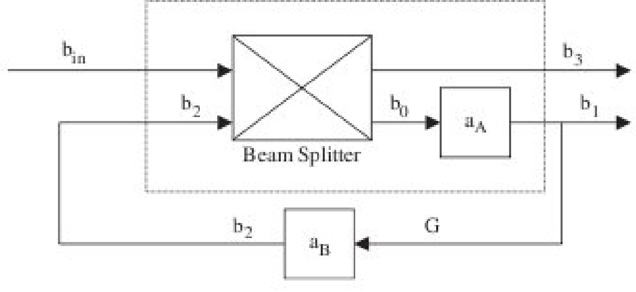

Figure 2. Design of a quantum mechanical feedback system.

The beam splitter is an optical device which allows us to perform the manipulation of quantum signals. The input field b_{in} is sent to one port of a beam splitter, which is chosen to have reflectivity α and transmitivity β, and b_2 is the feedback output of the cavity from B to the other input port of the beam splitter. Meanwhile, one of the transmitted fields from the beam splitter is sent back to the cavity A. Assume that the time of propagation between the systems is negligible as the feedback system does not include time delay in the closed-loop.

5. State space model of the feedback system

The mathematical modelling of the controlled process in the feedback system consists of describing the dynamic characteristics of the process along with the input–output relation of the system. Let us first derive the dynamics of the cavity A in the forward path of the composite feedback control system.

5.1 *Dynamics of the cavity in the feedback system*

In the feedback system an arbitrary operator r obeys the following evolution law

$$r(t+dt) = U^{+}(dt)r(t)U(dt), \tag{42}$$

where $U(dt)$ is a unitary operator given by

$$U(dt) = e^{-iHdt} \tag{43}$$

with

$$H = i\Bigg[\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}(a_A b_{in}^{+} - a_A^{+} b_{in}) + \frac{\beta}{(1+\alpha)}\sqrt{\gamma_B}(a_B b_{in}^{+} - a_B^{+} b_{in}) + \sqrt{\gamma_A\gamma_B}(a_A^{+} a_B - a_A a_B^{+})\Bigg]. \tag{44}$$

To derive the dynamics of the cavity A we take $r = a_A$ and write down equation (43) as

$$\begin{aligned} U(dt) &= \exp(-iHdt) \\ &= \exp\Bigg[\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}(a_A dB_{in} - a_A^{+} dB_{in}) \\ &\quad + \frac{\beta}{(1+\alpha)}\sqrt{\gamma_B}(a_B dB_{in}^{+} - a_B^{+} dB_{in}) \\ &\quad + \frac{1}{2}\sqrt{\gamma_A\gamma_B}(a_A^{+} a_B dt - a_A a_B^{+} dt)\Bigg] \end{aligned} \tag{45}$$

and

$$U^{+}(dt) = \exp(iH^{+}dt)$$
$$= \exp\left\{-\left[\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(a_A dB_{in} - a_A^{+} dB_{in}\right) + \frac{\beta}{(1+\alpha)}\sqrt{\gamma_B}\left(a_B dB_{in}^{+} - a_B^{+} dB_{in}\right) + \frac{1}{2}\sqrt{\gamma_A\gamma_B}\left(a_A^{+} a_B dt - a_A a_B^{+} dt\right)\right]\right\} \quad (46)$$

Now,

$$a_A(t+dt) = e^{-X} a_A e^{X}$$
$$= e^{(-1)X} a_A e^{-(-1)X}$$
$$= a_A - [X, a_A] + \frac{1}{2!}[X, [X, a_A]] + \cdots, \quad (47)$$

where

$$X = \left[\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(a_A dB_{in} - a_A^{+} dB_{in}\right) + \frac{\beta}{(1+\alpha)}\sqrt{\gamma_B}\left(a_B dB_{in}^{+} - a_B^{+} dB_{in}\right) + \frac{1}{2}\sqrt{\gamma_A\gamma_B}\left(a_A^{+} a_B dt - a_A a_B^{+} dt\right)\right]. \quad (48)$$

Now

$$[X, a_A] = \left[\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(a_A dB_{in} - a_A^{+} dB_{in}\right) + \frac{\beta}{(1+\alpha)}\sqrt{\gamma_B}\left(a_B dB_{in}^{+} - a_B^{+} dB_{in}\right) + \frac{1}{2}\sqrt{\gamma_A\gamma_B}\left(a_A^{+} a_B dt - a_A a_B^{+} dt\right), a_A\right]$$
$$= \frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dB_{in} - \frac{1}{2}\sqrt{\gamma_A\gamma_B}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) a_B dt. \quad (49)$$

Again

$$[X, [X, a_A]] = \left[\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(a_A dB_{in} - a_A^{+} dB_{in}\right) + \frac{\beta}{(1+\alpha)}\sqrt{\gamma_B}\left(a_B dB_{in}^{+} - a_B^{+} dB_{in}\right) + \frac{1}{2}\sqrt{\gamma_A\gamma_B}\left(a_A^{+} a_B dt - a_A a_B^{+} dt\right), \frac{\beta}{1+\alpha}\sqrt{\gamma_A} \times \left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dB_{in} - \frac{1}{2}\sqrt{\gamma_A\gamma_B}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) a_B dt\right]$$
$$= -\frac{\beta^2}{(1+\alpha)^2}\gamma_A a_A\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt - \frac{\beta^2}{(1+\alpha)^2}\sqrt{\gamma_A\gamma_B}\, a_B\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt. \quad (50)$$

Hence

$$a_A(t+dt) - a_A(t) = -\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dB_{in} + \frac{1}{2}\sqrt{\gamma_A\gamma_B}\, a_B\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt - \frac{\beta^2}{(1+\alpha)^2}\frac{\gamma_A}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) a_A dt - \frac{\beta^2}{2(1+\alpha)^2}\sqrt{\gamma_A\gamma_B}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) a_B dt. \quad (51)$$

Now

$$-\frac{\beta^2}{(1+\alpha)^2}\frac{\gamma_A}{2} = -\frac{(1-\alpha^2)}{(1+\alpha)^2}\frac{\gamma_A}{2}$$
$$= -\frac{1}{(1+\alpha)}\frac{\gamma_A}{2} + \frac{\alpha}{(1+\alpha)}\frac{\gamma_A}{2}$$
$$= -\frac{\gamma_A}{2} + \frac{\alpha}{(1+\alpha)}\gamma_A. \quad (52)$$

Then

$$\frac{\beta^2}{(1+\alpha)^2}\frac{\gamma_A}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) a_A dt$$
$$= -\frac{\gamma_A}{2} a_A\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt + \frac{\alpha}{(1+\alpha)}\gamma_A a_A\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt \quad (53)$$

and

$$1 - \frac{\beta^2}{(1+\alpha)^2} = \left[1 - \frac{1-\alpha^2}{(1+\alpha)^2}\right] = \frac{2\alpha}{1+\alpha}. \quad (54)$$

Therefore

$$\frac{1}{2}\sqrt{\gamma_A\gamma_B}\, a_B\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt - \frac{\beta^2}{2(1+\alpha)^2}\sqrt{\gamma_A\gamma_B}\, a_B$$
$$= \frac{\alpha}{1+\alpha}\sqrt{\gamma_A\gamma_B}\, a_B\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) dt. \quad (55)$$

Finally we get

$$\begin{aligned} a_A(t+dt) - a_A(t) = &-\frac{\beta}{(1+\alpha)}\sqrt{\gamma_A}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)dB_{in} \\ &+\frac{1}{2}\sqrt{\gamma_A\gamma_B}a_B\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)dt \\ &-\frac{\beta^2}{(1+\alpha)^2}\frac{\gamma_A}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)a_A\,dt \\ &-\frac{\beta^2}{2(1+\alpha)^2}\sqrt{\gamma_A\gamma_B}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)a_B\,dt \\ = &\left\{-\frac{\gamma_A}{2}a_A\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right. \\ &-\sqrt{\gamma_A}(\frac{\beta}{1+\alpha}b_{in}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ &-\sqrt{\gamma_A}\frac{\alpha}{1+\alpha}a_A\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ &\left.-\sqrt{\gamma_B}\frac{\alpha}{1+\alpha}a_B)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right\}dt. \end{aligned} \tag{56}$$

Hence

$$\begin{aligned} \dot{a}_A(t) = &-\frac{\gamma_A}{2}a_A(t)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ &-\sqrt{\gamma_A}\left\{\frac{\beta}{1+\alpha}b_{in}(t)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right. \\ &-\sqrt{\gamma_A}\frac{\alpha}{1+\alpha}a_A(t)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ &\left.-\sqrt{\gamma_B}\frac{\alpha}{1+\alpha}a_B(t)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right\}. \end{aligned} \tag{57}$$

This gives the state space model of the optical cavity A in the interacting Fock space.

Equation (57) representing the dynamics of the cavity a_A in the feedback system in interacting Fock space is a generalization of the cavity in the feedback system in the boson Fock space. This is due to the fact that the commutator $[A, A^*] = (\lambda_{N+1}/\lambda_N) - (\lambda_N/\lambda_{N-1})$ in interacting Fock space is not necessarily unity. In the case of boson Fock space the value of the commutator becomes unity. Then the dynamics of the cavity in the feedback system reduces to

$$\begin{aligned} \dot{a}_A(t) = &-\frac{\gamma_A}{2}a_A(t) - \sqrt{\gamma_A}\left\{\frac{\beta}{1+\alpha}b_{in}(t)\right. \\ &\left.-\sqrt{\gamma_A}\frac{\alpha}{1+\alpha}a_A(t) - \sqrt{\gamma_B}\frac{\alpha}{1+\alpha}a_B(t)\right\}. \end{aligned} \tag{58}$$

5.2 *Input–output relation of the feedback system*

The input signals b_{in} and b_2 to the beam splitter described in figure 2 are related to the output operators b_0 and b_3 by

$$\begin{bmatrix} b_3 \\ b_0 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}\begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}, \tag{59}$$

where α and β are real and satisfy $\alpha^2 + \beta^2 = 1$. From the input–output relation of each system, we have

$$\begin{aligned} b_1 &= \sqrt{\gamma_A}a_A + b_0 \\ b_2 &= \sqrt{\gamma_B}a_B + b_1. \end{aligned} \tag{60}$$

These relations determine each signal in the feedback loop. For, from (59) we have

$$\begin{bmatrix} b_3 \\ b_0 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}\begin{bmatrix} b_{in} \\ b_2 \end{bmatrix} = \begin{bmatrix} \alpha b_{in} + \beta b_2 \\ \beta b_{in} - \alpha b_2 \end{bmatrix}, \tag{61}$$

hence we get

$$\begin{aligned} b_3 &= \alpha b_{in} + \beta b_2 \\ b_0 &= \beta b_{in} - \alpha b_2. \end{aligned} \tag{62}$$

From (60) we now have

$$\begin{aligned} b_2 &= \sqrt{\gamma_B}a_B + b_1 \\ &= \sqrt{\gamma_B}a_B + \sqrt{\gamma_A}a_A + b_0. \end{aligned} \tag{63}$$

From (62) and (63) we get

$$b_0 = \beta b_{in} - \alpha\sqrt{\gamma_B}a_B - \alpha\sqrt{\gamma_A}a_A - \alpha b_0 \tag{64}$$

or

$$b_0 = \frac{\beta}{1+\alpha}b_{in} - \frac{\alpha}{1+\alpha}\left(\sqrt{\gamma_A}a_A + \sqrt{\gamma_B}a_B\right). \tag{65}$$

Similarly, we get

$$\begin{aligned} b_1 &= \sqrt{\gamma_A}a_A + b_0 \\ &= \frac{\beta}{1+\alpha}b_{in} + \frac{1}{1+\alpha}\sqrt{\gamma_A}a_A - \frac{\alpha}{1+\alpha}\sqrt{\gamma_B}a_B, \end{aligned} \tag{66}$$

again

$$\begin{aligned} b_2 &= \sqrt{\gamma_B}a_B + \sqrt{\gamma_A}a_A + b_0 \\ &= \frac{\beta}{1+\alpha}b_{in} + \frac{1}{1+\alpha}\left(\sqrt{\gamma_A}a_A + \sqrt{\gamma_B}a_B\right) \end{aligned} \tag{67}$$

and finally

$$\begin{aligned} b_3 &= \alpha b_{in} + \beta b_2 \\ &= b_{in} + \frac{\beta}{1+\alpha}\left(\sqrt{\gamma_A}a_A + \sqrt{\gamma_B}a_B\right). \end{aligned} \tag{68}$$

Each signal in the feedback loop can now be written as

$$\begin{aligned} b_0 &= \frac{\beta}{1+\alpha} b_{in} - \frac{\alpha}{1+\alpha}\left(\sqrt{\gamma_A}a_A + \sqrt{\gamma_B}a_B\right) \\ b_1 &= \frac{\beta}{1+\alpha} b_{in} + \frac{1}{1+\alpha}\sqrt{\gamma_A}a_A - \frac{\alpha}{1+\alpha}\sqrt{\gamma_B}a_B \\ b_2 &= \frac{\beta}{1+\alpha} b_{in} + \frac{1}{1+\alpha}\left(\sqrt{\gamma_A}a_A + \sqrt{\gamma_B}a_B\right) \\ b_3 &= b_{in} + \frac{\beta}{1+\alpha}\left(\sqrt{\gamma_A}a_A + \sqrt{\gamma_B}a_B\right). \end{aligned} \tag{69}$$

5.3 *Transfer function of the composite system*

We now describe the transfer function of the quantum feedback system from the dynamics and the input–output relation of the feedback system.

Using (57) the dynamics of the cavity A is rewritten as

$$\begin{aligned} \dot{a_A}(t) &= -\frac{\gamma_A}{2} a_A(t)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ &\quad - \sqrt{\gamma_A}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) b_0 \\ &= -\frac{\gamma_A}{2} a_A(t)\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ &\quad + \left[-\sqrt{\gamma_A}\beta\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right), \sqrt{\gamma_A}\alpha \right. \\ &\quad \left. \times\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right]\begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}. \end{aligned} \tag{70}$$

That is

$$\dot{x} = A'x + B'u, \tag{71}$$

where

$$\begin{aligned} x &= a_A \\ A' &= -\frac{\gamma_A}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ B' &= \left[-\sqrt{\gamma_A}\beta\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right), \sqrt{\gamma_A}\alpha\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right)\right] \\ u &= \begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}. \end{aligned} \tag{72}$$

The state equation of the dynamics of the cavity described by (71) and (72) is an operator equation with the coefficients as operators in respective Hilbert space.

The output equations are given by

$$b_3 = \alpha b_{in} + \beta b_2 = [\alpha, \beta]\begin{bmatrix} b_{in} \\ b_2 \end{bmatrix} \tag{73}$$

and

$$\begin{aligned} b_1 &= \sqrt{\gamma_A}a_A + b_0 \\ &= \sqrt{\gamma_A}a_A + [\beta, -\alpha]\begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}. \end{aligned} \tag{74}$$

In vector notation, we have

$$\begin{bmatrix} b_3 \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{\gamma_A} \end{bmatrix} a_A + \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}\begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}. \tag{75}$$

That is

$$y(t) = C'x(t) + D'u(t), \tag{76}$$

where

$$\begin{aligned} y(t) &= \begin{bmatrix} b_3 \\ b_1 \end{bmatrix} \\ C' &= \begin{bmatrix} 0 \\ \sqrt{\gamma_A} \end{bmatrix} \\ x(t) &= a_A(t) \\ D' &= \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix} \\ u(t) &= \begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}. \end{aligned}$$

The quantum feedback control system given by (70) and (76) in the interacting Fock space is represented as

$$\left[\begin{array}{c|c} A' & B' \\ \hline C' & D' \end{array}\right] = \left[\begin{array}{c|c} -\frac{\gamma_A}{2}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) & -\beta\sqrt{\gamma_A}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right),\ \alpha\sqrt{\gamma_A}\left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}}\right) \\ \hline \begin{matrix} 0 \\ \sqrt{\gamma_A} \end{matrix} & \begin{matrix} \alpha & \beta \\ \beta & -\alpha \end{matrix} \end{array}\right] \tag{77}$$

The state space dynamics of the composite feedback system represented by (77) is a generalization of the state space equation described in the recent publication by Yanagisawa and Kimura (2003a, b). Thus the study of the feedback system presents a more generalization of the well known facts of the bosonic mode.

In the bosonic mode the state space model given by equation (77) reduces to

$$\left[\begin{array}{c|c} A' & B' \\ \hline C' & D' \end{array}\right] = \left[\begin{array}{c|c} -\frac{\gamma_A}{2} & -\beta\sqrt{\gamma_A}, \alpha\sqrt{\gamma_A} \\ \hline \begin{matrix} 0 \\ \sqrt{\gamma_A} \end{matrix} & \begin{matrix} \alpha & \beta \\ \beta & -\alpha \end{matrix} \end{array}\right]. \tag{78}$$

The transfer function relation of the output function $Y(s)$ to the input function $U(s)$ in bosonic mode is given by

$$Y(s) = G(s)U(s), \tag{79}$$

where

$$Y(s) = \begin{bmatrix} b_3 \\ b_1 \end{bmatrix} \tag{80}$$

and

$$U(s) = \begin{bmatrix} b_{in} \\ b_2 \end{bmatrix}. \tag{81}$$

The transfer function $G(s)$ of the composite system is given by

$$\begin{aligned} G(s) &= C'(sI - A')^{-1}B' + D' \\ &= \begin{bmatrix} 0 \\ \sqrt{\gamma_A} \end{bmatrix}\left(sI + \frac{\gamma_A}{2}\right)^{-1} \\ &\quad \left[-\beta\sqrt{\gamma_A}, \alpha\sqrt{\gamma_A}\right] + \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix} \\ &= p(s)\begin{bmatrix} 0 & 0 \\ -\beta\gamma_A & \alpha\gamma_A \end{bmatrix} + \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix} \\ &= \begin{bmatrix} \alpha & \beta \\ -\beta\gamma_A p(s) + \beta & \alpha\gamma_A p(s) - \alpha \end{bmatrix}, \end{aligned} \tag{82}$$

where $p(s) = (sI + (\gamma_A/2))^{-1}$.

6. Conclusion

The study of the cavity quantum electrodynamics (QED) has received much attention in recent years by different authors (Berman 1994, Doherty *et al.* 2000). All the studies in respect of generation of photons have been made by the authors in the boson Fock space. The present study described in this paper is a generalization of all the existing model of the dynamics of the cavity in the interacting Fock space. The state and output equations of the cavity in interacting Fock space has been described in this paper.

The input–output relation of a cavity with a second cavity in the feedback loop can easily be described as

$$b_1(s) = M(s)b_{in}(s),$$

where $M(s)$ is the closed-loop transfer function of the form

$$M(s) = \frac{\beta G(s)}{1 + \alpha G(s)H(s)}.$$

with $G(s) = \gamma_A(s + (\gamma_A/2))^{-1} + 1$ representing the forward path gain (transfer) function and $H(s) = \gamma_B(s + (\gamma_B/2)^{-1} + 1$ representing the feedback transfer function of the closed-loop system.

The phase margin (PM) of the control system can easily be shown to be zero and the gain margin (GM) is $(1/\alpha)(>1)$ in terms of the parameters of the beam splitter.

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