Water wave scattering by two partially immersed nearly vertical barriers

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Abstract

This paper provides a mathematical investigation of the problem of scattering of surface water waves by two surface-piercing barriers that are almost vertical and are described by the same shape function in the context of linear theory by employing a simplified perturbational analysis. Green's identity is used to express the perturbation to the quantities of interest—the reflection and transmission coefficients—in terms of the solution to the unperturbed system. As in the case of single nearly vertical barrier, here also the perturbed transmission coefficient vanishes identically while the perturbed reflection coefficient is obtained in terms of a number of definite integrals involving shape function. When the two barriers are merged into a single barrier, the known result for a single barrier is recovered.

Keywords: Water wave scattering; Linear theory; Nearly vertical barriers; Perturbation analysis; Reflection and transmission coefficients

1. Introduction

Within the framework of linearised theory of water waves, the problems involving scattering of normally incident surface water wave trains in deep water by thin fixed plane vertical barriers, admit of exact solutions (cf.Ursel [1], Evans [2], Porter [3] for a single barrier, Levine and Rodemich [4], Jarvis [5] for two equal parallel barriers). A substantial amount of research work related to water wave scattering problems involving the thin vertical barriers has been carried out during the last six decades (see Mandal and Chakrabarti [6]). Problems involving thin curved barriers or inclined straight plane barriers do not admit of exact solutions but can be studied by some approximate methods. These have been studied by using hypersingular integral equation formulations (cf. Parson and Martin [7], [8], Midya et al [9], Kanoria and Mandal [10], Mandal and Gayen(Chowdhury) [11]). The hypersingular integral equation arising in each problem has been solved approximately and numerical estimates for the reflection and transmission coefficients are then obtained. When the barriers are slightly curved, which we call nearly vertical barriers, it is also not possible to find exact solutions, rather perturbed forms of the solution can be found. A problem associated with a surface-piercing nearly vertical single barrier was first handled by Shaw [12] using perturbational analysis which involved the solution of a singular integral equation. Evans [13], in a short note, gave an idea for computation of the wave amplitude produced by small oscillations of a partially immersed flexible plate which involved an application of

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Green's integral theorem. Following this idea, Mandal and Chakrabarti [14] determined the first order corrections to the reflection and transmission coefficients (R and T, respectively) for the problem of surface water wave scattering by a fixed nearly vertical barrier for its two configurations, viz. a partially immersed barrier and a completely submerged barrier extending infinitely downwards, using a different perturbational analysis.The case of nearly vertical thin plate submerged in deep water was investigated by Mandal and Kundu [15] employing Shaw's [12] method as well as the method used by Mandal and Chakrabarti [14].

In the present paper, the first order corrections R_1, T_1 to R and T, respectively for the problem of surface wave scattering by two fixed nearly vertical identical barriers, which are partially immersed to the same depth in infinitely deep water, have been determined by using the technique employed by Mandal and Chakrabarti [14]. Here we have used the exact solution of the double barrier problem given by Levine and Rodemich [4]. As in the case of single barrier, T_1 also vanishes here identically while R_1 is obtained in terms of definite integrals involving the shape function. The known result for a single partially immersed nearly vertical barrier is recovered when the two barriers are merged to a single barrier by making the distance between the two partially immersed edges of the barriers to tend to zero.

2. Statement of the problem

Assuming linear theory, we consider the two-dimensional problem of water wave scattering by two fixed nearly vertical partially immersed barriers in deep water. The position of the mean free surface is given by $y = 0$, the y-axis being chosen vertically downwards into the fluid region, and x-axis along the direction of an incoming train of surface waves.

Let the configurations of the barriers be described by $x = \pm a + \epsilon c(y)$, $0 < y < 1$ where ϵ is a small non-dimensional parameter giving a measure of maximum deviation of the curved barriers from the vertical and $c(y)$ is the shape function which is continuous and bounded in $(0, 1)$ satisfying $c(0) = c(1) = 0$. Assuming the motion in the fluid to be irrotational and simple harmonic in time t with angular frequency σ, it can be described by a velocity potential $Re\{\phi(x, y)e^{-i\sigma t}\}$.

Then $\phi(x, y)$ satisfies the Laplace's equation

$$\nabla^2\phi = 0 \quad \text{in the fluid region,} \tag{1}$$

the linearised free surface condition

$$K\phi + \phi_y = 0 \quad \text{on} \quad y = 0, \tag{2}$$

with $K = \sigma^2/g$, g being the acceleration due to gravity, the barrier conditions

$$\frac{\partial\phi}{\partial n} = 0 \quad \text{on} \quad x = \pm a + \epsilon c(y), 0 < y < 1 \tag{3}$$

where n denotes the normal to the surface of the curved barriers, the edge conditions

$$r^{1/2}\nabla\phi \quad \text{is bounded as} \quad r \to 0, \tag{4}$$

where r is the distance from the points $(\pm a, 1)$, the deep water conditions

$$\phi, \nabla\phi \to 0 \quad \text{as} \quad y \to \infty, \tag{5}$$

and finally, the infinity requirements given by

$$\phi(x, y) \sim \begin{cases} T\phi^{\text{inc}}(x, y) & \text{as } x \to \infty, \\ \phi^{\text{inc}}(x, y) + R\phi^{\text{inc}}(-x, y) & \text{as } x \to -\infty. \end{cases} \tag{6}$$

In the condition (6), R and T, respectively denote the (complex) reflection and transmission coefficients to be determined and

$$\phi^{\text{inc}}(x, y) = e^{-Ky+iKx} \tag{7}$$

denotes the incident wave potential propagating from the direction of $x = -\infty$.

3. Method of solution

For nearly vertical barriers, the parameter ϵ can be assumed to be very small. The boundary conditions (3) on nearly vertical barriers can be expressed as

$$\phi_x(\pm a \pm 0, y) - \epsilon \frac{\mathrm{d}}{\mathrm{d}y}\{c(y)\phi_y(\pm a \pm 0, y)\} + 0(\epsilon^2) = 0, \quad \text{for} \quad 0 < y < 1 \tag{8}$$

where ± 0 denote values on two sides of each barrier. The forms of the approximate boundary conditions (8) suggest that ϕ, R and T have the following perturbational expansions, in terms of the small parameter ϵ:

$$\begin{aligned} \phi(x, y; \epsilon) &= \phi_0(x, y) + \epsilon\phi_1(x, y) + o(\epsilon^2), \\ R(\epsilon) &= R_0 + \epsilon R_1 + o(\epsilon^2), \\ T(\epsilon) &= T_0 + \epsilon T_1 + o(\epsilon^2). \end{aligned} \tag{9}$$

Substituting the expansions (9) in the basic partial differential equation (1), the free surface condition (2), the approximate boundary conditions (8), the conditions (4)–(6), we find, after equating the coefficients of identical powers of ϵ from both sides of the results, that ϕ_0 and ϕ_1 satisfy the following two problems p_1 and p_2, respectively.

p_1. The function ϕ_0 satisfies

(i) $\nabla^2\phi_0 = 0 \quad \text{in} \quad y > 0, -\infty < x < \infty$
(ii) $K\phi_0 + \phi_{0y} = 0 \quad \text{on} \quad y = 0,$
(iii) $\phi_{0x} = 0 \quad \text{on} \quad x = \pm a, 0 < y < 1,$
(iv) $r^{1/2}\nabla\phi_0 \quad \text{is bounded as} \quad r = \{(x \pm a)^2 + (y-1)^2\}^{1/2} \to 0,$
(v) $\phi_0, \nabla\phi_0 \to 0 \quad \text{as} \quad y \to \infty,$
(vi) $\phi_0(x, y) \sim \begin{cases} T_0 e^{-Ky+iKx} & \text{as } x \to \infty, \\ e^{-Ky+iKx} + R_0 e^{-Ky-iKx} & \text{as } x \to -\infty. \end{cases}$

p_2. The function ϕ_1 satisfies

(i) $\nabla^2\phi_1 = 0 \quad \text{in} \quad y > 0, -\infty < x < \infty,$
(ii) $K\phi_1 + \phi_{1y} = 0 \quad \text{on} \quad y = 0,$
(iii) $\phi_{1x}(\pm a \pm 0, y) = \frac{\mathrm{d}}{\mathrm{d}y}\{c(y)\phi_{0y}(\pm a \pm 0, y)\}, 0 < y < 1,$
(iv) $r^{1/2}\nabla\phi_1 \quad \text{is bounded as} \quad r = \{(x \pm a)^2 + (y-1)^2\}^{1/2} \to 0,$
(v) $\phi_1, \nabla\phi_1 \to 0 \quad \text{as} \quad y \to \infty,$
(vi) $\phi_1(x, y) \sim \begin{cases} T_1 e^{-Ky+iKx} & \text{as } x \to \infty, \\ R_1 e^{-Ky-iKx} & \text{as } x \to -\infty. \end{cases}$

The problem p_1 corresponds to water wave scattering by two thin vertical partially immersed barriers. Its explicit solution was obtained by Levine and Rodemich [4] by using complex variable theory. His results are reproduced in Appendix Appendix A in an equivalent form for the purpose of their use in obtaining the first order corrections R_1 and T_1 to the reflection and transmission coefficients appearing in the problem p_2.

Without solving the problem p_2 fully, R_1, T_1 can be obtained by employing Evans's [13] idea. To find R_1, we apply Green's integral theorem to the functions $\phi_0(x, y)$ and $\phi_1(x, y)$ in the region bounded by the lines

$$\begin{aligned} &y = 0, \quad a \le x \le X; \qquad x = X, \quad 0 \le y \le Y; \qquad y = Y, \quad -X \le x \le X; \qquad x = -X, \quad 0 \le y \le Y; \\ &y = 0, \quad -X \le x \le -a; \qquad x = -a - 0, \quad 0 \le y \le 1; \qquad x = -a + 0, \quad 0 \le y \le 1; \\ &y = 0, \quad -a \le x \le a; \qquad x = a - 0, \quad 0 \le y \le 1; \qquad x = a + 0, \quad 0 \le y \le 1 \end{aligned}$$

and circles of small radius δ with centers at $(\pm a, 1)$ and ultimately make X, Y tend to infinity and δ tend to zero. Using arguments similar to Evans [13], we obtain

$$\begin{aligned} \mathrm{i}R_1 = &\int_0^1 \{\phi_0(a+0, y)\phi_{1x}(a+0, y) - \phi_0(a-0, y)\phi_{1x}(a-0, y)\}\mathrm{d}y \\ &+ \int_0^1 \{\phi_0(-a+0, y)\phi_{1x}(-a+0, y) - \phi_0(-a-0, y)\phi_{1x}(-a-0, y)\}\mathrm{d}y. \end{aligned} \tag{10}$$

Using the condition (*iii*) of p_2 in the relation (10), integrating by parts and using $c(0) = c(1) = 0$, we find that

$$\begin{aligned} \mathrm{i}R_1 = &\int_0^1 c(y)[\{\phi_{0y}(a-0, y) + \phi_{0y}(-a+0, y)\}\{\phi_{0y}(a-0, y) - \phi_{0y}(-a+0, y)\} \\ &- \{\phi_{0y}(a+0, y) + \phi_{0y}(-a-0, y)\}\{\phi_{0y}(a+0, y) - \phi_{0y}(-a-0, y)\}]\mathrm{d}y. \end{aligned} \tag{11}$$

Following Levine and Rodemich [4], $\phi_{0y}(x, y)$ is given by

$$\phi_{0y}(x, y) = \mathrm{e}^{-Ky}\int_0^y \mathrm{e}^{Kv} Ref(x+\mathrm{i}v)\mathrm{d}v - K\mathrm{e}^{-Ky}\lambda_{\pm}(x) \quad \text{for} \quad x > a \quad \text{and} \quad x < -a, \tag{12}$$

$$\phi_{0y}(x, y) = \mathrm{e}^{-Ky}\int_0^y \mathrm{e}^{Kv} Ref(x+\mathrm{i}v)\mathrm{d}v - K\mathrm{e}^{-Ky}\mu(x) \quad \text{for} \quad -a < x < a. \tag{13}$$

where the functions $f(z)$, $\lambda_{\pm}(x)$ and $\mu(x)$ are given in the Appendix A. Using (12) and (13), the relation (11) produces

$$\begin{aligned} \mathrm{i}R_1 = &\int_0^1 c(y)\left[\mathrm{e}^{-Ky}\int_0^y \mathrm{e}^{Kv}\{Ref(a-0+\mathrm{i}v) + Ref(-a+0+\mathrm{i}v)\}\mathrm{d}v - K\mathrm{e}^{-Ky}\{\mu(a)+\mu(-a)\}\right] \\ &\times \left[\mathrm{e}^{-Ky}\int_0^y \mathrm{e}^{Kv}\{Ref(a-0+\mathrm{i}v) - Ref(-a+0+\mathrm{i}v)\}\mathrm{d}v - K\mathrm{e}^{-Ky}\{\mu(a)-\mu(-a)\}\right]\mathrm{d}y \\ &- \int_0^1 c(y)\left[\mathrm{e}^{-Ky}\int_0^y \mathrm{e}^{Kv}\{Ref(a+0+\mathrm{i}v) + Ref(-a-0+\mathrm{i}v)\}\mathrm{d}v - K\mathrm{e}^{-Ky}\{\lambda_+(a)+\lambda_-(-a)\}\right] \\ &\times \left[\mathrm{e}^{-Ky}\int_0^y \mathrm{e}^{Kv}\{Ref(a+0+\mathrm{i}v) - Ref(-a-0+\mathrm{i}v)\}\mathrm{d}v - K\mathrm{e}^{-Ky}\{\lambda_+(a)-\lambda_-(-a)\}\right]\mathrm{d}y. \end{aligned} \tag{14}$$

The integrals appearing in (14) can be evaluated numerically, once the form of $c(y)$ is known.

As $a \to 0$, the two barriers merge to a single barrier. So if we make $a \to 0$ in (14), R_1 for a single barrier should be recovered. To show this, approximations of various quantities as $a \to 0$ are obtained in Appendix B. Using the results given in (B.16)–(B.18) in Appendix B in (14), and noting that the second integral in (14) contributes nothing, and using the contributions for the first integral in (14), we obtain as $a \to 0$,

$$\frac{\mathrm{i}R_1}{4K}(\pi I_1(K) + \mathrm{i}K_1(K)) \approx -K\int_0^1 c(y)\mathrm{e}^{-2Ky}\left(\int_1^y \frac{v\mathrm{e}^{Kv}}{(1-v^2)^{1/2}}\mathrm{d}v\right)\mathrm{d}y + \int_0^1 c(y)\frac{\mathrm{e}^{-Ky}}{(1-y^2)^{1/2}}\mathrm{d}y. \tag{15}$$

This result coincides with the result, obtained by Shaw [12] and Mandal and Chakrabarti [14] for a single partially immersed nearly vertical barrier.

Again, to find T_1, we use Green's integral theorem to the functions $\psi_0(x, y) = \phi_0(-x, y)$ and $\phi_1(x, y)$ in the region mentioned above to obtain finally,

$$\begin{aligned} \mathrm{i}T_1 &= \int_0^1 \{\psi_0(a+0, y)\phi_{1x}(a+0, y) - \psi_0(a-0, y)\phi_{1x}(a-0, y)\}\mathrm{d}y \\ &\quad + \int_0^1 \{\psi_0(-a+0, y)\phi_{1x}(-a+0, y) - \psi_0(-a-0, y)\phi_{1x}(-a-0, y)\}\mathrm{d}y \\ &= \int_0^1 \{\phi_0(-a-0, y)\phi_{1x}(a+0, y) - \phi_0(-a+0, y)\phi_{1x}(a-0, y)\}\mathrm{d}y \\ &\quad + \int_0^1 \{\phi_0(a-0, y)\phi_{1x}(-a+0, y) - \phi_0(a+0, y)\phi_{1x}(-a-0, y)\}\mathrm{d}y. \end{aligned}$$

Using the condition (iii) of p_2, then integrating by parts and using $c(0) = c(1) = 0$, we find that the above integral vanishes identically so that

$$T_1 \equiv 0. \tag{16}$$

Thus, the first order correction to the transmission coefficient vanishes identically for two nearly vertical barriers also, as was the case for a single nearly vertical barrier.

4. Conclusion

A simplified perturbational analysis together with appropriate use of Green's integral theorem is employed to obtain the first order corrections R_1 and T_1 to the reflection and transmission coefficients for two nearly vertical partially immersed barriers in deep water. While R_1 is obtained in terms of definite integrals involving the shape function describing the barriers, T_1 vanishes identically. If the two barriers are merged to the single barrier then the known result R_1 for a single barrier is recovered. Here the same shape function has been chosen to describe the configurations of the two barriers for simplicity. However there is no difficulty if two different shape functions $c_i(y)(i = 1, 2)$ such that $c_i(1) = 0$ and $|c_i(y)|$ is bounded for $0 < y < 1$ are chosen. The first order correction for R_1 will involve definite integrals involving $c_1(y)$ and $c_2(y)$ while T_1 will be identically zero.

It may be noted that the present problem can be handled by a hypersingular integral equation formulation as has been done by Mandal and Gayen(Chowdhury)[11] for two symmetric circular arc shaped plates when the form of the shape function $c(y)$ is known and ϵ is also prescribed. However, in solving the resulting hypersingular integral equation by collocation method, the evaluation of the elements of the matrix of the resulting linear system, although straight forward in principle, is quite cumbersome in practice. The present perturbational method appears to be simple in comparison to the hypersingular integral equation formulation.

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Appendix A

In this section, we mention those inevitable expressions of [4] which are used above and in Appendix B.

The function $f(z)$ in (12) and (13) is given by

$$f(z) = (C - D) f_1(z) + (C + D) f_2(z) \tag{A.1}$$

$$f_1(z) = \frac{3\beta\zeta^2 + \beta^2}{(\zeta^2 - \beta^2)^3} + \frac{2p\beta^2}{(\zeta^2 - \beta^2)^2} \tag{A.2}$$

$$f_2(z) = \frac{\zeta^3 + 3\beta^2\zeta}{(\zeta^2 - \beta^2)^3} + \frac{2p\beta\zeta}{(\zeta^2 - \beta^2)^2} \tag{A.3}$$

where C, D, p are constants (cf. [4]). z and ζ are related by

$$z = \frac{\alpha\gamma}{\beta^2}\int_0^{\zeta} \frac{\beta^2 - t^2}{(\alpha^2 - t^2)^{1/2}(\gamma^2 - t^2)^{1/2}}\mathrm{d}t, \tag{A.4}$$

where α, β, γ are determined by the following relations

$$\frac{\beta^2}{\gamma^2} = \frac{E(k)}{K(k)}, \tag{A.5}$$

$$\alpha[K(k)E(k,\theta) - E(k)F(k,\theta)] = E(k), \tag{A.6}$$

$$\frac{\pi}{2}\alpha = aE(k) \tag{A.7}$$

with

$$k = \left(1 - \frac{\alpha^2}{\gamma^2}\right)^{1/2} \quad \text{and} \quad \theta = \sin^{-1}\frac{1}{k}\left(1 - \frac{\beta^2}{\gamma^2}\right)^{1/2}, \tag{A.8}$$

$F(k,\theta)$, $E(k,\theta)$, $K(k)$ and $E(k)$ being elliptic integrals. Also $\lambda_{\pm}(x)$ and $\mu(x)$ in (12) and (13) are given by

$$\lambda_{\pm}(x) = -\frac{1}{K}\int_{\pm\infty}^{x} \sin K(x-u)f(u)\mathrm{d}u + \begin{cases} T_0\mathrm{e}^{\mathrm{i}Kx}, & \text{for } x > a \\ \mathrm{e}^{\mathrm{i}Kx} + R_0\mathrm{e}^{-\mathrm{i}Kx}, & \text{for } x < -a, \end{cases} \tag{A.9}$$

$$\mu(x) = -\frac{1}{K}\int_0^x \sin K(x-u)f(u)\mathrm{d}u - \frac{\cos K(a+x)}{K\sin 2Ka}\int_0^a \cos K(a-u)f(u)\mathrm{d}u$$
$$- \frac{\cos K(a-x)}{K\sin 2Ka}\int_{-a}^0 \cos K(a+u)f(u)\mathrm{d}u, \quad \text{for} \quad -a < x < a, \tag{A.10}$$

$$T_0 = \frac{1}{iK}\mathrm{e}^{-\mathrm{i}Ka}[(C-D)\{I_1(1) - \sin KaI_2(1)\} + (C+D)\{I_1(2) - i\cos KaI_2(2)\}], \tag{A.11}$$

$$R_0 = \mathrm{e}^{-2\mathrm{i}Ka} + \frac{1}{\mathrm{i}K}\mathrm{e}^{-\mathrm{i}Ka}[(C-D)\{I_1(1) - \sin KaI_2(1)\} - (C+D)\{I_1(2) - i\cos KaI_2(2)\}], \tag{A.12}$$

where

$$I_1(j) = \int_0^a \cos K(a-x)f_j(x)\mathrm{d}x, \tag{A.13}$$

$$I_2(j) = \int_0^{\infty} \mathrm{e}^{-Ky}f_j(\mathrm{i}y)\mathrm{d}y, \tag{A.14}$$

$$I_3(j) = \int_{C_1} [\cos K(a-z) - \cosh K]f_j(z)\mathrm{d}z \quad (j = 1, 2) \tag{A.15}$$

where C_1 is a curve from $a+0$ to $a-0$ in the cut half plane occupied by the fluid.

Appendix B

Approximation as $a \to 0$

Since

$$E(k) = \int_0^1 \frac{(1-k^2x^2)^{1/2}}{(1-x^2)^{1/2}}\mathrm{d}x.$$

use of the first mean value theorem of integral calculus produces

$$E(k) = \frac{\pi}{2}(1 - k^2\xi^2), \quad 0 < \xi < 1.$$

Thus, (A.7) gives

$$\alpha = a(1 - k^2\xi^2)^{1/2}.$$

so that

$$1 - \frac{\alpha^2}{\gamma^2} = \frac{\gamma^2 - a^2}{\gamma^2 - a^2\xi^2} \approx 1 \quad \text{as} \quad a \to 0$$

and hence

$$k \approx 1 \quad \text{as} \quad a \to 0.$$

Again since

$$K(k) \approx \ln \frac{4}{(1 - k^2)^{1/2}} \approx \infty \quad \text{as} \quad a \to 0, \tag{B.1}$$

we find from (A.5) that

$$\frac{\beta^2}{\gamma^2} \approx 0 \quad \text{as} \quad a \to 0. \tag{B.2}$$

Also from (A.8) that

$$\theta \approx \frac{\pi}{2} \quad \text{as} \quad a \to 0. \tag{B.3}$$

Thus we find from (A.7) and (B.2) that

$$\alpha \approx \frac{2a}{\pi}, \quad \beta^2 \approx 0 \quad \text{as} \quad a \to 0. \tag{B.4}$$

Now using (A.5) in (A.6), we find that

$$\frac{\beta^2}{\alpha} = \frac{\gamma^2 E(k, \theta)}{1 + \alpha F(k, \theta)}. \tag{B.5}$$

Since as $a \to 0, \quad k \approx 1 \quad \text{and} \quad \theta \approx \frac{\pi}{2}$, we find that

$$E(k, \theta) \approx 1. \tag{B.6}$$

Using (B.1), we find that as $a \to 0$

$$\alpha F(k, \theta) \approx \alpha K(k) \approx \alpha \ln\left(\frac{4\gamma}{\alpha}\right) \approx \alpha \ln\left(\frac{4\gamma_0}{\alpha}\right) \approx 0 \tag{B.7}$$

since, as $\gamma > 0$ always, we can write

$$\gamma \approx \gamma_0 \quad \text{as} \quad a \to 0 \tag{B.8}$$

where $\gamma_0 > 0$. Using (B.6)–(B.8) in (B.5), we find that

$$\frac{\beta^2}{\alpha} \approx \gamma_0^2. \tag{B.9}$$

Now from (A.4)

$$\frac{dz}{d\zeta} = \frac{\alpha\gamma}{\beta^2} \frac{\beta^2 - t^2}{(\alpha^2 - t^2)^{1/2}(\gamma^2 - t^2)^{1/2}}.$$

As $a \to 0$, using (B.4), (B.8) and (B.9) we find that

$$\frac{dz}{d\zeta} = \frac{1}{\gamma_0} \frac{\zeta}{(\zeta^2 - \gamma_0^2)^{1/2}}. \tag{B.10}$$

But we know that for a single barrier($a = 0$), (cf. Levine and Rodemich [4])

$$\frac{dz}{d\zeta} = \frac{\zeta}{(\zeta^2 - 1)^{1/2}}. \tag{B.11}$$

Comparing (B.10) and (B.11) we find

$$\gamma_0 = 1.$$

Using this and (B.4), (B.8) and (B.9), we find as $a \to 0$

$$\alpha \approx \frac{2a}{\pi}, \quad \beta \approx \left(\frac{2a}{\pi}\right)^{1/2}, \quad \gamma \approx 1. \tag{B.12}$$

The approximation in (B.12) are now used to find the approximations for $f_1(z)$, $f_2(z)$, $f(z)$ and the various integrals for $a \approx 0$. Form (A.2) and (A.3) we get

$$f_1(z) \approx \beta\left(\frac{3}{\zeta^4} + \zeta\right), \; f_2(z) \approx \frac{2}{\zeta^3} \quad \text{as} \quad a \to 0 \tag{B.13}$$

where ζ is given by the transformation (B.11). From (A.13)–(A.15) we have (cf. Levine and Rodemich [4])

$$I_1(1) - \sin Ka I_2(1) = -\frac{1}{2} K K_1(K, a)$$

$$I_1(2) - i \cos Ka I_2(2) = -\frac{1}{2} K K_2(K, a)$$

$$I_j(K, a) = \frac{2i}{\pi K} I_3(j), \quad j = 1, 2$$

where

$$K_j(K, a) = \frac{2}{K} \int_a^\infty \cos K(x - a) f_j(x) dx, \quad j = 1, 2.$$

As $a \to 0$, we find

$$K_1(K, a) \approx \frac{2\beta}{K} \int_0^\infty \cos Kx \left[\frac{3}{(1 + x^2)^2} + (1 + x^2)^{1/2}\right] dx,$$

$$I_1(K, a) \approx -\frac{2\beta}{\pi K} \int_0^1 (\cosh Ky - \cosh K) \left[\frac{3}{(1 - y^2)^2} + (1 - y^2)^{1/2}\right] dy, \tag{B.14}$$

$$K_2(K, a) \approx 4K_1(K), \quad I_2(K, a) \approx 2I_1(K) \tag{B.15}$$

where $I_1(K)$ and $K_1(K)$ are modified Bessel functions. From (A.11), we find after using the approximations (B.14) and (B.15) as $a \to 0$ that

$$T_0 \approx \frac{iK_1(K)}{\Delta_1}, \; R_0 \approx \frac{\pi I_1(K)}{\Delta_1}$$

where

$$\Delta_1 = \pi I_1(K) + iK_1(K).$$

Again from (A.1) we find that as $a \to 0$

$$f(z) \approx \frac{1}{\Delta_1}\frac{1}{(1+z^2)^{3/2}} \quad \text{for} \quad a>0, \qquad f(z) \approx -\frac{1}{\Delta_1}\frac{1}{(1+z^2)^{3/2}} \quad \text{for} \quad a<0 \tag{B.16}$$

so that from (A.9) and (A.10), we obtained as $a \to 0$,

$$\lambda_+(a)+\lambda_-(-a) \approx 2, \qquad \lambda_+(a)-\lambda_-(-a) \approx \frac{2}{K\Delta_1}\int_0^\infty \frac{\sinh Kv}{(1-v^2)^{3/2}}\mathrm{d}v - \frac{2\pi I_1(K)}{\Delta_1} \tag{B.17}$$

and

$$\mu(a)+\mu(-a) \approx 0, \qquad \mu(a)-\mu(-a) \approx -\frac{2}{\Delta_1 K^2}. \tag{B.18}$$

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